Skyrmions at high density

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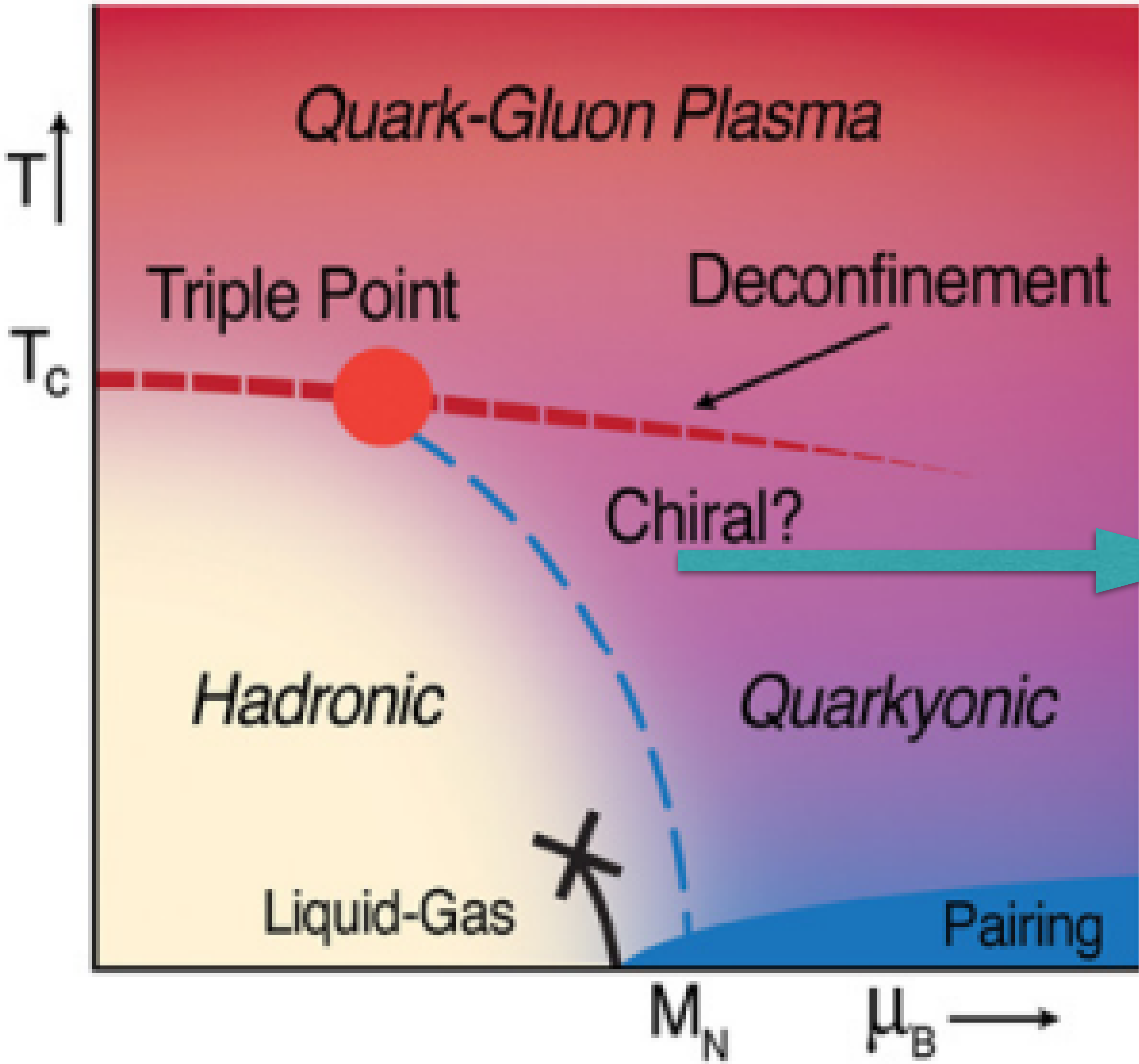


The phase diagram of strongly interacting matter

Matter at finite μ_B

–Lattice QCD has the sign problem

–Effective theories are necessary

Together with Hee-Jung Lee, Dong-Pil Min, Byung-Yoon Park and Mannque Rho we have studied over the years (2002-2010) the phase transition in the high μ_B density region by means the Skyrme model, and completions : Skyrme+ dilaton, Skyrme+dilaton +vector mesons. This work is being continued and extended nowadays using more sophisticated techniques: Hidden symmetry, Ads/CFT,…. Let me recall a picture of the discovered phase transition.

