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"Modern Problems in Nuclear and Elementary Particle Physics"
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Transport phenomena with holography

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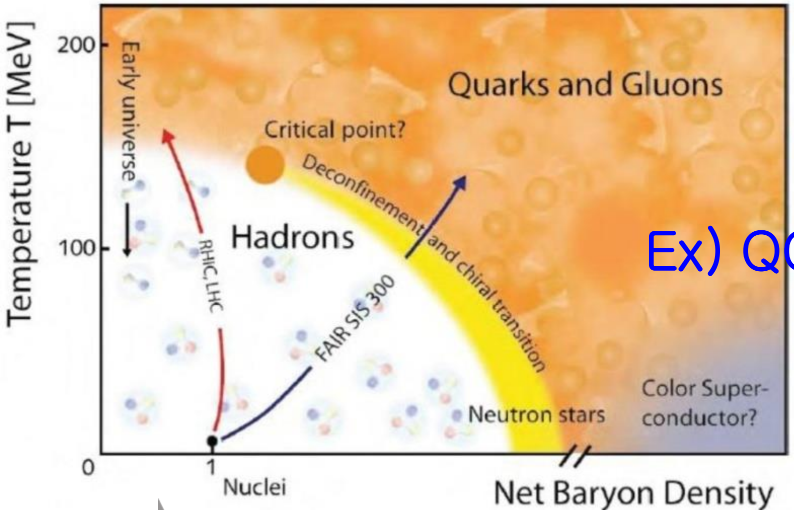
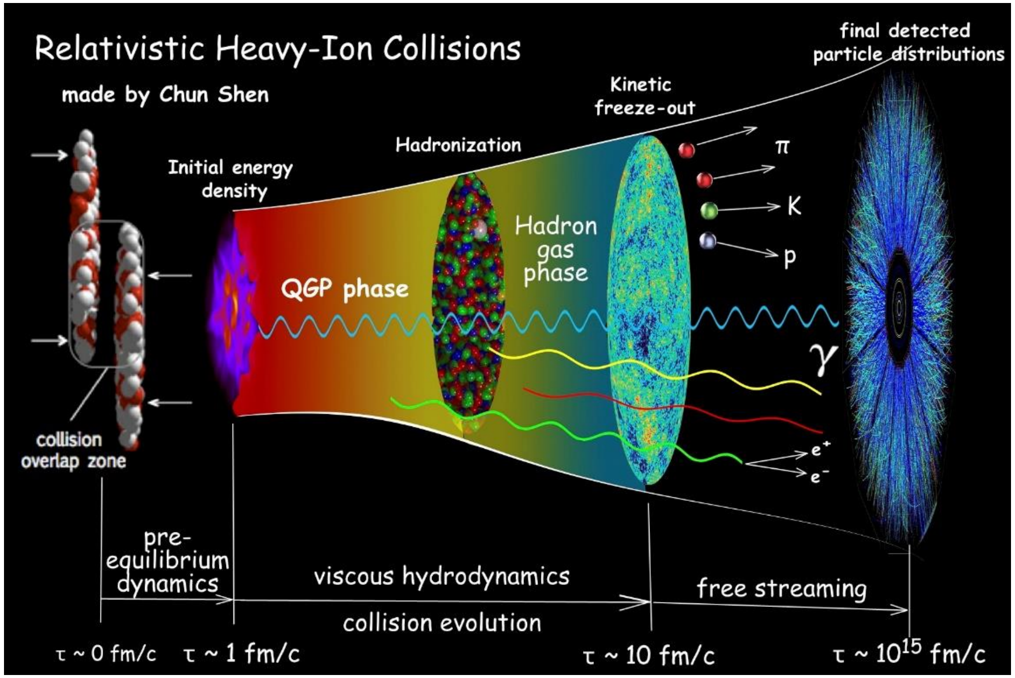
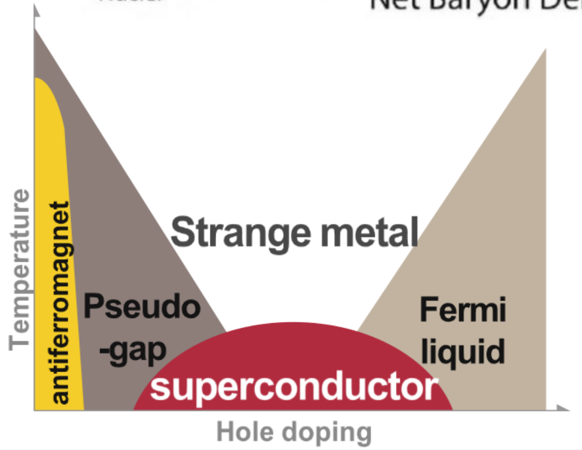
I. Introduction ：Motivation & Basics

How to understand the nonperturbative physics of the strongly interacting systems ?



