UNITARITY ANALYSES OF THE PION-NUCLEON ELASTIC SCATTERING

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- The pion-nucleon scattering
- A new unitarization approach

THE PION-NUCLEON SCATTERING

- The πN elastic scattering $\rightarrow$ one of the most fundamental and important processes in nuclear or hadron physics
- Decades of researching
- Problems to study
  - Low energy properties: πN σ-term, subthreshold expansions [C. Ditsche et. al. 2012 JHEP] [Y. H. Chen et. al. 2013 PRD]
  - Intermediate resonances: $\Delta(1232), N^*(1440), N^*(1535) \cdots$

$N^*(1535)$ AND $N^*(1440)$

- $N^*(1535)$: S_{11} πN channel ($L_{2I\ 2J}$ convention)

  [N. Kaiser et. al. 1995 PLB][J. Nieves et. al. 2000 PRD]

  - lies above the p wave first resonance $N^*(1440)$
  - large couple channel effects with πN and ηN
- $N^*(1440)$ (Ropper resonance): P_{11} channel, various puzzles
  - low mass, large decay width, coupling to σN channel...

    [O. Krehl et. al. 2000 PRC]
  - two-pole structure? [R. A. Arndt et. al. 1985 PRD]
  - complex branch cut in P_{11} channel? [S. Ceci et. al. 2011 PRC]
- A method is needed to exam the relevant channels carefully and to exhume more physics behind.

A NEW UNITARIZATION APPROACH

- Application of EFT
  - effective interactions
  - amplitudes calculated under perturbation theory
  - unitarization
  - fit to the experimental data
- Unitarization: reformation of the amplitudes from perturbation theory
- To generate singularity structures (poles corresponding to particles) correctly $\rightarrow$ Padé approximation, K-matrix $\cdots$

PKU FACTORIZATION

- PKU factorization $\rightarrow$ elastic scatterings $m_1+m_2 \rightarrow m_1+m_2$, factorized partial wave S [Z. G. Xiao and H. Q. Zheng 2001 NPA]

$$S(s) = \prod_b \frac{1-\mathrm{i}\rho(s)\frac{s}{s-s_\mathrm{L}}\sqrt{\frac{s_b-s_\mathrm{L}}{s_\mathrm{R}-s_b}}}{1+\mathrm{i}\rho(s)\frac{s}{s-s_\mathrm{L}}\sqrt{\frac{s_b-s_\mathrm{L}}{s_\mathrm{R}-s_b}}} \prod_v \frac{1+\mathrm{i}\rho(s)\frac{s}{s-s_\mathrm{L}}\sqrt{\frac{s'_v-s_\mathrm{L}}{s_\mathrm{R}-s'_v}}}{1-\mathrm{i}\rho(s)\frac{s}{s-s_\mathrm{L}}\sqrt{\frac{s'_v-s_\mathrm{L}}{s_\mathrm{R}-s'_v}}}$$
$$\prod_r \frac{M_r^2-s+\mathrm{i}\rho(s)sG_r}{M_r^2-s-\mathrm{i}\rho(s)sG_r} e^{2\mathrm{i}\rho(s)f(s)}$$

- s_b: bound states. s'_v: virtual states (second Riemann sheet). z_r: resonances (second Riemann sheet).
- $s_\mathrm{L} = (m_1-m_2)^2$, $s_\mathrm{R} = (m_1+m_2)^2$, $\rho(s) = \sqrt{(s-s_\mathrm{L})(s-s_\mathrm{R})}/s$

PKU FACTORIZATION

- Resonances

$$M_0^2 = \mathrm{Re}[z_0] + \mathrm{Im}[z_0]\frac{\mathrm{Im}[\sqrt{(z_0 - s_R)(z_0 - s_L)}]}{\mathrm{Re}[\sqrt{(z_0 - s_R)(z_0 - s_L)}]}$$

$$G_0 = \frac{\mathrm{Im}[z_0]}{\mathrm{Re}[\sqrt{(z_0 - s_R)(z_0 - s_L)}]}$$

- Background term $\rightarrow$ left hand cut and inelastic cut of the elastic channel (subtraction at $s = 0$)

$$f(s) = \frac{s}{2\pi \mathrm{i}} \int_L ds' \frac{\mathrm{disc} f(s')}{(s' - s)s'} + \frac{s}{2\pi \mathrm{i}} \int_{R'} ds' \frac{\mathrm{disc} f(s')}{(s' - s)s'}$$

- $f(0) \equiv 0$ [Z. Y. Zhou and H. Q. Zheng 2006 NPA]

PKU FACTORIZATION

- $f(s)$ perturbatively calculated, pole positions as parameters
- Advantages
  - rigorous and universal for two body elastic scatterings
  - separated S
  - low energy contributions are sensitive to (not too) distant poles
  - definite sign of the phase shifts $\rightarrow$ figuring out hidden contributions
- Applications
  - the $\pi\pi$ elastic scattering $\rightarrow$ existence of the σ particle

    [Z. G. Xiao and H. Q. Zheng 2001 NPA]
  - the πK elastic scattering $\rightarrow$ κ resonance

    [H. Q. Zheng et. al. 2004 NPA]
  - elastic scatterings with spin ??

πN SCATTERING

- πN diagonalized helicity structure

$$T^J_\pm = T^J_{++} \pm T^J_{+-}$$

$$\mathrm{Im}T^J_\pm = \rho(s)|T^J_\pm|^2$$

- Interpretation:

$$T^J_\pm = \langle F|\hat{T}|JJ_z+\rangle$$

with the final state

$$\langle F| = \langle JJ_z+| \pm \langle JJ_z-|$$

- $T^J_\pm$ final state parity $(-1)^{J\pm1/2}$ $\rightarrow$ orbital angular momentum $L = J \mp 1/2$

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LAGRANGIAN

- The Lagrangian are taken from [N. Fettes et. al. 2000 Ann. Phys.]
- $\mathcal{O}(p^1)$:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}\left(\mathrm{i}\not{D} - M + \frac{1}{2}g\not{u}\gamma^5\right)N$$

- $\mathcal{O}(p^2)$ ("$\langle\rangle$" stands for trace in isospin space):

$$\mathcal{L}_{\pi N}^{(2)} = c_1\langle\chi_+\rangle\bar{N}N - \frac{c_2}{4M_N^2}\langle u^\mu u^\nu\rangle(\bar{N}D_\mu D_\nu N + \text{h.c.}) + \frac{c_3}{2}\langle u^\mu u_\mu\rangle\bar{N}N - \frac{c_4}{4}\bar{N}\gamma^\mu\gamma^\nu\left[u^\mu, u^\nu\right]N$$