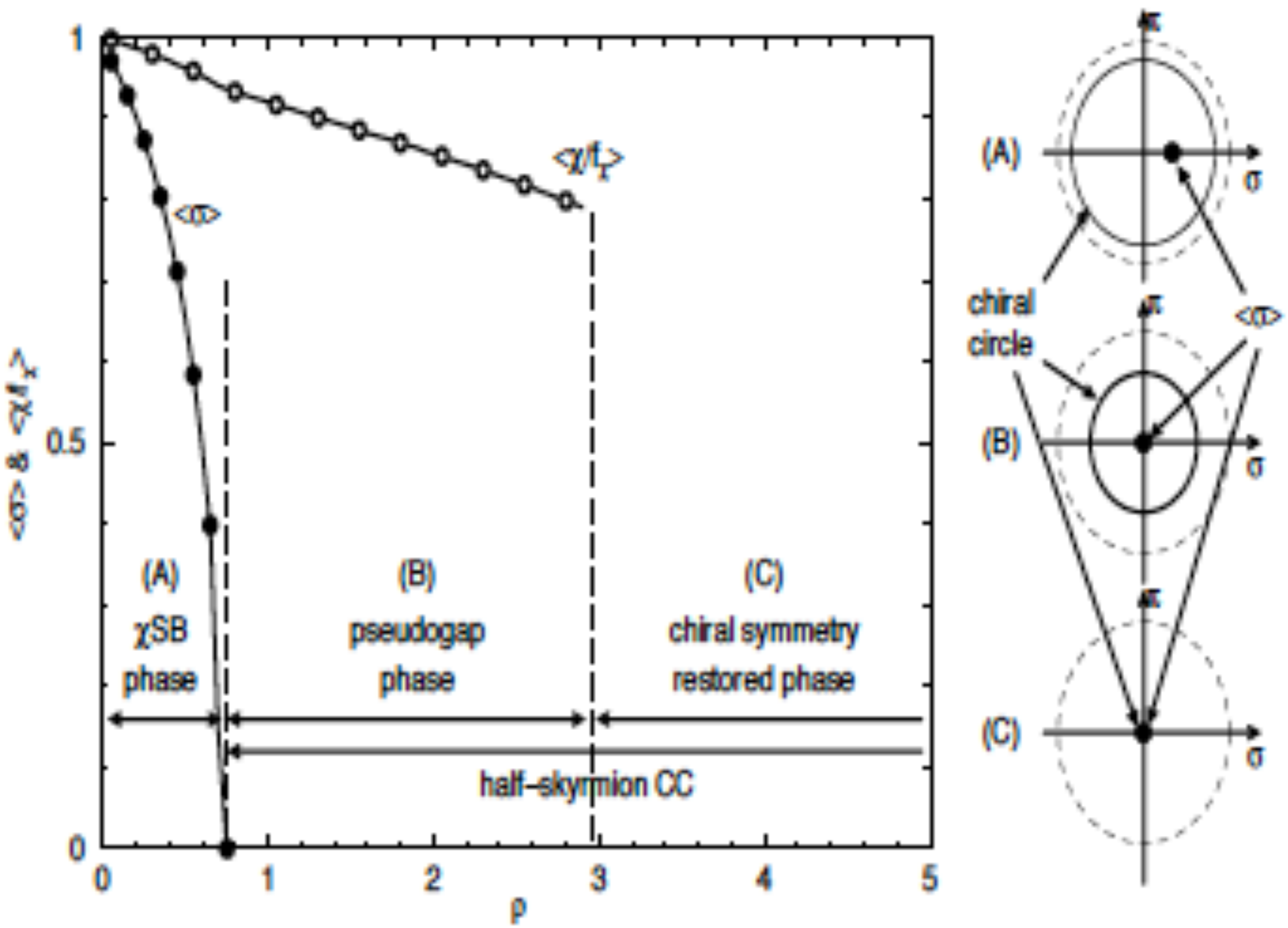
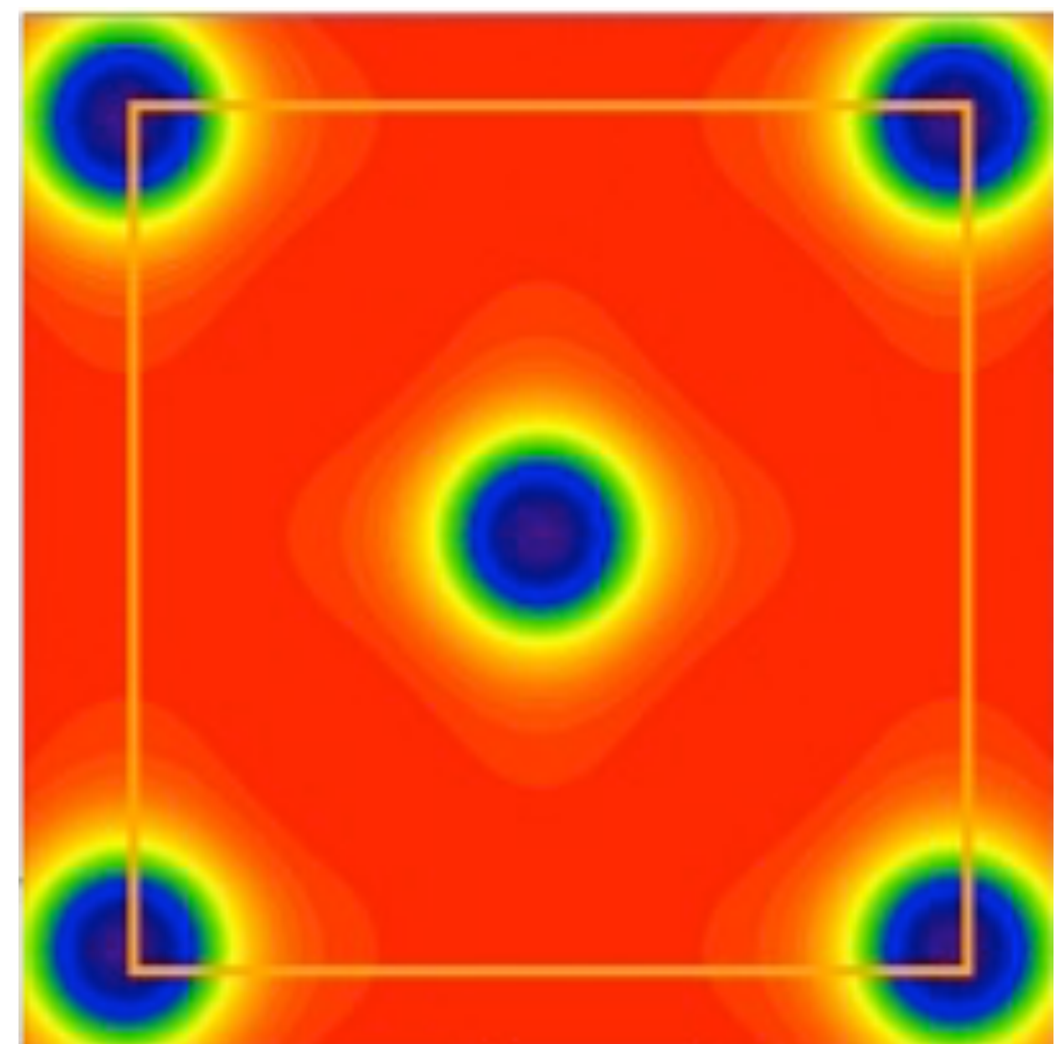


