

# Hidden-charm Pentaquarks in a Constituent Quark Model

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Joint Workshop

# Exotic Hadrons

- # Hadron is a color-singlet composite of quarks and gluons.

$$q-q^{\bar{b}a} \text{ (meson)}: 3 \otimes \bar{3} = \underset{\textcolor{purple}{\circlearrowleft}}{1} \oplus 8$$

$$q-q-q \text{ (baryon)}: 3 \otimes 3 \otimes 3 = \underset{\textcolor{purple}{\circlearrowleft}}{1} \oplus 8 \oplus 8 \oplus 10$$

- # and *MORE* ...

$$g-g \text{ (glueball)} : 8 \otimes 8 = \underset{\textcolor{purple}{\circlearrowleft}}{1} \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$$

$$q-q^{\bar{b}a}-g \text{ (hybrid)}: 3 \otimes \bar{3} \otimes 8 = \underset{\textcolor{purple}{\circlearrowleft}}{1} \oplus (3 \times 8) \oplus 10 \oplus \overline{10} \oplus 27$$

**$q^2-q^{\bar{b}a2}$  (tetra-quark):**

$$3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = (\underline{2} \times \underset{\textcolor{purple}{\circlearrowleft}}{1}) \oplus (4 \times 8) \oplus 10 \oplus \overline{10} \oplus 27$$

$$q^4-q^{\bar{b}a} \text{ (penta-quark)}: 3^4 \otimes \bar{3} = (\underline{3} \times \underset{\textcolor{purple}{\circlearrowleft}}{1}) \oplus \dots$$

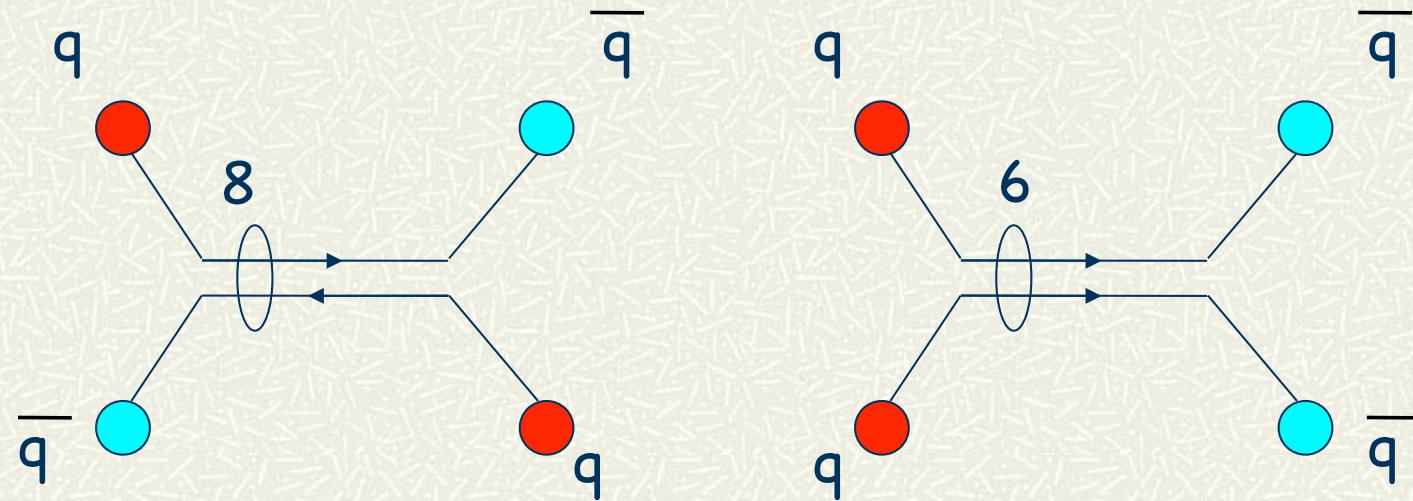
$$q^6 \text{ (di-baryon)}: 3^6 = (\underline{5} \times \underset{\textcolor{purple}{\circlearrowleft}}{1}) \oplus \dots$$

# Multi-Quark (MQ) dynamics

- # “Extrapolation” to MQ hadrons is not trivial.
- # “Color Confinement” is a key in the MQ dynamics.

Exotic Hadrons are “**Colorful**” ! (Lipkin@YKIS06)

$(qq^{\bar{b}ar})_8$  or  $(qq)_6$  are allowed only in the MQ hadrons.

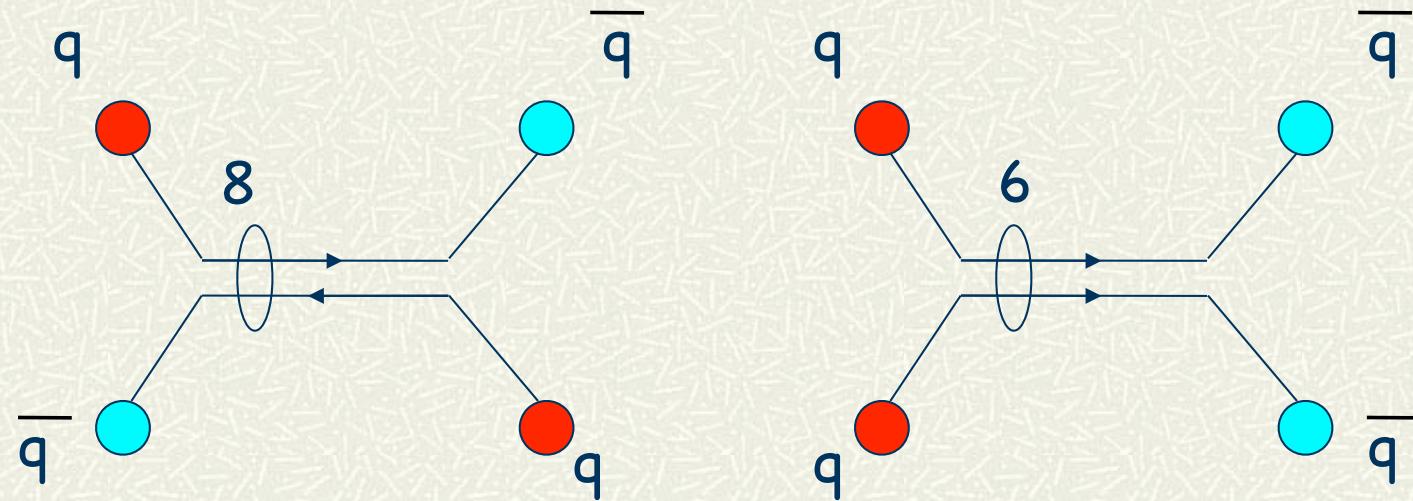


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*Novel Dynamics*

# What we learn from MQ hadrons?

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## # *CONFINEMENT* of Quarks

**What is the Mechanism and Dynamics of quark confinement?**

**Modeling of confinement**

**Bag model v.s. Potential model**

## # *COUPLINGS* of Resonances to multi-hadron final states

**Decay channels and widths of MQ hadrons**

**Mechanisms of the strong decays**

**how colors are recombined for decays**

**Possibility of narrow resonances**

**hidden color states as Feshbach resonances?**

# Bag Model

- # **MIT Bag Model:**  
Quarks (and gluons) are confined (and, in total, color-singlet) in a “Bag”. The bag is self-sustained by the “bag energy”.

- # **Two conditions at the bag surface**

- *No outflow of color from the surface*  $n \cdot j_c^\alpha|_{\text{surface}} = 0$

$$j_c^{\alpha\mu} = \bar{q}\gamma^\mu \frac{\lambda^\alpha}{2} q + (\text{gluon color current})$$

- *Pressure balance of two phases*  $P_{\text{in}} = P_{\text{out}}$

$P_{\text{in}} =$  (pressure by quarks and gluons)

$P_{\text{out}} =$  (pressure by the bag energy)

$$E_{\text{bag}} = BV$$

# Bag Model

## # Energy of the hadron containing massless quarks

$$E(R) = B \frac{4\pi R^3}{3} + \sum_i E_i = \frac{4\pi B R^3}{3} + \sum_i \frac{\omega_i}{R}$$

$$\frac{dE(R)}{dR} = B 4\pi R^2 - \frac{\sum_i \omega_i}{R^2} = 0 \longrightarrow R(n) = \left( \frac{n\omega}{4\pi B} \right)^{1/4}$$

$$E_n = E(R(n)) = (\text{const}) \times B^{1/4} n^{3/4}$$

- #  **$E_n$  is a convex function of  $n$ , that is  $E_{2n} < 2E_n$ . If there is no other interaction, the binding energy is larger as the size of the system gets larger.**
- # **The energy scale is  $B^{1/4} \sim 200$  MeV. It is not surprising to have a bound state of binding energy  $\sim 100\text{-}200$  MeV.**

# Potential Model

## # Two-body confinement forces

### ■ Force without color-cluster saturation is *no good.*

$$V = \sum_{i < j} v(r_{ij}) \longrightarrow \langle V \rangle \sim \frac{n(n-1)}{2} \langle v \rangle \sim \text{gravity}$$

### ■ Spin-independent color-saturated force is linear in $n$ .

$$V = - \sum_{i < j} (\lambda_i^c \cdot \lambda_j^c) v(r_{ij}) \longrightarrow \langle V \rangle \sim \frac{8}{3} n \langle v \rangle$$