LAGRANGIAN

- $\mathcal{O}(p^3)$:

$$
\begin{aligned}
\mathcal{L}_{\pi N}^{(3)} = & -\frac{d_1+d_2}{4M}\left\{\bar{N}\left[u_\mu, h_\nu^{\ \mu}\right](D^\nu N) + \text{h.c.}\right\} \\
& + \frac{d_3}{12M_N^3}\left\{\bar{N}\left[u_\mu, \left[D_\nu, u_\rho\right]\right](D^{\mu\nu\rho} N) + \text{h.c.}\right\} \\
& + \frac{d_5}{2M}\left\{\mathrm{i}\bar{N}\left[\chi_-, u_\mu\right](D^\mu N) + \text{h.c.}\right\} \\
& + \frac{d_{14}-d_{15}}{8M}\left\{\mathrm{i}\bar{N}\sigma^{\mu\nu}\left\langle\left[D_\rho, u_\mu\right]u_\nu - u_\mu\left[D_\nu, u_\rho\right]\right\rangle(D^\rho N) + \text{h.c.}\right\} \\
& + \frac{d_{16}}{2}\bar{N}\gamma^\mu\gamma^5\langle\chi_+\rangle u_\mu N + \frac{\mathrm{i}d_{18}}{2}\bar{N}\gamma^\mu\gamma^5\left[D_\mu, \chi_-\right]N
\end{aligned}
$$

CONVENTIONS

- Conventions

$$D_\mu = \partial_\mu + \Gamma_\mu$$
$$\Gamma_\mu = \frac{1}{2}\left[u^\dagger(\partial_\mu - \mathrm{i}r_\mu)u + u(\partial_\mu - \mathrm{i}l_\mu)u^\dagger\right]$$
$$u_\mu = \mathrm{i}\left[u^\dagger(\partial_\mu - \mathrm{i}r_\mu)u - u(\partial_\mu - \mathrm{i}l_\mu)u^\dagger\right]$$
$$\chi_\pm \equiv u^\dagger\chi u^\dagger \pm u\chi^\dagger u$$
$$\chi = 2B_0(s + ip)$$
$$h_\nu^{\ \mu} = \left[D_\nu, u^\mu\right] + \left[D^\mu, u_\nu\right]$$
$$D^{\mu\nu\rho} = D^\mu D^\nu D^\rho + D^\nu D^\mu D^\rho + D^\nu D^\rho D^\mu + \cdots$$

During the scattering process calculation $2B_0 s \to 2B_0 m_q = m_\pi^2$, and other sources are set to 0

ISOSPIN DECOMPOSITION

- Symmetric part vs. anti-symmetric part

$$T(\pi^a + N_\mathrm{i} \to \pi^{a'} + N_\mathrm{f}) = \chi_\mathrm{f}^\dagger \left(\delta^{aa'} T^S + \frac{1}{2}\left[\tau^{a'}, \tau^a\right] T^A \right) \chi_\mathrm{i}$$

- Isospin channels

$$T^{I=1/2} = T^S + 2T^A$$

$$T^{I=3/2} = T^S - T^A$$

HELICITY STRUCTURE

- Spin decomposition

$$T^{S,A} = \bar{u}(p',s')\left[A^{S,A}(s,t) + \frac{1}{2}(\not{q}+\not{q}')B^{S,A}(s,t)\right]u(p,s)$$
$$= \bar{u}(p',s')\left[D^{S,A}(s,t) + \frac{i\sigma^{\mu\nu}q_\nu q'_\mu}{2M}B^{S,A}(s,t)\right]u(p,s)$$

where $D = A + (s-u)B/(4M_N)$

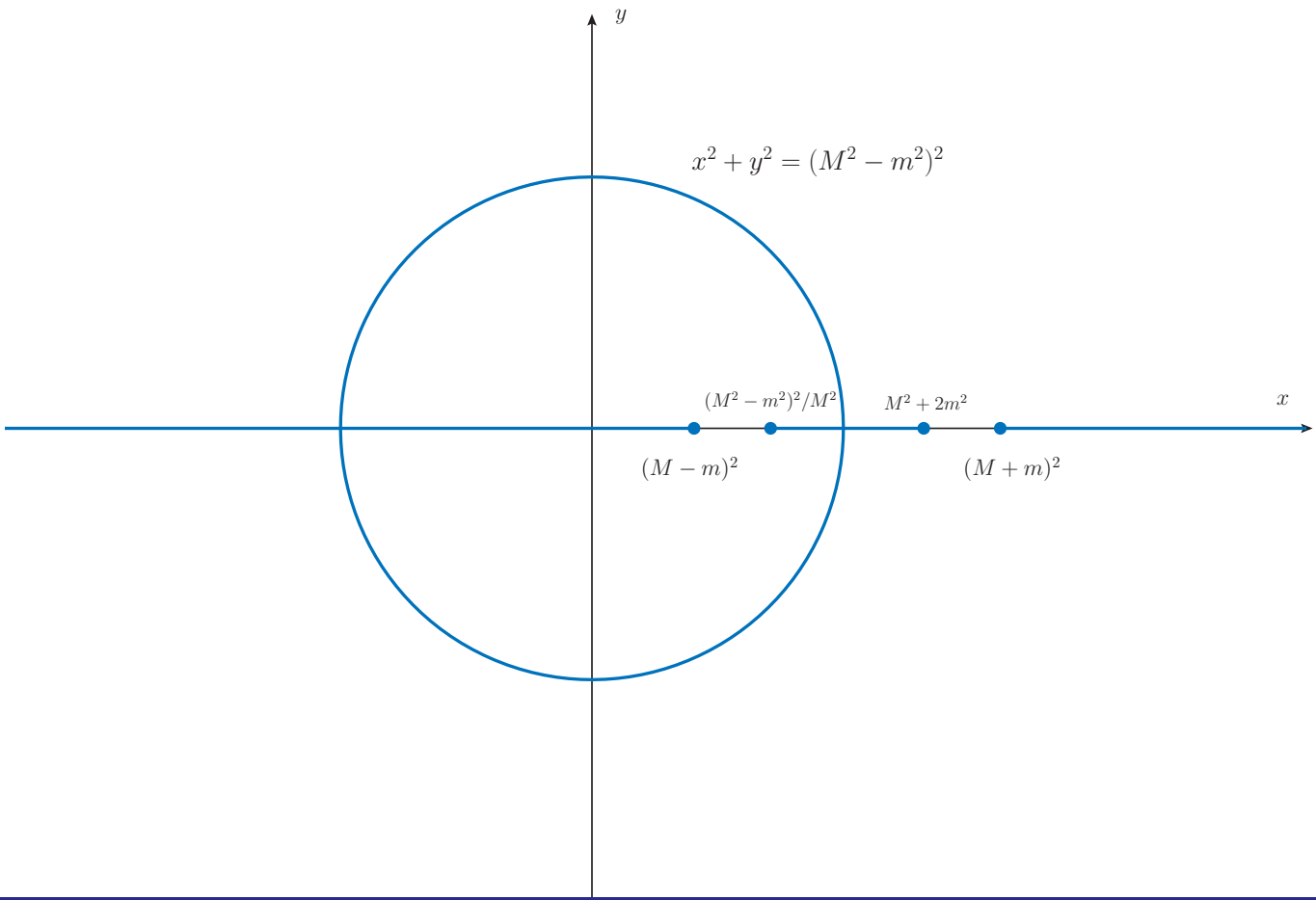
- Helicity amplitudes ($z_s = \cos\theta$)

$$T_{++} = (\frac{1+z_s}{2})^{\frac{1}{2}}[2M_N A(s,t) + (s - m_\pi^2 - M_N^2)B(s,t)]$$
$$T_{+-} = -(\frac{1-z_s}{2})^{\frac{1}{2}}s^{-\frac{1}{2}}[(s - m_\pi^2 + M_N^2)A(s,t) + M_N(s + m_\pi^2 - M_N^2)B(s,t)]$$