Ex) QCD





Ex) Strongly correlated condensed matter systems

Holography Principle (AdS/CFT correspondence) :
3+1 dim. Quantum Field Theory ⇔ 4+1 Gravity Theory

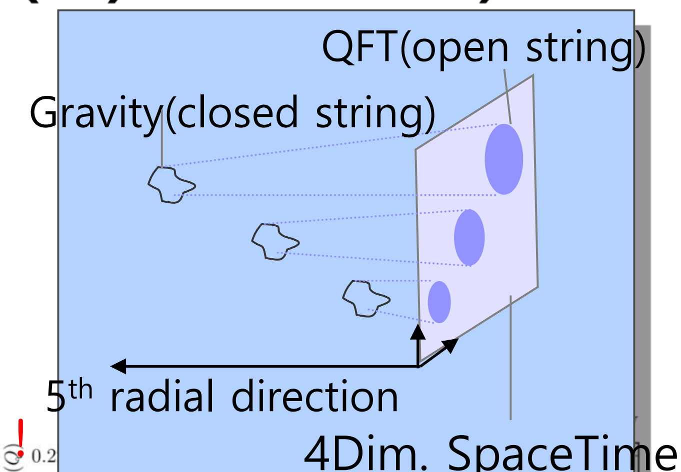
- New methodology for strongly interacting systems ?

Contents

II. Holography Principle (AdS/CFT Correspondence)

"2nd revolution of the string theory (1994)

(Strongly Interacting) Quantum Field Theory
in a given space time dimension (Ex):3+1=4 dim)
can be equivalently described by
the (classical) gravity theory
in one higher dimensional spacetime (Ex): 4+1=5dim).



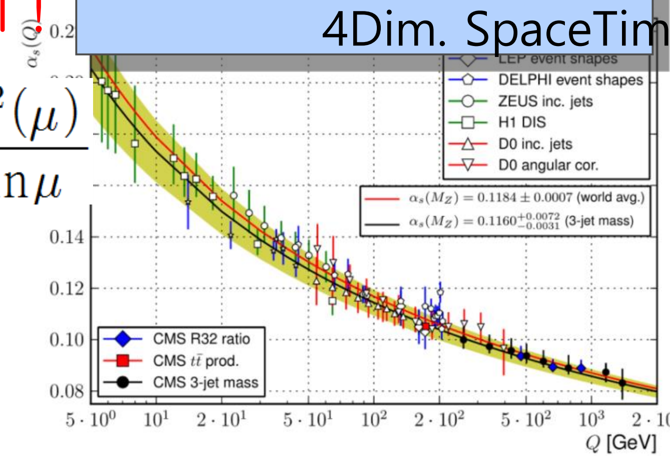
Question : 4 = 5 ?

3+1 dim. QFT (large Nc)
⇔
4+1 dim. Effective Gravity description

Naïve Answer : Coupling constants are running in QFT !

Energy scale in QFT corresponds to
the parameter in extra "dimension"
or radial direction in AdS5 space

$$\beta(g^2) = \frac{dg^2(\mu)}{d\ln\mu}$$



$$g_s = e^{\phi(r)} = g^2_{YM}(\mu)$$

New Paradigm for the Strongly Interacting Quantum System

‘Size’ L of the 5dim is proportional to the coupling constant λ of the 4 dim.

4Dim QFT	Perturbative : Easy	**Nonperturbative : Hard**
Coupling constant λ	$\lambda \ll 1$	$\lambda \gg 1$
Size of the paramenter L	$L \ll 1$	$L \gg 1$
5Dim parameter	Quantum Gravity : Hard	**Classical Gravity: “Easy”**

Strongly Interacting Quantum System $(\lambda >> 1)$

$\Leftrightarrow$

Classical Gravity $(L >> 1)$

New Methodolgoy : can use the 5 dim. classical gravity description for the 4dim. strongly interacting system.

