11th APCTP-BLTP JINR-PINP NRC KI-SPbSU Joint Workshop

"*Modern problems in nuclear and elementary particle physics*"

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Doubly Magic Drip-line Nucleus ^{48}Si

— Novel Phenomena & New Physics —

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Collaborators: J. J. Li, N. Van Giai, J. Margueron, et. al.

OUTLINE

1. **Introduction and Motivation**
   - **Covariant Density Functional theory**
   - **Challenges in Nuclear Physics**
   - **New Magicity and Bubbles in Ca and Si isotopes**
2. **Covariant density functional theory with Fock terms**
   - **Relativstic Hartree-Fock (RHF) theory**
   - **Relativistic Hartree-Fock-Bogoliubov (RHFB) theory**
3. **New Physics in determining the magicity of ^{48} Si**
   - **Bubble and magic shells**
   - **Self-consistent tensor force effects in magicity**
   - **Neutron and/or proton crossing-shell excitations**
4. **Conclusions and Perspectives**

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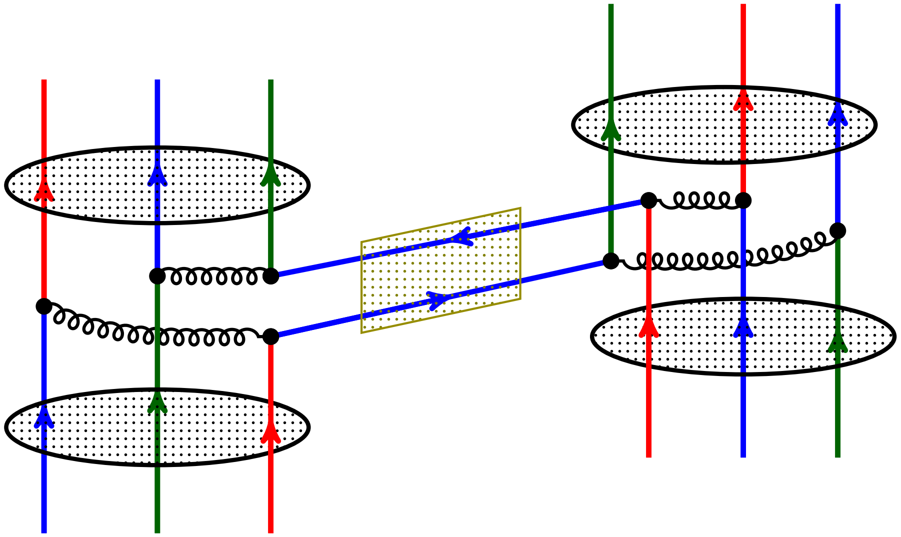
Nuclear Force: Meson Exchange Diagram

- Nuclear Force: meson exchange diagram

Yukawa (1935)



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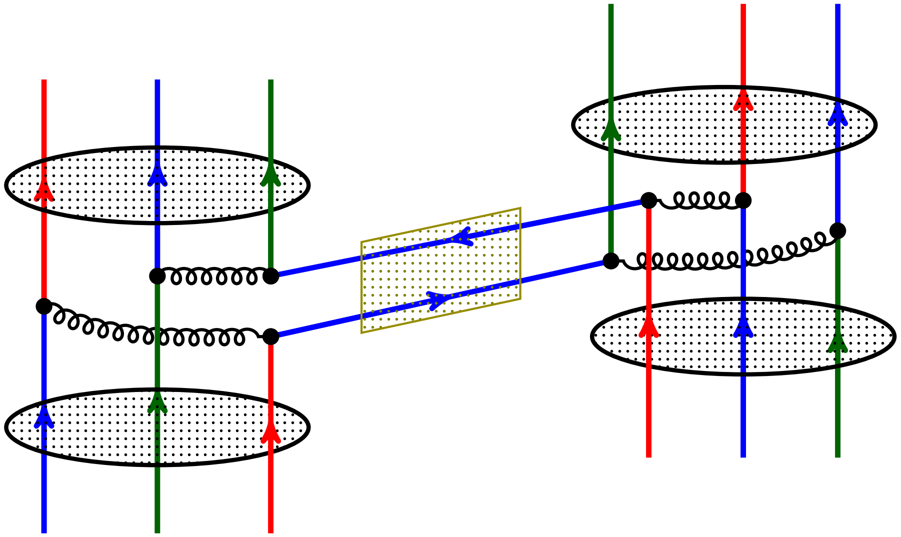

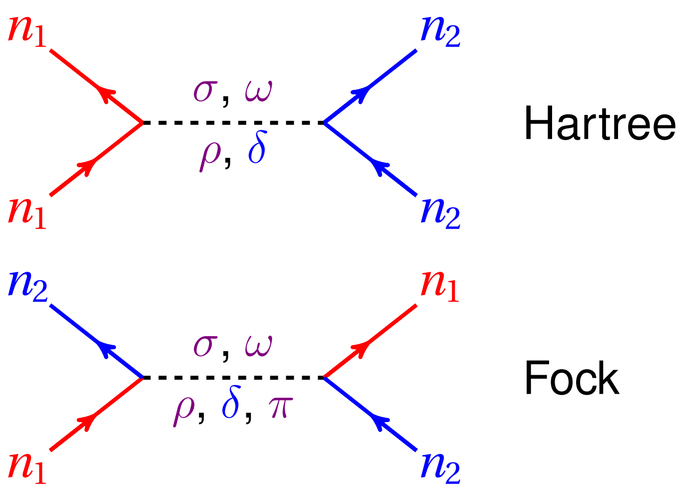

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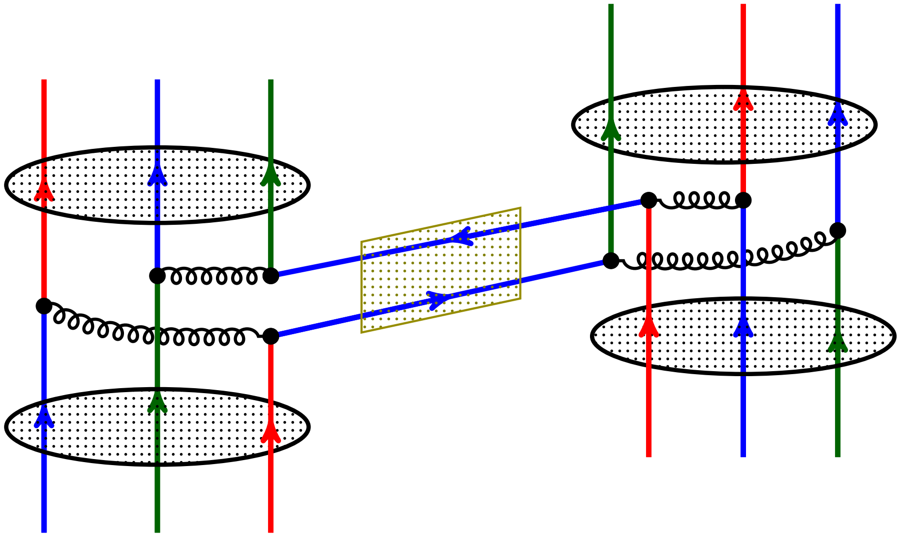

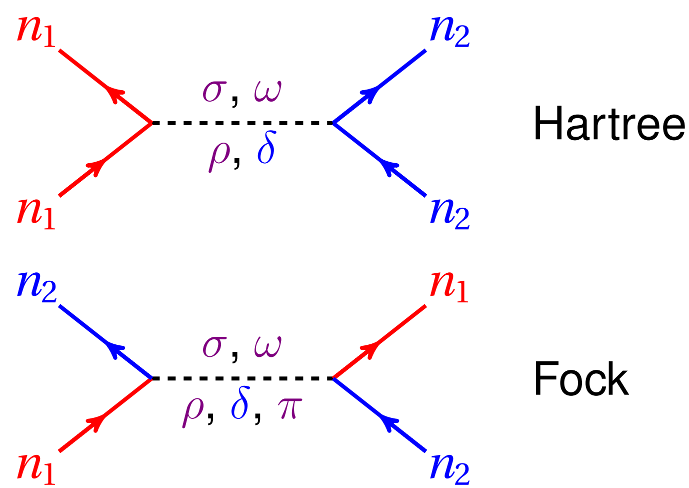

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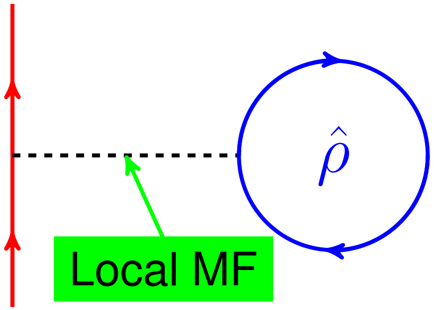
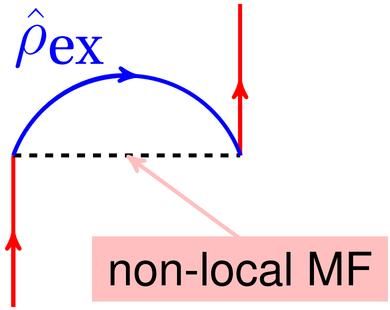
— Being consistent with principle of density functional theory





$\hat{\rho}$: local density

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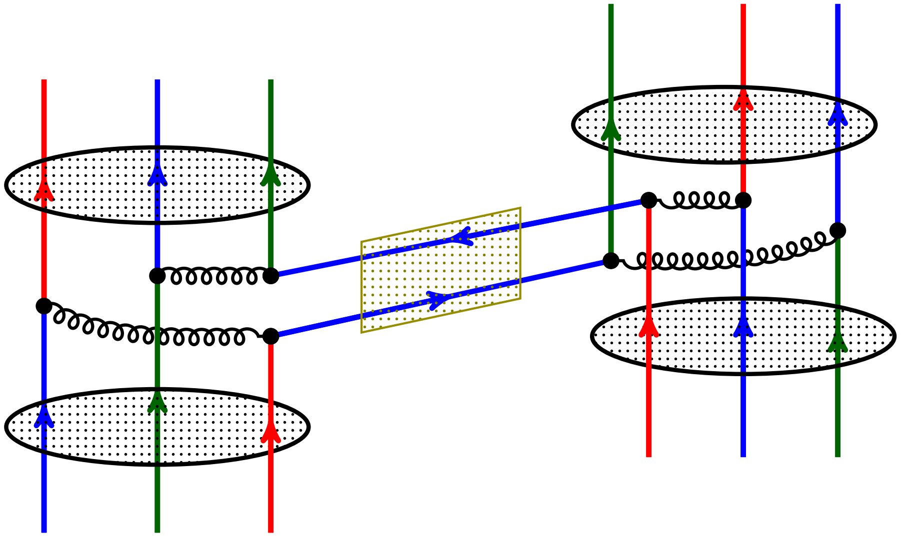

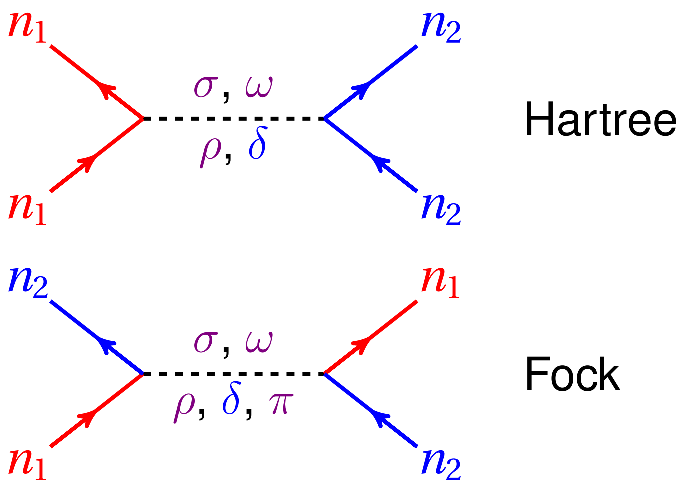

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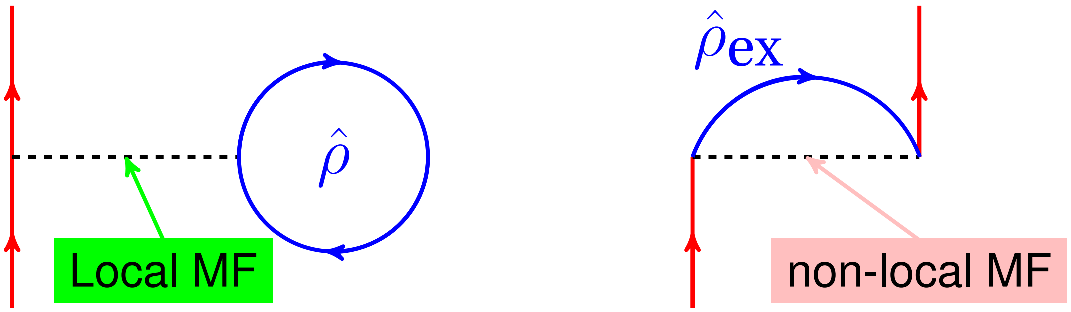

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- Complicated nuclear in-medium effects: non-perturbative nuclear force?

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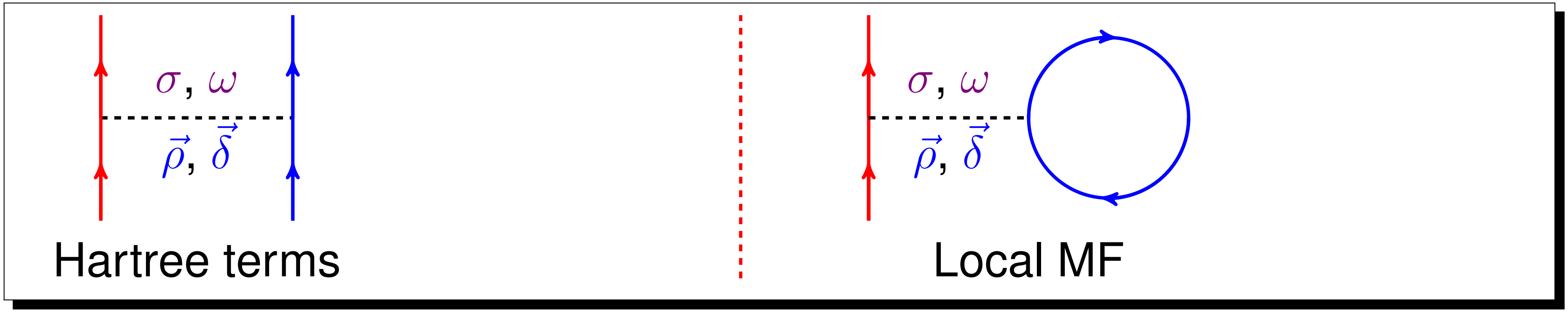
  - Walecka(1974), Serot(1986), Reihard(1989), Ring(1996), Bender(2003), Meng(2006)
  - ☞ Natural treatment of spin-orbit coupling: Covariant framework
  - ☞ Limited by Hartree approach, important ingredients, e.g., tensor force, are missing.



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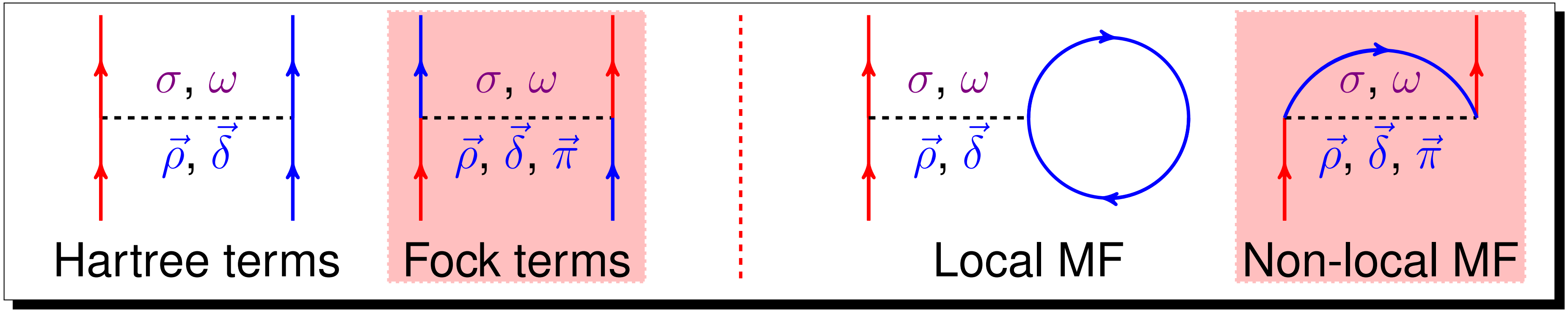
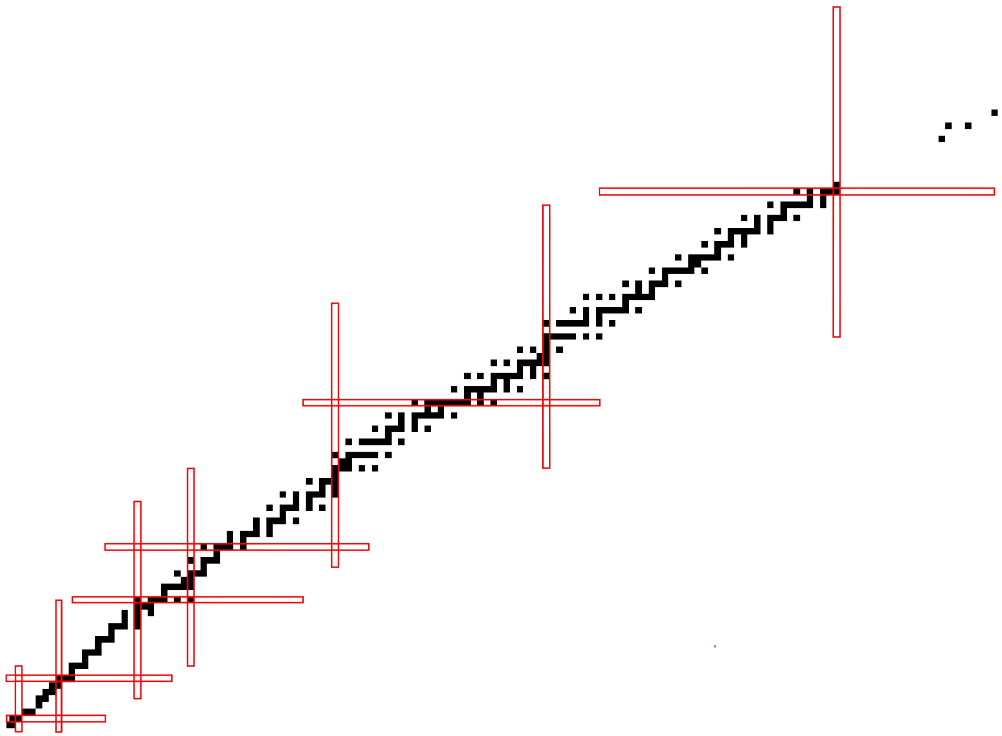
- CDF theory with Fock terms: relativistic Hartree-Fock (RHF) theory
  - Bouyssy (1987), Bernardos (1993), Shi (1995), Marcos (2004), Long (2006-2010).
  - Maintain the advantages of RMF theory, and include the tensor force naturally.
  - Non-local Fock terms are not easy to handle.