- Partial wave integration

$$T^J_{++} = \frac{1}{32\pi}\int_{-1}^{1} dz_s T_{++}(s,t(s,z_s))d^J_{-1/2,-1/2}(z_s)$$
$$T^J_{+-} = \frac{1}{32\pi}\int_{-1}^{1} dz_s T_{+-}(s,t(s,z_s))d^J_{1/2,-1/2}(z_s)$$

BRANCH CUT STRUCTURE OF PARTIAL WAVE πN ELASTIC SCATTERING AMPLITUDE



PHASE SHIFT IN TERMS OF POLES AND CUTS

- Aim at analysing phase shift data
- PKU factorization $\rightarrow$ conventionally additive phase shift
- Phase shift contributions
  - bound states $\rightarrow$ negative phase shift
  - virtual states (usually hidden !) $\rightarrow$ positive phase shift
  - resonances $\rightarrow$ positive phase shift
  - left hand cut $\rightarrow$ (at most time) negative phase shift
    (proved in quantum mechanical potential scatterings
    [T. Regge 1958 Nuovo Cimento])

πN TREE LEVEL PHASE SHIFTS

- Tree level left hand cut
  - $(-\infty, 0] \rightarrow$ kinematic
  - $[(M_N^2 - m_\pi^2)^2/M_N^2, M_N^2 + 2m_\pi^2] \rightarrow u$ channel nucleon exchange
- The latter is numerically very small.
- The main contribution of $f(s)$

$$f(s) = \frac{s}{\pi} \int_{-\infty}^{0} \frac{\sigma(w)dw}{w(w-s)}$$

- The dispersion spectrum function

$$\sigma(w) = \mathrm{Im}\left\{\frac{\ln(S_{\mathrm{tree}})}{2i\rho(w)}\right\} = -\frac{\ln|1 + 2i\rho(w)T_{\mathrm{tree}}|}{2\rho(w)}$$

The contribution is always negative

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TREE-LEVEL QUALITATIVE ANALYSIS

- Focus on s and p waves
- Values of the constants

$$F = 0.0924\text{GeV},\ g = 0.1267$$

$$M_N = 0.9383\text{GeV},\ m_\pi = 0.1396\text{GeV}$$

- $\mathcal{O}(p^2)$ K-Matrix phase shift: $T = T_{\text{tree}}/(1 - i\rho T_{\text{tree}}),\ \delta = \arctan(\rho T_{\text{tree}})$
- A rough K-Matrix fit to S_{11}, S_{31}, P_{11}, P_{31} channels, $\sqrt{s} = 1.082 \sim 1.158$ GeV, $\chi^2/\text{d.o.f.} = 3.624$.

$$c_1 = -0.642,\ c_2 = 1.559,\ c_3 = -2.812,\ c_4 = 1.560$$

POLES

- Pole positions are taken from [A.V. Anisovich et. al. 2012 Eur. Phys. J. A]
- Poles in experiments$\rightarrow$ the third Riemann sheet

$$\sqrt{s_p}^{\text{III}} = M_p - i(\Gamma_{\pi N} + \Gamma_{\text{inel.}})/2$$

"inel.": inelastic channels

- In πN single channel analyses, second Riemann sheet poles are needed
- Shadow poles in the second Riemann sheet (rough approximation)