AdS/CFT Dictionary

Witten 98; Gubser, Klebanov, Polyakov 98

Parameters (g_s , R) $\boxed{4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N}$ (g_{YM}^2 , N)

Partition function of bulk gravity theory (semi-classial)

$$Z_{str}[\phi_0(x)] = \int_{\phi_0} \mathcal{D}\phi \exp(-S[\phi, g_{\mu\nu}])$$

$$= e^{-S(\phi_{cl}[\phi_0])}$$

$$\phi(t, \mathbf{x};\, u = \infty) = u^{\Delta-4}\phi_0(t, \mathbf{x})$$

ϕ_0 bdry value of the bulk field

Generating functional of bdry QFT for operator $\mathcal{O}$

$$Z[\phi_0(x)] = \left\langle \exp \int_{boundary} d^d x \phi_0 \mathcal{O} \right\rangle$$

$$= \int \mathcal{D}[\Phi] \exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\}$$

ϕ_0 (t,x) : source of the bdry $\mathcal{O}$

$$\boxed{Z_{str}[\phi_0(x)] = Z[\phi_0(x)]}$$

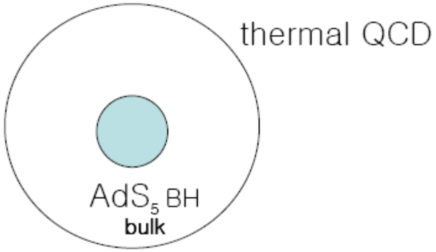
- ϕ : scalar → $S = \int d^4x du \sqrt{-g}\left(g^{ab}\partial_a\phi\partial_b\phi - m^2\phi^2\right)$ $\phi(u) \sim u^{4-\Delta}\phi_0 + u^{\Delta}\langle\mathcal{O}\rangle$
- Correlation functions by $\frac{\delta^n Z_{\rm string}}{\delta\phi_0(t_1,\mathbf{x}_1)\cdots\delta\phi_0(t_n,\mathbf{x}_n)} = \left\langle T\,\mathcal{O}(t_1,\mathbf{x}_1)\cdots\mathcal{O}(t_n,\mathbf{x}_n)\right\rangle_{\rm field\ theory}$
- 5D bulk field Φ(t, x, u) ←→ Operator $\mathcal{O}$ (t, x)
  w/ 5D mass $E(\lambda, J_1, J_2, \cdots)$ w/ Operator dimesion $\Delta(\lambda, J_1, J_2, \cdots)$
- 5D gauge symmetry ←→ Current (global symmetry)
- Radial coord. r (= 1/u) in the bulk is proportional to the energy scale E of QFT

- Temperature ↔
  Black hole geometry
  $$T = T_{Hawking}$$

E. Witten (1998)

$$ds_5^2 = \frac{1}{z^2}\left(f^2(z)dt^2 - (dx^i)^2 - \frac{1}{f^2(z)}dz^2\right),$$

$$f^2(z) = 1 - (\frac{z}{z_T})^4 \qquad T = \frac{1}{\pi z_T}$$



- Flavor degrees of Freedom
  by adding probe brane

$$y(\rho) = M_q + \frac{<\bar{\psi}\psi>}{\rho^2} + \cdots \qquad (\rho >> 1)$$

- Chemical potential or density
  by turning on U(1) gauge field on the probe brane

$$A_\mu \leftrightarrow < J^\mu > = \bar{\psi}\gamma^\mu\psi$$

$$A_t = \mu + \frac{Q}{\rho^2} + \cdots \qquad (\rho >> 1)$$

III. Holographic QCD & CMT

Witten '98

Goal : Using the 5 dim. dual gravity, study 4dim. strongly interacting QCD & CMT such as spectra & Phases, etc.

Needs the dual geometry !.

