A. V. Karpishkov<sup>1,2</sup>, M. A. Nefedov<sup>1</sup>, and V. A. Saleev<sup>1,2</sup>

<sup>1</sup> Samara National Research University <sup>2</sup> Joint Institute for Nuclear Research

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#### Outline

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Summary

Single Spin Asymmetries in charmonium production with ISI and FSI in CGI model Generalized Parton Model (GPM) and it's application to calculation of SSA

# Generalized Parton Model (GPM) and it's application to calculation of SSA

Single Spin Asymmetries in charmonium production with ISI and FSI in CGI model Generalized Parton Model (GPM) and it's application to calculation of SSA Factorization formula for the GPM

#### Factorization schemes in different $p_T$ -regions

The traditional Collinear Parton Model (CPM) is applicable in a region of high- $p_T$  production

$$\mu \sim p_T \gg \Lambda_{QCD},$$

so we can neglect influence of small intrinsic  $\mathbf{q}_{\mathbf{T}}$  of initial partons ( $\langle q_{\mathbf{T}}^2 \rangle \simeq 1 \text{ GeV}^2$ ).

But if we're interested in particle production in a region of  $p_T \simeq \sqrt{\langle q_T^2 \rangle} \ll \mu$ , we should take into account intrinsic  $q_T$ . It can be done within TMD approach, factorization for which has been proven in the limit  $q_T \ll \mu$  [J. Collins, Camb. Monogr., Part. Phys. Nucl. Phys. Cosmol. 32, 1-624 (2011)]. In our case, the hard scale  $\mu$  is given by charmonium mass  $m_C = 3.1 \div 3.7$  GeV. So we can use phenomenological TMD-ansatz, a so called Generalized Parton Model (GPM), initial partons in which are on-shell:

$$q_{\mu} = xP_{\mu}^{+} + yP_{\mu}^{-} + q_{T\mu}, (q_{\mu})^{2} = 0, \qquad (1)$$

and a factorized prescription for TMD parton distribution functions (PDFs) is used:

$$F_a(x, q_T, \mu_F) = f_a(x, \mu_F)G_a(q_T),$$
 (2)

where  $f_a(x,\mu_F)$  – corresponding CPM PDF,  $G_a(q_T)$  – Gaussian distribution  $G_a(q_T) = \exp(-q_T^2/\left\langle q_T^2 \right\rangle_a)/(\pi \left\langle q_T^2 \right\rangle_a).$ 

Single Spin Asymmetries in charmonium production with ISI and FSI in CGI model Generalized Parton Model (GPM) and it's application to calculation of SSA Factorization formula for the GPM

#### Factorization formula for the GPM

Within the GPM we can write the following expression for the differential cross-section of  $2 \rightarrow 1$  hard subprocess  $g(q_1) + g(q_2) \rightarrow C(k)$ :

$$d\sigma(pp \to \mathcal{C}X) = \int dx_1 \int d^2 \mathbf{q_{1T}} \int dx_2 \int d^2 \mathbf{q_{2T}} \times F_g(x_1, q_{1T}, \mu_F) F_g(x_2, q_{2T}, \mu_F) d\hat{\sigma}(gg \to \mathcal{C}), \quad (3)$$

where  $C = J/\psi, \psi(2S)$  or  $\chi_c(1P)$ , and

$$d\hat{\sigma}(gg \to \mathcal{C}) = (2\pi)^4 \delta^{(4)}(q_1 + q_2 - k) \frac{|M(gg \to \mathcal{C})|^2}{2x_1 x_2 s} \frac{d^4k}{(2\pi)^3} \delta_+(k^2 - m_{\mathcal{C}}^2).$$
(4)

In a case of  $2 \to 2$  subprocess  $g(q_1) + g(q_2) \to C(k) + g(q_3)$ ,  $C = J/\psi, \psi(2S)$  in formula (3)  $d\hat{\sigma}(gg \to C)$  must be replaced by:

$$d\hat{\sigma}(gg \to \mathcal{C}g) = (2\pi)^4 \delta^{(4)}(q_1 + q_2 - k - q_3) \frac{\overline{|M(gg \to \mathcal{C}g)|^2}}{2x_1 x_2 s} \frac{d^3k}{(2\pi)^3 2k_0} \frac{d^4q_3}{(2\pi)^3} \delta_+(q_3^2).$$
(5)