Fig. 7. Average values of $\sigma = \frac{1}{2}\mathrm{Tr}(U)$ and χ/f_χ of the lowest energy crystal configuration at a given baryon number density.

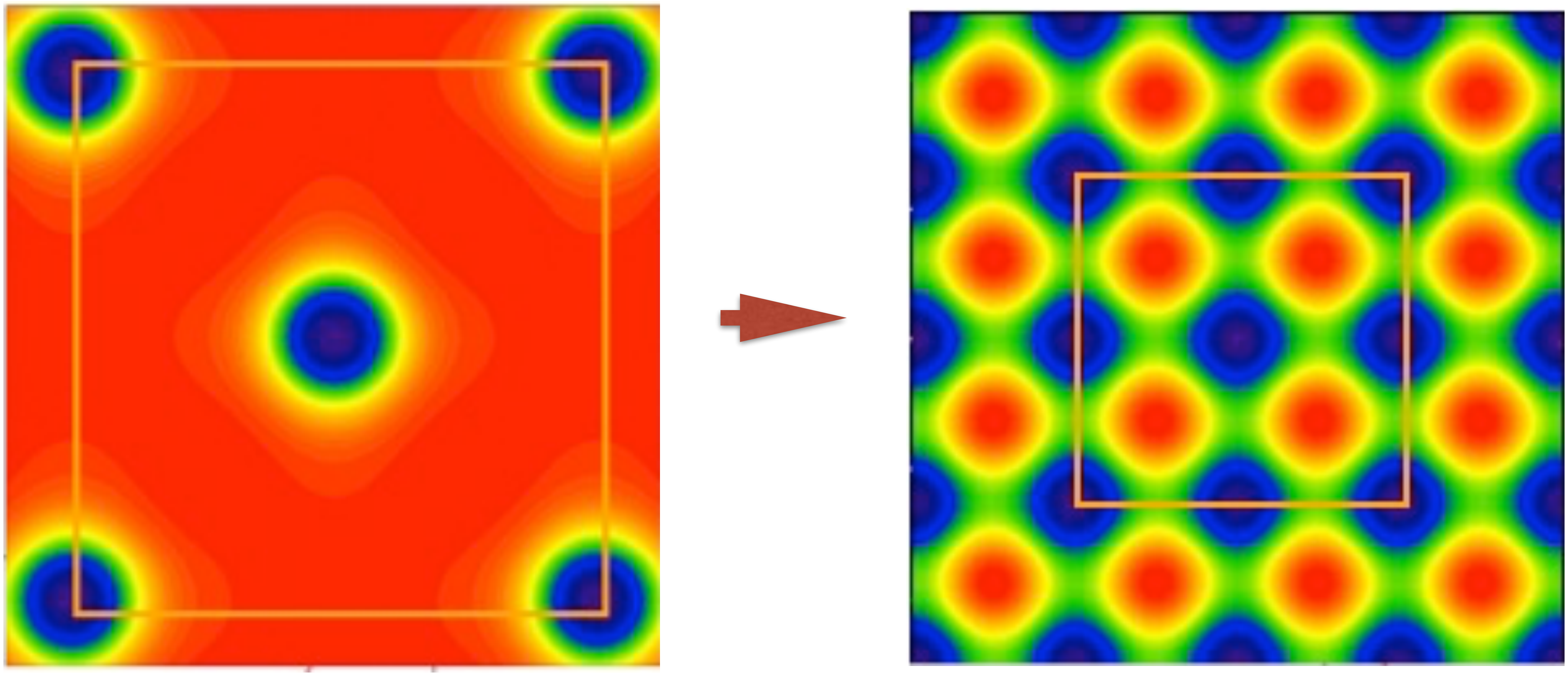
I will not discuss the details of this beautiful scenario but show my particular understanding of one feature using work done in collaboration with Alessandro Drago, and Valentina Mantovani-Sarti. For details: Phys.Lett. B728, 323 and Int.J.Mod.Phys. E26 (2017) 1740029.