Ex) Confinement –Deconfinement

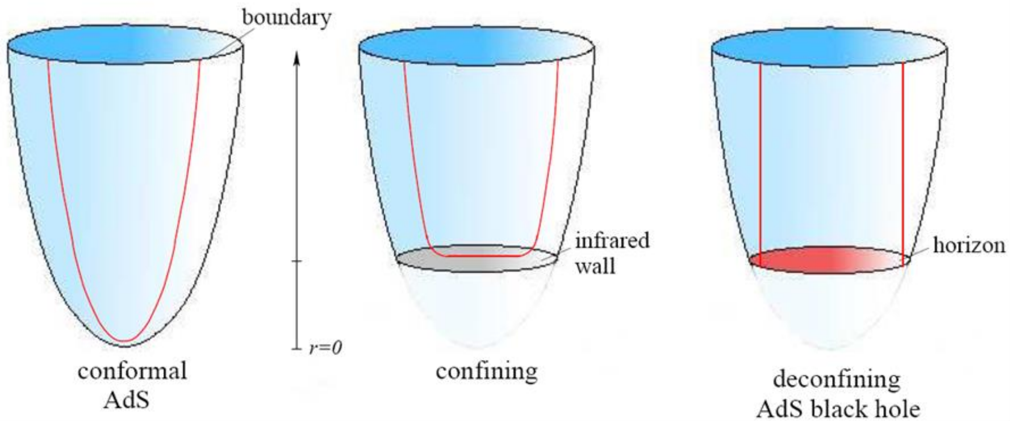
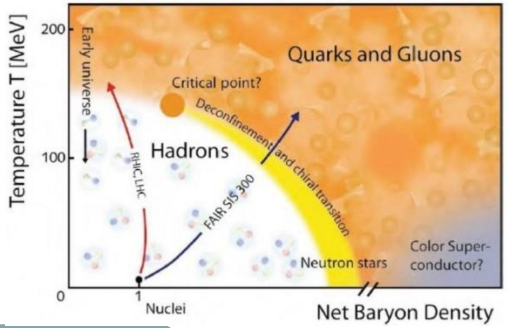


Figure from Erdmenger et.al, EPJA (2008)

Hawking–Page phase transition

=Transition of bulk geometry



The geometry is described by the following action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{g}\,(-\mathcal{R}+2\Lambda)$$

$\Lambda = -\frac{6}{R^2}$
cosmological constant

The geometry with smaller action is the stable one for given T.

$$\Delta S \;=\; \lim_{\epsilon\to 0}(S_{AdSBH}-S_{tAdS}) \;=\; \frac{\pi z_h R^3}{\kappa^2}\left(\frac{1}{z_{IR}^4}-\frac{1}{2z_h^4}\right)$$

> 0 for T < Tc
< 0 for T>Tc

[Herzog , Phys.Rev.Lett.98:091601,2007]

Low T QCD Phase transition High T

	Confinement	Deconfinement
QCD (4Dim)	Hadron	Quark-Gluon
Gravity (5Dim)	Thermal AdS	AdS Black Hole

Hawking–Page phase transition

Holographic QCD for finite chemical potential

Low T QCD Phase transition High T

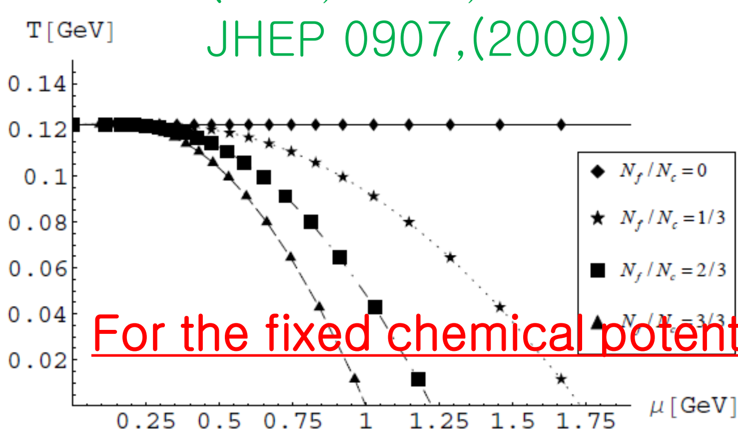
	Confinement	Deconfinement
QCD (4Dim)	Hadron	Quark-Gluon
Gravity (5Dim)	thermal & charged AdS	RNAdS Black Hole