Four-momenta of initial partons are on mass-shell  $(q_1^2 = q_2^2 = 0)$  and have longitudinal (along the Z-axis) and transverse parts:

$$q_{1}^{\mu} = \left(x_{1}\frac{\sqrt{s}}{2} + \frac{\mathbf{q}_{1T}^{2}}{2\sqrt{sx_{1}}}, \mathbf{q}_{1T}, x_{1}\frac{\sqrt{s}}{2} - \frac{\mathbf{q}_{1T}^{2}}{2\sqrt{sx_{1}}}\right)^{\mu}, \quad (6)$$
$$q_{2}^{\mu} = \left(x_{2}\frac{\sqrt{s}}{2} + \frac{\mathbf{q}_{2T}^{2}}{2\sqrt{sx_{2}}}, \mathbf{q}_{2T}, -x_{2}\frac{\sqrt{s}}{2} + \frac{\mathbf{q}_{2T}^{2}}{2\sqrt{sx_{2}}}\right)^{\mu}. \quad (7)$$

Single Spin Asymmetries in charmonium production with ISI and FSI in CGI model Generalized Parton Model (GPM) and it's application to calculation of SSA Polarized production. SSA

#### Single Spin Asymmetry

In inclusive process  $p^{\uparrow}p \to \mathcal{C}X \ \mathcal{C} = J/\psi, \chi_c, \psi(2S))$  SSA is defined as:

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\Delta\sigma}{2d\sigma}.$$
(8)

The numerator and denominator of  $A_N$  have the form:

$$d\sigma \propto \int dx_1 \int d^2 q_{1T} \int dx_2 \int d^2 q_{2T} F_g(x_1, q_{1T}, \mu_F) F_g(x_2, q_{2T}, \mu_F) d\hat{\sigma}(gg \to \mathcal{C}X), \quad (9)$$
  
$$d\Delta \sigma \propto \int dx_1 \int d^2 q_{1T} \int dx_2 \int d^2 q_{2T} [\hat{F}_g^{\uparrow}(x_1, \mathbf{q}_{1T}, \mu_F) - \hat{F}_g^{\downarrow}(x_1, \mathbf{q}_{1T}, \mu_F)] \times F_g(x_2, q_{2T}, \mu_F) d\hat{\sigma}(gg \to \mathcal{C}X), \quad (10)$$

where  $\hat{F}_{g}^{f,\downarrow}(x,q_T,\mu_F)$  is the distribution of unpolarized gluon (or quark) in polarized proton.

Following the Trento conventions [A. Bacchetta, U. DAlesio, M. Diehl and C. A. Miller, Phys. Rev. D **70**, 117504 (2004)], the gluon Sivers function (GSF) can be introduced as

$$\Delta \hat{F}_{g}^{\uparrow}(x_{1}, \mathbf{q}_{1T}, \mu_{F}) \equiv \hat{F}_{g}^{(\uparrow)}(x_{1}, \mathbf{q}_{1T}, \mu_{F}) - \hat{F}_{g}^{(\downarrow)}(x_{1}, \mathbf{q}_{1T}, \mu_{F}) = = \Delta^{N} F_{g}^{\uparrow}(x_{1}, \mathbf{q}_{1T}^{2}, \mu_{F}) \cos(\phi_{1}) \equiv \left(-2\frac{q_{1T}}{M_{p}}\right) F_{1T}^{\perp g}(x_{1}, \mathbf{q}_{1T}^{2}, \mu_{F}) \cos(\phi_{1}).$$
(11)

Moreover, GSF must satisfy the positivity bound  $\forall x_1, q_{1T}$ :

$$\left|\Delta^{N}F_{g}^{\uparrow}(x_{1},\mathbf{q}_{1T}^{2},\mu_{F})\right| \leq 2F_{g}(x_{1},q_{1T},\mu_{F}).$$

$$(12)$$

Single Spin Asymmetries in charmonium production with ISI and FSI in CGI model Generalized Parton Model (GPM) and it's application to calculation of SSA Including ISI and FSI – Color Gauge Invariant formulation of GPM

#### Single Spin Asymmetry in the CGI-GPM framework



Figure 1 : LO diagrams for the process  $p^{\uparrow}p \rightarrow J/\psi X$ , assuming a color-singlet production mechanism, within the GPM (a) and the CGI-GPM (b), (c). It turns out that only initial state interactions depicted in (b) contribute to the SSA. Figure is from [D'Alesio *et.al.*, Phys. Rev. D **96**, 036011 (2017)].