$$\sqrt{s_p}^{\text{II}} = M_p - i(\Gamma_{\pi N} - \Gamma_{\text{inel.}})/2$$

POLES

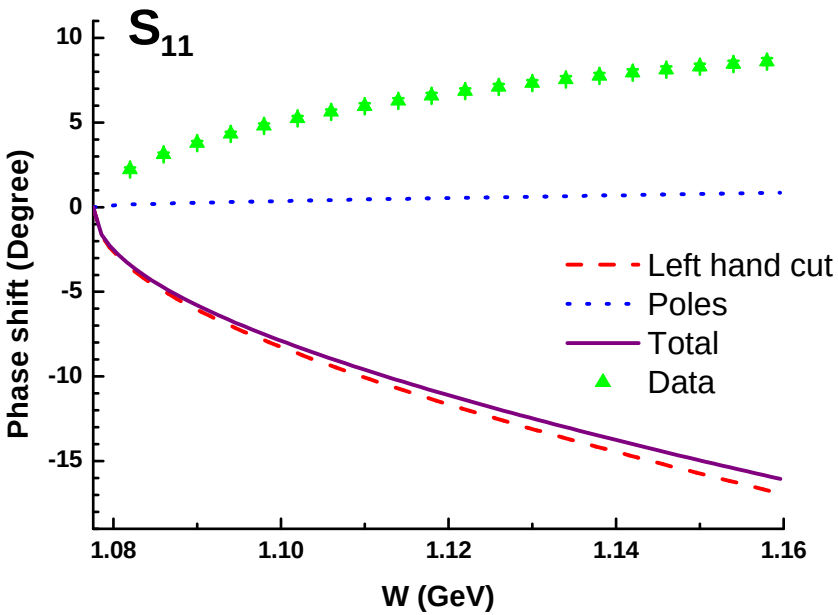
- Intermediate particles

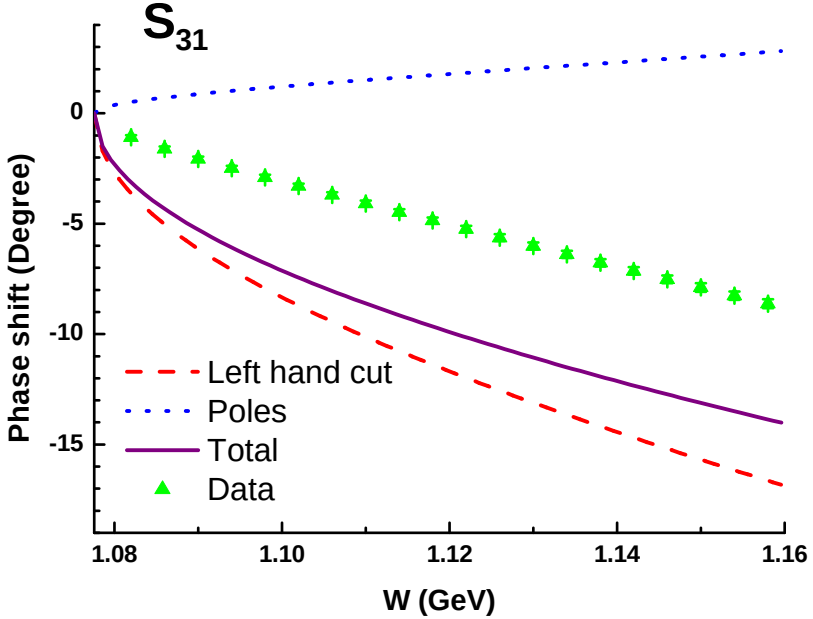
Channel	$I(J^P)$	Intermediate particles
S_{11}	$\frac{1}{2}(\frac{1}{2}^-)$	$N^*(1535), N^*(1650), N^*(1895)$
S_{31}	$\frac{3}{2}(\frac{1}{2}^-)$	$\Delta(1620), \Delta(1900)$
P_{11}	$\frac{1}{2}(\frac{1}{2}^+)$	$N, N^*(1440), N^*(1710), N^*(1880)$
P_{31}	$\frac{3}{2}(\frac{1}{2}^+)$	$\Delta(1910)$
P_{13}	$\frac{1}{2}(\frac{3}{2}^+)$	$N^*(1720), N^*(1900)$
P_{33}	$\frac{3}{2}(\frac{3}{2}^+)$	$\Delta(1232), \Delta(1600), \Delta(1920)$

TREE LEVEL PHASE SHIFTS

$L_{2I\ 2J}$ convention, $W = \sqrt{s}$, data: green triangles

[R. A. Arndt et. al. (GWU group) 2006 PRC]





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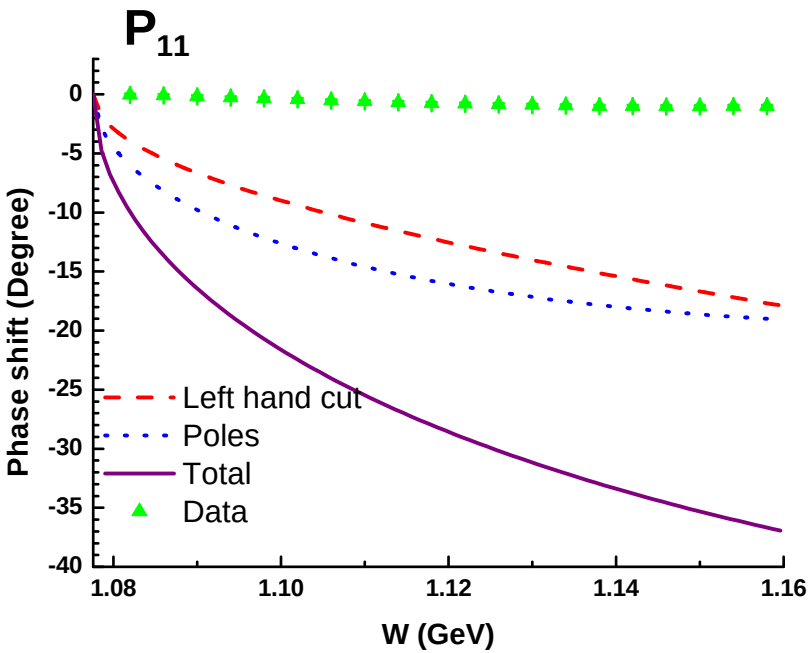

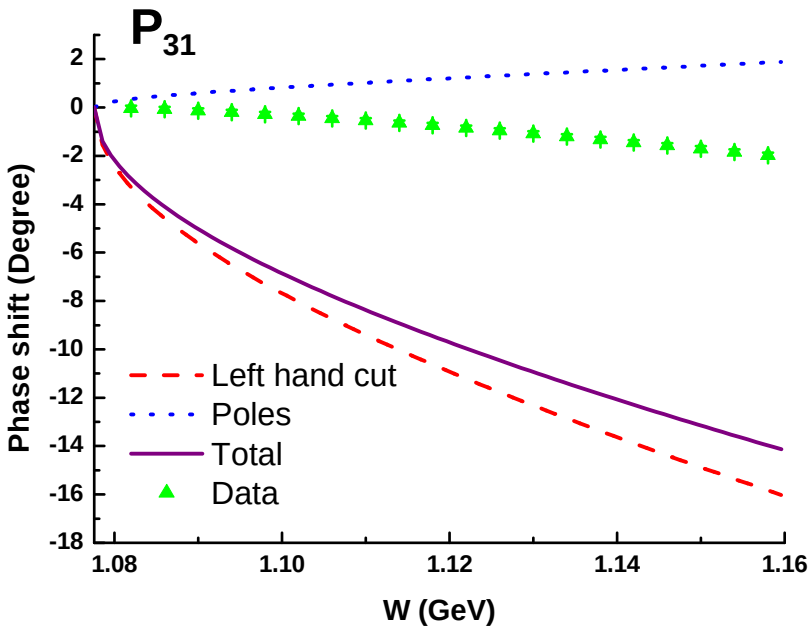

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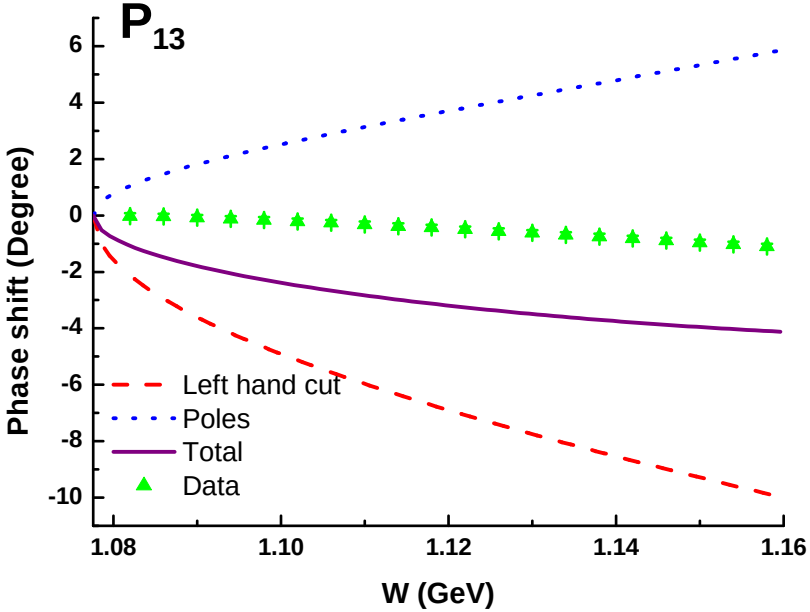
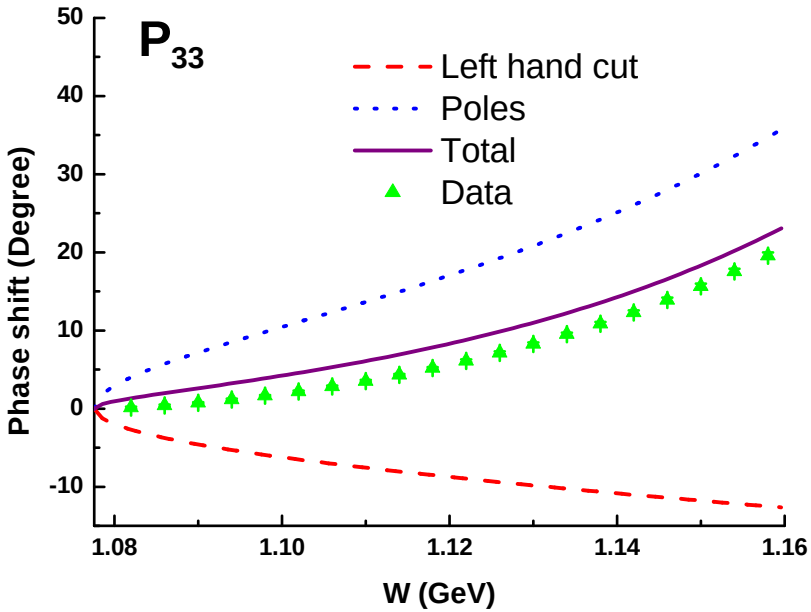