The Skyrme model contains topological solitons: skyrmions. They are well localized objects and at low densities one can simple put skyrmions into a volume and make skyrmion matter.

The groundstate of skyrmion matter is a crystal formed of well localized skyrmions, i.e. B=1 particles in a FCC lattice.

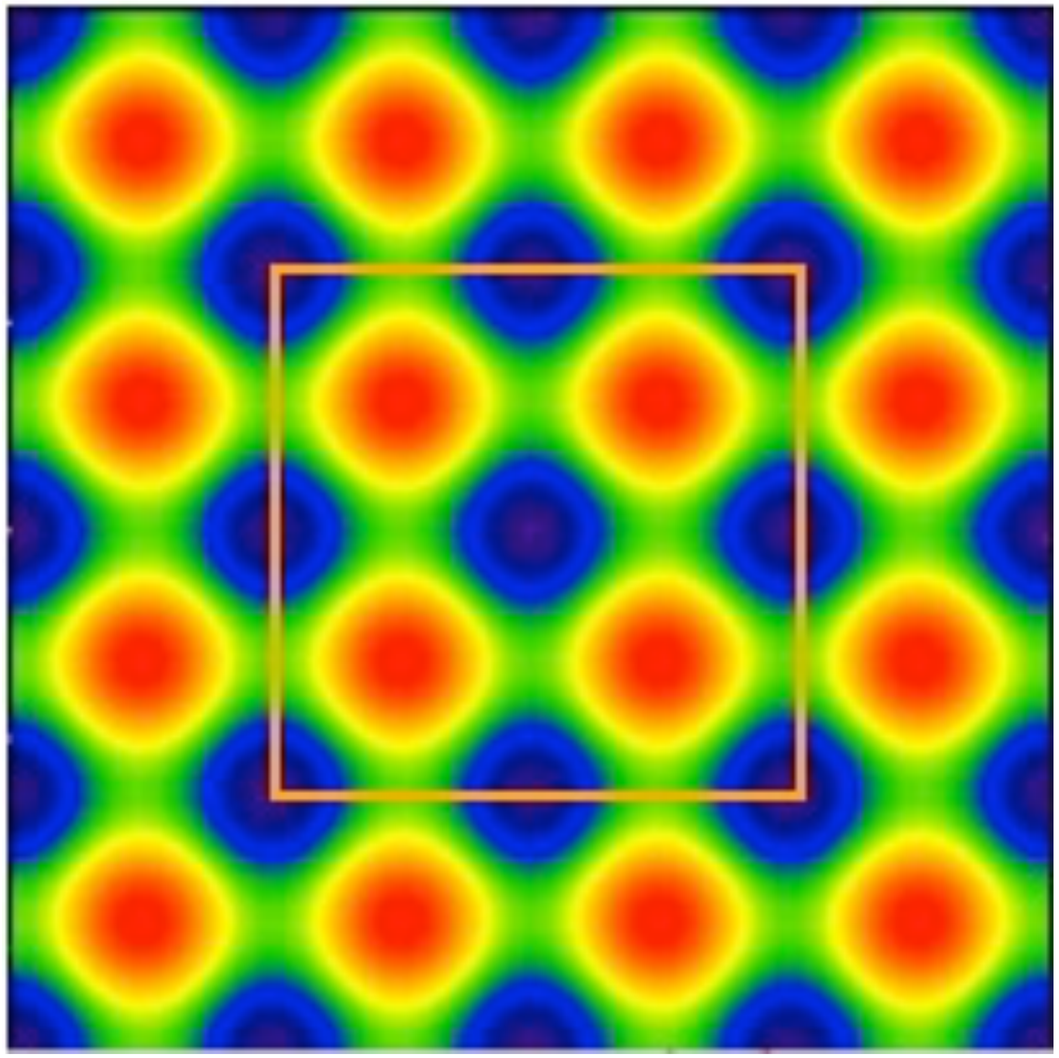


We can now pump in more skyrmions and increase the density. A phase transition occurs into a new kind of crystal also formed of well localized objects but this time they are half-skyrmions, i.e. B=1/2 particles in a CC lattice



Local baryon number densities at low density (left) and beyond the critical density (right) with massive pions. For high density the system is half-skyrmion in a CC crystal configuration.