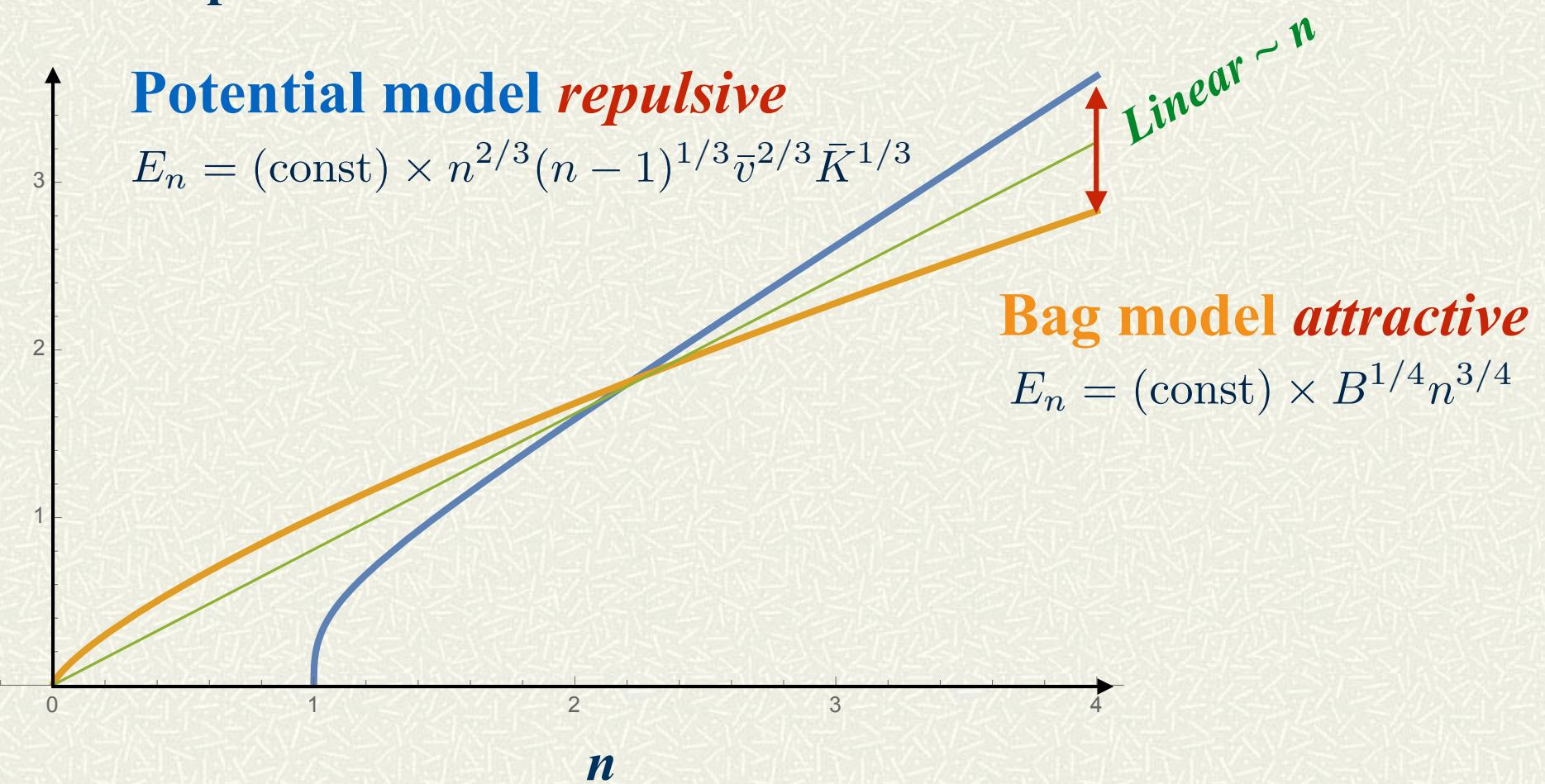
### ■ Radius $R$ determined by the energy minimum

$$E(R) = \langle K + V \rangle \sim \frac{n-1}{R^2} \bar{K} + n \bar{v} R$$

$$E_n = (\text{const}) n^{2/3} (n-1)^{1/3} \bar{v}^{2/3} \bar{K}^{1/3}$$

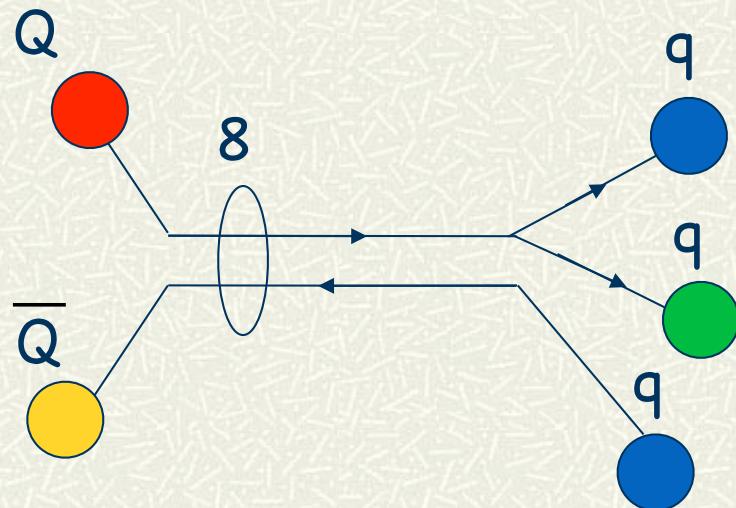
# Bag model v.s. Potential model

## # $n$ dependences



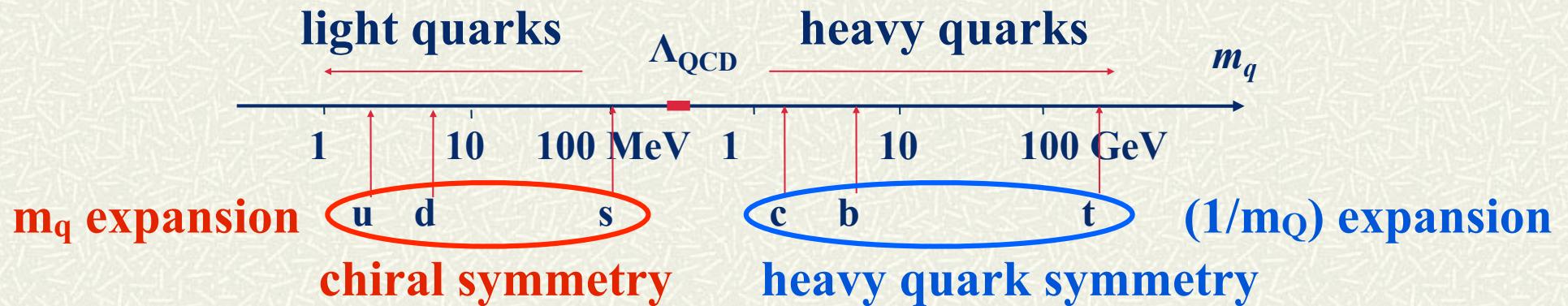
# Exotic MQ states

- # To look for “stable” (or narrow) multi-quark states, we consider “colorful” configurations.
- # *Hidden Charm Pentaquarks* are cases in which the *color-octet* “baryon” might be stabilized with the help of *color-octet* heavy “quarkonium”.

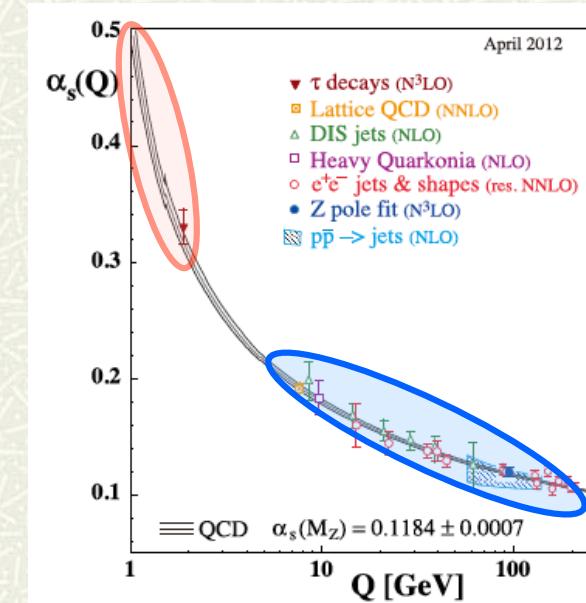


# Heavy Quark

- # QCD Lagrangian is flavor independent, but the coupling constant runs.

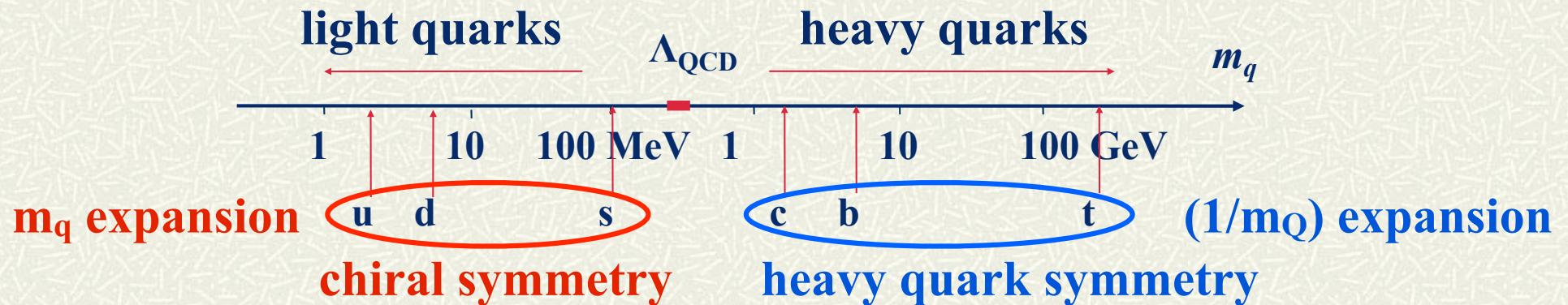


- # Light quarks are nonperturbative/ relativistic.
- # Heavy quarks are perturbative/ non-relativistic.