New Challenges and Opportunities

- Nuclides: On earth ($\sim$300),



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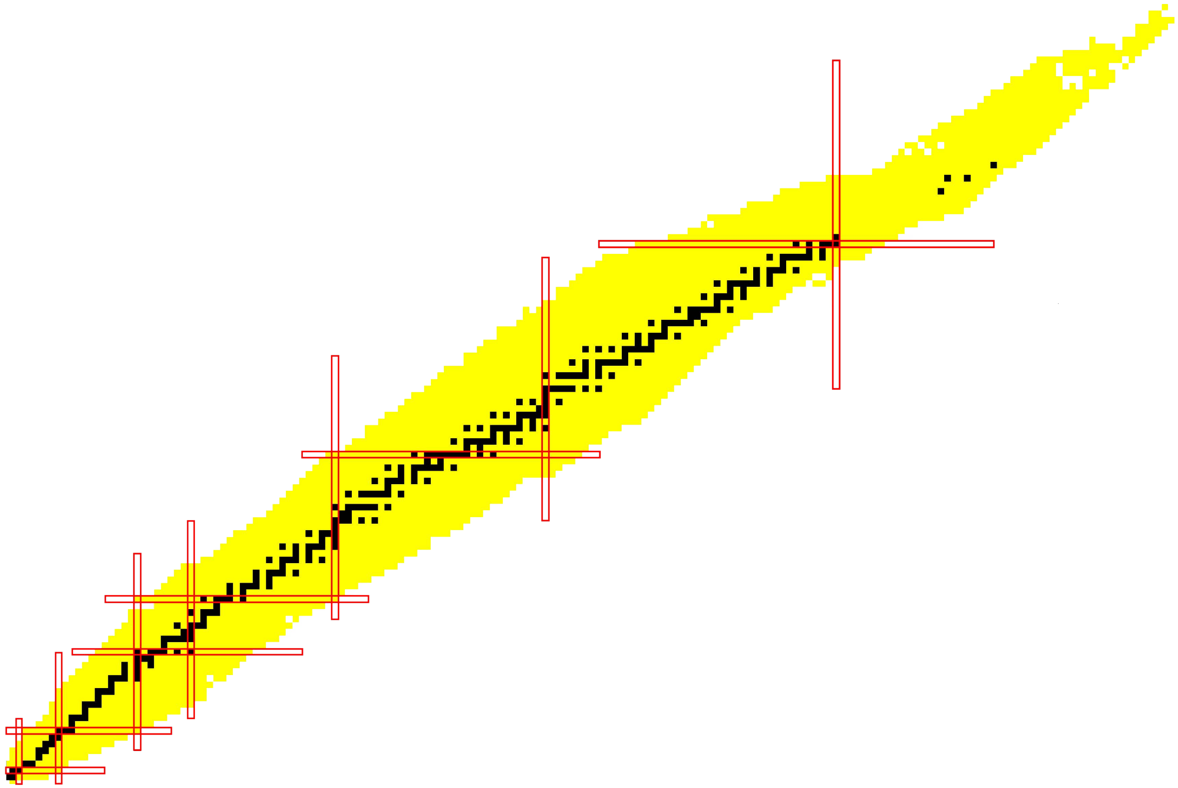

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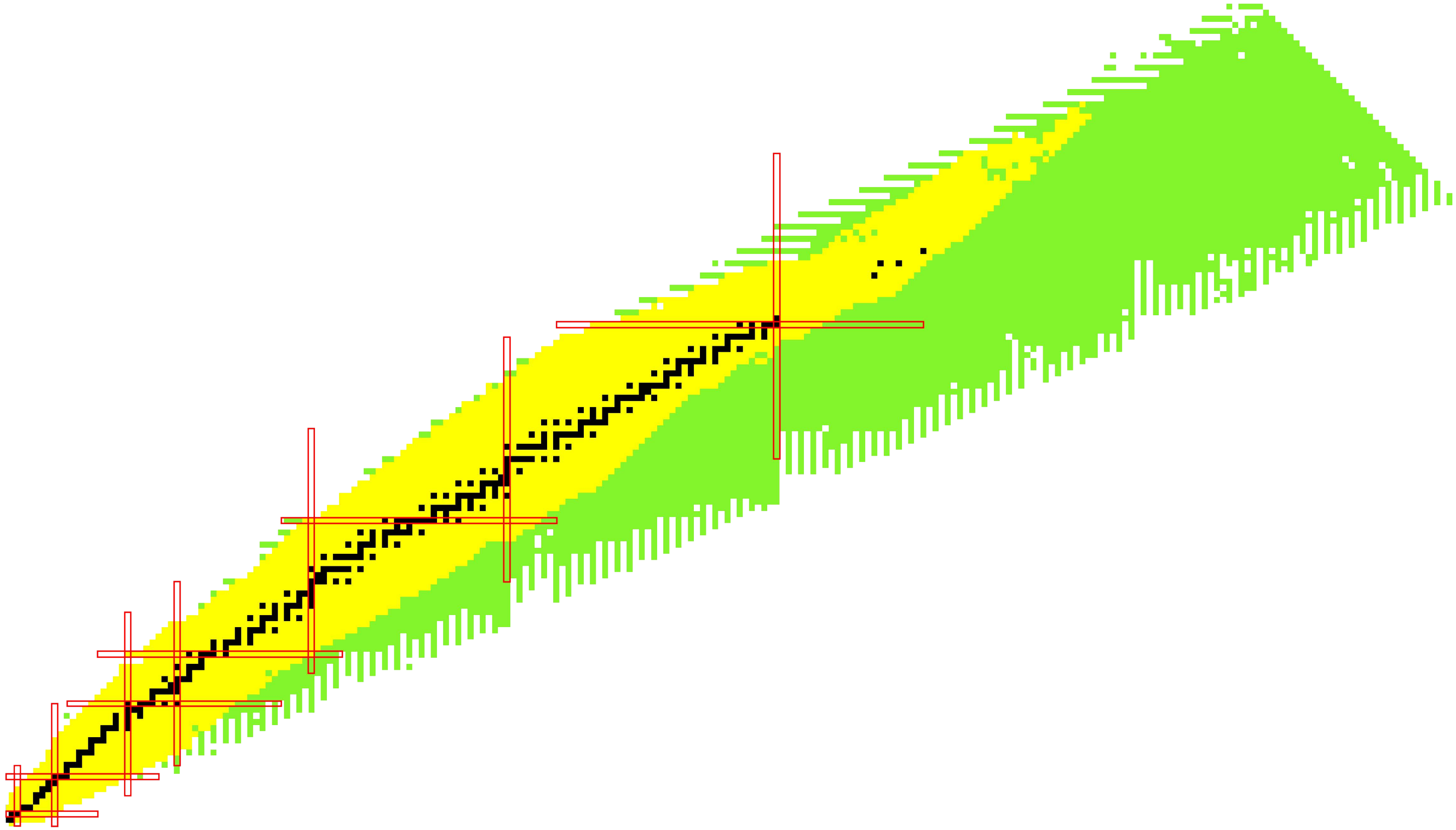

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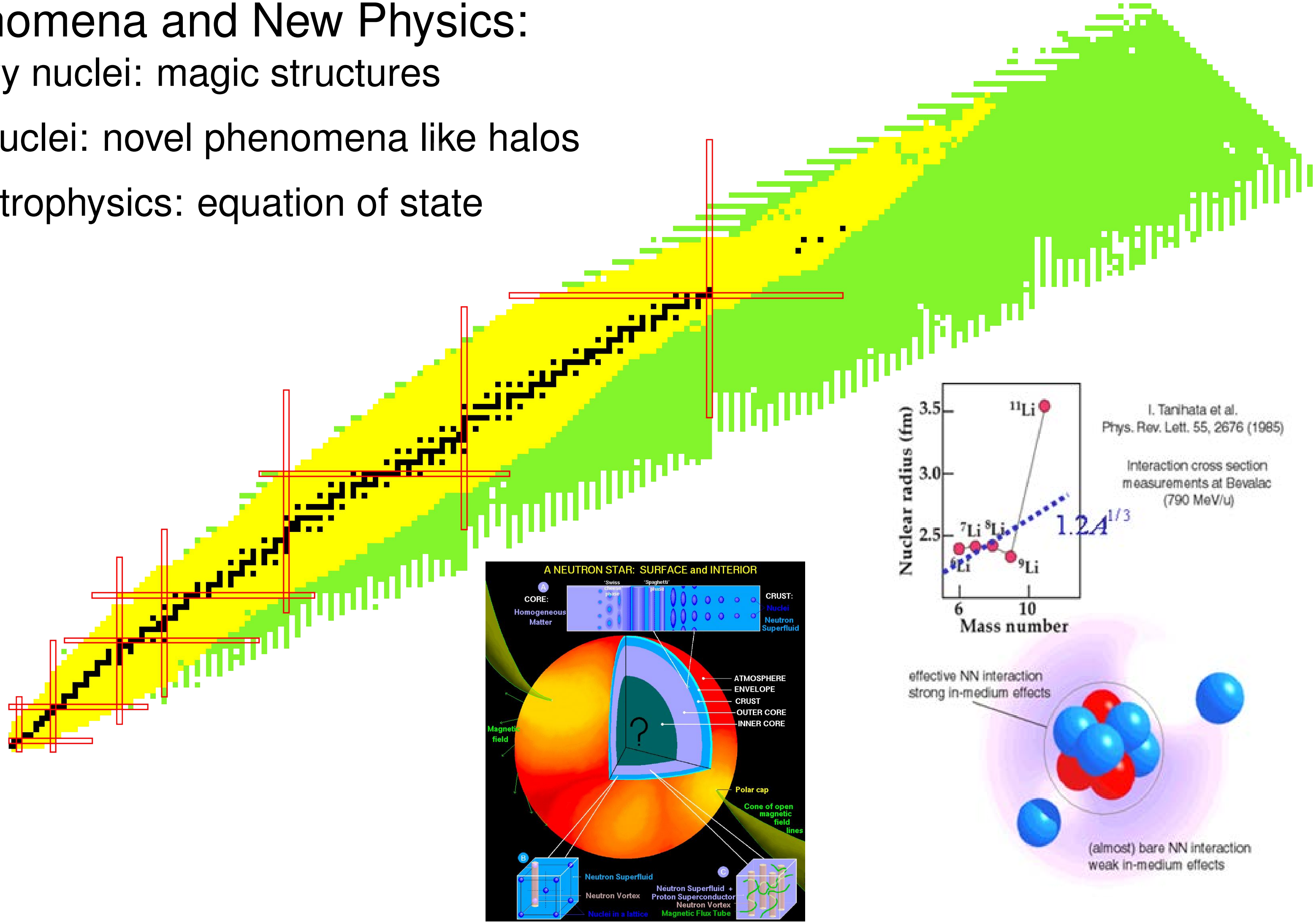

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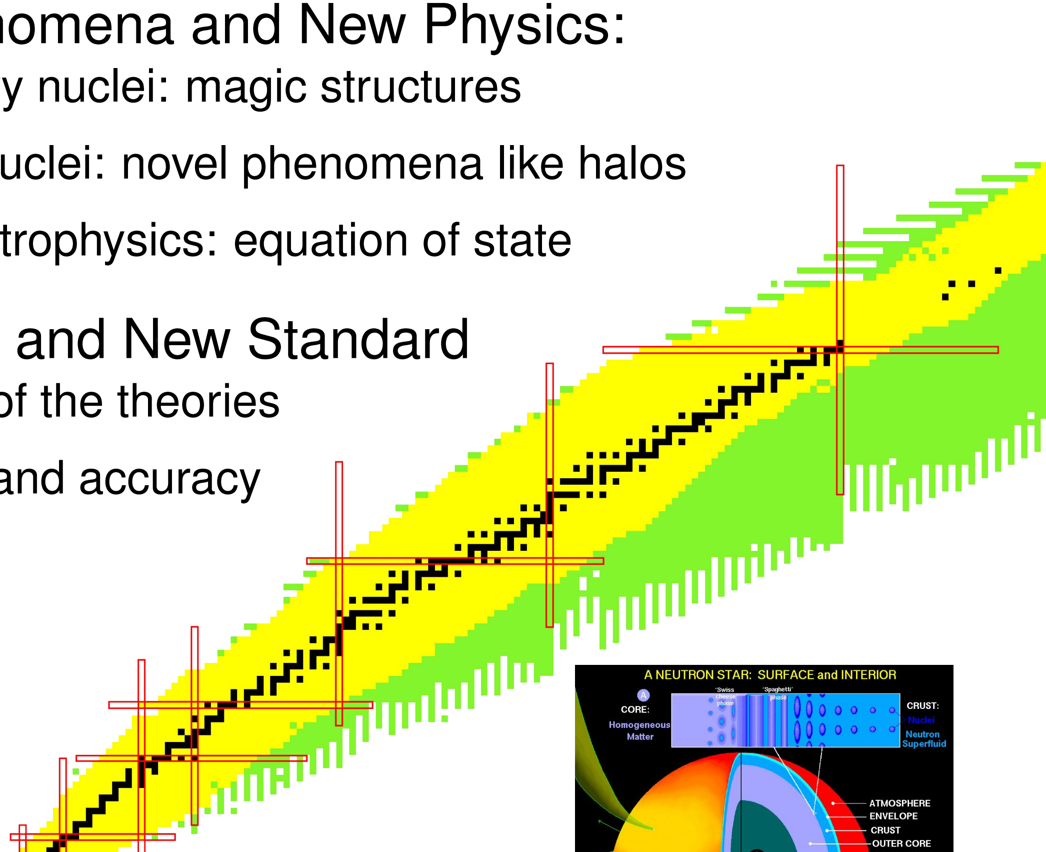

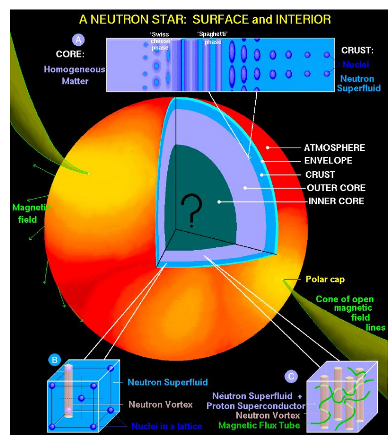

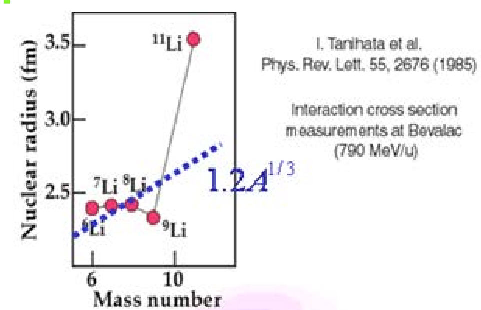

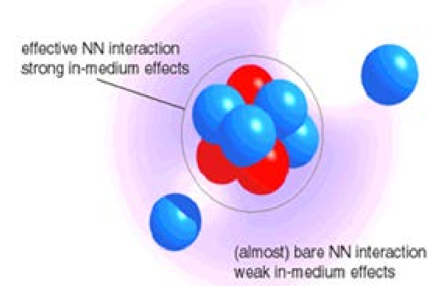

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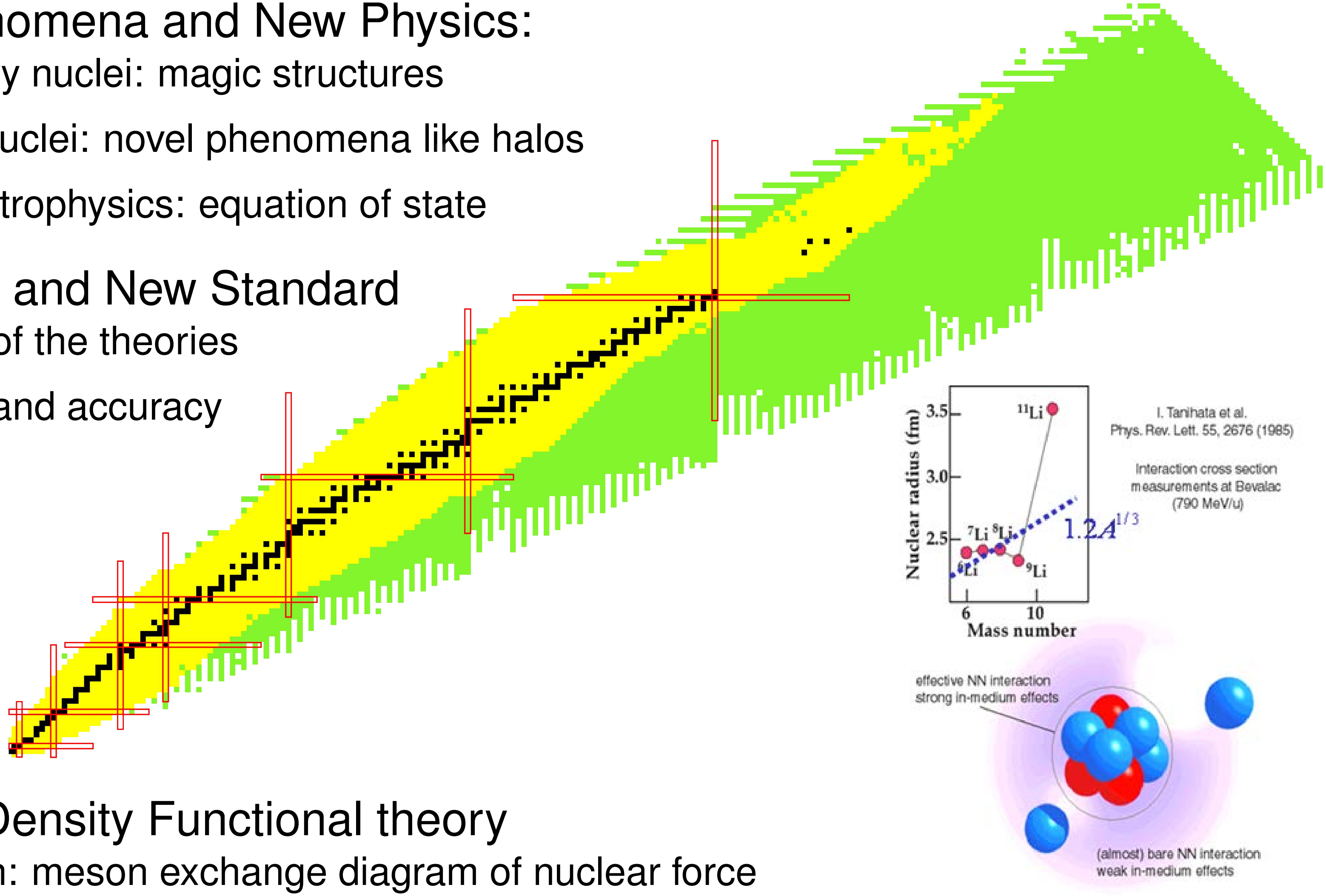
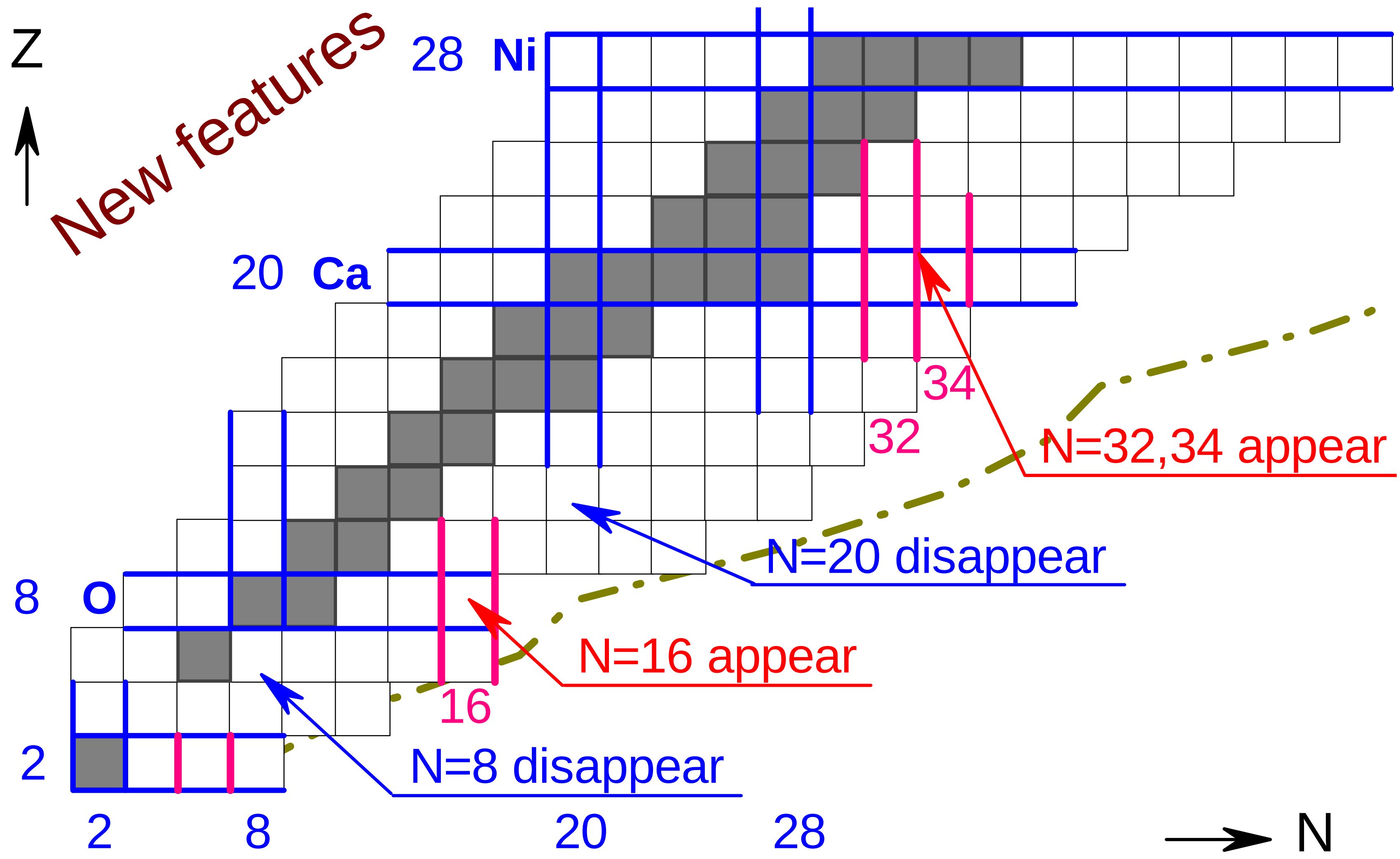


- Covariant Density Functional theory
  - ☞ Foundation: meson exchange diagram of nuclear force
  - ☞ Reliability: relativity and spirit of density functional theory

Magicity in neutron-rich mid-mass nuclei



Ozawa (2000); Sorlin (2008); Hoffman (2008); Steppenbeck (2013, 2015); Wienholtz (2013).