Hawking−Page transition

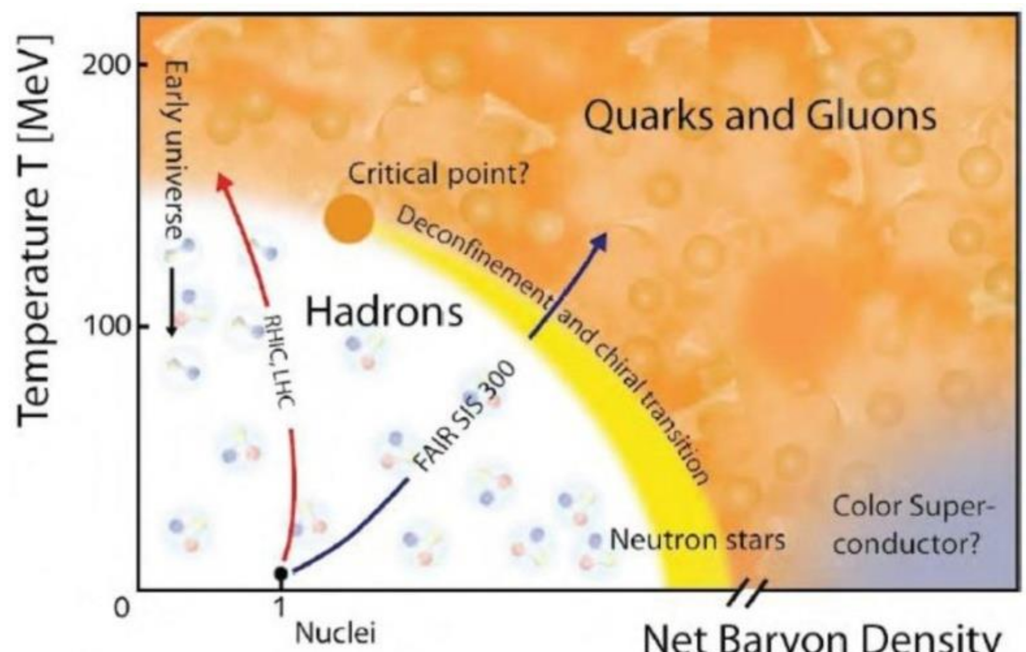
⇓ (q = 0)

Gravity (5Dim)	Thermal AdS	AdS Black Hole

(BHL, Park ,Sin
JHEP 0907,(2009))



For the fixed chemical potential



Light meson spectra in the hadronic phase

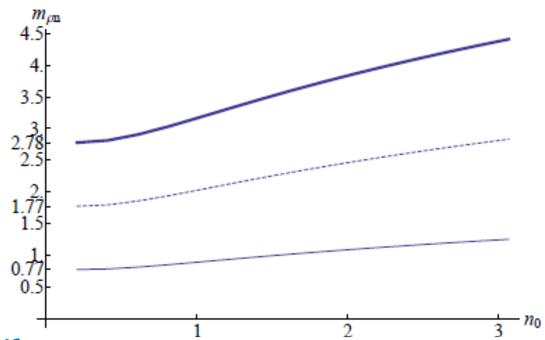
Jo, BHL, Park,Sin
JHEP 2010, arXiv:0909.3914

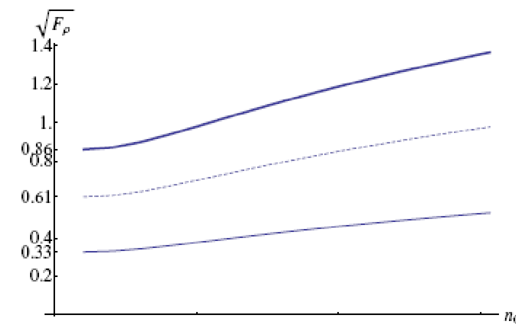
Turn on the fluctuation in bulk corresponding the meson spectra in QCD

$$\Delta S = \int d^5x\sqrt{G}\ \mathrm{Tr}\left[|DX|^2 - \frac{3}{R^2}|X|^2 + \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right] \quad M_X^2 = -3/l^2$$

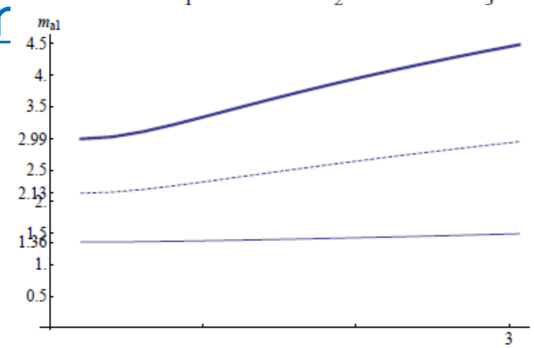
X is the dual to the quark bilinear operator $<\bar{q}\, q>$.