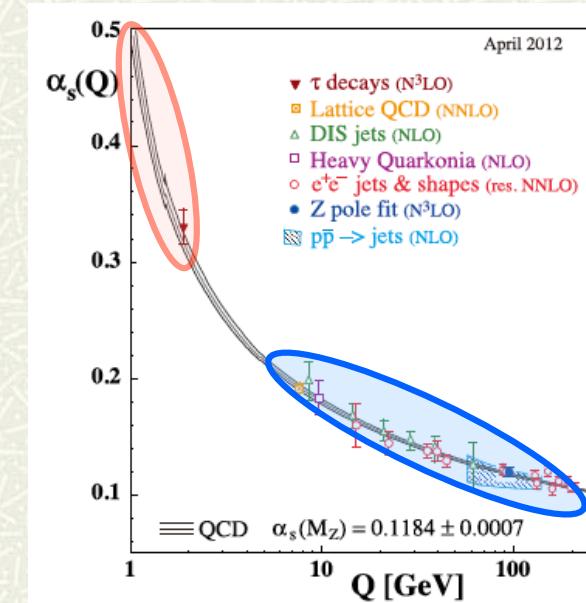
# Heavy Quark

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- # Light quarks are nonperturbative/ relativistic.
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**Heavy Quark Helps!**



# Charmonium

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- # The quark model gives very good guidelines to classify and interpret the hadron spectrum.

The charmonium spectrum is a textbook example.  
*“hydrogen atom” in QCD*

# Charmonium

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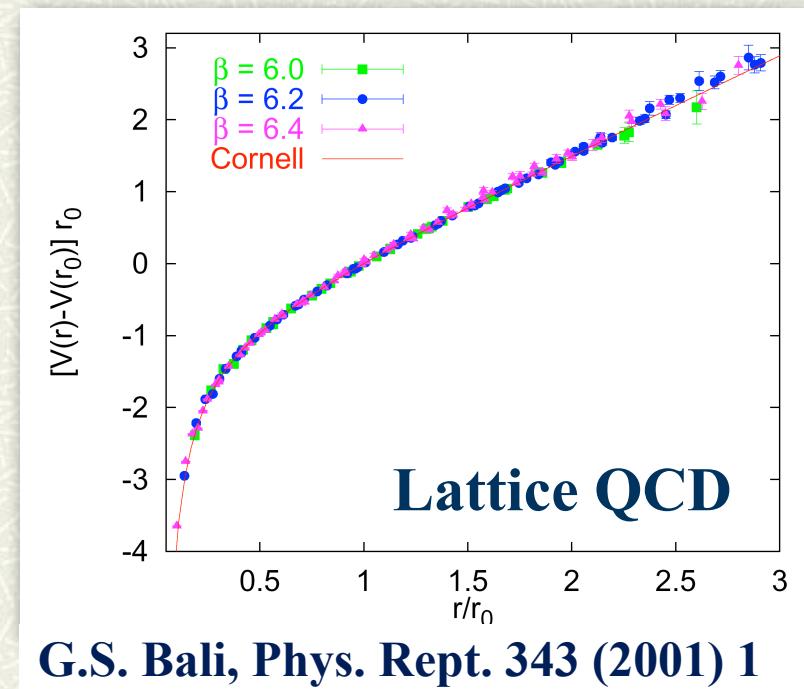
The charmonium spectrum is a textbook example.  
*“hydrogen atom” in QCD*

- # The Hamiltonian with a Linear + Coulomb potential

$$V(r) = -\frac{e}{r} + \sigma r$$

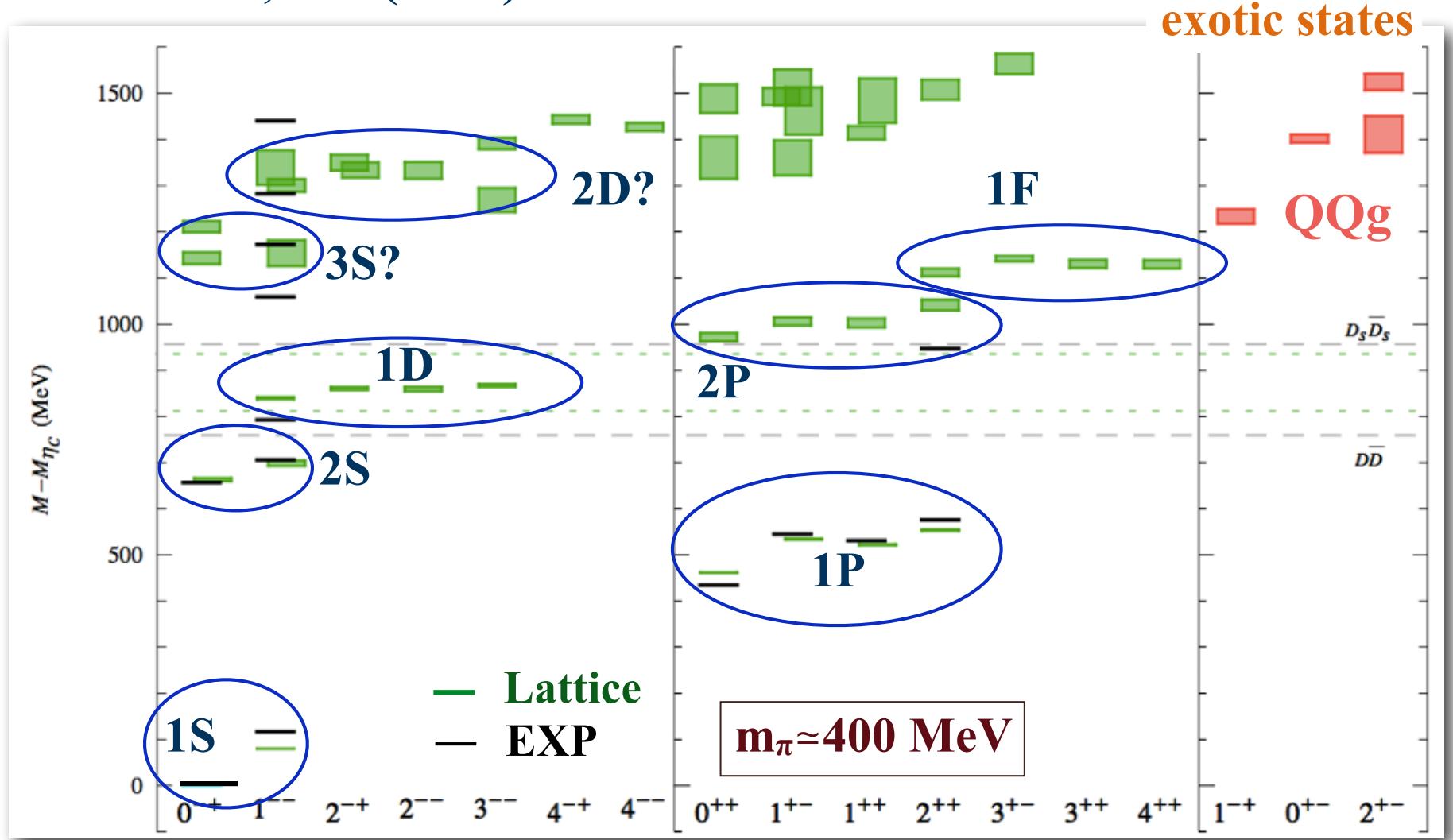
E. Eichten, et al., PRL 34 (1975) 369

gives a good fit to the 1S, 1P, 2S, ...  
charmonium (and bottomonium)  
states.



# Charmonium

Liuming Liu, et al. (Hadron Spectrum Collaboration)  
JHEP 07, 126 (2012)

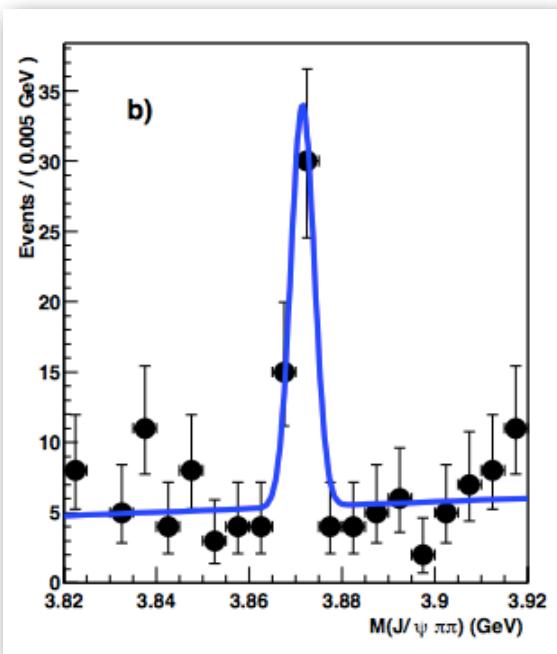


# HQ Exotic Hadrons

- # X(3872) found in 2003 by Belle (KEK)  
→ *not reproduced by lattice QCD using only  $q\text{-}q^{\bar{b}ar}$  operators.*
- # Z(3900), Z(4430) etc. : charged hidden charm states

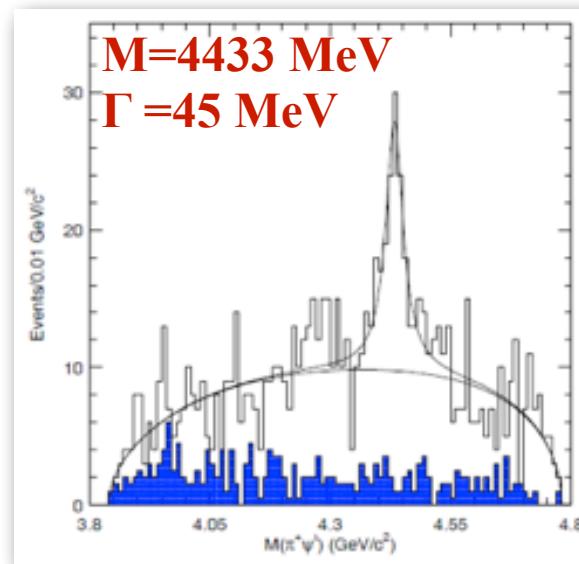
X(3872)