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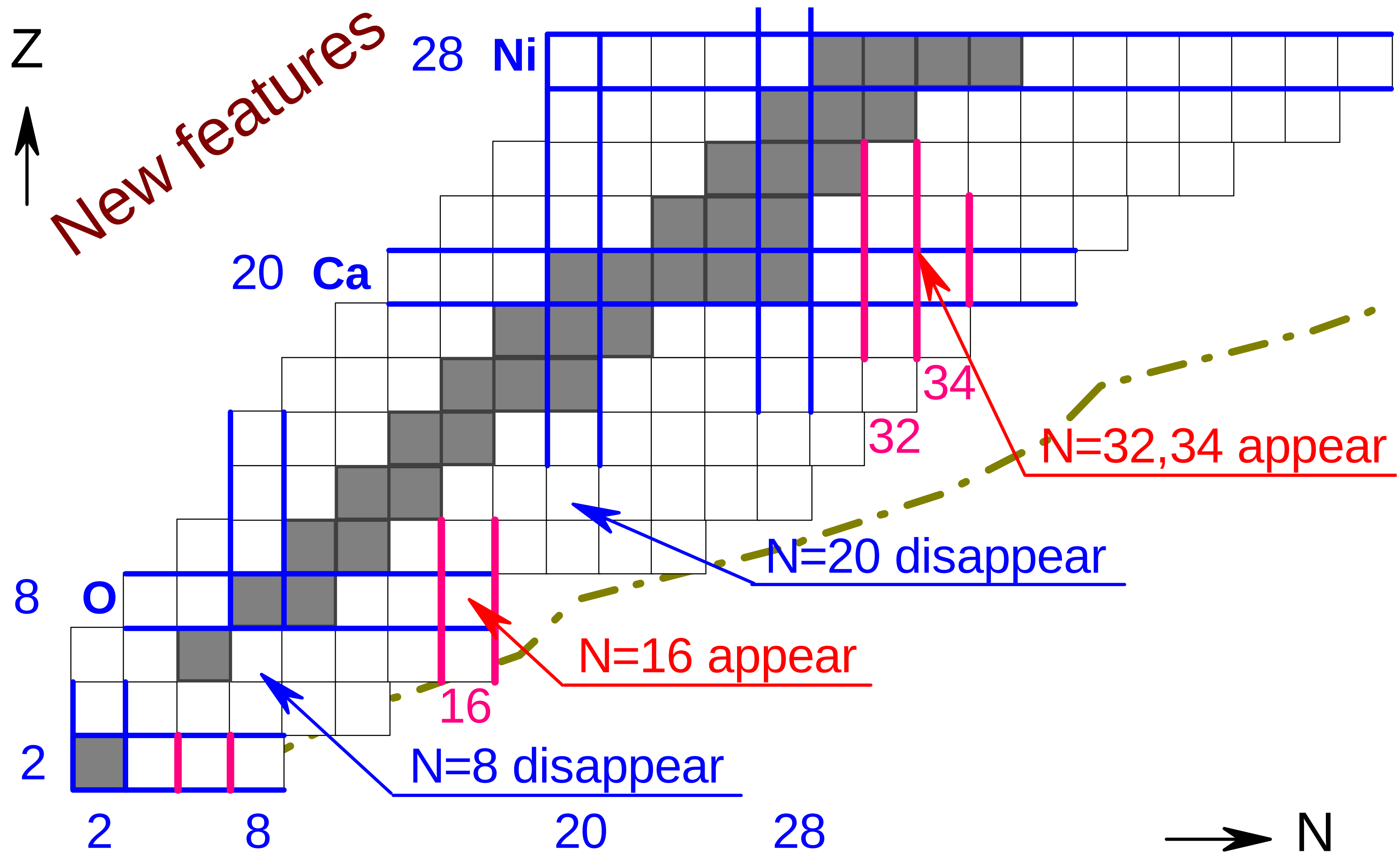

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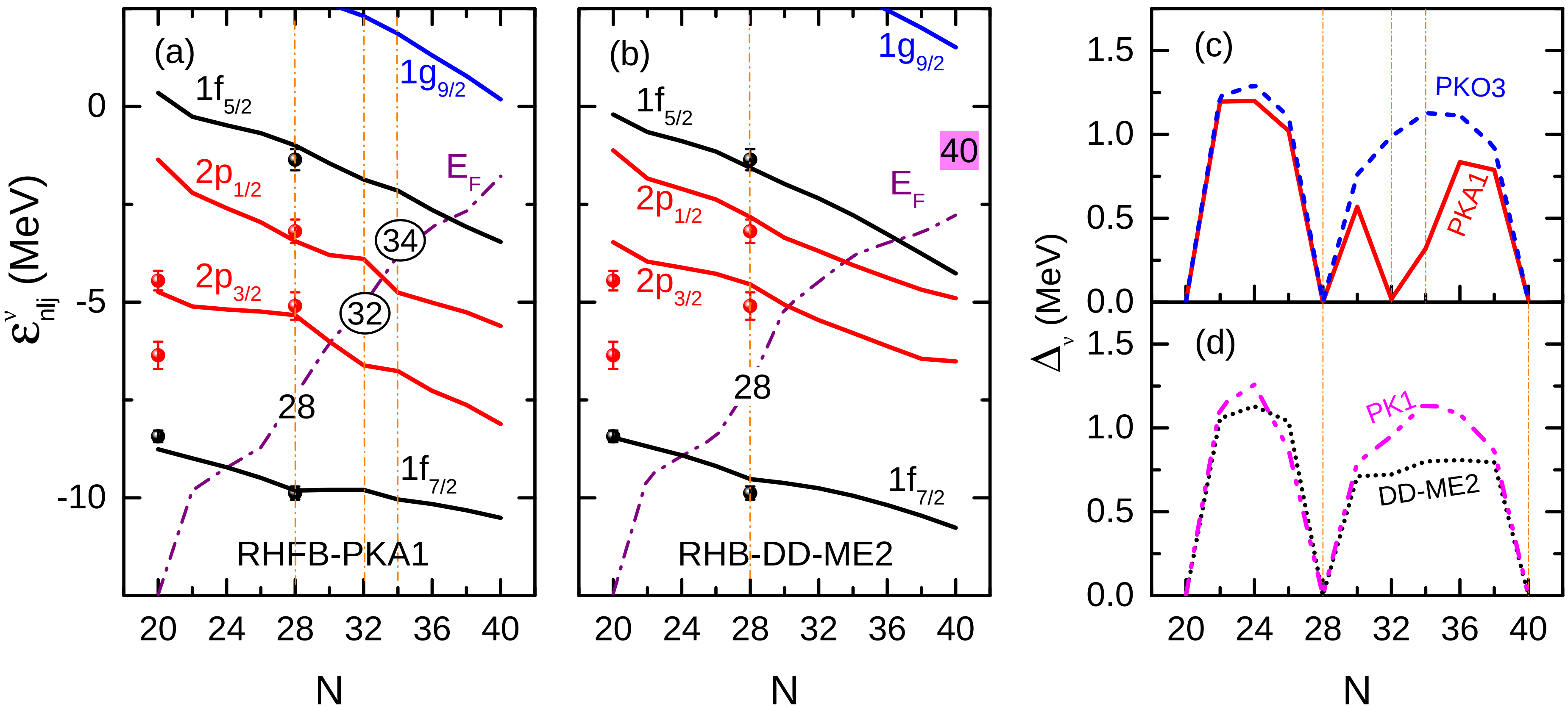
Mechanism in determining magic structures: tensor force?

New Magicity: $N = 32$ & 34



— Li, Margueron, LONG, Giai, PLB 753, 97 (2016) —

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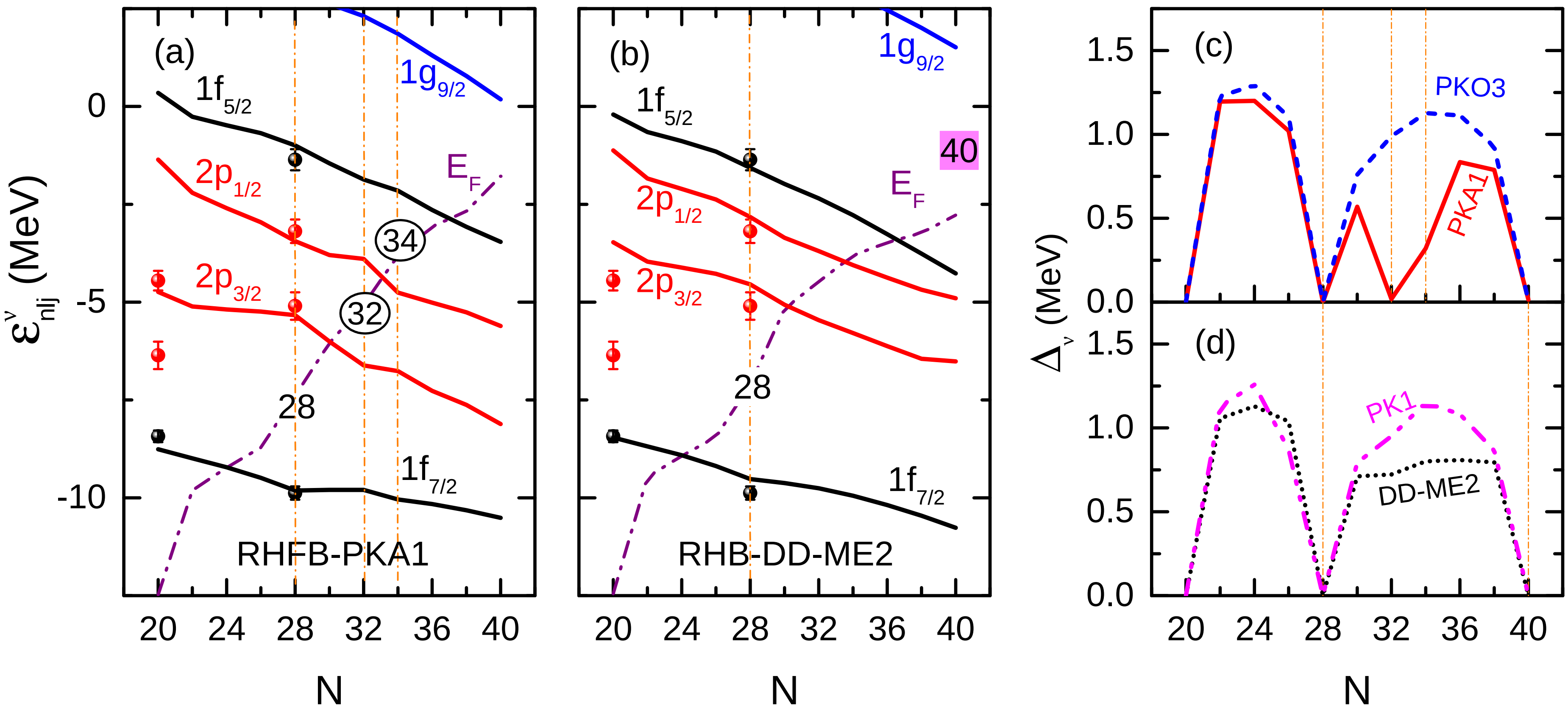

- ^{52}Ca and ^{54}Ca are the newly found doubly magic nuclei in Ca isotopes.
- RHFB+PKA1 can well reproduce the magicity at $N = 32$ & 34, which shows the reliability of the model.

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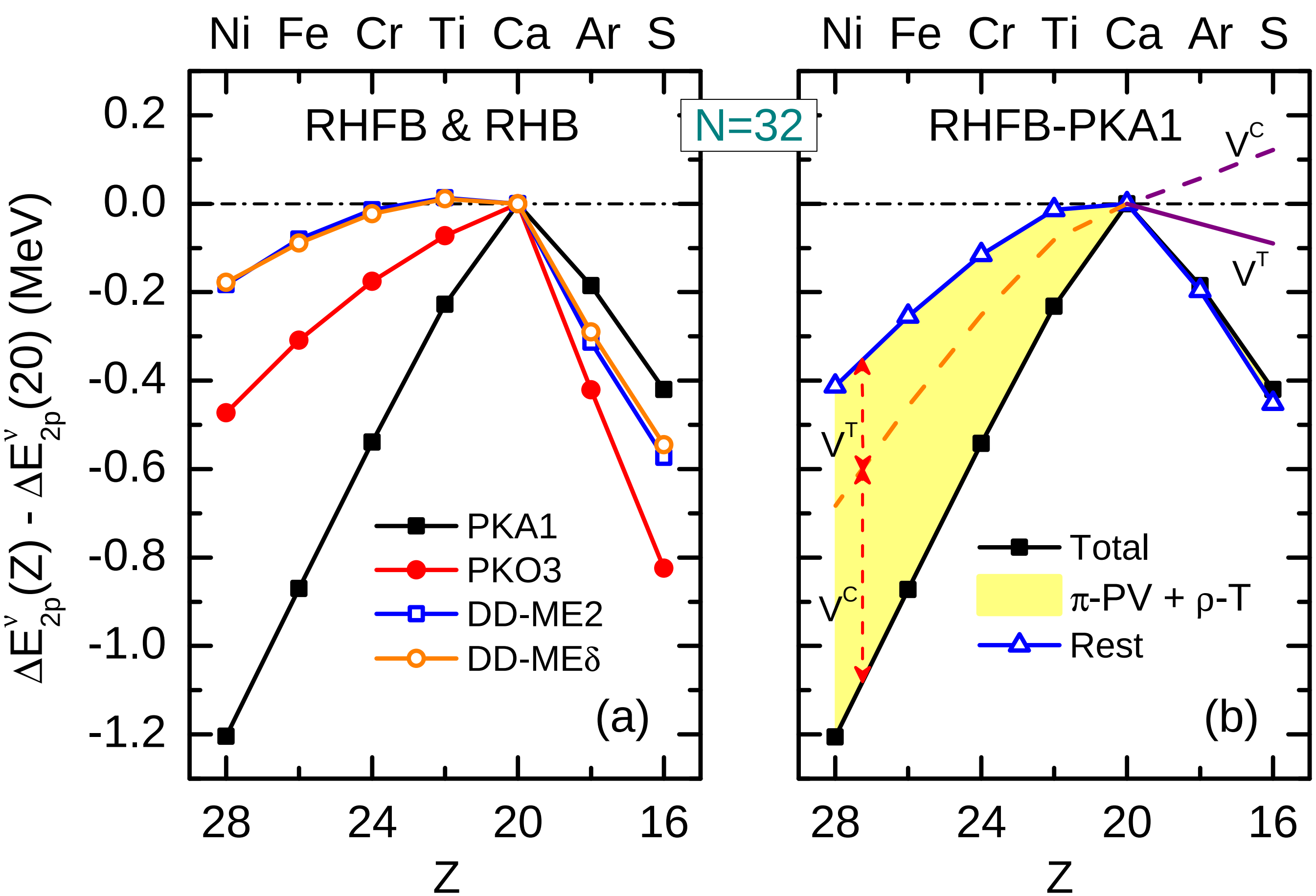
Physics related to new magicity



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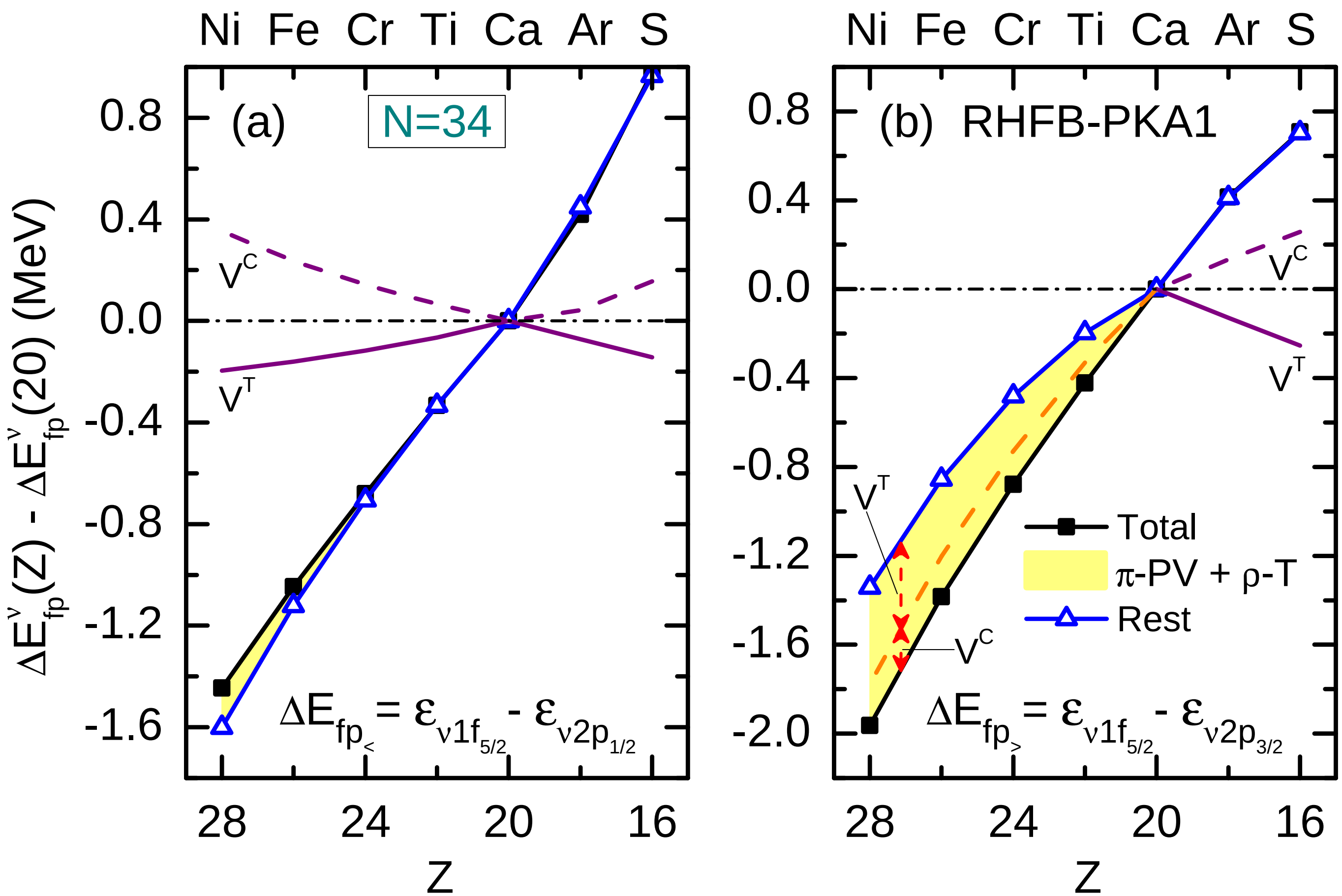

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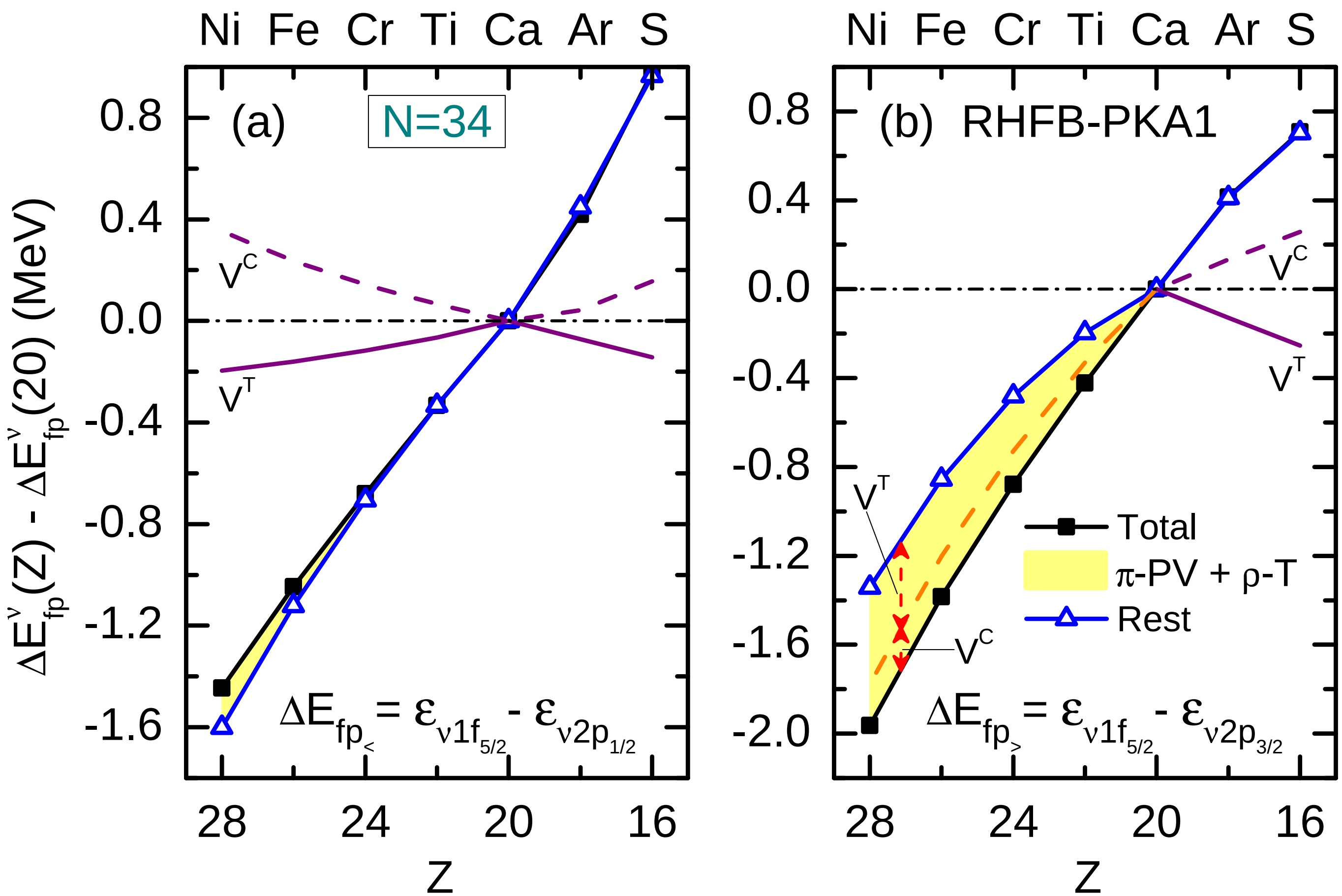


- $N = 32$: isovector ρ-T & π-PV couplings are the key physics.
- $N = 34$: isovector ρ-T & π-PV couplings are not so significant any more.