In GPM (Fig. 1 (a)) we can write the numerator of the asymmetry as follows:

$$d\Delta\sigma \propto \left(-2\frac{q_{1T}}{M_p}\right) F_{1T}^{\perp g}(x_1, \mathbf{q}_{1T}^2, \mu_F) \cos(\phi_1) \otimes F_g(x_2, q_{2T}, \mu_F) \otimes H_{gg \to cd}^U, \quad (13)$$
  
where  $H_{gg \to cd}^U = \overline{|M(gg \to cd)|^2}.$ 

Single Spin Asymmetries in charmonium production with ISI and FSI in CGI model Generalized Parton Model (GPM) and it's application to calculation of SSA Including ISI and FSI – Color Gauge Invariant formulation of GPM

#### Single Spin Asymmetry in the CGI-GPM framework



Figure 2: LO diagrams for the process  $p^{\uparrow}p \rightarrow J/\psi X$ , assuming a color-singlet production mechanism, within the GPM (a) and the CGI-GPM (b), (c). It turns out that only initial state interactions depicted in (b) contribute to the SSA. Figure is from [D'Alesio *et.al.*, Phys. Rev. D **96**, 036011 (2017)].

Formally, the numerator of the asymmetry in the CGI-GPM approach ([L. Gamberg and Z. B. Kang, Phys. Lett. B **696**, 109 (2011)]) can be obtained from eq. (13) by with the substitution:

$$F_{1T}^{\perp g} H_{gg \to J/\psi g}^{U} \to \frac{C_{I}^{(f)} + C_{F_{c}}^{(f)}}{C_{U}} F_{1T}^{\perp g(f)} H_{gg \to J/\psi g}^{U} + \frac{C_{I}^{(d)} + C_{F_{c}}^{(d)}}{C_{U}} F_{1T}^{\perp g(d)} H_{gg \to J/\psi g}^{U} \equiv \\ \equiv F_{1T}^{\perp g(f)} H_{gg \to J/\psi g}^{Inc(f)} + F_{1T}^{\perp g(d)} H_{gg \to J/\psi g}^{Inc(d)}.$$
(14)

Single Spin Asymmetries in charmonium production with ISI and FSI in CGI model Generalized Parton Model (GPM) and it's application to calculation of SSA Including ISI and FSI – Color Gauge Invariant formulation of GPM

#### Color factors and Feynman rules in the CGI-GPM framework



Figure 3 : CGI-GPM color rules for the eikonal three-gluon (a), quark-gluon (b) and antiquark-gluon (c) vertices. The color projectors for the gluon (d) and the quark Sivers functions (e) are shown as well. The eikonal gluon has color index c. Figure is from [D'Alesio *et. al.*, Phys. Rev. D **96**, 036011 (2017)]. The color factors are:

$$\mathcal{T}_{aa'}^c = \mathcal{N}_{\mathcal{T}} T_{aa'}^c, \mathcal{D}_{aa'}^c = \mathcal{N}_{\mathcal{D}} D_{aa'}^c, \mathcal{Q}_{ij}^c = \mathcal{N}_{\mathcal{Q}} t_{ij}^c, \tag{15}$$

where  $T_{cb}^a \equiv -if_{acb}$ ,  $D_{bc}^a \equiv d_{abc}$ ,  $\mathcal{N}_{\mathcal{T}} = \frac{1}{Tr[T^cT^c]} = 1/(N_c(N_c^2 - 1))$ ,  $\mathcal{N}_{\mathcal{D}} = \frac{1}{Tr[D^cD^c]} = 1/((N_c^2 - 4)(N_c^2 - 1))$ ,  $\mathcal{N}_{\mathcal{Q}} = \frac{1}{Tr[t^ct^c]} = 2/(N_c^2 - 1)$ . So, correspondingly, for the *f*- and *d*-type GSF, the relative color factor is therefore calculated from Fig. 1(b) as follows:

$$C_I^{(f)} = -\frac{1}{2}C_U, C_I^{(d)} = 0.$$
(16)