Belle



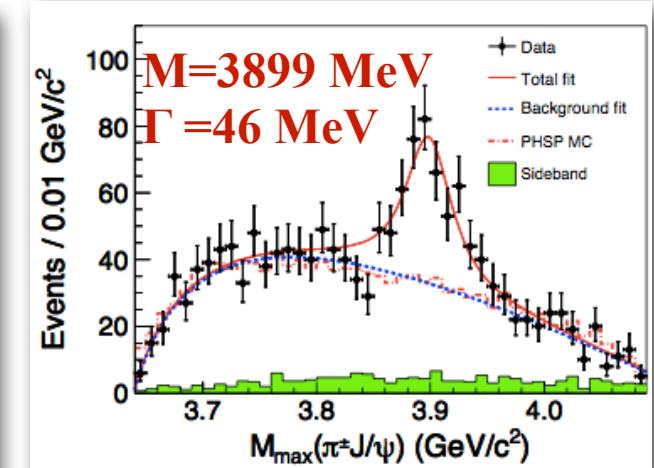
Z<sub>c</sub><sup>+</sup>(4430)

Belle



Z<sub>c</sub><sup>+</sup>(3900)

BES III



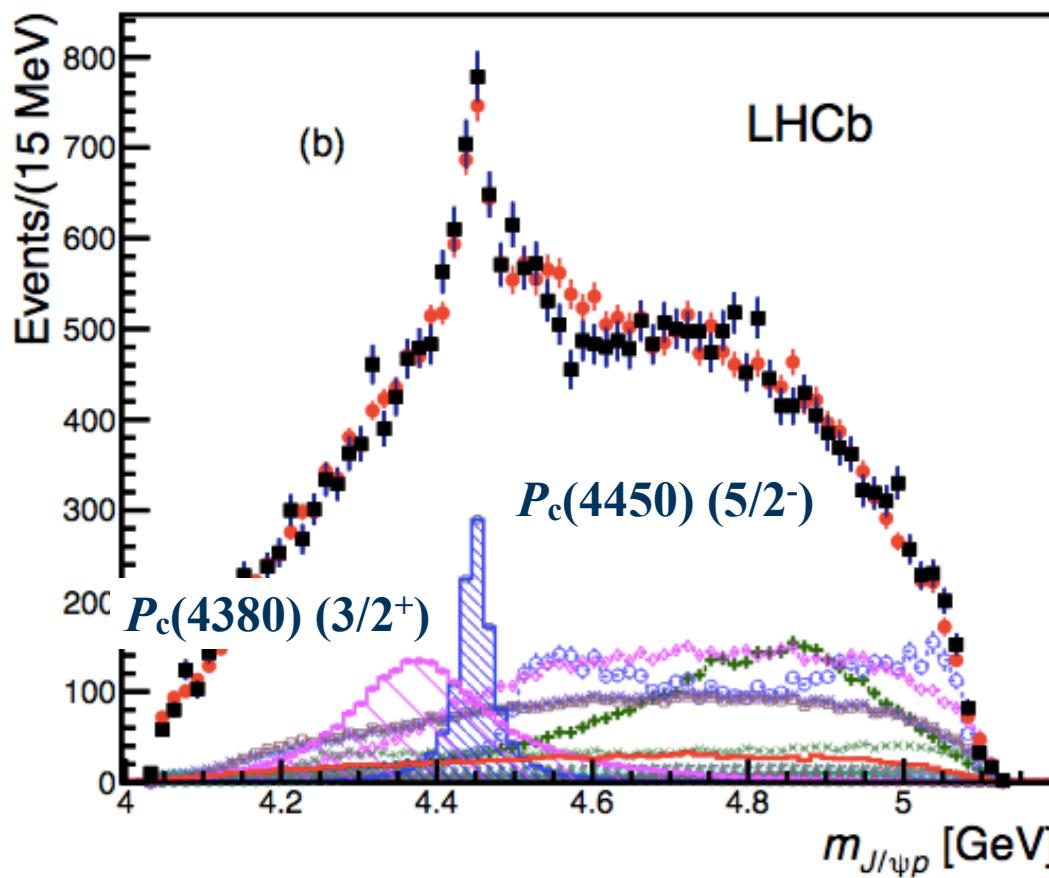
PRL 110 (2013) 252001

PRL 100 (2008) 142001

PRL 91 (2003) 262001

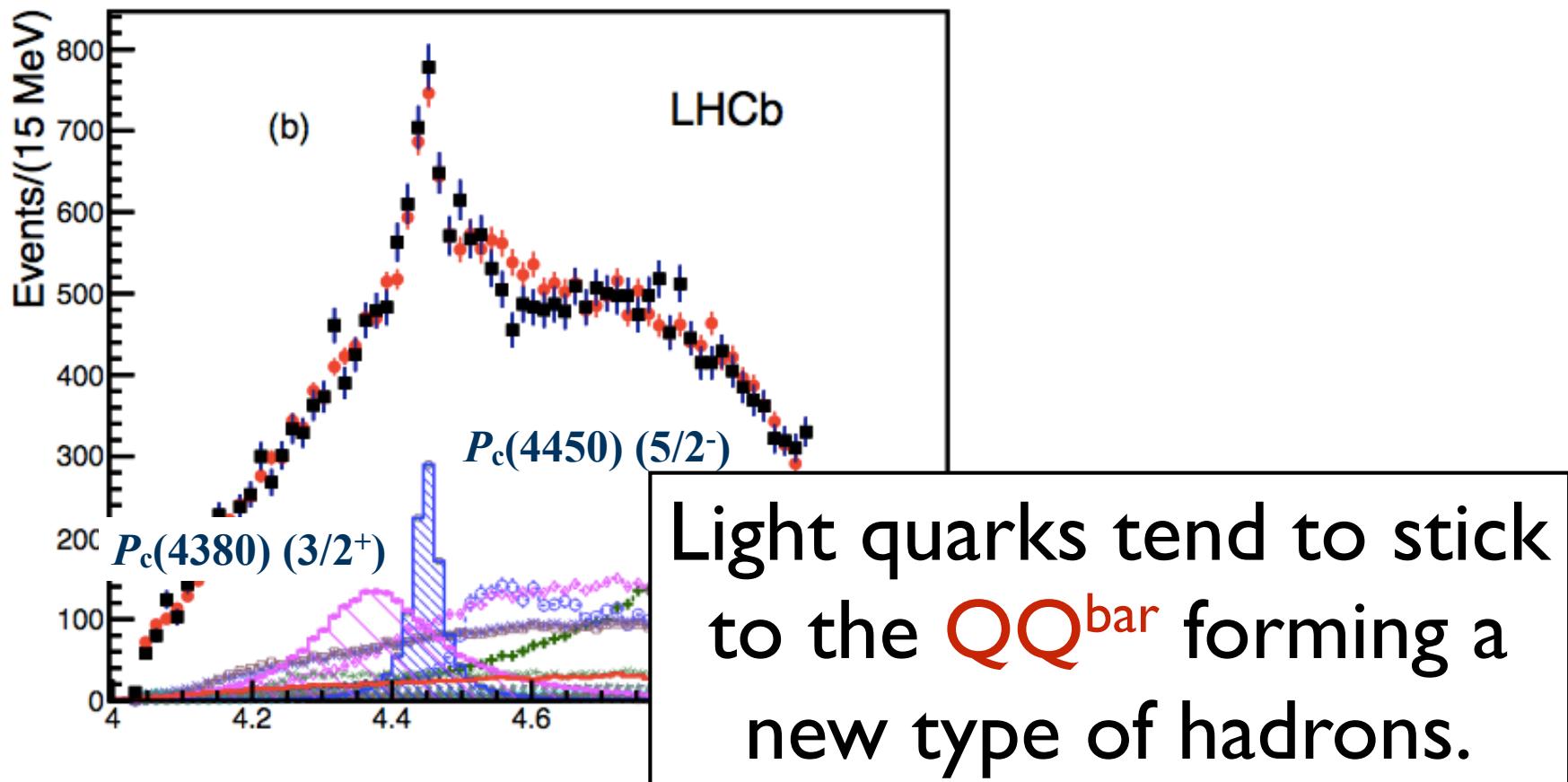
# *Hidden Charm Pentaquark $P_c$*

- #  $P_c \rightarrow J/\psi + p$  ( $cc\bar{u}ud$ )  
LHCb (*PRL 115 (2015) 07201*) found two penta-quark states with hidden  $cc$ .



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- #  $P_c \rightarrow J/\psi + p$  ( $cc\bar{u}ud$ )  
LHCb (*PRL 115 (2015) 07201*) found two penta-quark states with hidden  $cc$ .



# *Hidden Charm Pentaquark $P_c$*

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## Constituent quark model analyses

- # Study of  $qqq\bar{c}^{\text{bar}}c$  five quark system with three kinds of quark-quark hyperfine interaction,  
**S.G. Yuan, K.W. Wei, J. He, H.S. Xu, B.S. Zou,**  
**Eur. Phys. J. A 48 (2012) 61**
- # The hidden charm pentaquarks are the hidden color-octet uud baryons?  
**Sachiko Takeuchi, Makoto Takizawa, PL B764 (2017) 254–259**
- # Flavor-singlet charm pentaquark  
**Yoya Irie, Makoto Oka, Shigehiro Yasui,**  
**arXiv:1707.04544.**
- # Hidden-charm pentaquark with strangeness  
**Sachiko Takeuchi, et al., in preparation.**

# *Color Magnetic Interaction*

## # Color-Magnetic Interaction

$$V_{\text{CMI}} = -\alpha \sum_{i < j} (\vec{\lambda}_i \cdot \vec{\lambda}_j) (\vec{\sigma}_i \cdot \vec{\sigma}_j) f(r_{ij}) \quad f(r_{ij}) \sim \delta(r_{ij})$$