It remains some mystery on the physics that triggers the $N = 34$ shell.

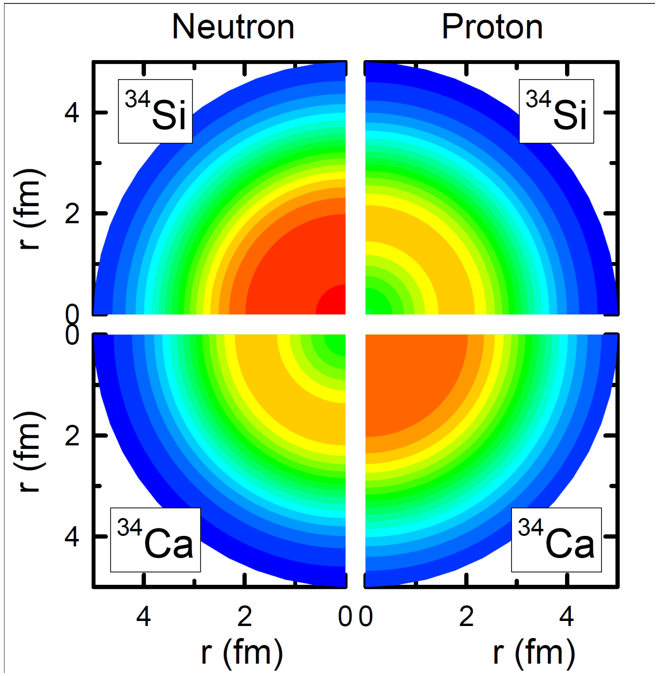
New drip-line magic nucleus: ^{48}Si

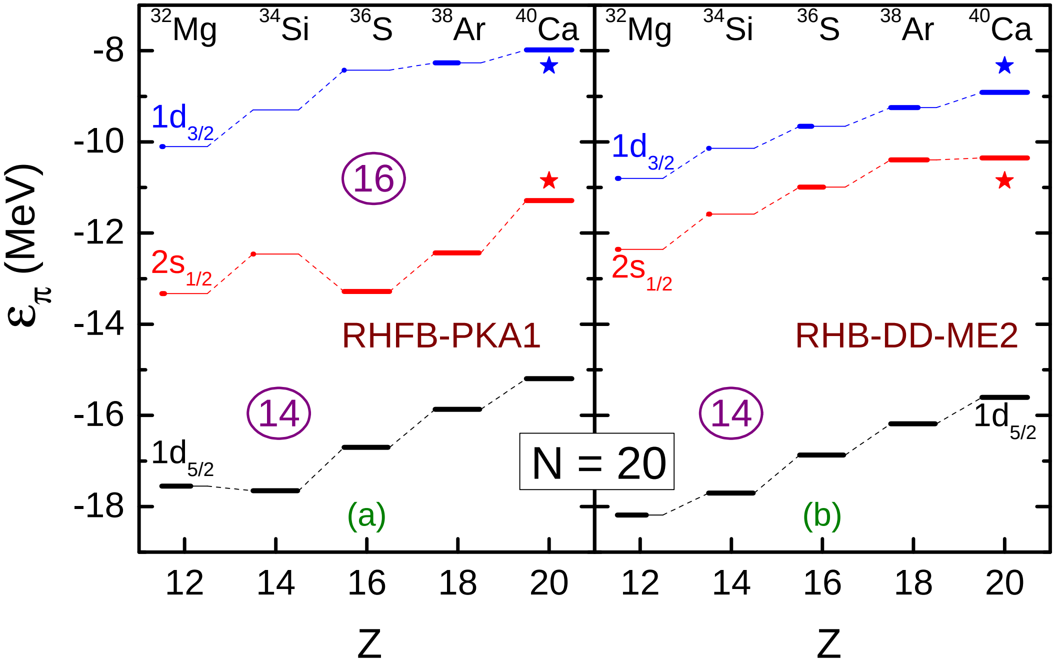
Table 1: Energy gap corresponding to $N = 32$ & 34 in the Ni, Ca and Si isotopes. Results are provided by the calculations with RHFB-PKA1, RHFB-PKO3 and RHB-DD-ME2 models.

Force	$\Delta E(i, i')$	N	Ni	Ca	Si
PKA1	$(\nu 2p_{1/2}, \nu 2p_{3/2})$	32	1.51	**2.72**	0.81
	$(\nu 1f_{5/2}, \nu 2p_{1/2})$	34	1.04	**2.60**	**4.3**
PKO3	$(\nu 2p_{1/2}, \nu 2p_{3/2})$	32	1.22	1.69	0.68
	$(\nu 1f_{5/2}, \nu 2p_{1/2})$	34	-1.72	0.77	**2.72**
DD-ME2	$(\nu 2p_{1/2}, \nu 2p_{3/2})$	32	1.58	1.76	0.92
	$(\nu 1f_{5/2}, \nu 2p_{1/2})$	34	-1.23	1.21	**3.18**

- For ^{52,54}Ca, only PKA1 shows distinct shells $N = 32$ & 34, whereas for ^{60}Ni and ^{46}Si all the models present similar values of $\Delta_{\nu 2p}$.
- For ^{48}Si: all the models shows distinct shell $N = 34$, and therefore ^{48}Si can be referred as the new magic drip-line nucleus.

Bubble structure predicted in ^{34}Si & ^{34}Ca





- Proton/neuron semi-bubbles occur in mirror systems ^{34}Si/^{34}Ca, since $N/Z = 14$ shells prevent valence protons or neutrons to occupy $2s_{1/2}$ orbit.

—J. J. Li, W. H. Long, J. L. Song, and Q. Zhao, Phys. Rev. C 93, 054312 (2016)

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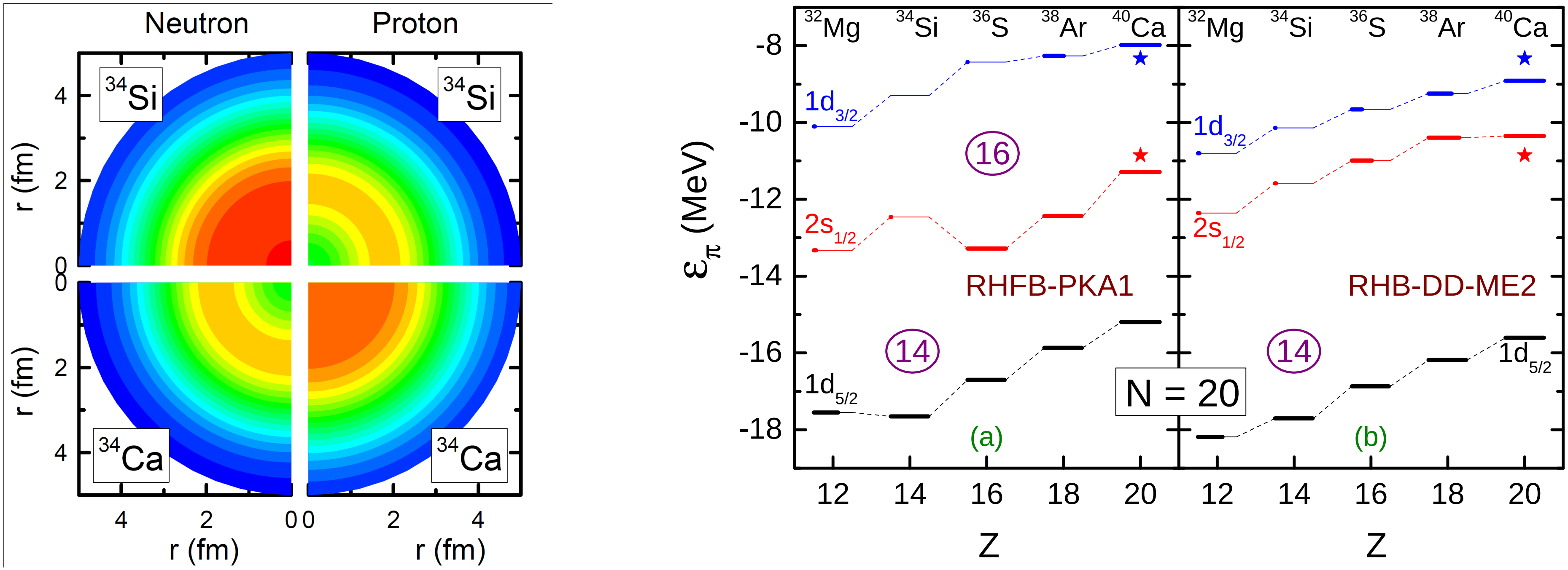

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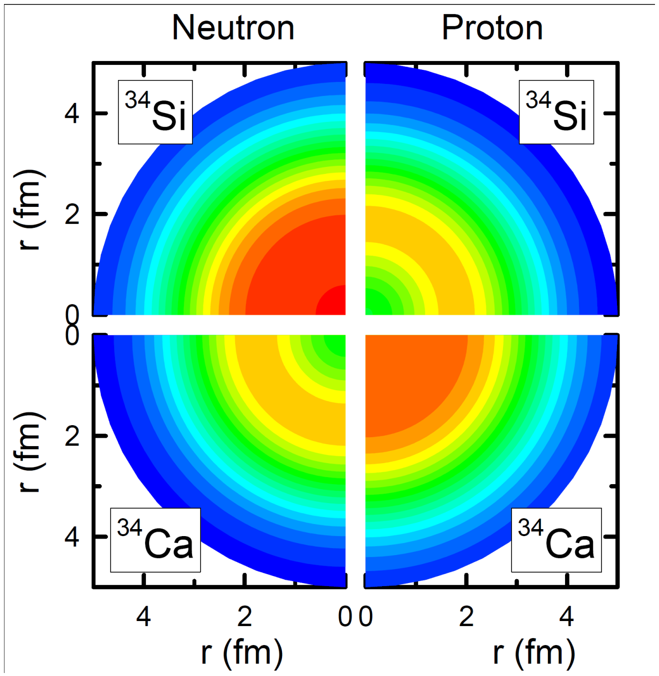

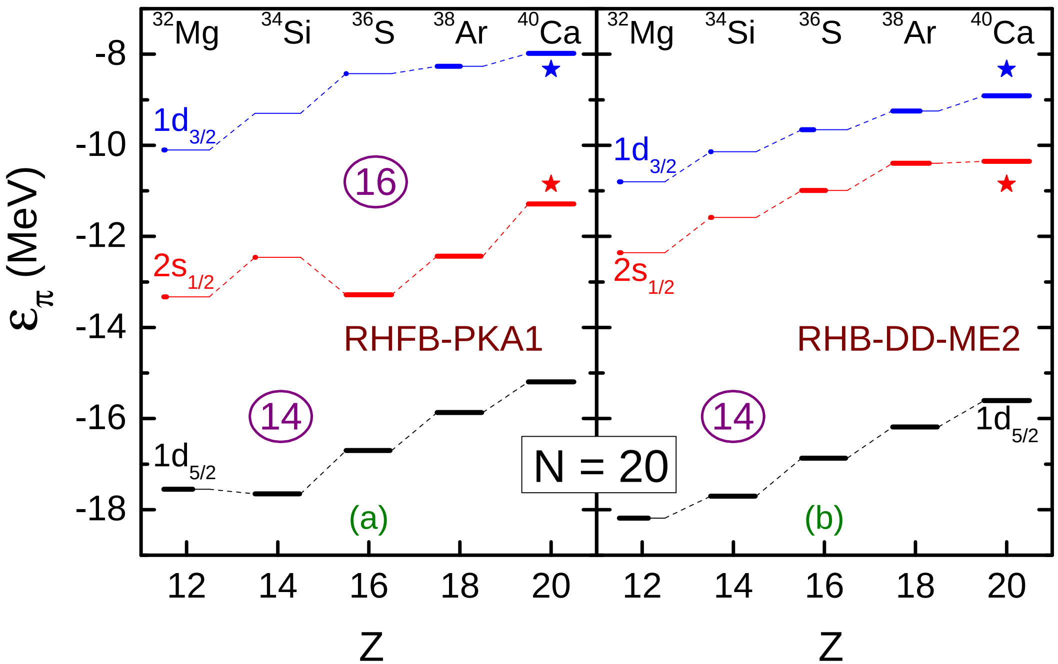

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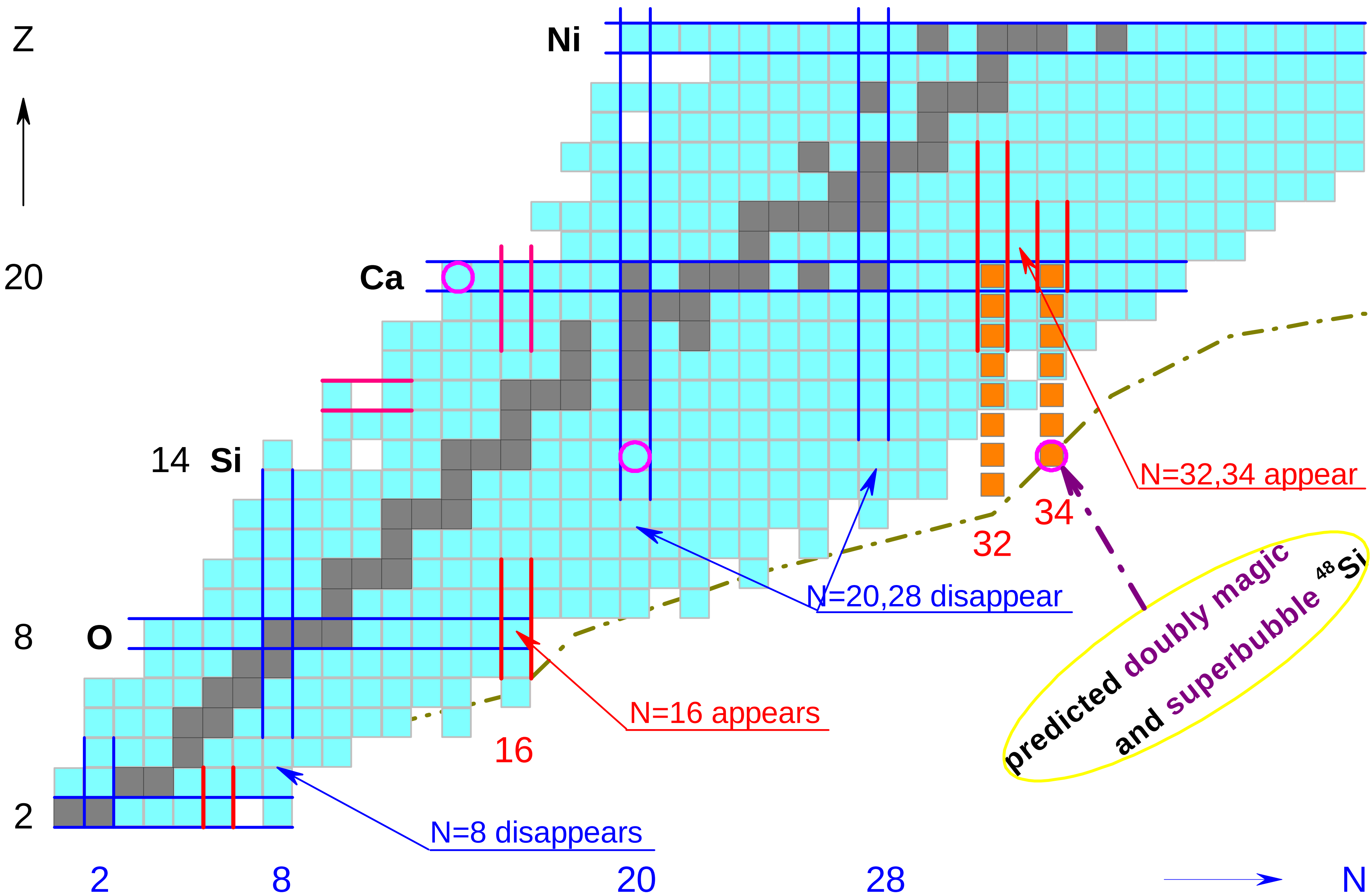
Does the proton bubble structure still remain in ^{48}Si?

Studying Object: ^{48}Si



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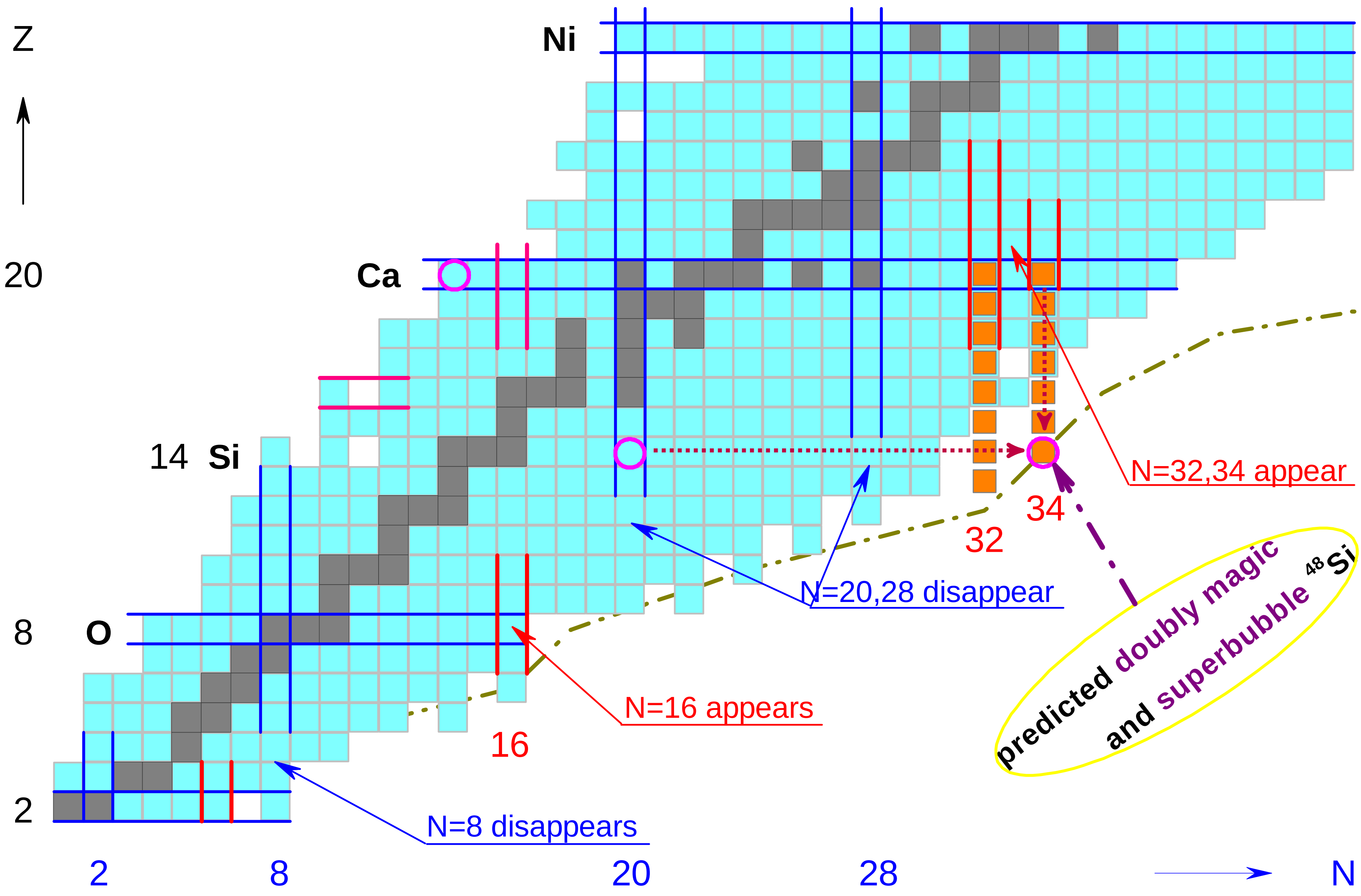


OUTLINE

RHF Hamiltonian

A. Bouyssy (1987)

- Effective Hamiltonian for nuclei: $\phi = \sigma$-S; ω-V, A-V, ρ-V; ρ-VT, ρ-T; π-PV

$$H = \int d\boldsymbol{x}\, \bar{\psi}\, (-i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + M)\, \psi + \frac{1}{2}\int d\boldsymbol{x} d\boldsymbol{x}' \bar{\psi}(\boldsymbol{x})\bar{\psi}(\boldsymbol{x}')\Gamma_\phi D^\phi \psi(\boldsymbol{x}')\psi(\boldsymbol{x})$$