This description fits well into the so called quarkyonic phase where, before deconfinement, nuclear matter should undergo structural changes involving the restoration of fundamental symmetries of QCD



However, the half-skyrmion configuration is difficult to envisage from a quarkish point of view since the quarks have baryon number 1/3 and the only way to make baryon number 1/2= 1/3 +1/6.

To try to understand this feature we were inspired by the Cheshire Cat principle and used a quark model model previously used in collaboration with Alessandro Drago, Phys.Rev. C86 (2012) 015211, and Byung-Yoon Park, Int.J.Mod.Phys. A28 (2013) no.27, 1350136, Few Body Syst. 54 (2013) 513 for other purposes: The Chiral Dilaton Model (CMD).

Baryons are made of quarks interacting with pion, sigma and dilaton fields (extension of the Friedberg and Lee, non topological soliton model).
The corresponding Lagrangian density reads:

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu\partial_\mu - g_\pi(\sigma + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5)]\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}) - V(\sigma,\boldsymbol{\pi}). \tag{1}$$

The potential is given by,

$$V(\sigma,\boldsymbol{\pi}) = \lambda_1^2(\sigma^2+\boldsymbol{\pi}^2) - \lambda_2^2\ln(\sigma^2+\boldsymbol{\pi}^2) - \sigma_0 m_\pi^2\sigma, \tag{2}$$

where,

$$\lambda_1^2 = \frac{1}{2}\frac{B\delta\phi_0^4 + m_\pi^2\sigma_0^2}{\sigma_0^2} = \frac{1}{4}(m_\sigma^2 + m_\pi^2),$$
$$\lambda_2^2 = \frac{1}{2}B\delta\phi_0^4 = \frac{\sigma_0^2}{4}(m_\sigma^2 - m_\pi^2).$$

Here σ is the scalar-isoscalar field, $\boldsymbol{\pi}$ is the pseudoscalar-isotriplet meson field, ϕ is the dilaton field and ψ describes the isodoublet quark fields. This Lagrangian density, besides being invariant under chiral symmetry, is also invariant under another fundamental symmetry in QCD, scale invariance, which is spontaneously broken

A word of caution at this point. The half-skymion phase describing a rigid crystal structure should not be confused with the quarkyonic phase we are to describe. A mechanism will come out that might be analogous to the one occurring in the more complicated but closer related chiral bag model.

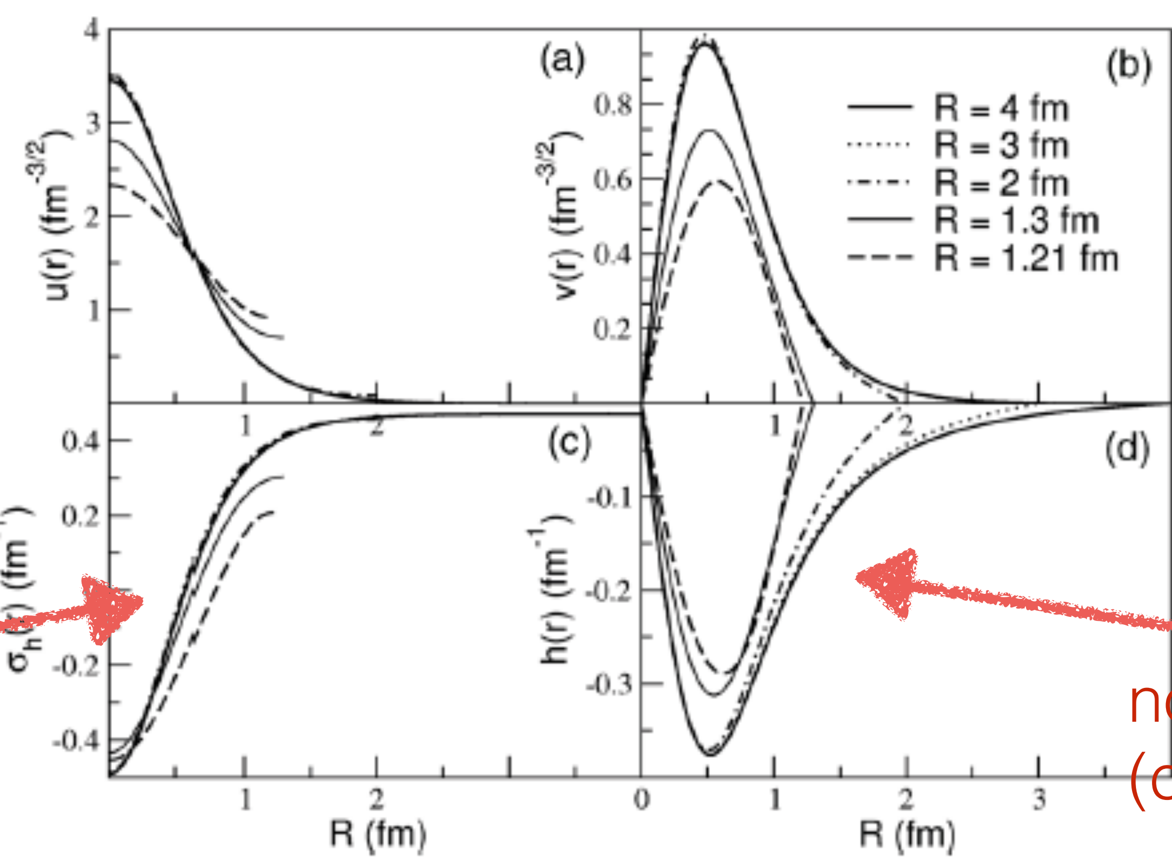
This model has a hedgehog solution with B=1 and Grand-Spin, **G=I+S,**
G=0+ which resembles the skyrmion in the chiral bag model but the pion field is non topological.