**prefers color-spin symmetric states**

$$\langle V_{\text{CMI}} \rangle_{(0s)^N} = \alpha \langle f(r) \rangle_{0s} \Delta_{\text{CM}} = V_0 \Delta_{\text{CM}}$$

$$\Delta_{\text{CM}} \equiv \left\langle - \sum_{i < j} (\vec{\lambda}_i \cdot \vec{\lambda}_j) (\vec{\sigma}_i \cdot \vec{\sigma}_j) \right\rangle$$

$$\Delta_{\text{CM}} = 8N - 2C_2[SU(6)_{cs}] + \frac{4}{3}S(S+1) + C_2[SU(3)_c]$$

$$C_2[SU(g)]([f_1, f_2, \dots, f_g]) = \sum_i f_i(f_i - 2i + g + 1) - \frac{N^2}{g}$$

$$C_2[\text{singlet}] = 0$$

# *Color Magnetic Interaction*

- # CMI prefers color-spin symmetric states, i.e. flavor antisymmetric states.

$$\Delta_{\text{CM}} = 8N - 2C_2[SU(6)_{cs}] + \frac{4}{3}S(S+1) + C_2[SU(3)_c]$$

$$\Delta_{\text{CM}}(\mathbf{10}) - \Delta_{\text{CM}}(\mathbf{8}) = 8 - (-8) = 16$$

$$M(\Delta) - M(N) = 16V_0 \sim 300 \text{ MeV}$$

$$V_0 \sim 300/16 \sim 19 \text{ MeV}$$

$$\Delta_{\text{CM}}(H) - 2\Delta_{\text{CM}}(\Lambda) = -24 - 2(-8) = -8 \quad \mathbf{H} (\Lambda\Lambda + \mathbf{N}\Xi + \Sigma\Sigma, \mathbf{S=0})$$

$$\Delta_{\text{CM}}(D_\Delta) - 2\Delta_{\text{CM}}(\Delta) = 16 - 2 \times 8 = 0 \quad \mathbf{D}_\Delta (\Delta\Delta, \mathbf{I=0}, \mathbf{S=3})$$

# H dibaryon

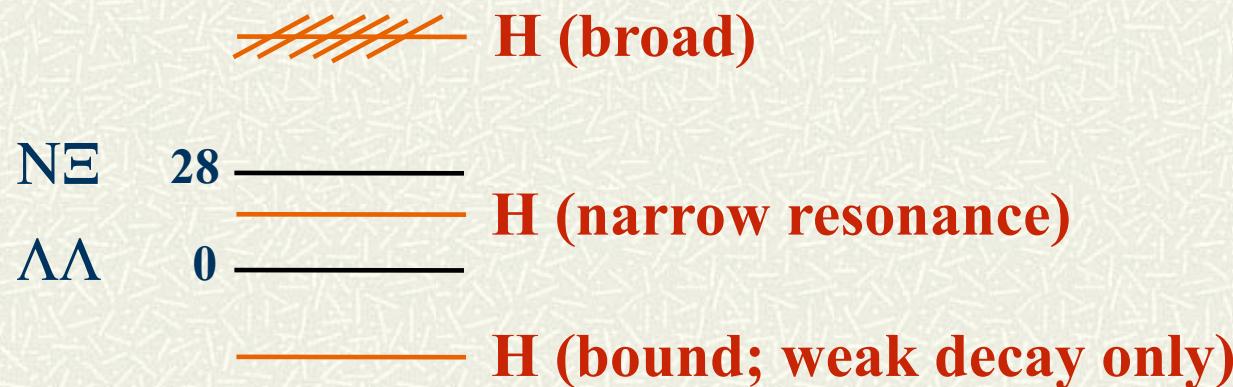
## H dibaryon

$u^2d^2s^2$  ( $S = -2$ ,  $J=0^+$   $I=0$ ) predicted by Jaffe (1977) in the MIT bag.

CMI prefers  
symmetric color-spin states  $\Leftrightarrow$  antisymmetric flavor state  $\rightarrow F=1$

$\Sigma\Sigma$  150 —————

$$|F=1\rangle = -\sqrt{\frac{1}{8}}|\Lambda\Lambda\rangle + \sqrt{\frac{4}{8}}|N\Xi\rangle + \sqrt{\frac{3}{8}}|\Sigma\Sigma\rangle$$



# H dibaryon

R.L. Jaffe, *Phys. Rev. Lett.* 38 (1977) 195.

TABLE I. Quantum numbers and masses of S-wave dibaryons.

SU(6) <sub>c,s</sub> representation	$C_6$	$J$	SU(3) <sub>f</sub> representation	Mass in the limit $m_s = 0$ (MeV)
490	144	0	1	1760
896	120	1, 2	8	1986
280	96	1	10	2165
175	96	1	10*	2165
189	80	0, 2	27	2242
35	48	1	35	2507
1	0	0	28	2799

- # The flavor singlet state ( $u^2d^2s^2$ ) is most favored by the  $C_6 = C_2[\text{SU}(6)]$ .

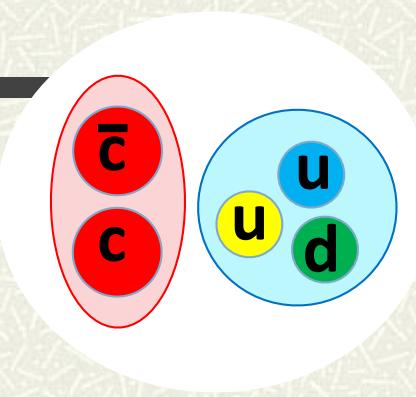
# Flavor Singlet Pentaquark $P_{cs}$

- # color 1  $c\bar{c}$   $\Delta_{CM} \equiv \left\langle - \sum_{i < j} (\vec{\lambda}_i \cdot \vec{\lambda}_j) (\vec{\sigma}_i \cdot \vec{\sigma}_j) \right\rangle$   
 $56 = (8, 1/2) + (10, 3/2)$   
 $(8, 1/2) \quad \Delta_{CM} = -8 \quad c\bar{c} \text{ uud (udd)} = \eta_c/J/\psi + p$   
 $(10, 3/2) \quad \Delta_{CM} = 8$

- # color 8  $c\bar{c}$  (Hidden Color States)  
 $70 = (1, 1/2) + (8, 1/2) + (8, 3/2) + (10, 1/2)$   
 $(1, 1/2) \quad \Delta_{CM} = -14 \quad P_{cs} = c\bar{c} \text{ uds} = \eta_8/\psi_8 + \Lambda_8(\text{singlet})$   
 $(8, 1/2) \quad \Delta_{CM} = -2 \quad \eta_8/\psi_8 + N_8$

The most favored state with  $c\bar{c}$  by CMI may not be  $J/\psi + p$ .

- #  $P_{cs}$  family ( $I=0$ , Str= -1)  
 $(c\bar{c})_{8,J=1} + (uds)_{8,J=1/2} \quad J^\pi = 1/2^-, 3/2^-$   
 $(c\bar{c})_{8,J=0} + (uds)_{8,J=1/2} \quad J^\pi = 1/2^-$



# *Flavor Singlet Pentaquark $P_{cs}$*

## # Potential Quark Model

**Linear confinement with color Casimir dependence**

$$V_{\text{conf}} = \sum_{i < j} -\sigma(\lambda_i \cdot \lambda_j) r_{ij}$$

**Coulomb electric interaction from one-gluon-exchange**

$$V_{\text{Coulomb}} = \sum_{i < j} \frac{\alpha_s}{4r_{ij}} (\lambda_i \cdot \lambda_j)$$

**Color magnetic spin-spin interaction from OGE**

$$V_{\text{CMI}} = -\frac{\alpha_s}{4} \sum_{i < j} \frac{\pi}{m_i m_j} (\lambda_i \cdot \lambda_j) \left[ 1 + \frac{2}{3} \sigma_i \cdot \sigma_j \right] \delta(r_{ij})$$

**Non-relativistic quarks with**

$$m(u, d) = 313 \text{ MeV} \quad m(s) = 522 \text{ MeV}$$

# *Instanton Induced Interaction*

**Instanton** : Classical solution of 4-dim. Euclidian QCD

Light quarks couple  
with instanton



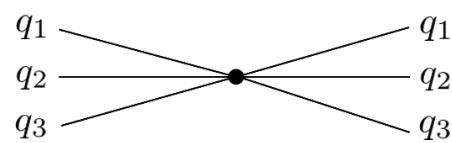
Effective point-like interaction  
of light quarks (KMT)

**3-body interaction**

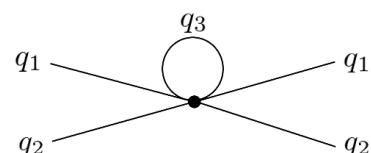
$$V_{\text{III}3} = V_0 \frac{189}{40} \sum_{(ijk)} \mathcal{A}_3^f \left[ 1 - \frac{1}{7} (\sigma_i \cdot \sigma_j + \sigma_j \cdot \sigma_k + \sigma_k \cdot \sigma_i) \right] \delta(r_{ij}) \delta(r_{jk})$$

**2-body interaction**

$$V_{\text{III}2} = U_0^{(2)} \frac{15}{8} \sum_{i < j} \mathcal{A}_2^f \frac{1}{m_i m_j} \left[ 1 - \frac{1}{5} \sigma_i \cdot \sigma_j \right] \delta(r_{ij})$$



The 3-body III is *repulsive* for  
flavor singlet u-d-s systems

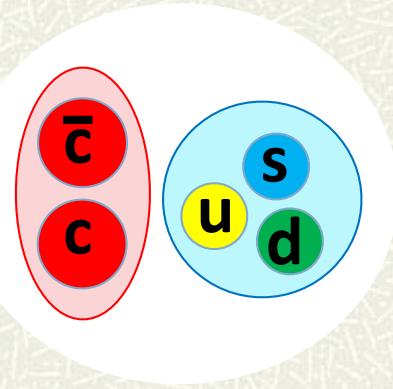


**2-body III**

G. 't Hooft, Phys. Rev. Lett 37 (1976) 8, Phys. Rev. D14 (1976) 3432  
S. Takeuchi, M. Oka, PRL 63 (1989) 1780, NPA547 (1992) 283c-288c

# Flavor Singlet Pentaquark $P_{cs}$

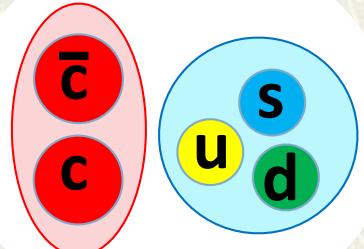
## # $P_{cs}$ family ( $I=0$ , Str= -1)