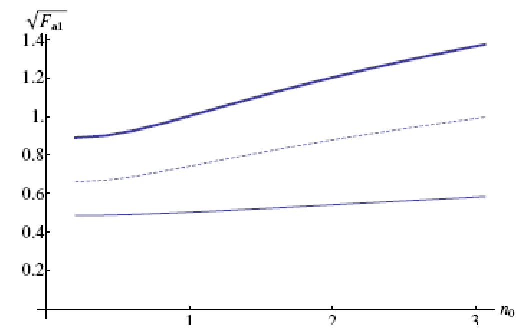
1. Vector meson



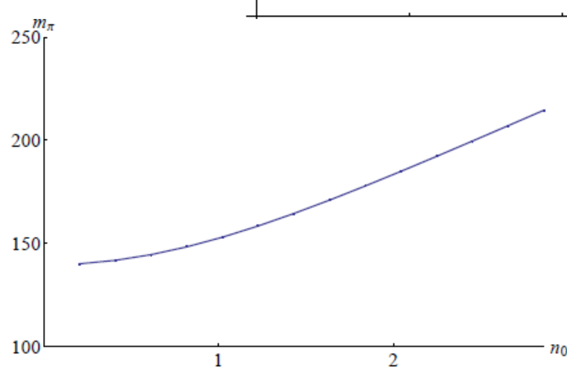


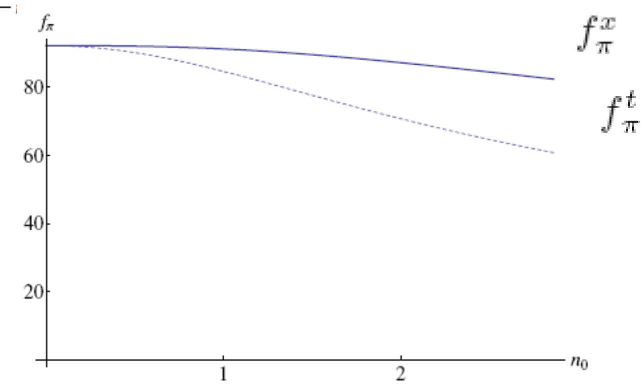
2. Axial vector mesor





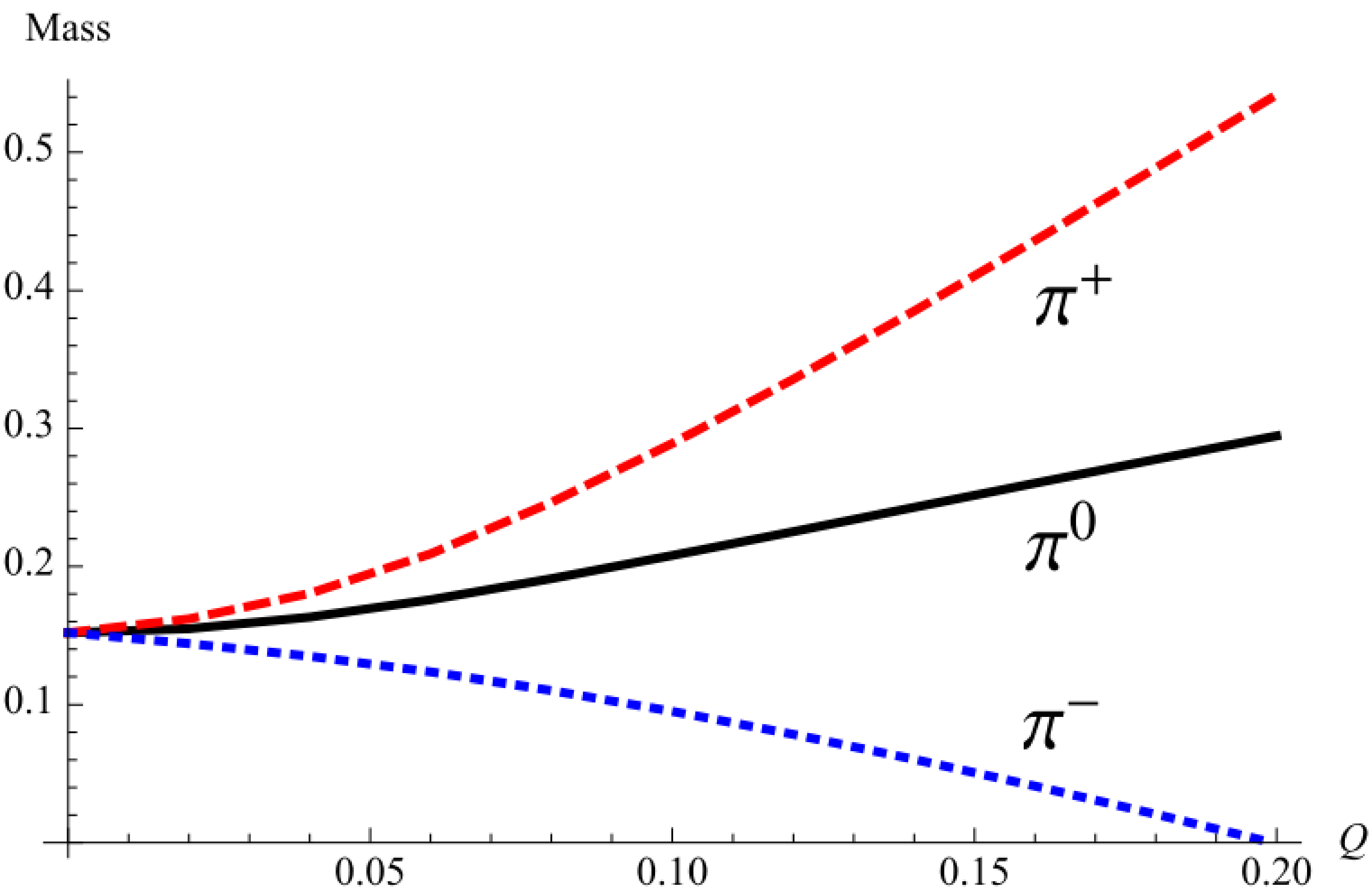
3. pion





Nuclear medium (with isospin 1/2 : protons & neutrons)

B-HL, S. Mamedov, S. Nam, C. Park, JHEP 1308 ,045 (2013) arXiv:1305.7281v2



The π meson mass splitting for $\alpha = 1/2$,
(mq= 2.383MeV, $\sigma = (304\text{MeV})^3$ Nc =3 and R=1)

Transport Properties

Linear response theory (Kubo Formula)

$$\mathfrak{G}^R_{\mathcal{O}_A\mathcal{O}_B} = \frac{\delta\langle\mathcal{O}_\mathcal{A}\rangle}{\delta\Phi_B^{(0)}},$$

$$\sigma(\omega) = \frac{\mathfrak{G}^R_{J_iJ_i}(\omega)}{i\omega}, \quad \alpha(\omega)T = \frac{\mathfrak{G}^R_{Q_iJ_i}(\omega)}{i\omega}, \quad \bar{\kappa}(\omega) = \frac{\mathfrak{G}^R_{Q_iQ_i}(\omega)}{i\omega}.$$

Note : $Z[\phi_0(x)] = \left\langle \exp\int_{boundary} d^dx\phi_0\mathcal{O} \right\rangle = \int \mathcal{D}[\Phi]\exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\}$