- Types of the interaction: $\Gamma_\phi(x, x')$

$$\Gamma_{\sigma\text{-S}} \equiv -\, g_\sigma(x)g_\sigma(x'), \qquad \Gamma_{A\text{-V}} \equiv \frac{e^2}{4}\left[\gamma_\mu\left(1-\tau_3\right)\right]_x\left[\gamma^\mu\left(1-\tau_3\right)\right]_{x'}, \tag{1}$$

$$\Gamma_{\omega\text{-V}} \equiv \left(g_\omega\gamma_\mu\right)_x\left(g_\omega\gamma^\mu\right)_{x'}, \qquad \Gamma_{\pi\text{-PV}} \equiv \frac{-1}{m_\pi^2}\left(f_\pi\vec{\tau}\gamma_5\gamma_\mu\partial^\mu\right)_x\cdot\left(f_\pi\vec{\tau}\gamma_5\gamma_\nu\partial^\nu\right)_{x'}, \tag{2}$$

$$\Gamma_{\rho\text{-V}} \equiv \left(g_\rho\gamma_\mu\vec{\tau}\right)_x\cdot\left(g_\rho\gamma^\mu\vec{\tau}\right)_{x'}, \qquad \Gamma_{\rho\text{-T}} \equiv \frac{1}{4M^2}\left(f_\rho\sigma_{\nu k}\vec{\tau}\partial^k\right)_x\cdot\left(f_\rho\sigma^{\nu l}\vec{\tau}\partial_l\right)_{x'}, \tag{3}$$

$$\Gamma_{\rho\text{-VT}} \equiv \frac{1}{2M}\left(f_\rho\sigma^{k\nu}\vec{\tau}\partial_k\right)_x\cdot\left(g_\rho\gamma_\nu\vec{\tau}\right)_{x'} + \left(g_\rho\gamma_\nu\vec{\tau}\right)_x\cdot\frac{1}{2M}\left(f_\rho\sigma^{k\nu}\vec{\tau}\partial_k\right)_{x'} \tag{4}$$

- Yukawa propagator $D_\phi\left(\boldsymbol{x},\boldsymbol{x}'\right)$ — neglecting retardation effects

$$D_\phi\left(\boldsymbol{x},\boldsymbol{x}'\right) = \frac{1}{4\pi}\frac{e^{-m_\phi|\boldsymbol{x}-\boldsymbol{x}'|}}{|\boldsymbol{x}-\boldsymbol{x}'|}, \qquad D_A\left(\boldsymbol{x},\boldsymbol{x}'\right) = \frac{1}{4\pi}\frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|} \tag{5}$$

RHF energy density functional (EDF) A. Bouyssy(1987)

- Solutions of Dirac Eq.: $\left\{\varepsilon_k > 0, c_k, c_k^\dagger \text{ (Fermi sea)};\ \varepsilon_l < 0, c_l, c_l^\dagger \text{ (Dirac sea)}\right\}$
- Quantizing nucleon spinor:

$$\psi = \sum_k \psi_k(\boldsymbol{x}) e^{-i\varepsilon_k t} c_k + \sum_l \psi_l(\boldsymbol{x}) e^{-i\varepsilon_l t} d_l^\dagger,$$

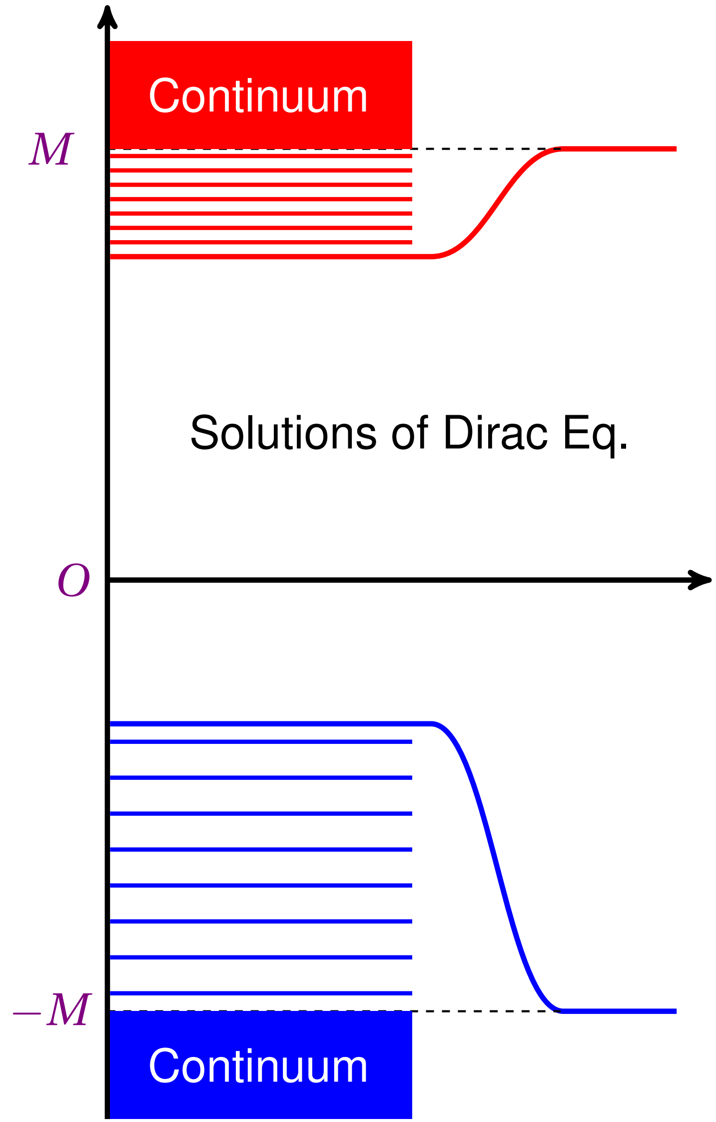
Ground state with no-sea approximation

$$|\Phi_0\rangle = \prod_{i=1}^{A} c_i^\dagger |0\rangle ,$$

RHF EDF: expectation of H referring to $|\Phi_0\rangle$

$$E = \langle\Phi_0| H |\Phi_0\rangle = \langle\Phi_0| T |\Phi_0\rangle + \sum_\phi \langle\Phi_0| V_\phi |\Phi_0\rangle$$

T and V_ϕ are the kinetic energy and potential energy terms, respectively.



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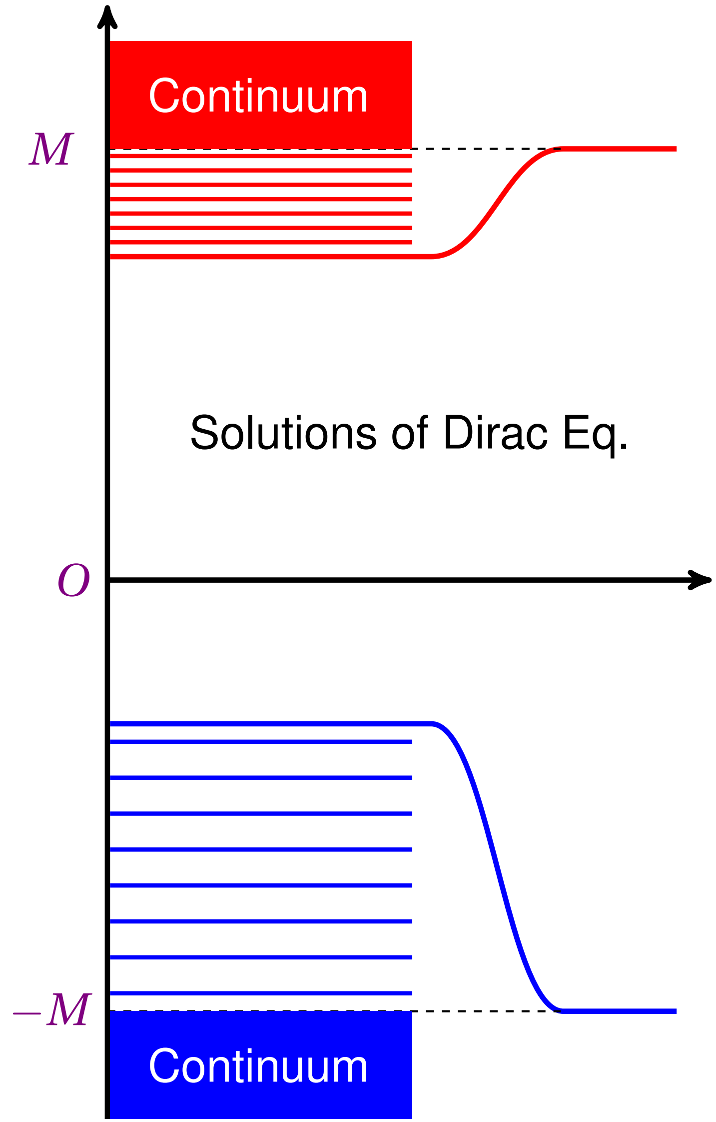
Fock

$$V_\phi = \frac{1}{2} \int d\boldsymbol{x} d\boldsymbol{x}' \sum_{\alpha\beta;\alpha'\beta'} c_\alpha^\dagger c_\beta^\dagger c_{\beta'} c_{\alpha'}$$

Hartree

$$\times \bar\psi_\alpha(\boldsymbol{x}) \bar\psi_\beta(\boldsymbol{x}') \Gamma_\phi D_\phi \psi_{\beta'}(\boldsymbol{x}') \psi_{\alpha'}(\boldsymbol{x}),$$



Spherical RHF equation

W.H. Long (2010)

- Variation of RHF energy functional E: integro-differential Dirac Eq.

$$\int d\boldsymbol{r}' h(\boldsymbol{r},\boldsymbol{r}')\psi_\alpha(\boldsymbol{r}') = \varepsilon_a\psi_\alpha(\boldsymbol{r}), \qquad \psi_\alpha(\boldsymbol{r}) = \frac{1}{r}\begin{pmatrix} iG_a\mathcal{Y}^{l_a}_{j_a m_a}(\hat{\boldsymbol{r}}) \\ -F_a\mathcal{Y}^{l'_a}_{j_a m_a}(\hat{\boldsymbol{r}}) \end{pmatrix} \tag{6}$$

where $\mathcal{Y}^l_{jm} = \sum_{\mu\sigma} C^{jm}_{l\mu\frac{1}{2}\sigma} Y_{l\mu}\chi_{\frac{1}{2}\sigma}$, single-particle Hamiltonian $h = h^{\rm kin} + h^{\rm D} + h^{\rm E}$:

$$h^{\rm kin}(\boldsymbol{r},\boldsymbol{r}') = [\boldsymbol{\alpha}.\boldsymbol{p} + \beta M]\,\delta(\boldsymbol{r}-\boldsymbol{r}'), \tag{7a}$$

$$h^{\rm D}(\boldsymbol{r},\boldsymbol{r}') = [\Sigma_T(\boldsymbol{r})\gamma_5 + \Sigma_0(\boldsymbol{r}) + \beta\Sigma_S(\boldsymbol{r})]\,\delta(\boldsymbol{r}-\boldsymbol{r}'), \tag{7b}$$

$$h^{\rm E}(\boldsymbol{r},\boldsymbol{r}') = \begin{pmatrix} Y_G(\boldsymbol{r},\boldsymbol{r}') & Y_F(\boldsymbol{r},\boldsymbol{r}') \\ X_G(\boldsymbol{r},\boldsymbol{r}') & X_F(\boldsymbol{r},\boldsymbol{r}') \end{pmatrix} \tag{7c}$$

- Local mean fields $\Sigma_{\rm S}$, Σ_0, and $\Sigma_{\rm T}$: functionals of local densities

$$\Sigma_{\rm S} = g_\sigma\sigma,\ \ \Sigma_{\rm T} = \frac{f_\rho}{2M}\left(\rho^{\rm VT} + \rho^{\rm T}\right)\tau_3,\ \ \Sigma_0 = g_\omega\omega + g_\rho\left(\rho^{\rm V} + \rho^{\rm TV}\right)\tau_3 + e\frac{1-\tau_3}{2}A + \Sigma_R$$

Hartree mean fields: σ, ω, $\rho^{\rm V}$, A, $\rho^{\rm TV}$ and $\rho^{\rm VT}$, $\rho^{\rm T}$; rearrangement term Σ_R。

Exchange (Fock) Potentials

W.H. Long (2010)

- Non-local MFs: functionals of the non-local densities

$$X_{G_a}^{(\phi)}(r,r') = \sum_b \mathscr{T}_{ab}^{\phi}\, \hat{j}_b^2\, g_\phi(r) g_\phi(r') \underline{F_b(r) G_b(r')} \mathscr{R}_{ab}^{X_G}(m_\phi; r, r'),$$

$$X_{F_a}^{(\phi)}(r,r') = \sum_b \mathscr{T}_{ab}^{\phi}\, \hat{j}_b^2\, g_\phi(r) g_\phi(r') \underline{F_b(r) F_b(r')} \mathscr{R}_{ab}^{X_F}(m_\phi; r, r'),$$

$$Y_{G_a}^{(\phi)}(r,r') = \sum_b \mathscr{T}_{ab}^{\phi}\, \hat{j}_b^2\, g_\phi(r) g_\phi(r') \underline{G_b(r) G_b(r')} \mathscr{R}_{ab}^{Y_G}(m_\phi; r, r'),$$

$$Y_{F_a}^{(\phi)}(r,r') = \sum_b \mathscr{T}_{ab}^{\phi}\, \hat{j}_b^2\, g_\phi(r) g_\phi(r') \underline{G_b(r) F_b(r')} \mathscr{R}_{ab}^{Y_F}(m_\phi; r, r'),$$

$\mathscr{T}_{ab}^{\phi}$: $\delta_{\tau_a\tau_b}$ (isoscalar) and $2 - \delta_{\tau_a\tau_b}$ (isovector).

- The underlined terms can be taken as the non-local density component.
- σ-scalar coupling:

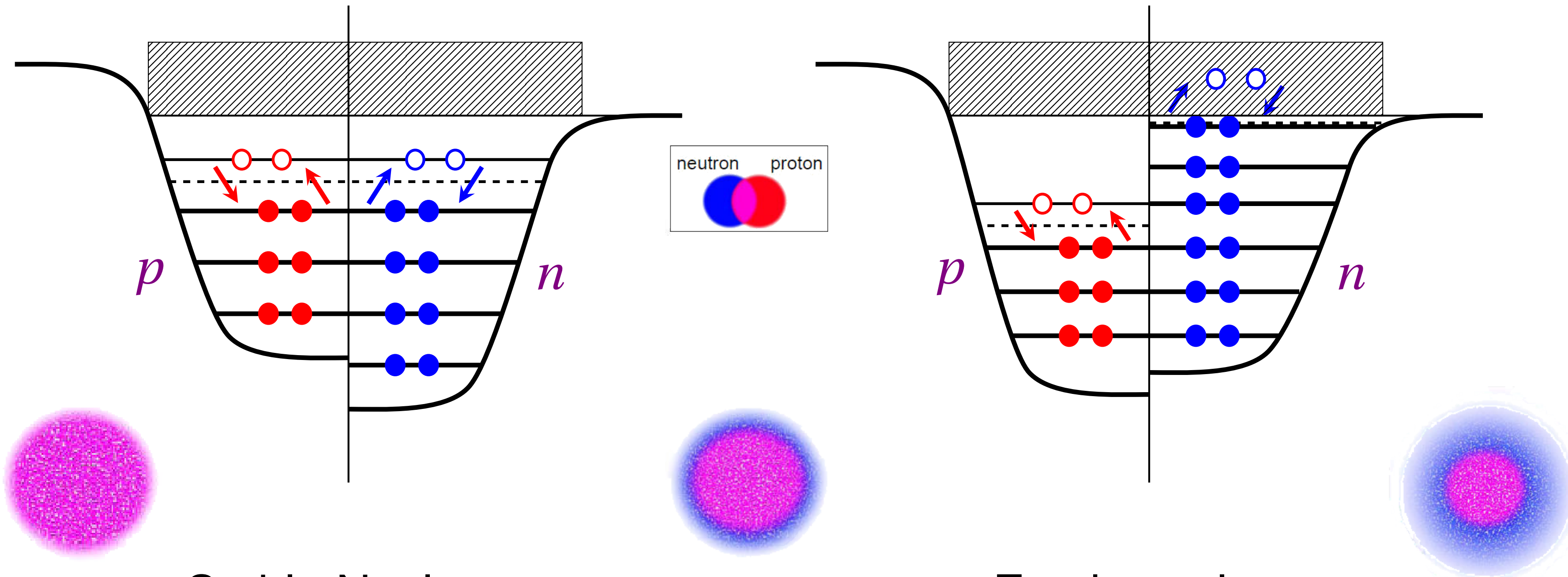
$$\mathscr{R}^{Y_G} = \mathscr{R}^{X_F} = -\mathscr{R}^{Y_F} = -\mathscr{R}^{X_G} = \mathscr{R}^{(\sigma)}$$

$$\mathscr{R}_{ab}^{(\sigma)}(m_\sigma; r, r') = \frac{1}{4\pi} \sum_L{}' C_{j_a \frac{1}{2} j_b -\frac{1}{2}}^{L0} C_{j_a \frac{1}{2} j_b -\frac{1}{2}}^{L0} R_{LL}(m_\sigma; r, r'). \quad (8)$$

The prime in Eq. (8) requires $L + l_a + l_b$ be even.

Unstable nuclei: continuum effects

- Unstable exotic nuclei reveal lots of new physics: weakly bound mechanism, continuum, halo, etc.



Stable Nucleus

Exotic nucleus

- Bogoliubov scheme: unified treatment of pairing and mean field effects

— J. Dobaczewski. NPA 422, 103 (1984); J. Meng, NPA 635, 3 (1998).

- Bogoliubov scheme also has the advantages in exploring superheavy nuclei.

— J.J. Li, W.H. LONG, J. Margueron, N. Van Giai, PLB 732, 169 (2014).

RHFB theory

W.H. Long (2010)

- Bogoliubov transformation: particle $\{c_\alpha, c_\alpha^\dagger\}$ ⟶ quasi-particle $\{\beta_\alpha, \beta_\alpha^\dagger\}$

$$\begin{pmatrix} c_\alpha \\ c_\alpha^\dagger \end{pmatrix} = \mathcal{W} \begin{pmatrix} \beta_\alpha \\ \beta_\alpha^\dagger \end{pmatrix} = \begin{pmatrix} \psi_U & \psi_V^* \\ \psi_V & \psi_U^* \end{pmatrix} \begin{pmatrix} \beta_\alpha \\ \beta_\alpha^\dagger \end{pmatrix}, \qquad \begin{aligned} \beta_\alpha &= \psi_U^\dagger c_\alpha + \psi_V^\dagger c_\alpha^\dagger \\ \beta_\alpha^\dagger &= \psi_V^T c_\alpha + \psi_U^T c_\alpha^\dagger \end{aligned} \tag{9}$$

where ψ_U and ψ_V quasi-particle spinors, and $\mathcal{W}^\dagger \mathcal{W} = 1$.