And in CSM of the heavy quark-antiquark pair to the FSI, depicted in Fig. 1(c):

$$C_{F_c}^{(f)} = C_{F_c}^{(d)} = 0. (17)$$

Single Spin Asymmetries in charmonium production with ISI and FSI in CGI model SSA in charmonium production at RHIC and NICA

## SSA in charmonium production at RHIC and NICA

SSA in charmonium production at RHIC and NICA

Numerical results. Comparison to PHENIX data

#### PHENIX-2018 data, $1.2 \le |y| \le 2.2, \sqrt{S} = 200$ GeV.



Figure 4: SSA  $A_N^{J/\psi}$  as function of  $x_F$  at  $\sqrt{s} = 200$  GeV. The theoretical results are obtained in NRQCD with D'Alesio *et al.*, SIDIS1 and SIDIS2 parameterizations of GSFs. Left panel: GPM-prediction. Right panel: CGI-GPM-prediction. Experimental data are from Ref. [C. Aidala *et al.* [PHENIX], Phys. Rev. D **98**, 012006 (2018)].

SSA in charmonium production at RHIC and NICA

Numerical results. Comparison to PHENIX data

#### PHENIX-2018 data, $-2.2 \le y \le -1.2$ , $\sqrt{S} = 200$ GeV.



Figure 5 : NRQCD predictions for SSA  $A_N^{J/\psi}$  as function of  $J/\psi$ -transverse momentum at  $\sqrt{s} = 200$  GeV. The theoretical results are obtained with D'Alesio *et al.*, SIDIS1 and SIDIS2 parameterizations of GSFs. Left panel: GPM-prediction. Right panel: CGI-GPM-prediction. Experimental data are from Ref. [C. Aidala *et al.* [PHENIX], Phys. Rev. D **98**, 012006 (2018)].

SSA in charmonium production at RHIC and NICA

Numerical results. Comparison to PHENIX data

#### PHENIX-2018 data, $1.2 \le y \le 2.2, \sqrt{S} = 200$ GeV.



Figure 6: NRQCD predictions for SSA  $A_N^{J/\psi}$  as function of  $J/\psi$ -transverse momentum at  $\sqrt{s} = 200$  GeV. The theoretical results are obtained with SIDIS1 and D'Alesio et al. parameterizations of GSFs. Left panel: GPM-prediction. Right panel: CGI-GPM-prediction. Experimental data are from Ref. [C. Aidala *et al.* [PHENIX], Phys. Rev. D **98**, 012006 (2018)].

SSA in charmonium production at RHIC and NICA

Numerical results. Predictions for NICA

#### Predictions for SSA at NICA (D'Alesio), $|y| \leq 3$ , $\sqrt{S} = 24$ GeV.



Figure 7 : Comparison of predictions for SSA  $A_N^{J/\psi}$  as function of  $x_F$  at  $\sqrt{s} = 24$  GeV in NRQCD (solid histogram) and ICEM (dashed histogram) approaches. Left panel: GPM-prediction. Right panel: CGI-GPM-prediction. The D'Alesio *et al.* parametrisation of GSFs is used.

SSA in charmonium production at RHIC and NICA

Numerical results. Predictions for NICA

#### Predictions for SSA at NICA (D'Alesio), $|y| \leq 3$ , $\sqrt{S} = 24$ GeV.



Figure 8 : Comparison of predictions for SSA  $A_N^{J/\psi}$  as function of  $p_T$  at  $\sqrt{s} = 24$  GeV in NRQCD (solid histogram) and ICEM (dashed histogram) approaches. Left panel: GPM-prediction. Right panel: CGI-GPM-prediction. The D'Alesio *et al.* parametrisation of GSFs is used.

SSA in charmonium production at RHIC and NICA

Numerical results. Predictions for NICA

#### Predictions for SSA at NICA (SIDIS1), $|y| \leq 3$ , $\sqrt{S} = 24$ GeV.



Figure 9 : Comparison of predictions for SSA  $A_N^{J/\psi}$  as function of  $x_F$  at  $\sqrt{s} = 24 \text{ GeV}$ in NRQCD (solid histogram) and ICEM (dashed histogram) approaches. Left panel: GPM-prediction. <u>Right panel</u>: CGI-GPM-prediction. The SIDIS1 parametrisation of GSFs is used.