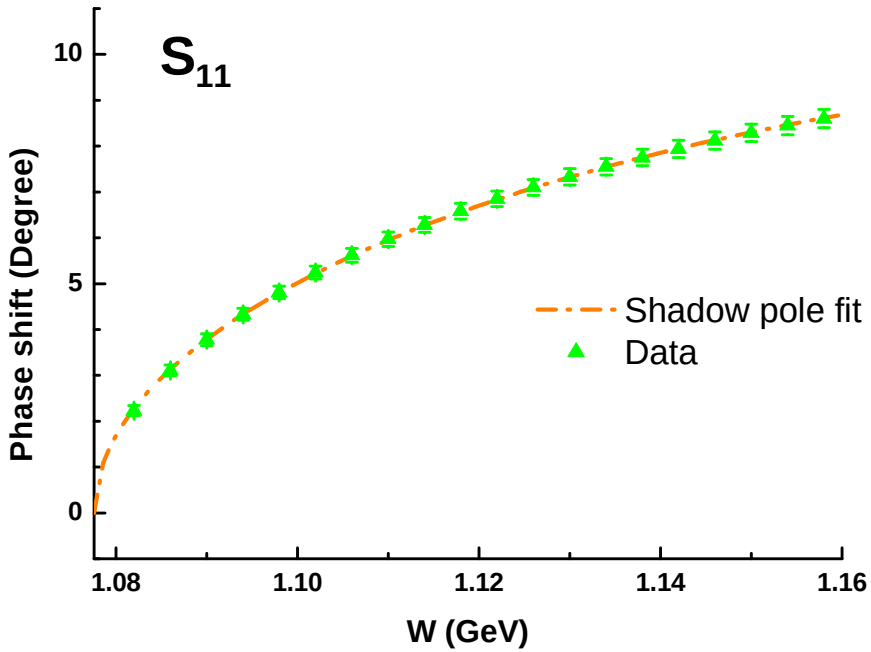


REMARKS

- In some channels (especially S_{11} and P_{11}) huge differences are found between "known poles+ left hand cut" and the data $\rightarrow$ missing positive contributions
- Possible interpretations
  - one loop contributions may be notable (but unlikely to distort the tree level results totally)
  - shadow poles in Riemann sheet II may significatively differ from the rough approximation $\rightarrow$ special properties of $N^*(1535)$ and $N^*(1440)$?
  - hidden poles? (virtual states or crazy resonances below threshold)
  - contributions from complex branch cuts?

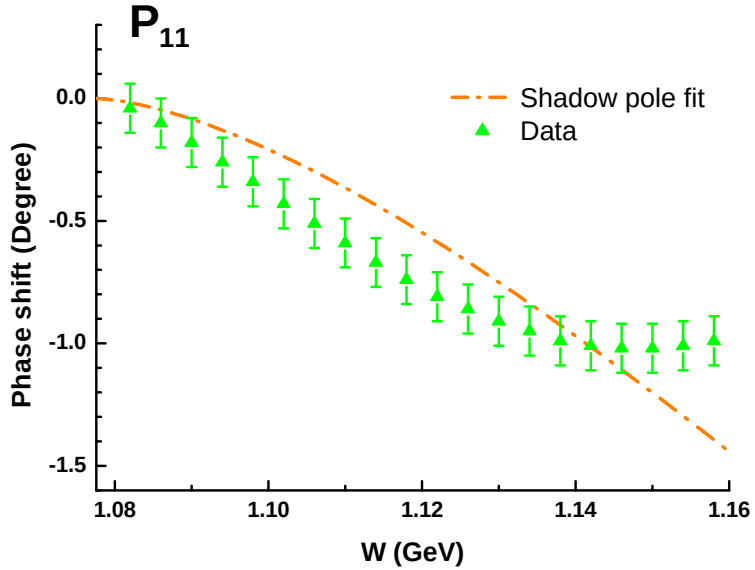
FINDING $N^*(1535)$ SHADOW POLE

- $N^*(1535)$ pole position as the fit parameter
- $\chi^2/\text{d.o.f.} = 0.017$, pole position in sheet II: $M = 0.880$ GeV, $\Gamma = 540$ MeV