- RHFB Equation: chemical potential λ for preserving the particle number

$$\int d\boldsymbol{r}' \begin{pmatrix} h(\boldsymbol{r},\boldsymbol{r}') & \Delta(\boldsymbol{r},\boldsymbol{r}') \\ -\Delta(\boldsymbol{r},\boldsymbol{r}') & h(\boldsymbol{r},\boldsymbol{r}') \end{pmatrix} \begin{pmatrix} \psi_U(\boldsymbol{r}') \\ \psi_V(\boldsymbol{r}') \end{pmatrix} = \begin{pmatrix} \lambda + E & 0 \\ 0 & \lambda - E \end{pmatrix} \begin{pmatrix} \psi_U(\boldsymbol{r}) \\ \psi_V(\boldsymbol{r}) \end{pmatrix} \tag{10}$$

where h is RHF single-particle Hamiltonian and pairing potential Δ reads,

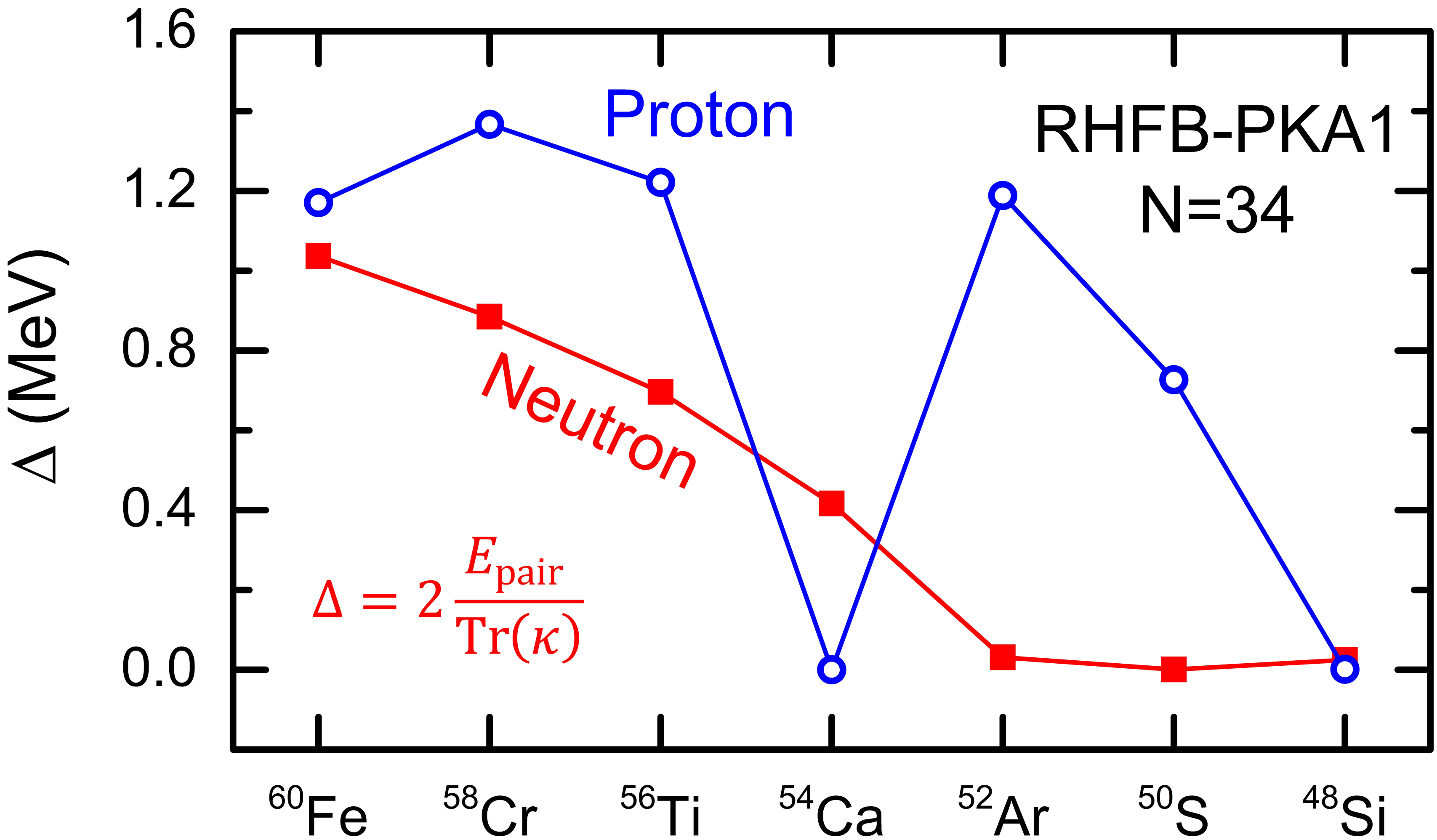
$$\Delta_\alpha(\boldsymbol{r},\boldsymbol{r}') = -\frac{1}{2}\sum_\beta V^{pp}_{\alpha\beta}(\boldsymbol{r},\boldsymbol{r}')\,\kappa_\beta(\boldsymbol{r},\boldsymbol{r}'), \qquad \kappa_\alpha(\boldsymbol{r},\boldsymbol{r}') = \psi^*_{V_\alpha}(\boldsymbol{r})\psi_{U_\alpha}(\boldsymbol{r}') \tag{11}$$

- In practice, such integral-differential equation is more convenient to be solved with the help of Dirac Woods-Saxon Basis.

—S.-G. Zhou, J. Meng, P. Ring, PRC 68, 034323 (2003).

OUTLINE

Pairing gaps along isotonic chain of $N = 34$



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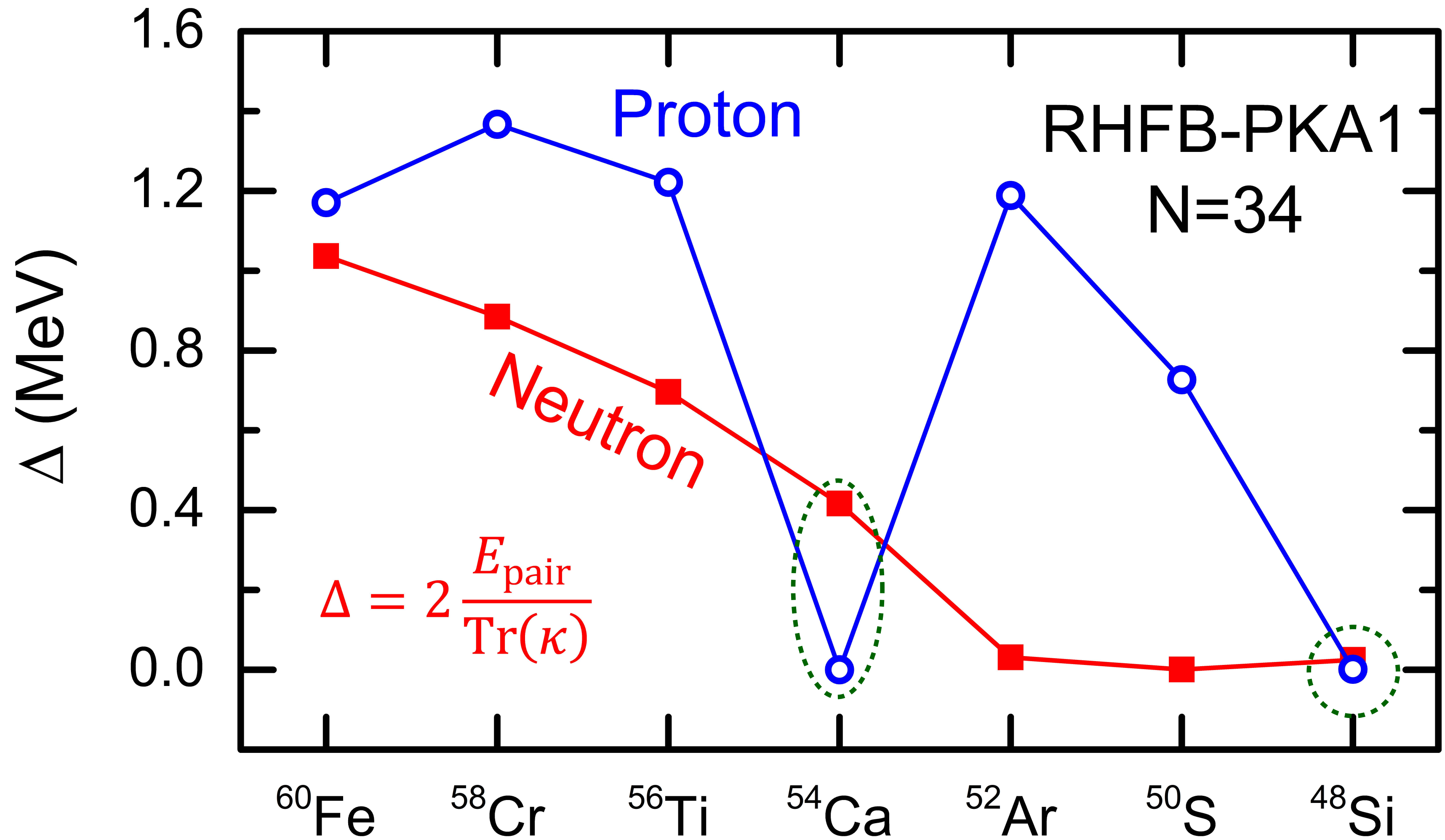
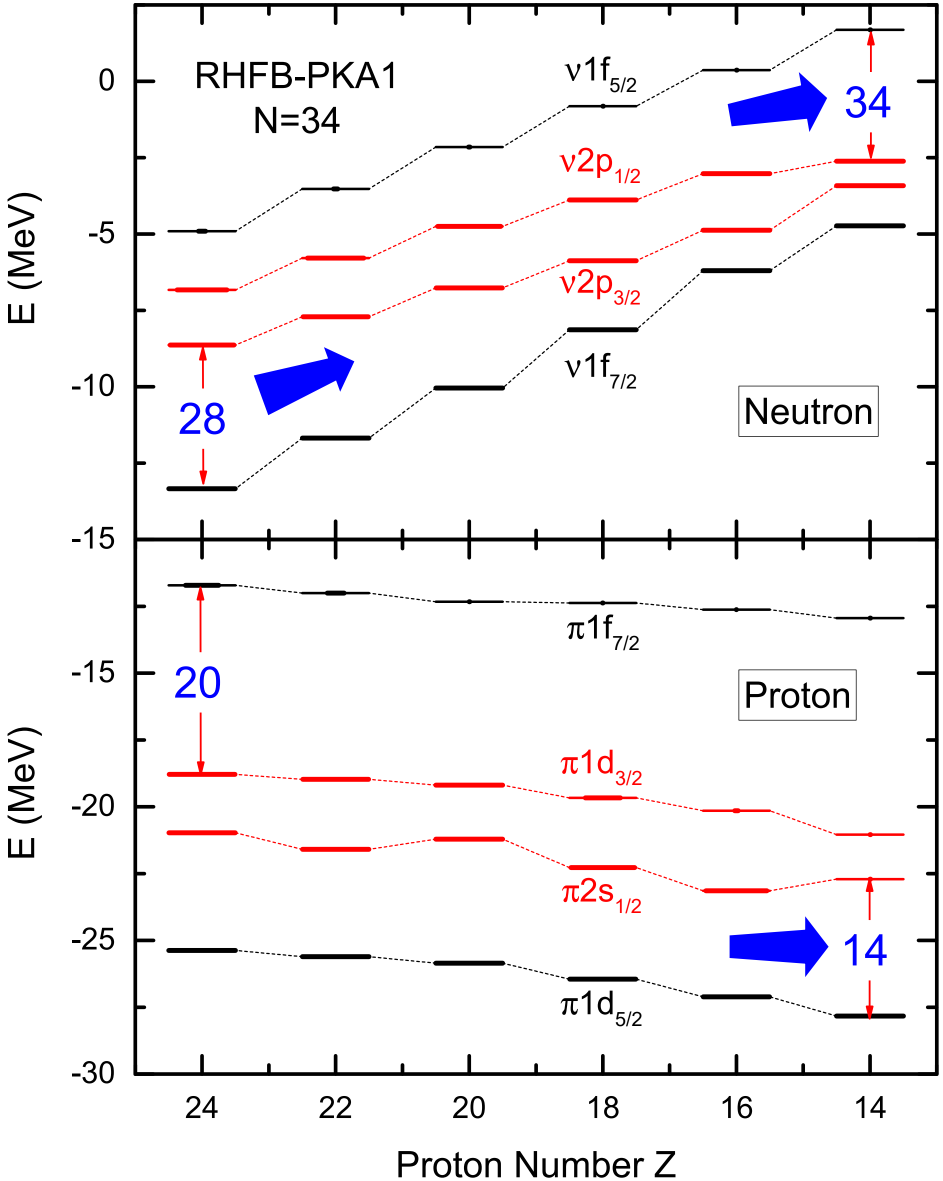


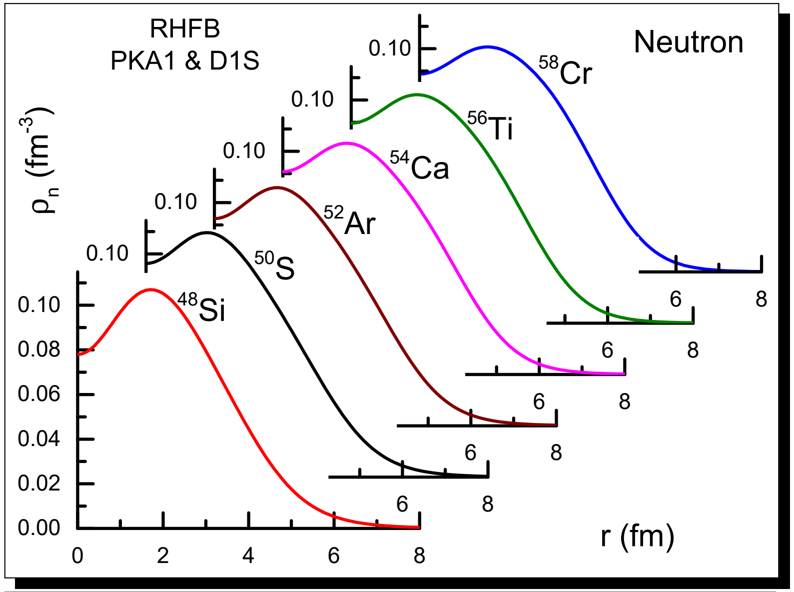
The prediction is consistent with other model calculations, like shell model.

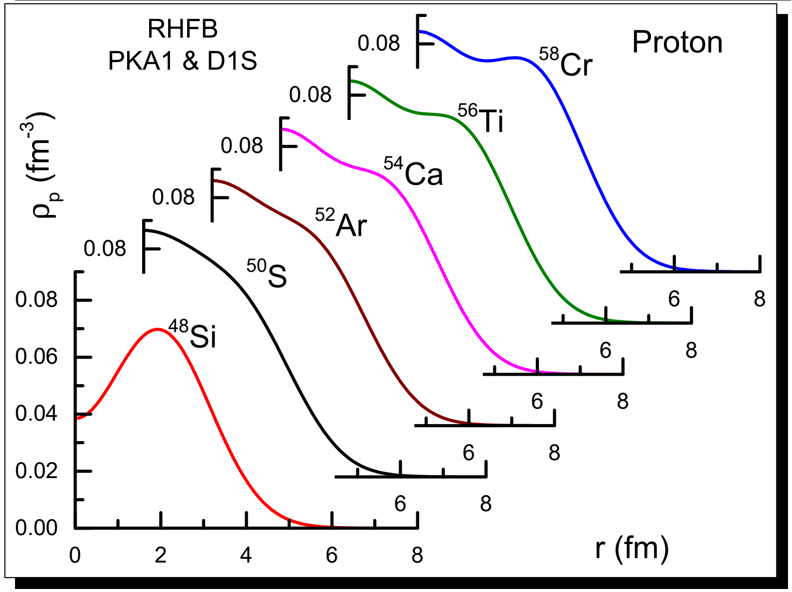
— D. Steppenbeck, S. Takeuchi, N. Aoi, P. Doornenbal, M. Matsushita, et al., PRL 114, 252501 (2015).

— Y. Utsuno, T. Otsuka, Y. Tsunoda, N. Shimizu, M. Honma, et al., JPS Conf. Proc. 6, 010007 (2015).

Magicity and Bubbles

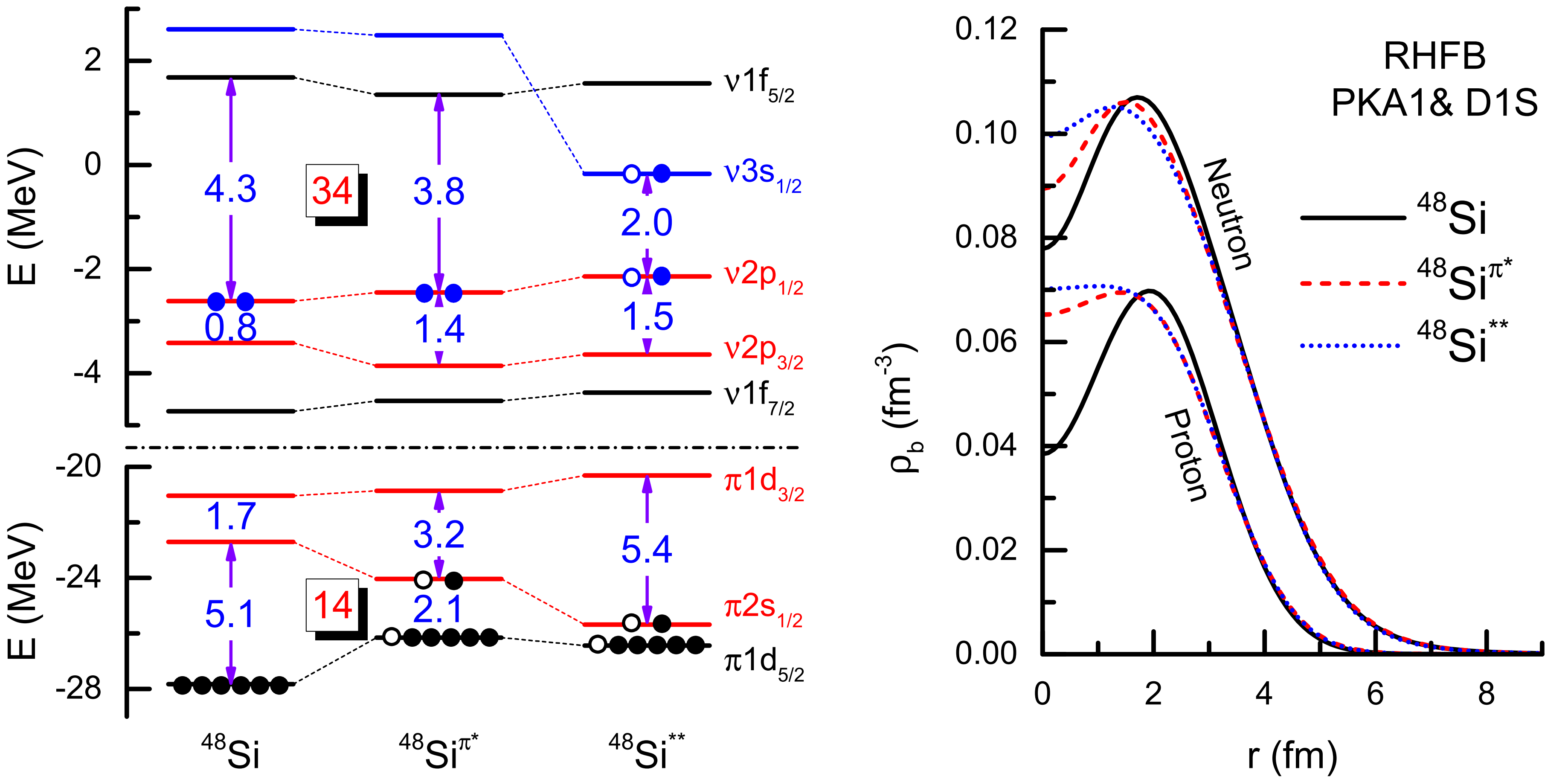






Bubble structure $\xrightarrow{\text{quench } \Delta_{SO} \text{ of } p \text{ orbits}}$ enhanced shells

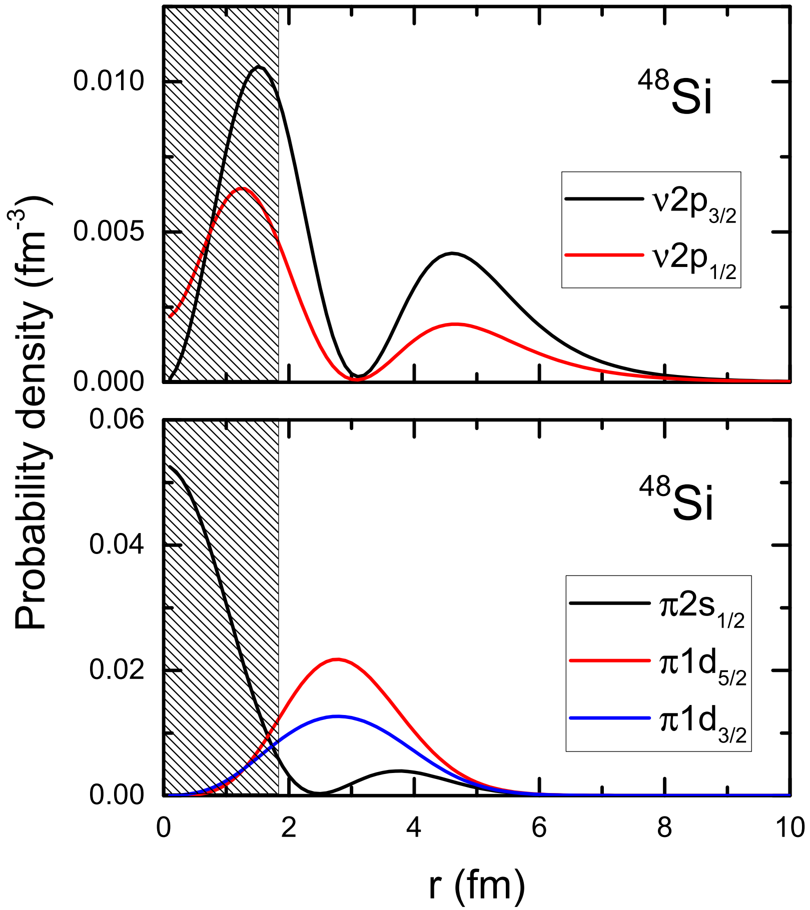
Shells enhanced by bubble structure

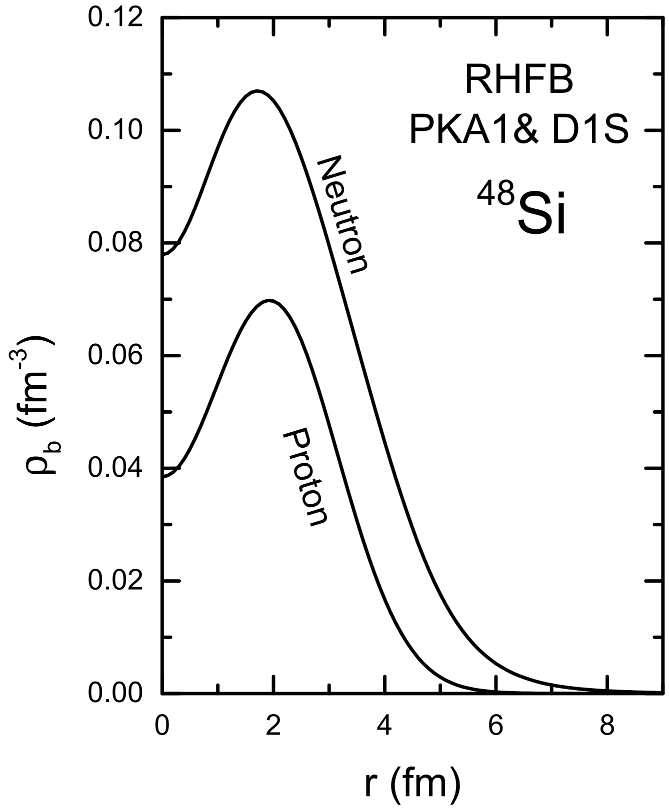


^{48}Si** ⟶ ^{48}Si$^{\pi*}$ ⟶ ^{48}Si (super bubble candidate)

Bubbles: SO splitting $\Delta_{\nu 2p}$ is quenched distinctly, leading to enhanced neutron shell $N = 34$. Similarly, the pseudo-spin splitting of $\Delta_{\pi 1\tilde{p}} = E_{\pi 2s_{1/2}} - E_{1d_{3/2}}$ is also compressed much to give the proton shell $Z = 14$.