The self-consistent $B = 1$ solution using the hedgehog Ansatz is given by,

$$\sigma_{B=1}(\mathbf{r}) = \sigma(r), \quad \boldsymbol{\pi}_{B=1}(\mathbf{r}) = \pi(r)\hat{r}$$

$$\psi_{B=1}(\mathbf{r}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} u(r) \\ i\boldsymbol{\sigma}\cdot\hat{r}v(r) \end{pmatrix} \frac{1}{2}(|u\downarrow\rangle - |d\uparrow\rangle). \tag{2}$$

The solution is stabilized, *not* by a topological constraint, *but* by the energy which becomes lower than three free quark masses. The solution for the the sigma field developes a bag-like spatial structure where the quark fields become localized.



bag-like structure
(only the quarks
carry baryon number)

non topological pion field
(carries no baryon number)

FIG. 2. Upper and lower components of the Dirac spinor [(a) and (b)] and the σ and pion fields [(c) and (d)] in the model without vector mesons, as functions of the cell radius R.

Our aim is to describe finite density in this model.

We use the Wigner-Seitz approximation: This approximation consists in replacing the cubic lattice (as in the Skyrme model) by a spherical symmetric one where each baryon is centered on a spherical shell of radius R. The finite density effects are provided by the requirement of specific boundary conditions on the fields at the origin and at the surface of the sphere.
For periodic potentials the spinor eigenfunctions must satisfy Bloch's theorem:

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\Phi_{\boldsymbol{k}}(\boldsymbol{r}),$$

where **k** is the crystal momentum and $\Phi_{\boldsymbol{k}}(\boldsymbol{r})$ is a spinor that has the same periodicity of the lattice.

The periodic potential generated by the meson field configurations in which the quarks move leads to the formation of a band structure with energy bands and gaps. Our soliton lattice acts as a color insulator and the quarks are well localized in each cell.
Concerning the filling of the band, when working at mean field level the relevant quantum number is the Grand-Spin G= S+I and the lower band corresponds to G=0+. The only degeneracy remaining is color and therefore three quarks per soliton completely fill the band.

What happens at high density?

We just discussed that the Skyrme model at moderate temperatures and high density exhibits a phase transition from an FCC crystal of skyrmions to a CC crystal of half-skyrmions and chiral symmetry is restored. Is there a similar scenario in a model with quarks?

In the Skyrme model description a delocalization of baryon number takes place. We can imagine that this mechanism translates into a modification of the baryon density profile in the CDM.

In the Figure we plot quark eigenvalue as a function of cell Radius (inverse density)