SSA in charmonium production at RHIC and NICA

Numerical results. Predictions for NICA

#### Predictions for SSA at NICA (SIDIS1), $|y| \leq 3$ , $\sqrt{S} = 24$ GeV.



Figure 10 : Comparison of predictions for SSA  $A_N^{J/\psi}$  as function of  $p_T$  at  $\sqrt{s} = 24$  GeV in NRQCD (solid histogram) and ICEM (dashed histogram) approaches. Left panel: GPM-prediction. Right panel: CGI-GPM-prediction. The SIDIS1 parametrisation of GSFs is used.

SSA in charmonium production at RHIC and NICA

Numerical results. Predictions for NICA

#### Predictions for SSA at NICA (SIDIS2), $|y| \leq 3$ , $\sqrt{S} = 24$ GeV.



Figure 11: Comparison of predictions for SSA  $A_N^{J/\psi}$  as function of  $x_F$  at  $\sqrt{s} = 24$  GeV in NRQCD (solid histogram) and ICEM (dashed histogram) approaches. Left panel: GPM-prediction. Right panel: CGI-GPM-prediction. The SIDIS2 parametrisation of GSFs is used.

SSA in charmonium production at RHIC and NICA

Numerical results. Predictions for NICA

## Predictions for SSA at NICA: NRQCD vs. ICEM (D'Alesio), $|y| \leq 3, \sqrt{S} = 24$ GeV.



Figure 12 : Comparison of predictions for SSA  $A_N^{J/\psi}$  as function of  $x_F$  at  $\sqrt{s} = 24$  GeV in NRQCD (left panel) and ICEM (right panel) approaches. The D'Alesio *et al.* parametrisation of GSFs is used. Estimation of errors for  $A_N^{J/\psi}$  are provided by [I. Denisenko].

SSA in charmonium production at RHIC and NICA

Numerical results. Predictions for NICA

### Predictions for SSA at NICA: NRQCD vs. ICEM (SIDIS1), $|y| \leq 3$ , $\sqrt{S} = 24$ GeV.



Figure 13 : Comparison of predictions for SSA  $A_N^{J/\psi}$  as function of  $x_F$  at  $\sqrt{s} = 24$  GeV in NRQCD (left panel) and ICEM (right panel) approaches. The SIDIS1 parametrisation of GSFs is used. Estimation of errors for  $A_N^{J/\psi}$  are provided by [I. Denisenko].

SSA in charmonium production at RHIC and NICA

Numerical results. Predictions for NICA

### Predictions for SSA at NICA: NRQCD vs. ICEM (SIDIS2), $|y| \leq 3, \sqrt{S} = 24$ GeV.



Figure 14 : Comparison of predictions for SSA  $A_N^{J/\psi}$  as function of  $x_F$  at  $\sqrt{s} = 24$  GeV in NRQCD (left panel) and ICEM (right panel) approaches. The SIDIS1 parametrisation of GSFs is used. Estimation of errors for  $A_N^{J/\psi}$  are provided by [I. Denisenko].

#### Summary

- The CGI-GPM formalism, which includes effects of ISIs and FSIs under a one-gluon exchange approximation, can reproduce the twist-3 collinear factorization formalism (see [L. Gamberg and Z. B. Kang, Phys. Lett. B **696**, 109 (2011)] and [D'Alesio *et.al.*, Phys. Rev. D **96**, 036011 (2017)] for details).
- The Color Gauge Invariant formulation of the GPM is able to reproduce the expected opposite relative sign of the Sivers asymmetries, due to the effects of FSIs and ISIs.
- In CGI-GPM within both the frameworks of CSM and ICEM the process  $p^{\uparrow}p \rightarrow \mathcal{H}X$  of *direct production* of  $J/\psi(\psi')$  is sensitive to *f*-type GSF. While the  $\chi_c$  production within the CSM is sensitive to *d*-type GSF and to *f*-type within the ICEM.
- In the CSM it is better to measure separately direct and feed-down contributions to the  $J/\psi$  asymmetry (at least for D'Alesio and SIDIS2 parametrizations). While in the ICEM it is better to look at the *sum* of both contributions.

### Thank you for your attention!