Shells enhanced by bubble structure





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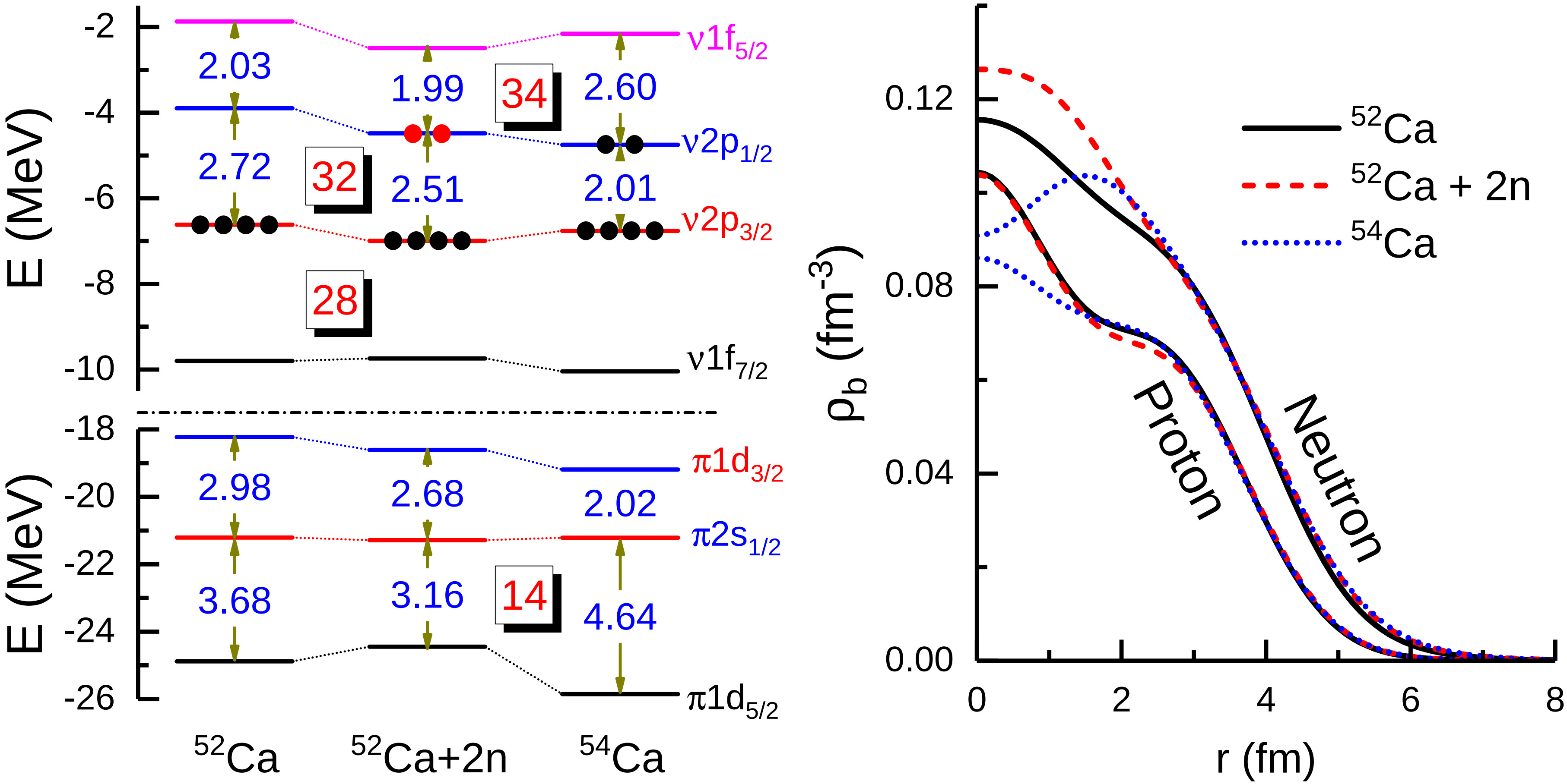
From ^{52}Ca to ^{54}Ca: Polarization determining the shell



- In ^{52}Ca central-bumped density profiles enhance the $\nu 2p$ splittings, whereas in ^{54}Ca dramatic central-depressed ones reduce $\Delta_{\mathrm{SO}}^{\nu 2p}$ distinctly.

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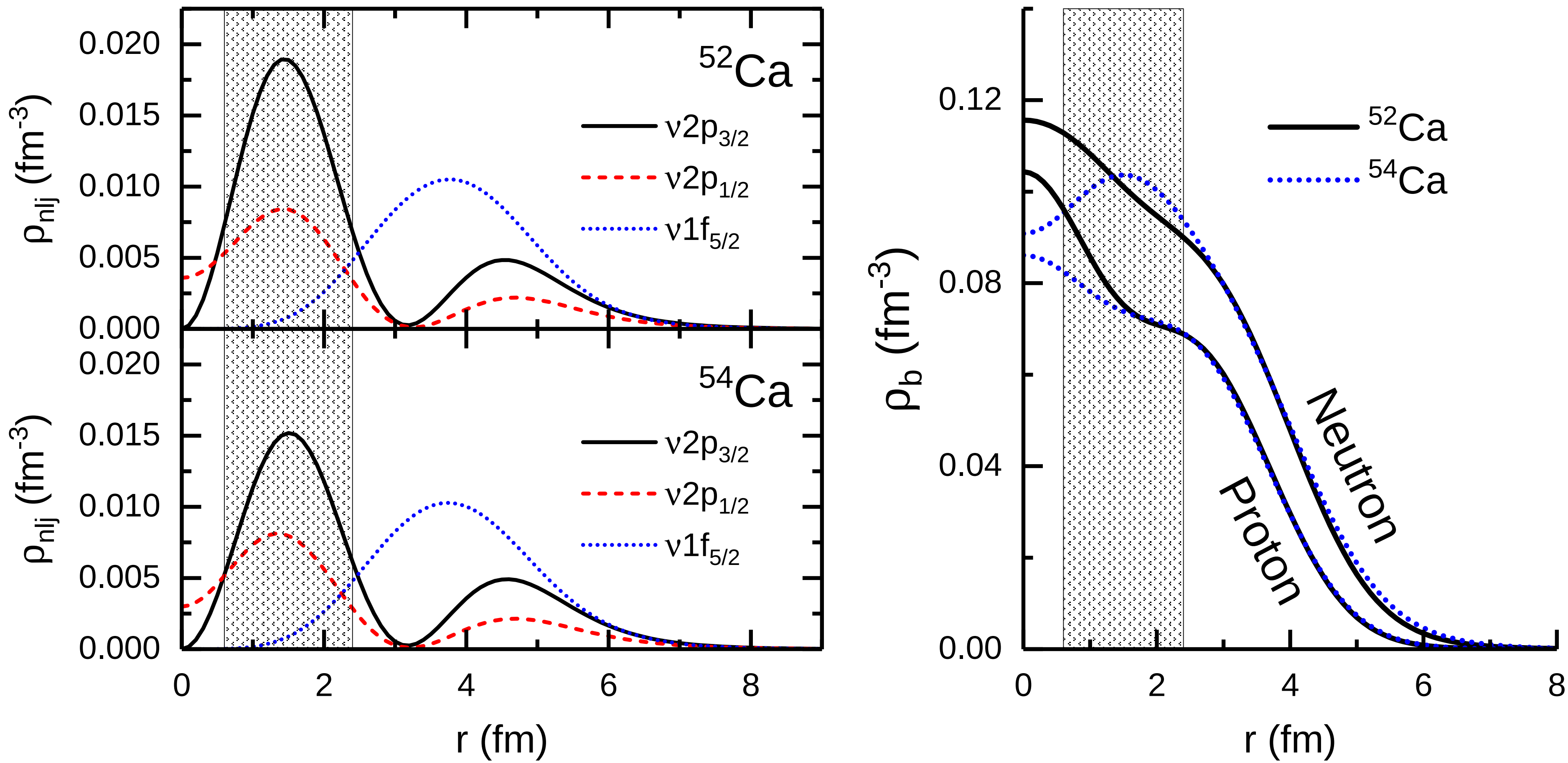

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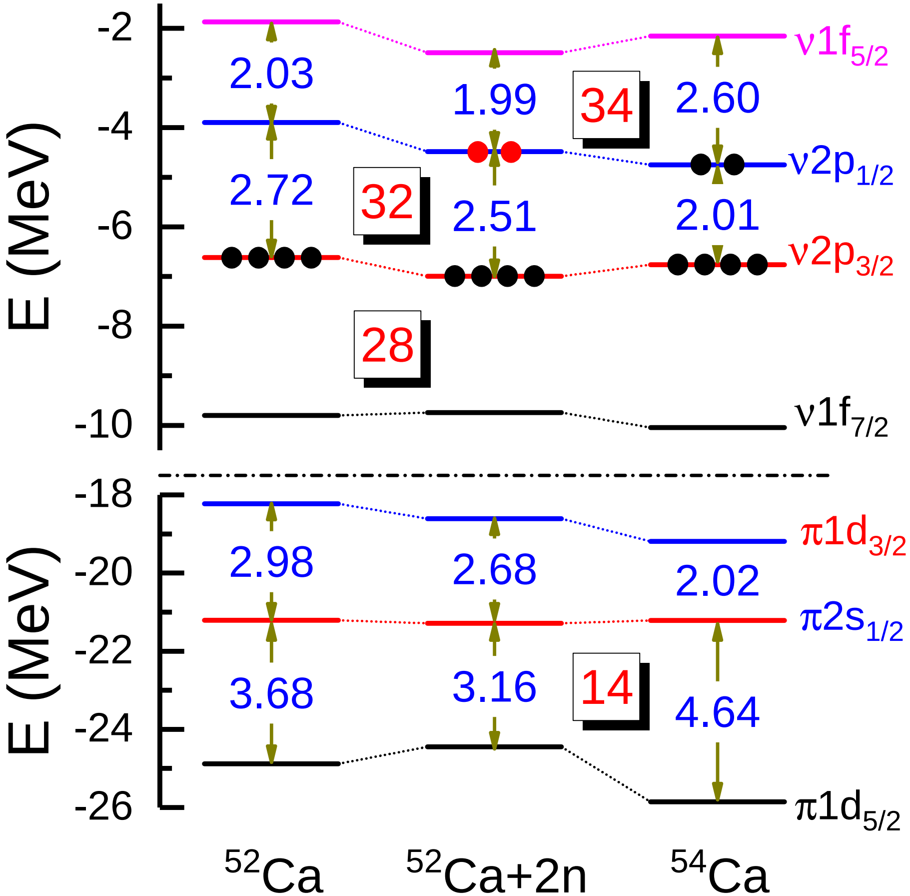
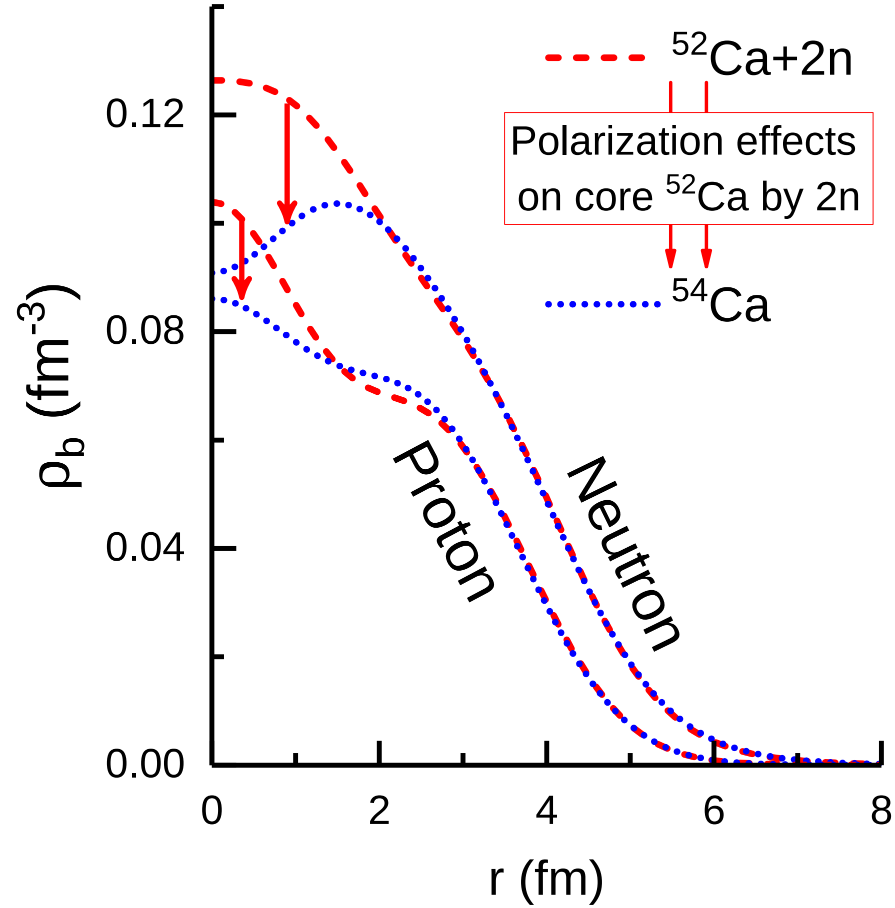
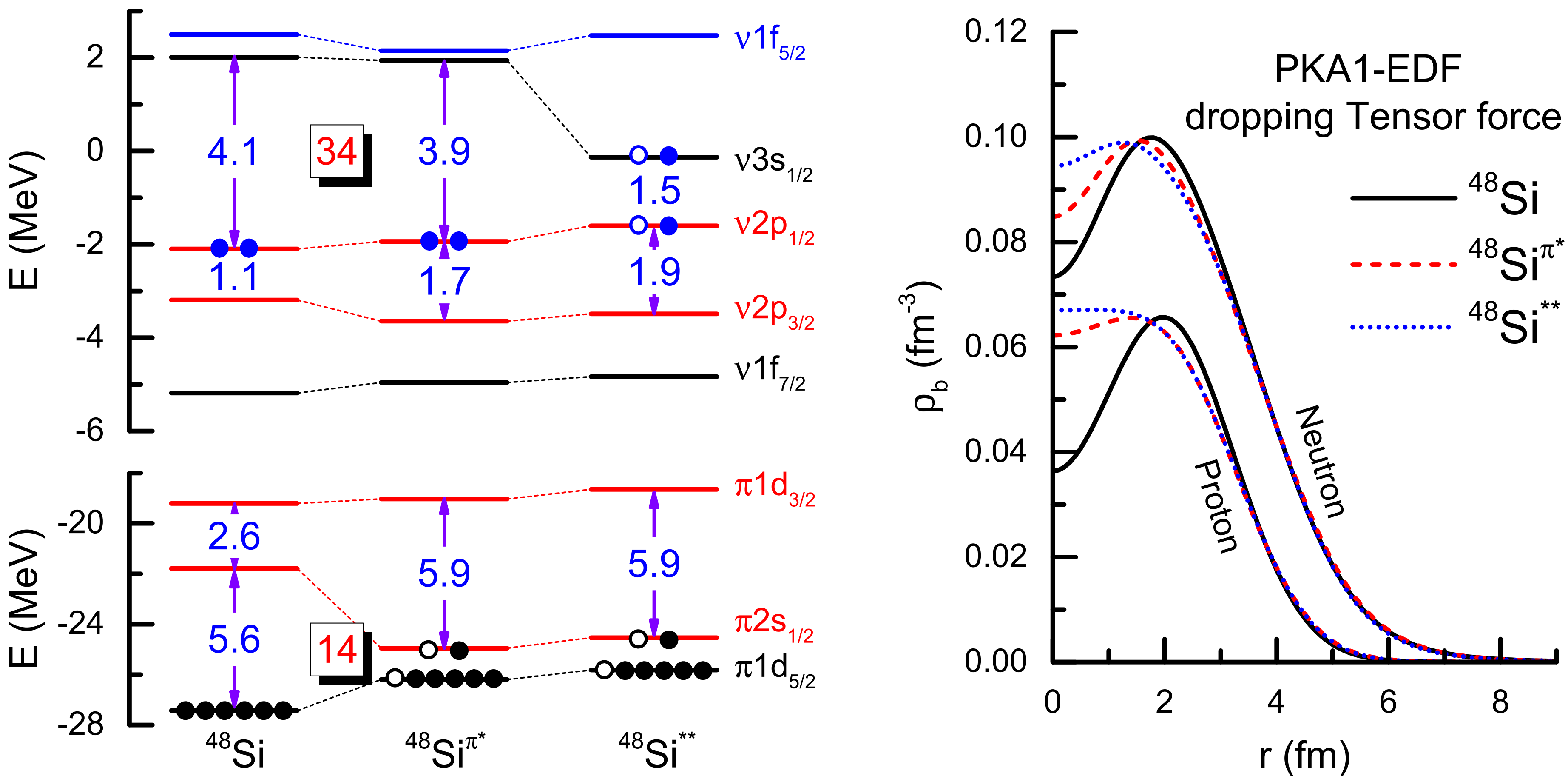




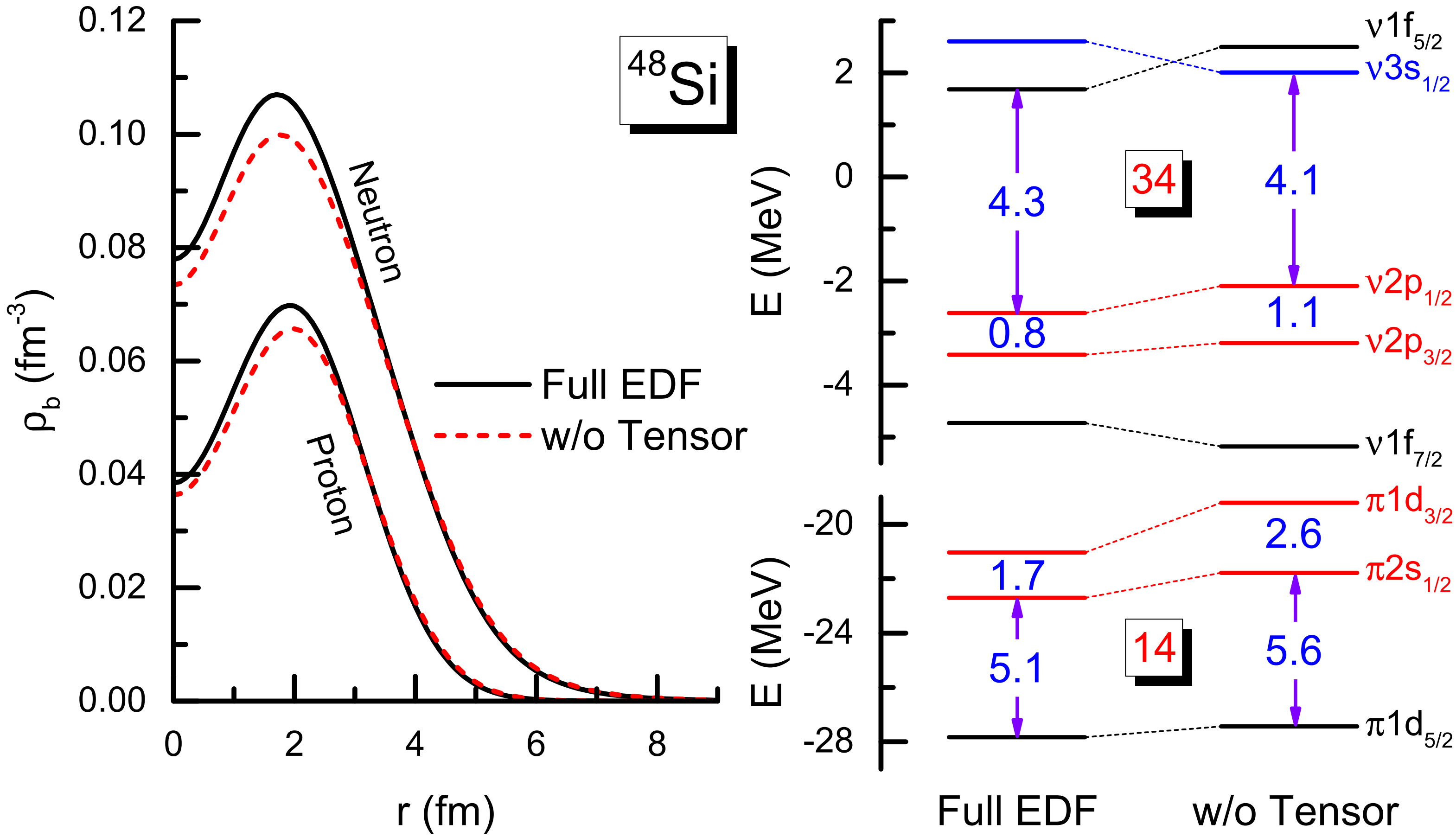
- In ^{52}Ca central-bumped density profiles enhance the $\nu 2p$ splittings, whereas in ^{54}Ca dramatic central-depressed ones reduce $\Delta_{\text{SO}}^{\nu 2p}$ distinctly.
- ^{52}Ca + $2n$ ⟶ ^{54}Ca: distinct $2n$-polarization effects on the core ^{52}Ca, which are not found by other CDFs (do not support ^{52}Ca as magic nucleus).