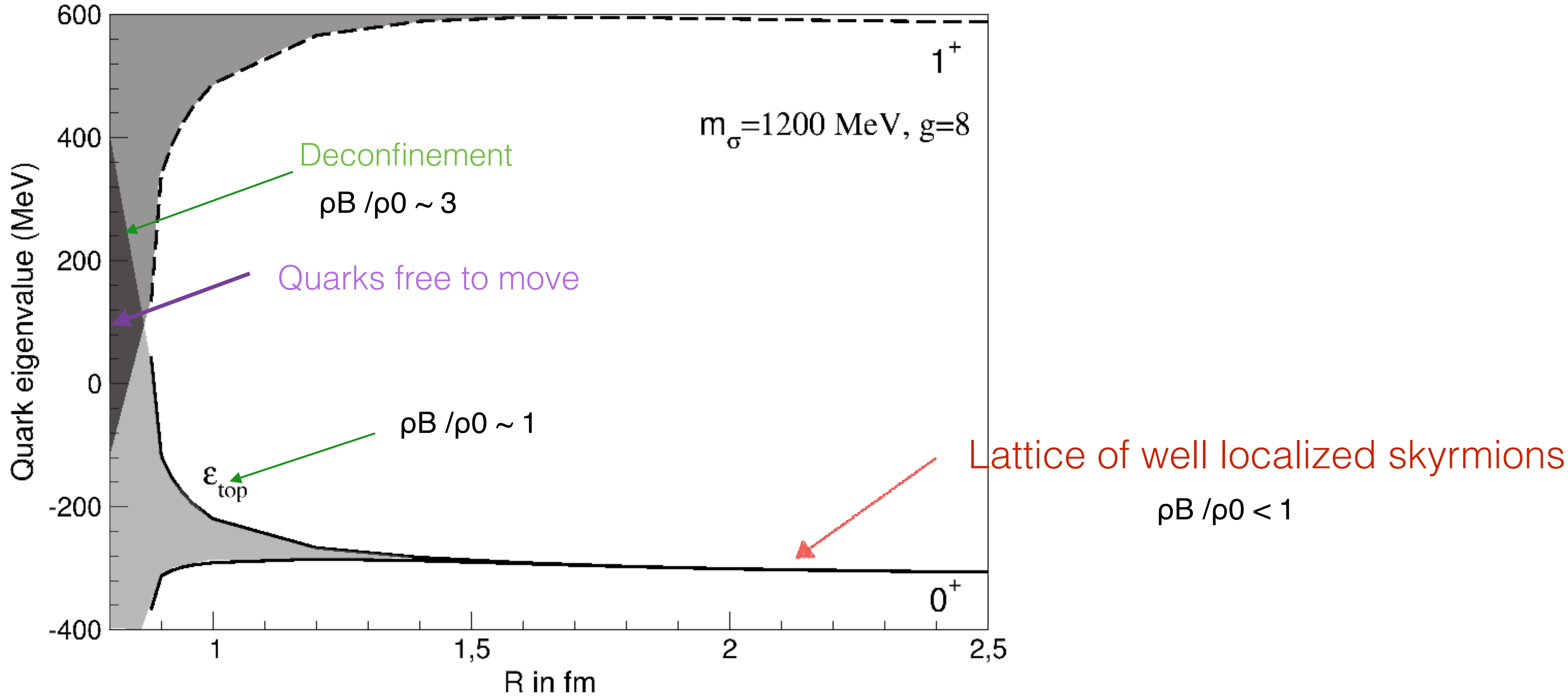


Figure 2. Quark eigenvalue as a function of the cell radius R in the CDM. The shaded area represents the band estimated following Ref. [36]. The first excited state 1^+ and the corresponding lower part of the band are also shown.

At low density the system behaves as a lattice of well localized solitons with quarks and meson fields confined in each cell. As some density the band gets wider and quarks from neighboring cites become free to move along the lattice and between the lower and the excited band.

The figure shows two distinct phenomena:

i)on the one hand the exchange of quarks of the G=0+ band and the G=1+ band at densities above ρB / ρ0 ~ 3 which can be interpreted as deconfinement since the crystal becomes a color conductor.

ii) the plunging of the excited stated into the lower band and the sharing between neighboring cells which is at the origin of delocalization of baryon number carried only by the quarks as will be shown next.

We show the density profiles as a function of density

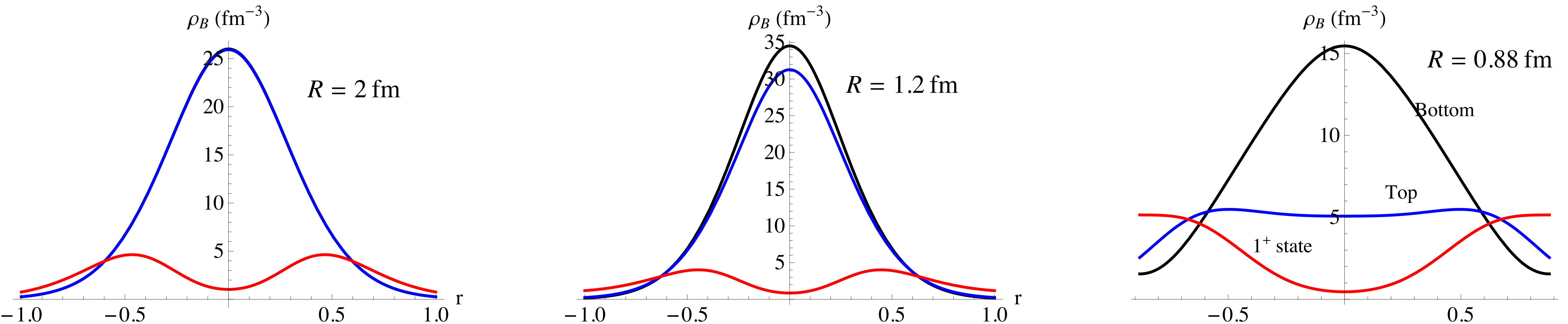


Figure 3. Baryon density profiles inside the Wigner-Seitz cell, for several values of R_{WS}. The profiles are shown for the ground state 0^+ (black line), the top of the lower band (blue line) and the first excited state $G = 1^+$ (red line). The three panel are chosen in order to show, going from the left to right, the confined $B = 1$ phase ($R_{WS} \geq 2$ fm), the delocalized $B = 1/2$ phase ($R_{WS} \leq 1.2$ fm) and finally the deconfined phase ($R_{WS} \leq 0.9$ fm).

i)At low density the solitons are well localized inside the cell, all the quarks occupy the lower state. Color insulator.

ii)As the density increases and the band gets wider, the G=1+ band plunges into the G=0+ band. The quarks are free to move to the upper band whose baryon density profile shows a valley in the center and two bumps close to the edges. This mechanism provides a delocalization of baryon number from the center to the edges.

iii)As we reach even higher densities the band broadens and the baryon densities for G=0+ and the top of the band do not vanish anymore at the boundaries of the cell allowing a continuous flow of quarks between the cells. This scenario represents deconfinement, since now the quarks are free to migrate everywhere in the lattice. Color conductor.