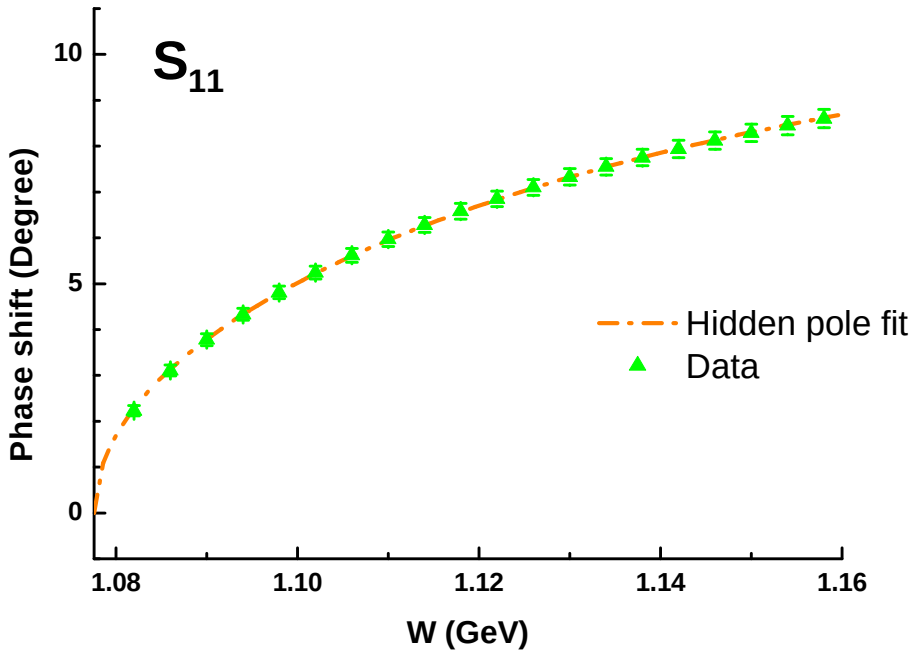
FINDING $N^*(1440)$ SHADOW POLE

- p wave scatterings $\rightarrow$ threshold behavior $\delta(k) \sim o(k^3)$, the k^1 term must vanish (k: 3-momentum)
- Fit under the constraint: $\chi^2/\text{d.o.f.} = 3.738$, $M = 0.896$ GeV, $\Gamma = 14.6$ MeV



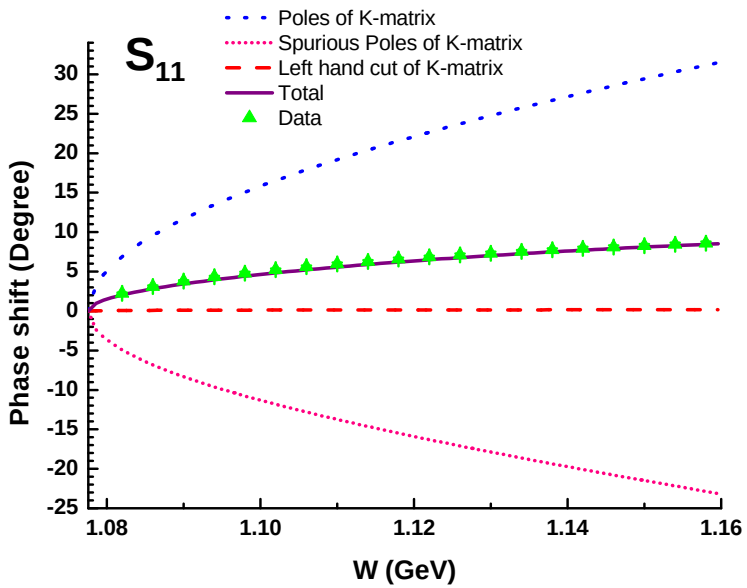
HIDDEN POLES?

- P_{11}: cannot fit the data well with a single virtual state or a resonance below threshold.
- S_{11}: $\chi^2/\text{d.o.f.} = 0.020$, $M = 0.874$ GeV, $\Gamma = 533$ MeV



How does K-Matrix fit the data well?

- K-Matrix phase shift $\rightarrow$ spurious poles(resonances in Riemann sheet I)$\rightarrow$ causality violation
- Cancellation of poles and spurious poles, e.g. S_{11}



SUMMARY

- A new unitarization approach separating phase shift contributions is used to the πN elastic scattering.
- In tree level, S_{11} and P_{11} channels contain a significant disagreement between "known poles $+$ cut" and the experiment, missing a large positive contribution.
- Second sheet shadow poles of $N^*(1535, 1440)$ may lie below the threshold.
- The K-matrix method may be helpful to determine the coupling constants, but in tree level is not reliable in dynamically generating poles.

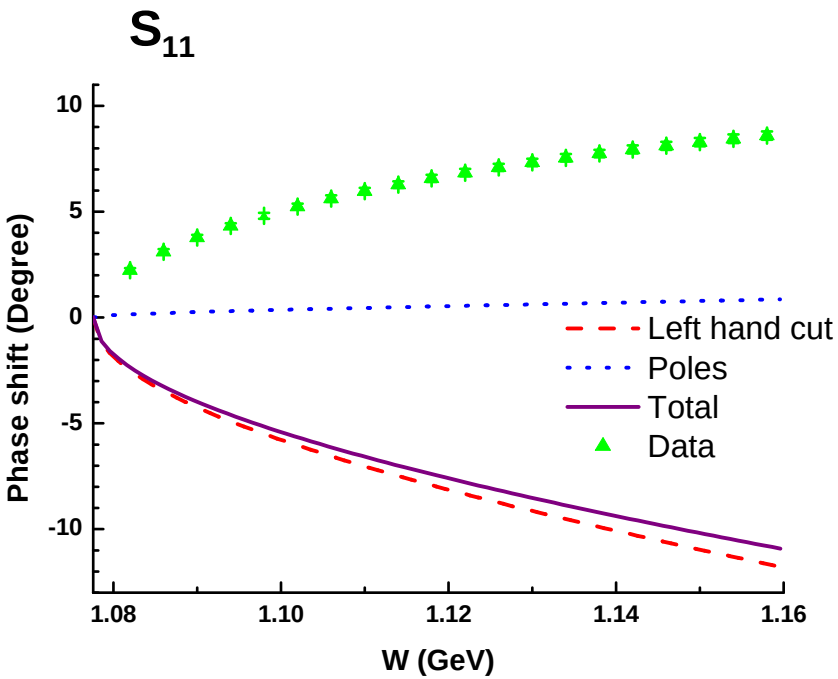
Outlook

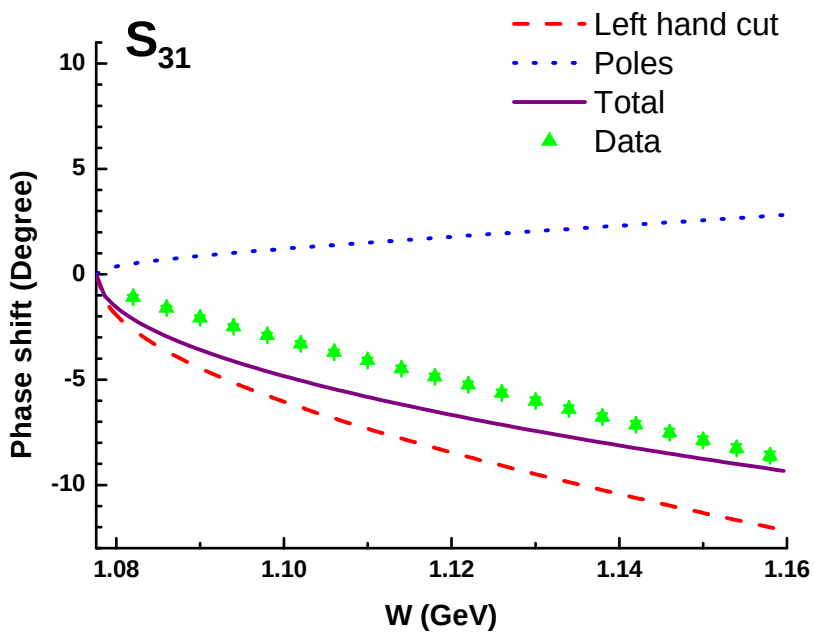
- The work has just begun!
- One loop EOMS calculation (Repetition of [Y. H. Chen et. al. 2013 PRD])
- Handling more complicated left hand cut
- Exam S_{11} and P_{11} channels further: shadow poles? hidden poles? $N^*(1440)$ double-pole? complex branch cuts?
- More quantitative analyses of the phase shift data

Thank you !!