$(I, J^P)$	octet type (8)					singlet type (1)						
		component	color	spin	flavor	isospin		component	color	spin	flavor	isospin
$(0, 1/2^-)$	$P_{cs8}$	$c\bar{c}$	<b>8</b>	0	—	—	$P_{cs1}$	$c\bar{c}$	<b>1</b>	0	—	—
		$uds$	<b>8</b>	$1/2$	<b>1</b>	0		$uds$	<b>1</b>	$1/2$	<b>8</b>	0
$(0, 1/2^-)$	$P'_{cs8}$	$c\bar{c}$	<b>8</b>	1	—	—	$P'_{cs1}$	$c\bar{c}$	<b>1</b>	1	—	—
		$uds$	<b>8</b>	$1/2$	<b>1</b>	0		$uds$	<b>1</b>	$1/2$	<b>8</b>	0
$(0, 3/2^-)$	$P^*_{cs8}$	$c\bar{c}$	<b>8</b>	1	—	—	$P^*_{cs1}$	$c\bar{c}$	<b>1</b>	1	—	—
		$uds$	<b>8</b>	$1/2$	<b>1</b>	0		$uds$	<b>1</b>	$1/2$	<b>8</b>	0

# Flavor Singlet Pentaquark $P_{cs}$

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$(I, J^P)$	octet type (8)						singlet type (1)					
		component	color	spin	flavor	isospin		component	color	spin	flavor	isospin
$(0, 1/2^-)$	$P_{cs8}$	$c\bar{c}$	<b>8</b>	0	—	—	$P_{cs1}$		<b>1</b>	0	—	—
		$uds$	<b>8</b>	$1/2$	<b>1</b>	0			$1/2$	<b>8</b>	0	
$(0, 1/2^-)$	$P'_{cs8}$	$c\bar{c}$	<b>8</b>	1	—	—	$P'_{cs1}$	$c\bar{c}$		—	—	—
		$uds$	<b>8</b>	$1/2$	<b>1</b>	0		$uds$		—	—	—
$(0, 3/2^-)$	$P^*_{cs8}$	$c\bar{c}$	<b>8</b>	1	—	—	$P^*_{cs1}$	$c\bar{c}$	<b>1</b>	1	—	—
		$uds$	<b>8</b>	$1/2$	<b>1</b>	0		$uds$	<b>1</b>	$1/2$	<b>8</b>	0

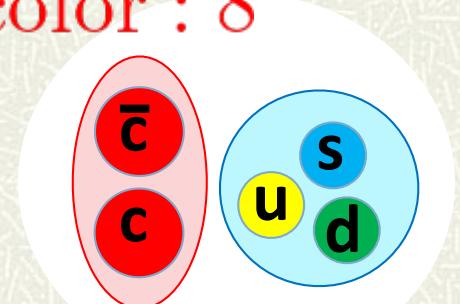
No bound state

# Flavor Singlet Pentaquark $P_{cs}$

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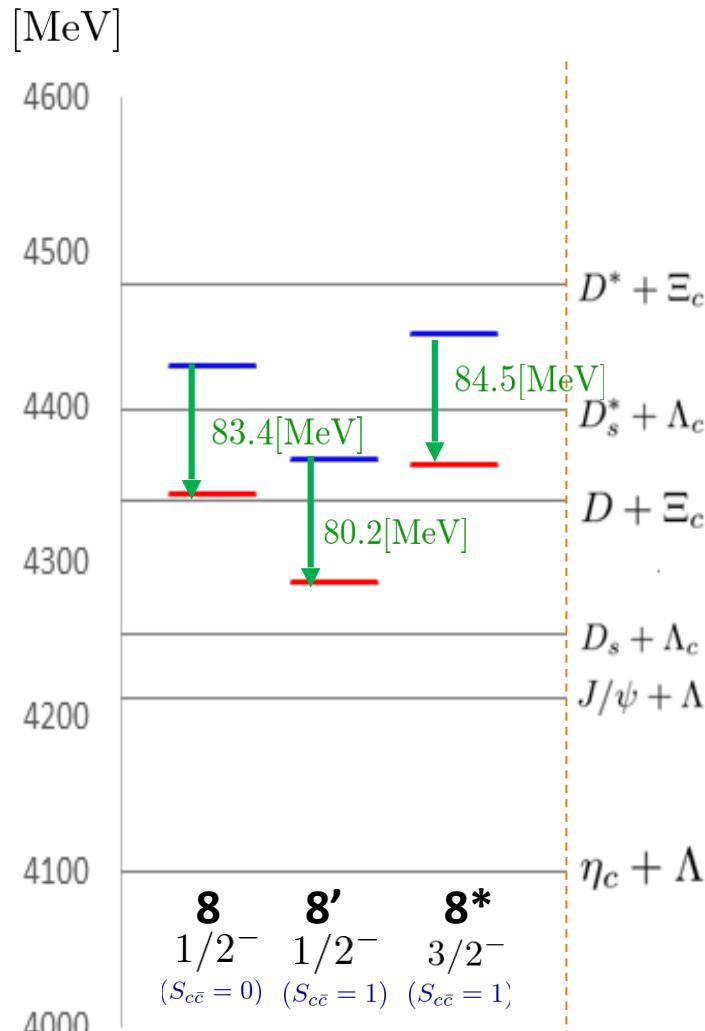
$(I, J^P)$		octet type (8)				
		component	color	spin	flavor	isospin
$(0, 1/2^-)$	8	$c\bar{c}$	8	0	—	—
		$uds$	8	1/2	1	0
$(0, 1/2^-)$	8'	$c\bar{c}$	8	1	—	—
		$uds$	8	1/2	1	0
$(0, 3/2^-)$	8*	$c\bar{c}$	8	1	—	—
		$uds$	8	1/2	1	0

color : 8



color : 8  
flavor : 1

# *Energy Spectrum (Preliminary)*



*Y. Irie, S. Yasui, M. Oka*  
*arXiv:1707.04544*

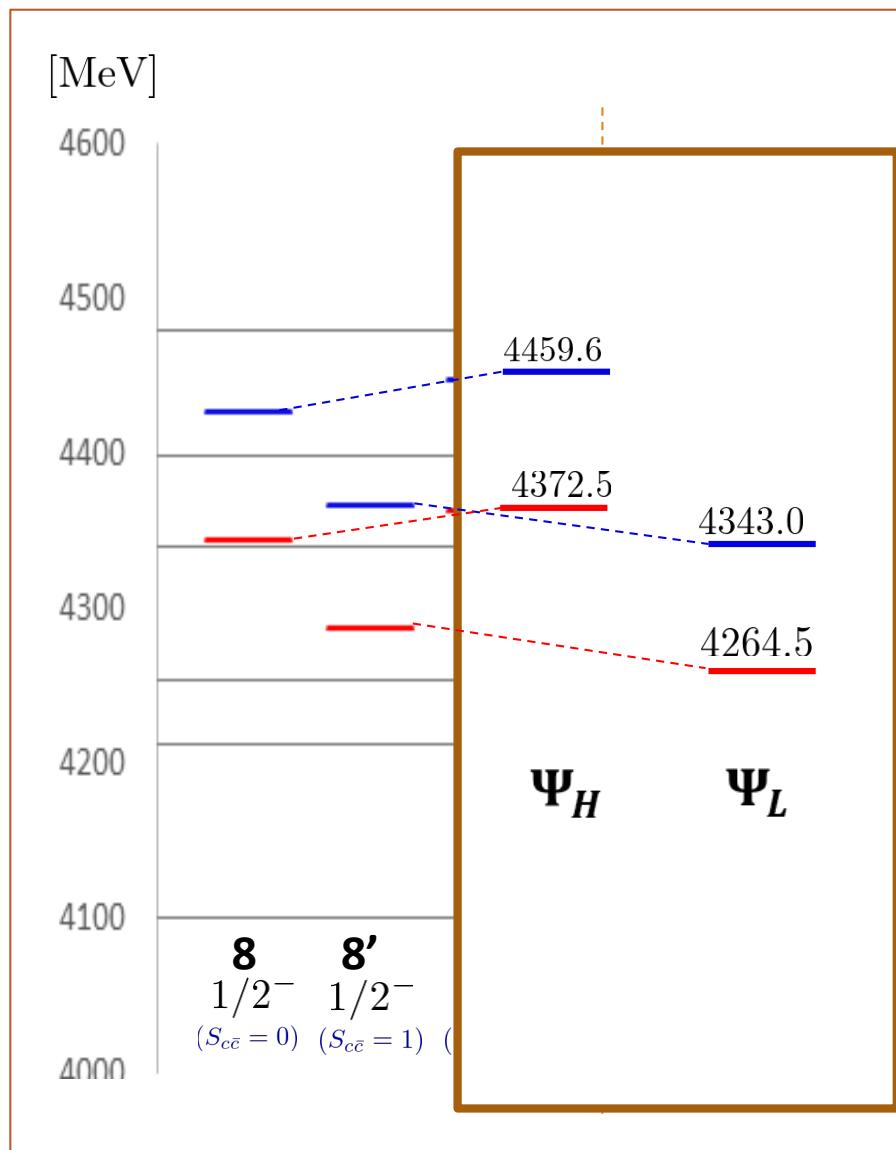
A variational method is used for a qualitative evaluation of the spectrum.

The lowest energy state is  $8'$ .

$$(1/2^-, S_{c\bar{c}} = 1)$$

The instanton induced interaction lowers the masses by about 80 MeV.