Caveat: The Wigner-Seitz approximation although very useful and widely used shows limitations compared to the skyrmion-lattice, since it does not take into account the long-range correlation between neighboring baryons coming from the pions nor does it allow the inclusion of different isospin configurations

Schematic representation of the delocalization process

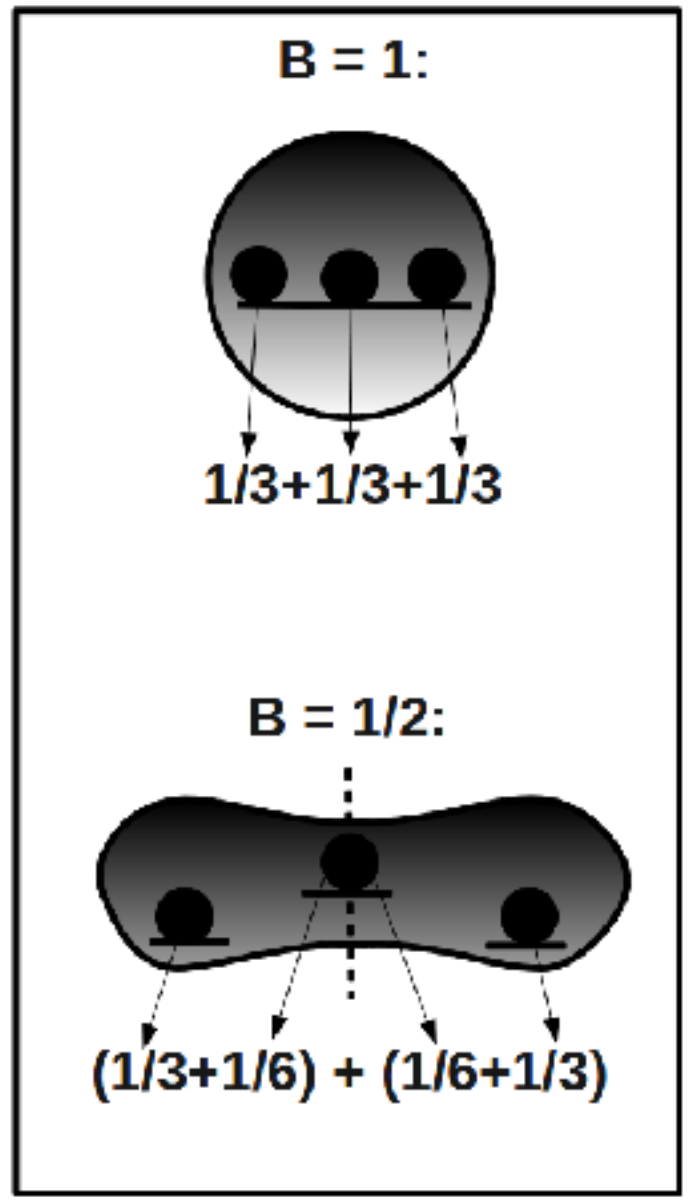


Figure 4. Representation of the delocalization process. In the upper part, the quarks in the $B = 1$ soliton lie in the ground state 0^+ and the baryon density presents a maximum at the center of the cell (upper panel in Fig. 3). As the radius shrinks, some quarks may jump into the first excited level 1^+, leading to a delocalization of baryon number as shown in the lower part of the figure. At the same time, as the radius decreases, the band gets wider and quarks are shared between neighbouring cells. The interplay between these two mechanisms leads to the $B = 1/2$ phase.

Conclusions

We have used a Lagrangian with quark and meson fields based on chiral symmetry and scale invariance to analyze the realization of a B= 1/2 phase at large baryon densities in skyrmionic matter. In the Skyrme model the half-skyrmion phase is strictly connected to topological modifications of the pion field since B is carried by the skyrmion. In this work the so-called quarkyonic phase leads to a similar mechanism of delocalization., i.e. by a modification of the baryon density profile in terms of quark fields.

To describe matter at high densities we have used the Wigner-Seitz approximation in which solitonic matter is described by a lattice of spherical shells of radius R. Finite density effects are obtained by imposing boundary conditions for the fields at the cell boundaries. The presence of a periodic potential generated by the sigma and pion fields leads to a band structure.

The phase transition takes place because the broadening of the bands allow quarks to move into the excited band. This phenomenon allows the baryon number profile within a cell to change. This delocalization generates the analog of the half-skyrmion phase. Ultimately the system becomes a color conductor and deconfinement is reached.

Summarizing the half-skyrmion phase is characterized in our description by the existence of two quark types the G=0+ and the G=1+ with different baryon number profiles.

To conclude: a word of caution the half-skymion phase describing a rigid crystal structure should not be confused with the quarkyonic phase have described. The delocalization mechanism that arises is the seed for the quarkyonic interpretation of the half-skyrmion phase within the spirit of the Cheshire Cat picture.

Acknowledgements

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