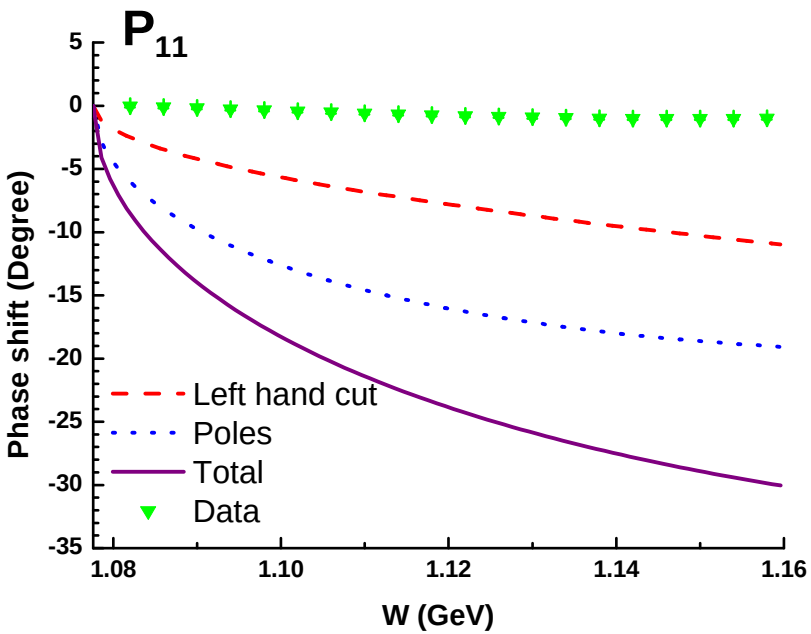
Back up

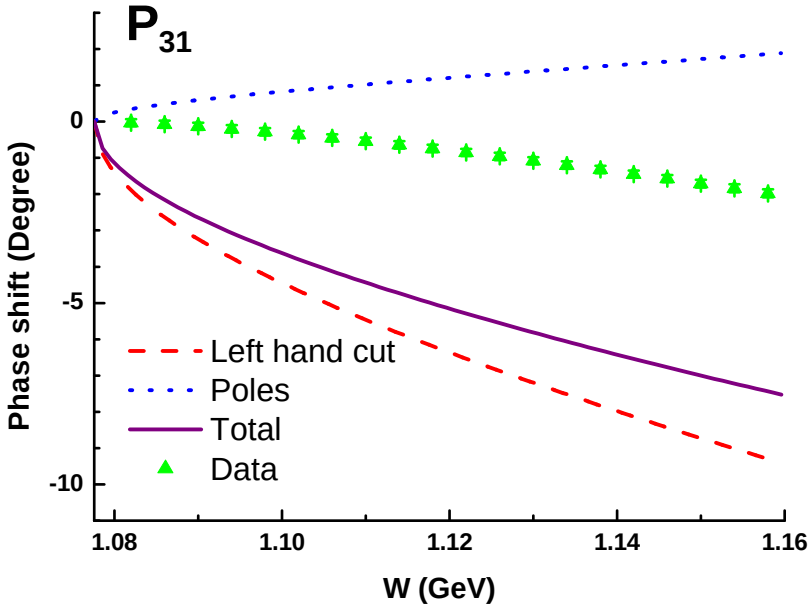
$\mathcal{O}(p^1)$ PHASE SHIFTS



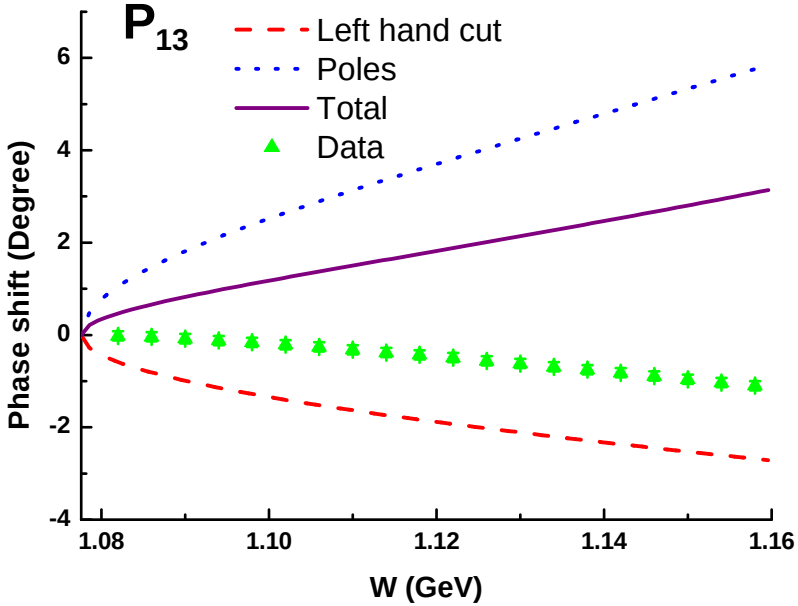


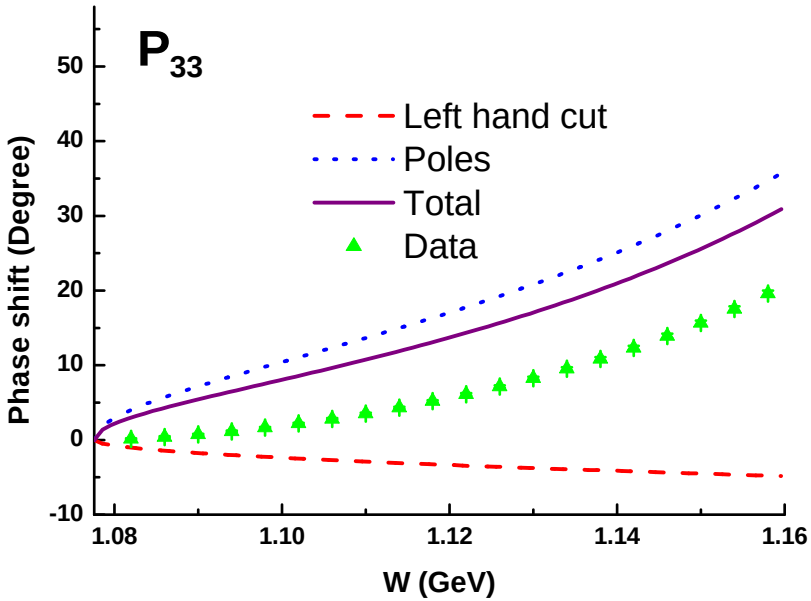
$\mathcal{O}(p^1)$ PHASE SHIFTS



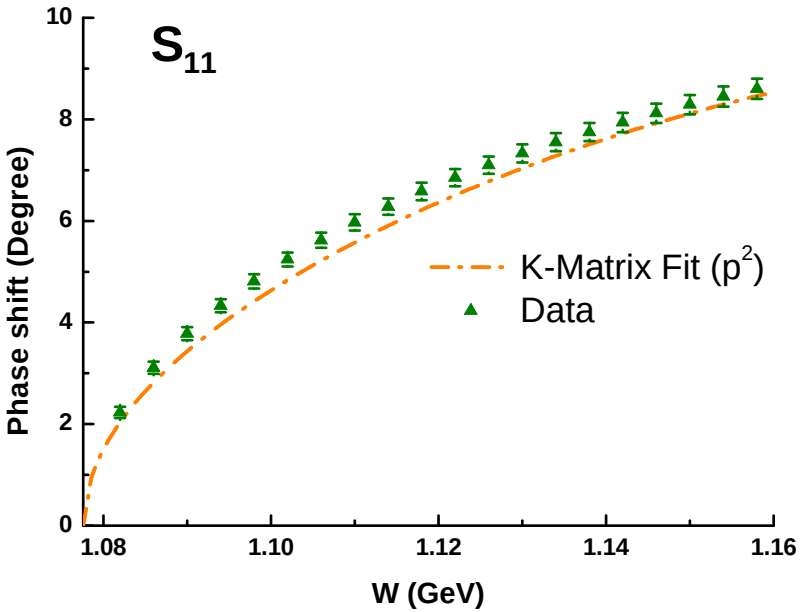