# *Energy Spectrum (Preliminary)*



*Y. Irie, S. Yasui, M. Oka*  
*arXiv:1707.04544*

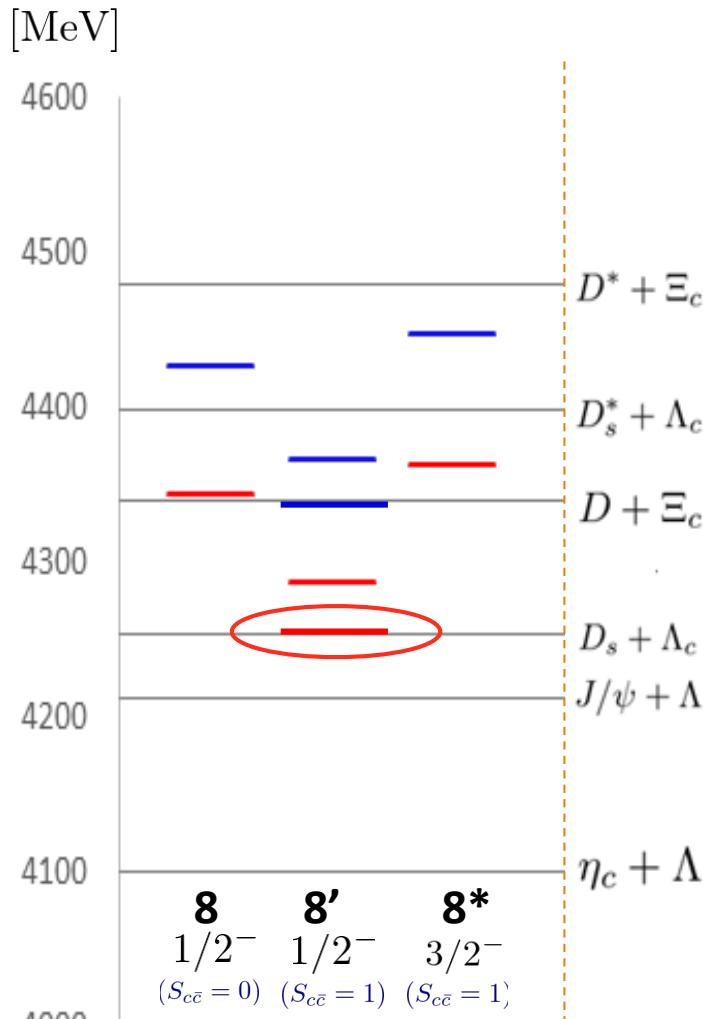
A variational method is used for a qualitative evaluation of the spectrum.

The lowest energy state is  $8'$ .  
( $1/2^-$ ,  $S_{c\bar{c}} = 1$ )

The instanton induced interaction lowers the masses by about 80 MeV.

Two  $1/2^-$  states mix by the CMI.

# Decays



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$\eta_c(J/\psi) + \Lambda$

Flavor SU(3) : suppressed

$D_s + \Lambda_c$

(barely) allowed

$8^*$  : D-wave decay

$D_s^* + \Lambda_c$

$8^*$  : S-wave decay

With Instantons

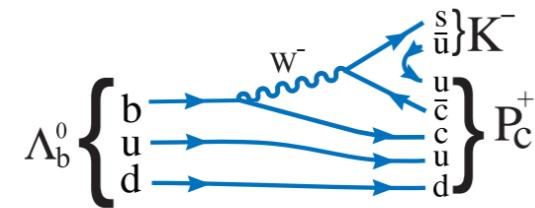
→ forbidden

# Production

$Pc(4380), Pc(4450)$

$$\Lambda_b^0(bud) \rightarrow P_c^+ + K^-$$

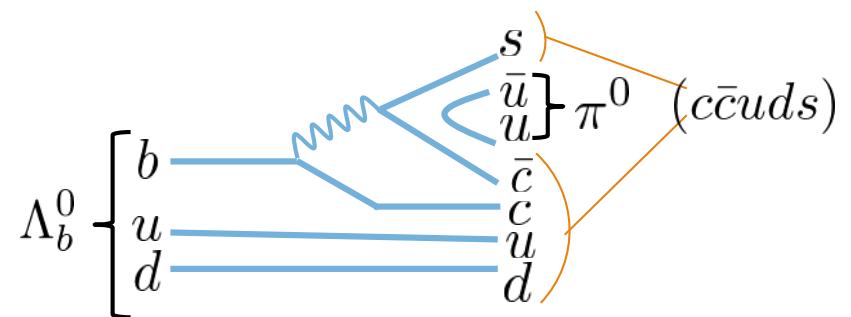
$$P_c^+ \rightarrow J/\Psi + p$$



R. Aaij *et al.* (LHCb Collaboration)  
Phys. Rev. Lett. 115, 072001 – Published 12 August 2015

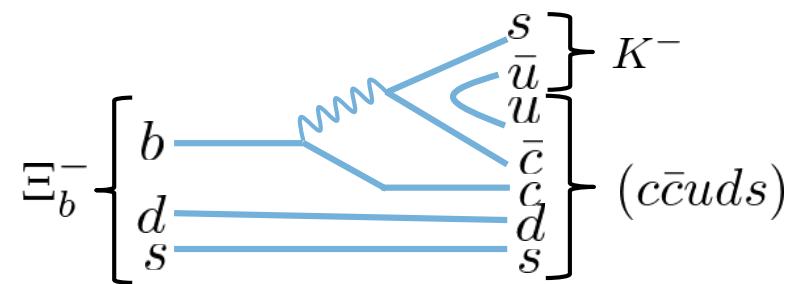
$$\Lambda_b^0(bud) \rightarrow (c\bar{c}uds) + \underline{\pi^0}$$

no charge



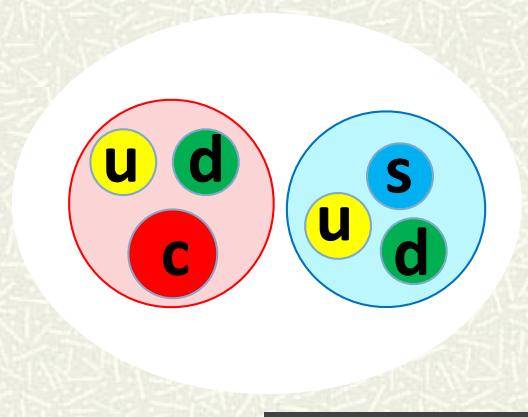
$$\Xi_b^- (bds) \rightarrow (c\bar{c}uds) + \underline{K^-}$$

minus charge!



# Conclusion

- # Exotic (MQ) hadrons can be keys for understanding the mechanisms of  
**CONFINEMENT – novel color configurations**  
**HADRON COUPLINGS/ INTERACTIONS**
- # Pentaquarks  
Hidden-charm pentaquarks  
 $P_c = c\bar{c}qqq$  (flavor octet),  
 $P_{cs} = c\bar{c}uds$  (flavor singlet)
- # Other possibilities  
 $P_{c\bar{s}} = c\bar{s}qqq$  (Diakonov)
- # Hexaquarks (aka Dibaryon)  
 $H = q^6 = (uuddss)$  (flavor singlet)  
 $H_c = (cuudds) = (cud uds)$  (flavor 3bar)

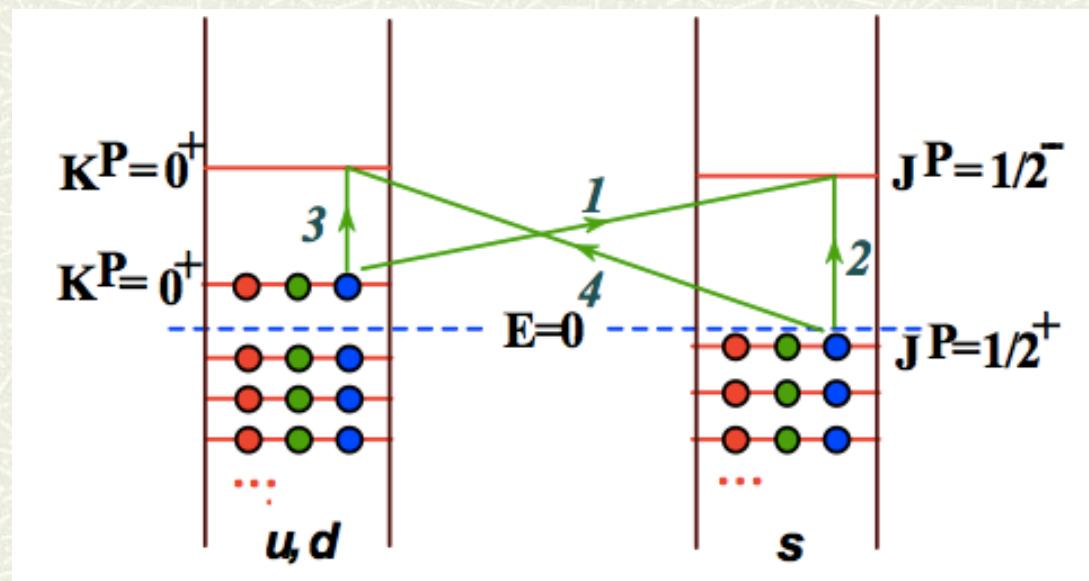


D. Diakonov, “Prediction of new charmed and bottomed exotic pentaquarks”, ArXiv: 1003.2157 and PTP S186 (2010) 99.

Chiral mean field at large  $N_c$  QCD indicates the conserved grand spin,  $K=I+J$ .

The lowest-lying baryon (hedgehog state) consists of three quarks (**R,B,G**) in the  $K=0$  orbit.

Three “exotic” baryons can be assigned to the excited states.