Dropping tensor force terms in Fock diagram



- Similar systematics is found as the calculations with full EDF.
- Bubble structure, instead of tensor force, is the key physics in determining the magicity.

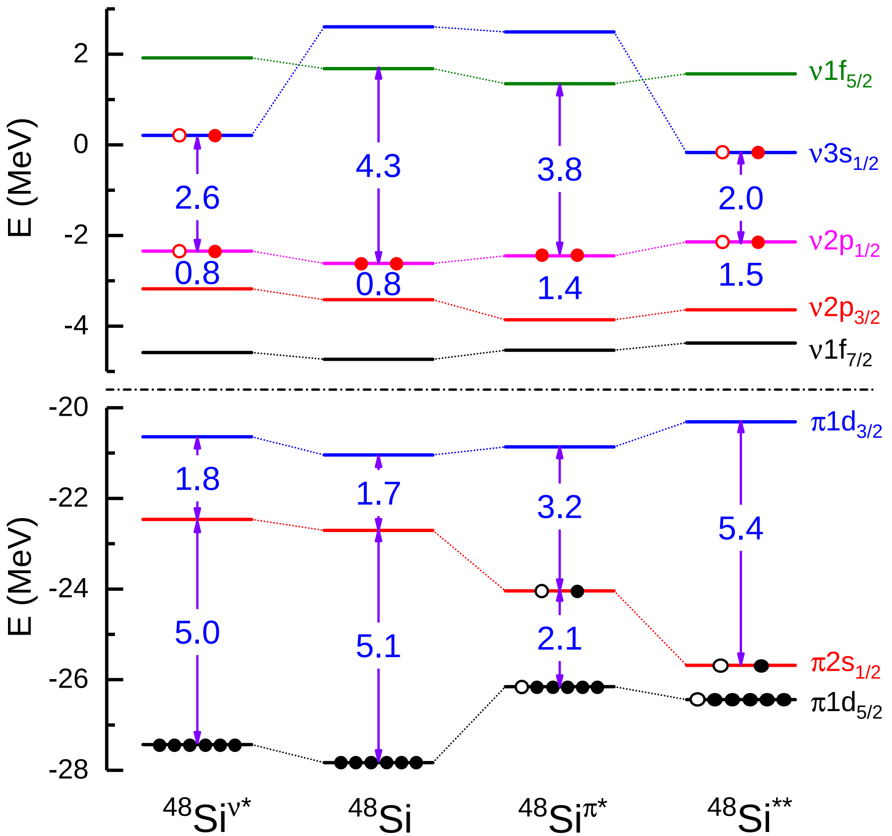
Tensor effects in ground states



- The proton and neutron bubbles remain as well even after removing the tensor force component in Fock terms.

- Nuclear tensor force tends to quench the proton shell $Z = 14$ while enlarge the neutron one $N = 34$ with a few percent.

Crossing-shell excitations of ^{48}Si



- Neutron (proton) crossing-shell excitations reduce the shell itself distinctly.

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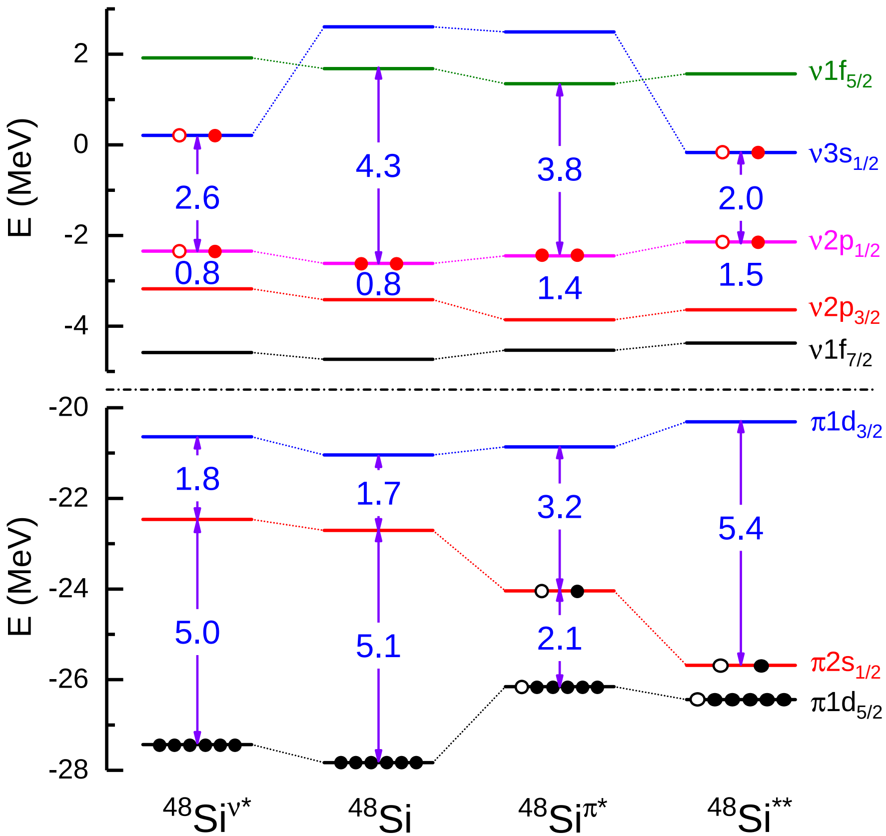
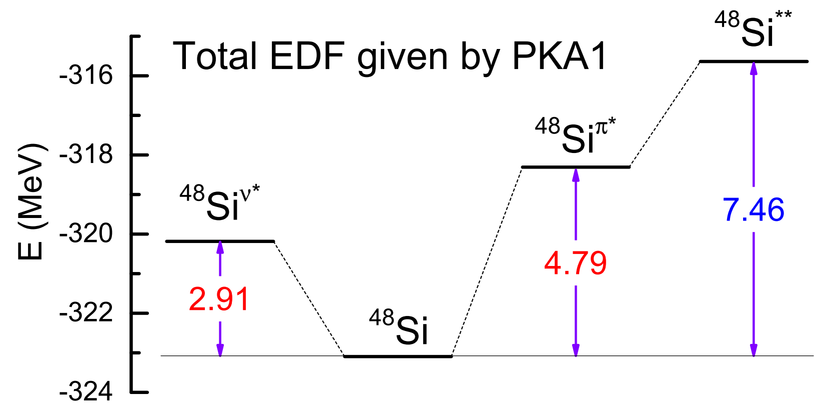
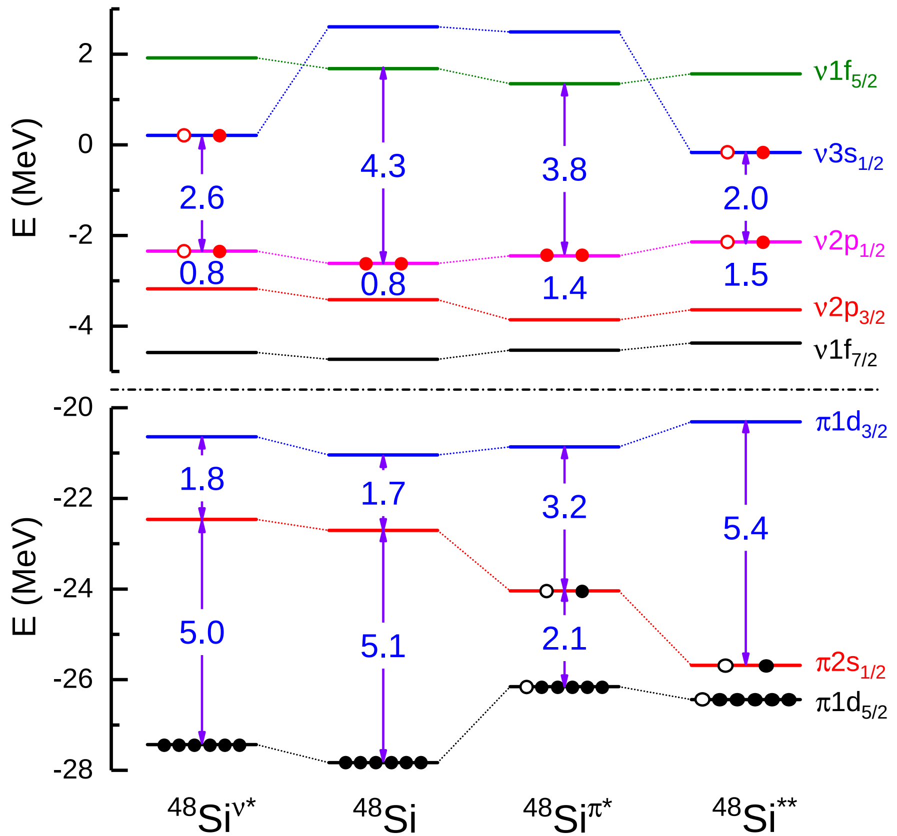


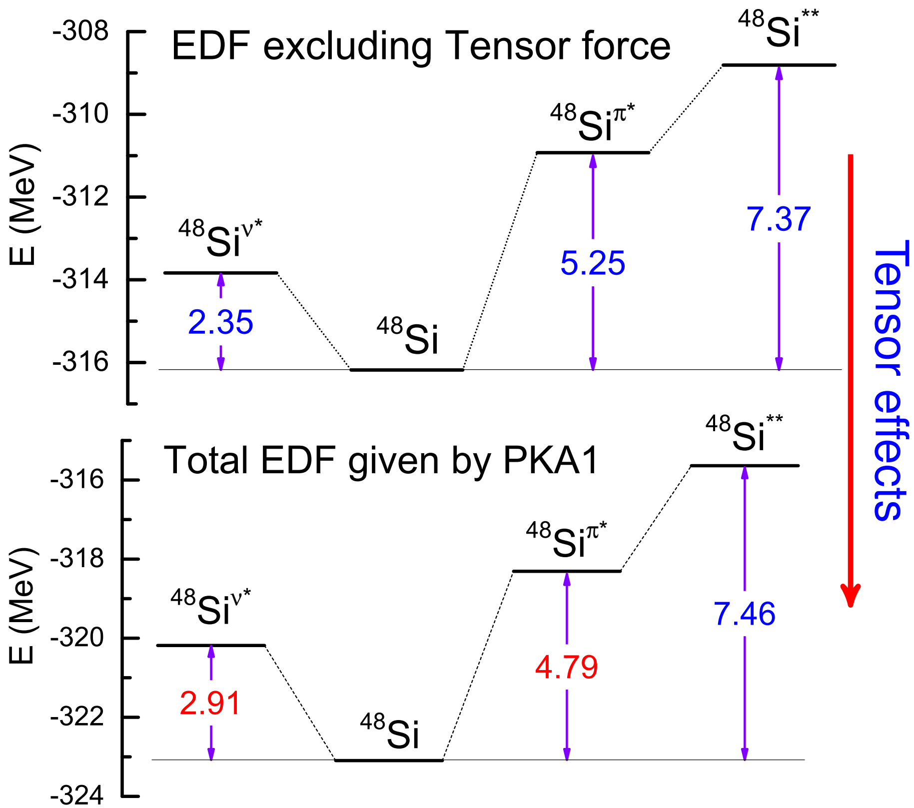


- Neutron (proton) crossing-shell excitations reduce the shell itself distinctly.
- Neutron and proton crossing-shell excitation energies are soundable.

$$^{48}\mathrm{Ca} \xrightarrow{4.42\ \mathrm{MeV}} {}^{48}\mathrm{Ca}^{\nu*} \qquad {}^{208}\mathrm{Pb} \xrightarrow{4.89\ \mathrm{MeV}} {}^{208}\mathrm{Pb}^{\nu*}$$

Crossing-shell excitations of ^{48}Si





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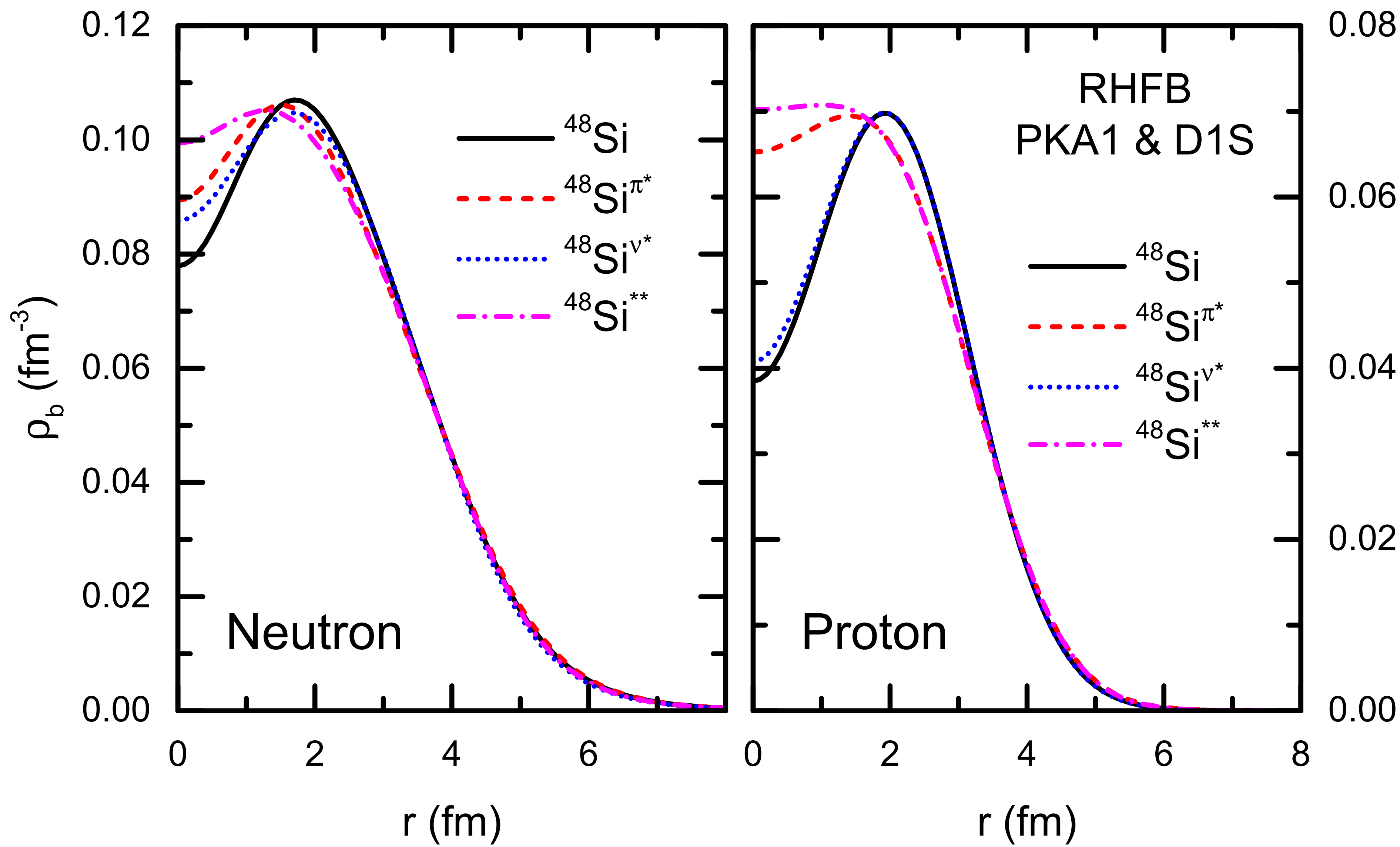
- Tensor force plays opposite roles in neutron and proton excitations.

Bubble structure in excited states



Proton bubble is sensitive to proton crossing-shell excitation.

OUTLINE

Conclusions and Perspectives

- Starting from the existing magicities, namely $Z = 14$ in ^{34}Si and $N = 34$ in ^{54}Ca, new doubly magic drip line nucleus ^{48}Si is predicted by the relativistic energy density functionals.

- Magicities $N = 32, 34$ and $Z = 14$, as well as the relevant physics, are discussed by using RHFB-PKA1 model.
  - Magicity $N = 32$ can be well reproduced by PKA1, in which the isovector π-PV and ρ-T couplings are crucial and the tensor force components are also certainly significant.
  - Magicity $N = 34$, arising just after $N = 32$, results from the quenched SO splitting of $\nu 2p$ orbits by the neutron semi-bubble in ^{54}Ca, in fact, the $2n$ polarization effects.
  - ^{34}Si is identified as doubly magic proton bubble nuclide by experiments, which also indicates the existence of proton shell $Z = 14$.
  - Both neutron shell $N = 34$ and proton one $Z = 14$ become more distinct with the occurrence of dual bubble structures in ^{48}Si, which certainly weaken the coupling with central distributed orbits, like s and p orbits.

- Perspective: $2n$ polarization effects and the nature of nuclear force.

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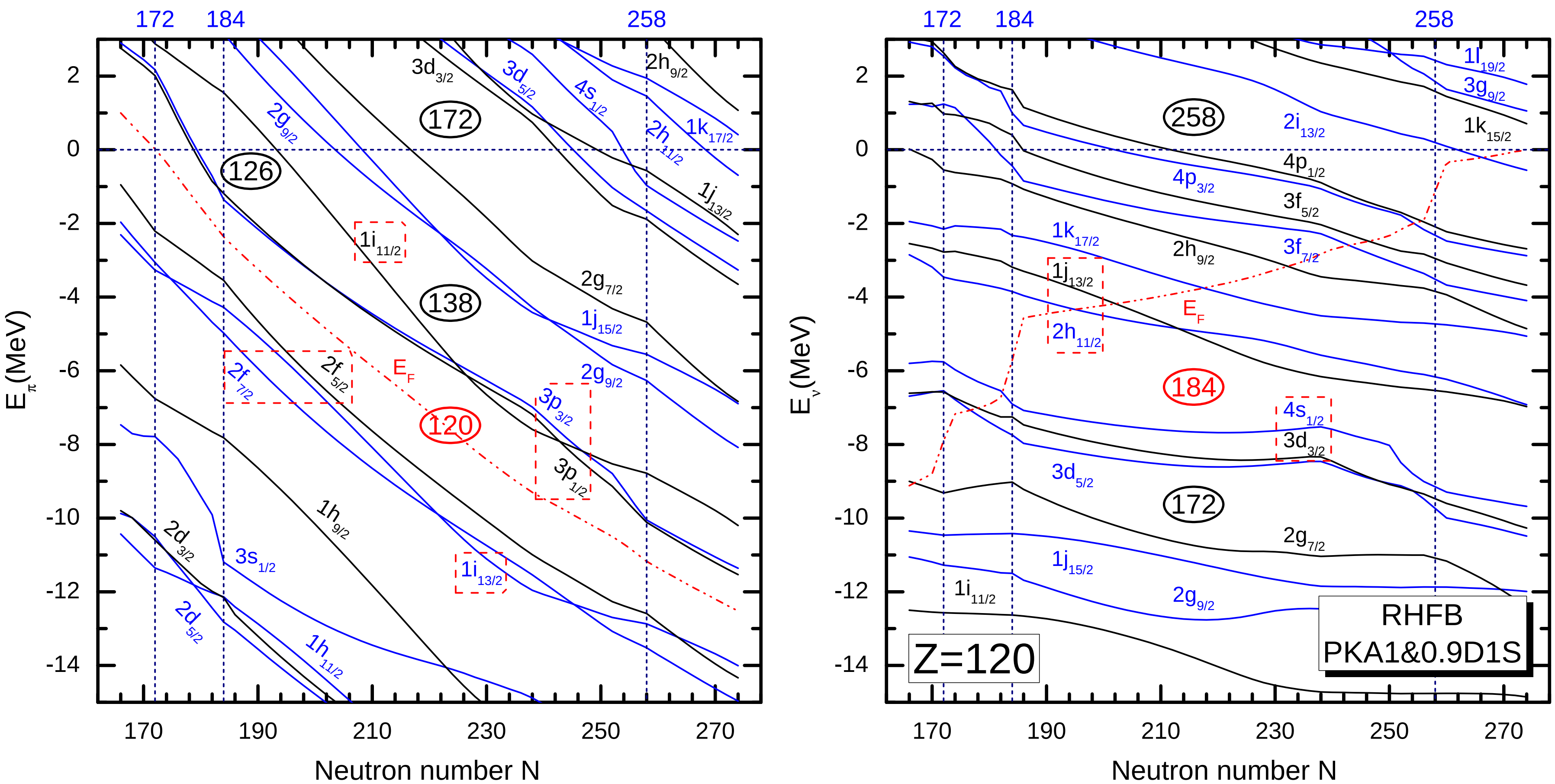
Thank you for your attention!

Group Photo

Thank you for your attention!

Similar mechanism in Superheavy magicity



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