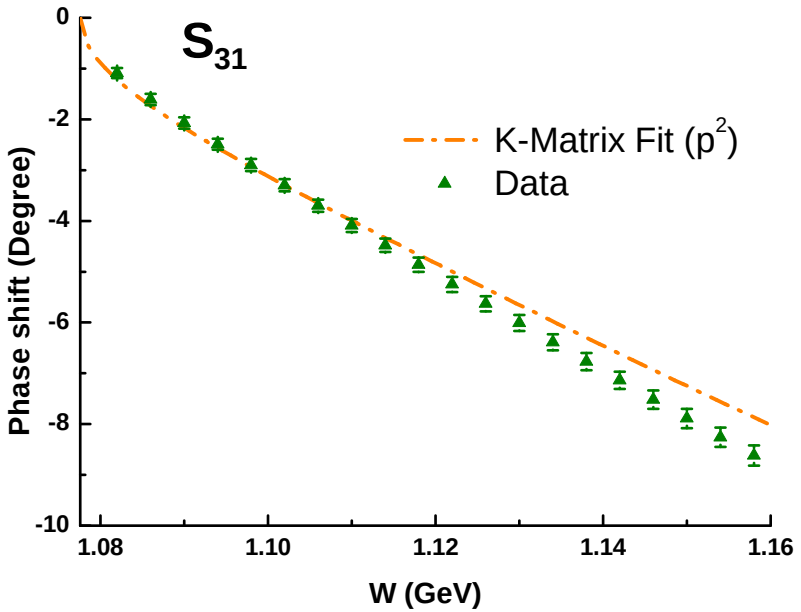
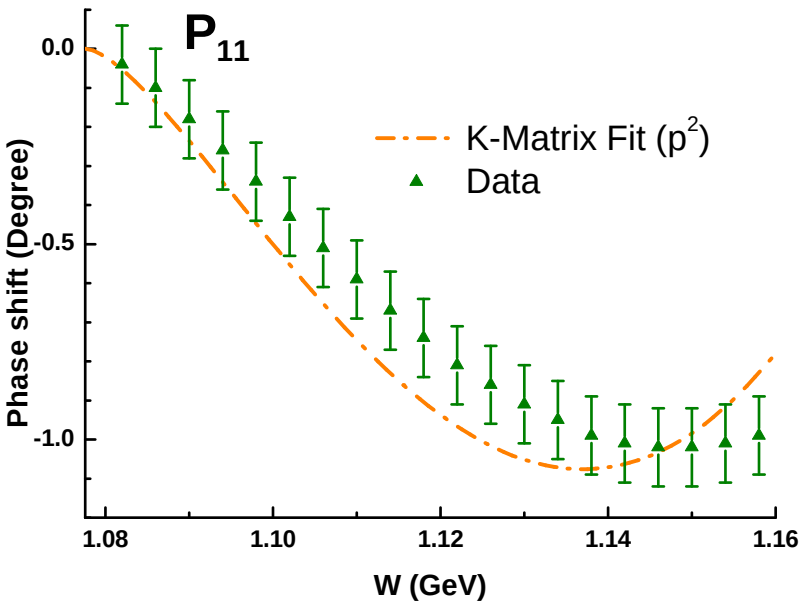
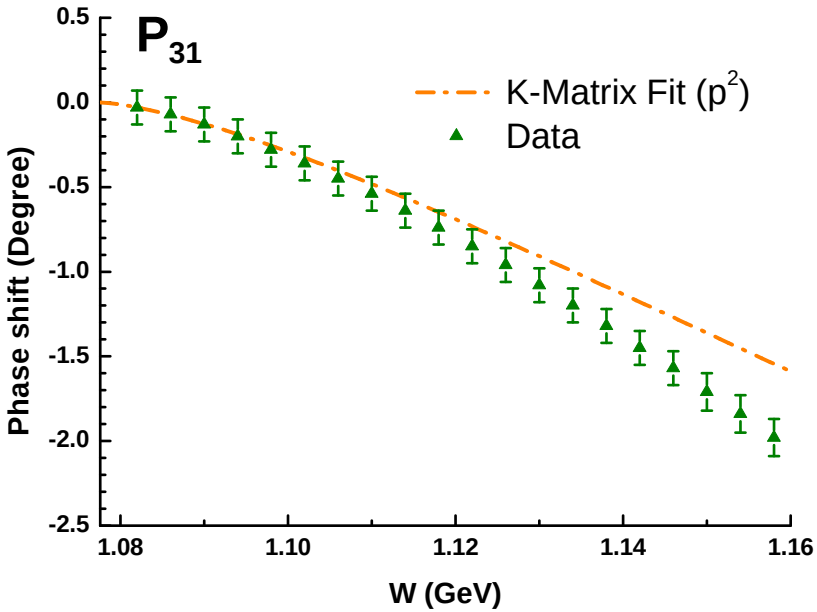
$\mathcal{O}(p^1)$ PHASE SHIFTS





$\mathcal{O}(p^2)$ K-MATRIX FIT

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