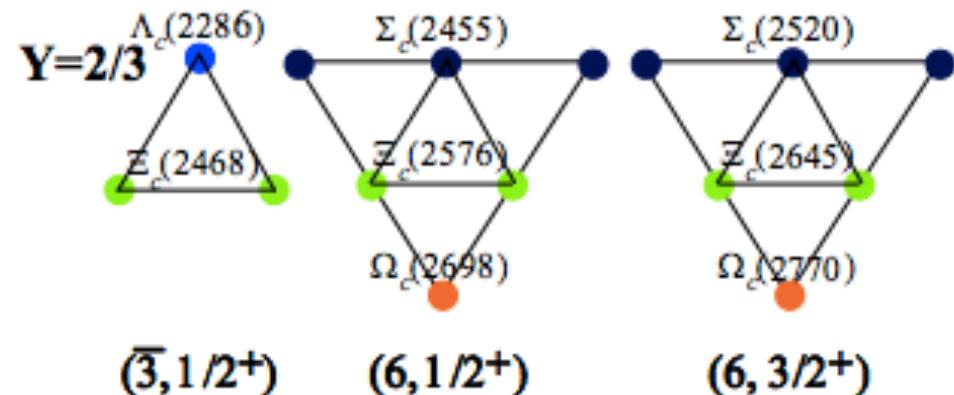
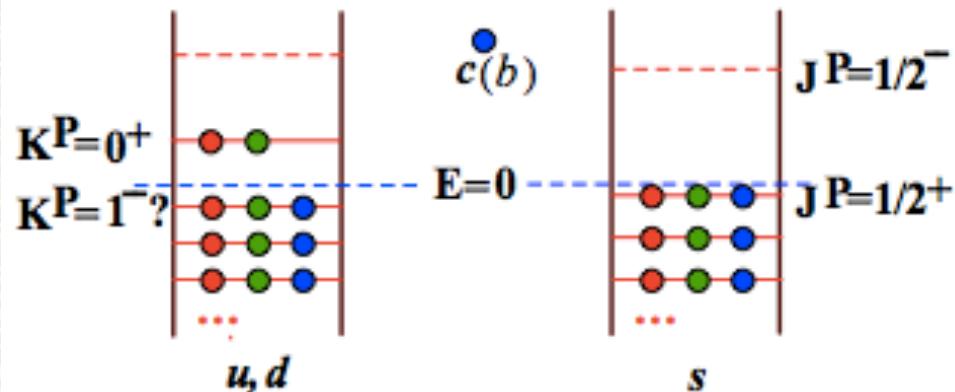
1:  $N + (su^{\bar{b}a}) = \Lambda(1405)$

2:  $N + (ss^{\bar{b}a}) = N^*(1535)$

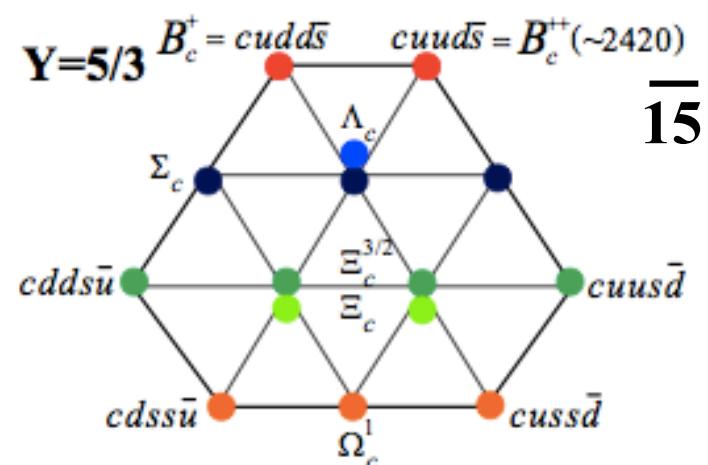
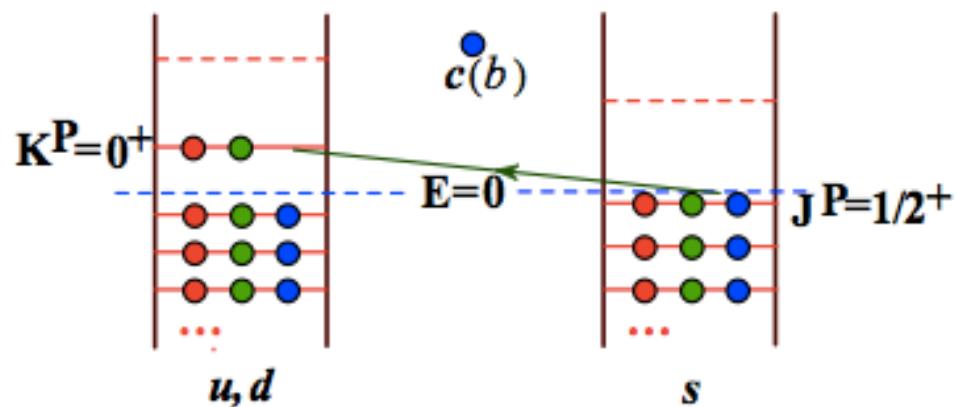
3:  $N^*(1440)$

4:  $N + (us^{\bar{b}a}) \rightarrow \Theta^+(1570)$

## Charmed baryons: ground states

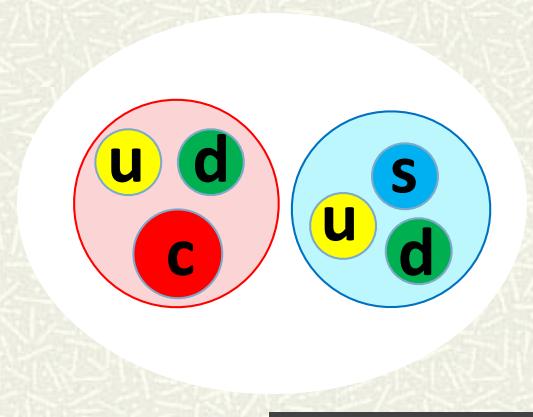


Charmed pentaquark  $cqq\bar{q}s^{\bar{q}}$  @  $m(\Lambda_c)+130 \text{ MeV} \sim 2420 \text{ MeV}$   
below  $\Lambda_c K$  threshold, Cabibbo allowed weak decay to  $\Lambda/\Sigma+K$

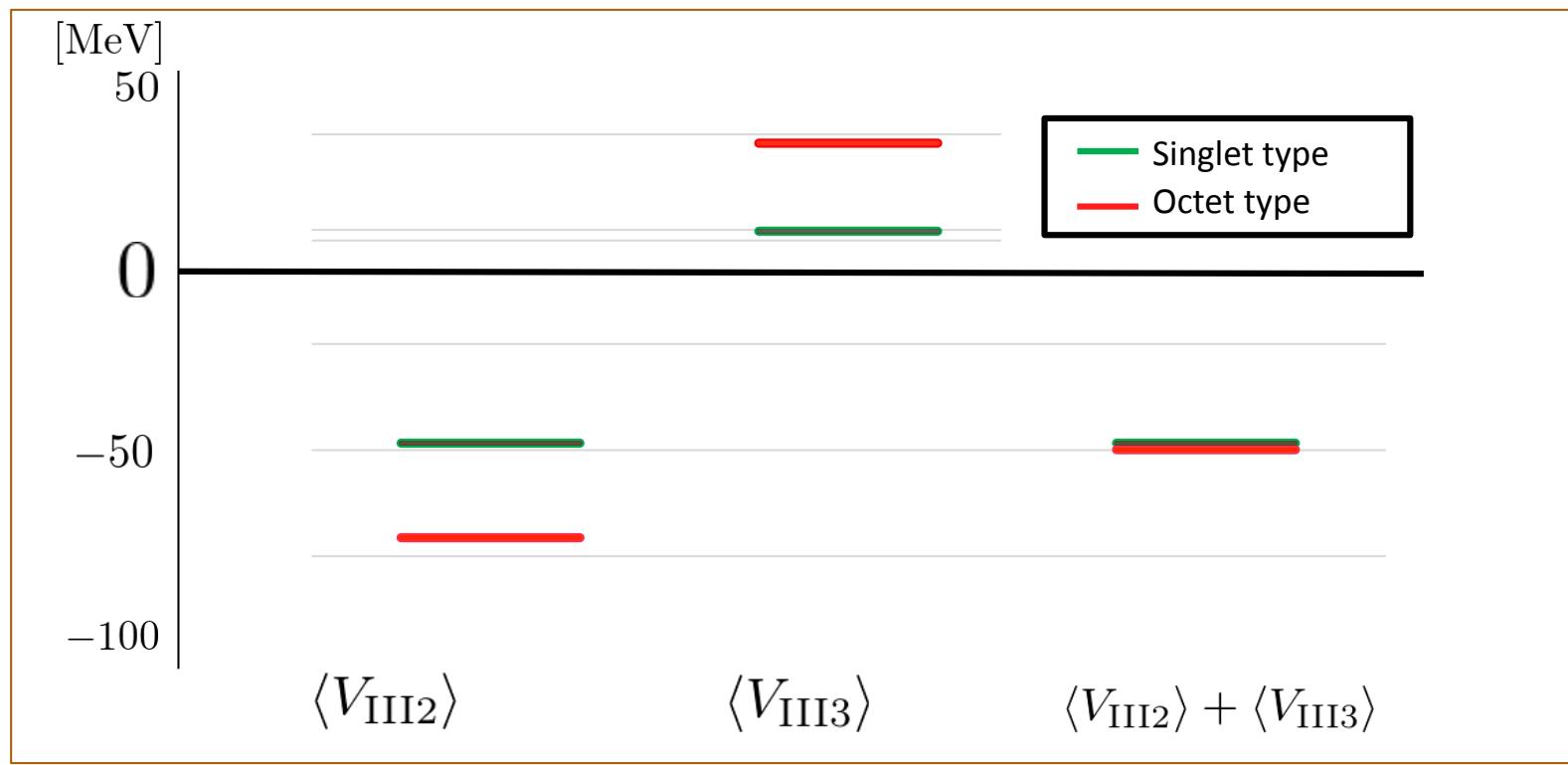


# Conclusion

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# *Contributions of III*



**Two-Body III**      **Three-Body III**      **Total**

more attractive  
in color octet      repulsive in color  
octet      Net effects are  
almost the same

# Contributions of CMI

$$V_{\text{CMI}} = -\frac{\alpha_s}{4} \sum_{i < j} \frac{\pi}{m_i m_j} (\lambda_i \cdot \lambda_j) [1 + \frac{2}{3} \sigma_i \cdot \sigma_j] \delta(r_{ij})$$

The CMI between the HQ and LQ modifies the masses.