Ex) $\langle T^{\mu\nu}(x)\rangle = \left.\frac{\delta W[g]}{\delta g_{\mu\nu}(x)}\right|_{g_{\mu\nu}=\eta_{\mu\nu}}, \quad \langle T^{\mu\nu}(x)T^{\lambda\rho}(y)\rangle = \left.\frac{\delta^2 W[g]}{\delta g_{\mu\nu}(x)\,\delta g_{\lambda\rho}(y)}\right|_{g_{\mu\nu}=\eta_{\mu\nu}}$

- Need the (retarded) two-point functions for the transport properties.
- Calculate the two-point function from the bulk gravity side (holography principle) to get the transport properties of the boundary quantum physics

Ex) the shear & bulk viscosities are defined as $\frac{\partial s}{\partial t} = \frac{\eta}{T}\left[\partial_i v_j + \partial_j v_i - \frac{2}{3}(\partial\cdot v)\delta_{ij}\right]^2 + \frac{\zeta}{T}(\partial\cdot v)^2$

Bulk viscosity → $\zeta = -\frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}ImG_R(\omega,0),$

Relevant Operator

Source	Operator
Background Metric	$T_{\mu\nu}$
Background EM	J^{μ}
Dynamical field ϕ	$\mathcal{O}$

Ex:

$$e^{iS[\Phi_{(0)}+\delta\Phi_{(0)}]} = \langle \exp\left(i\int d^dx\sqrt{-g_{(0)}}\delta\Phi_{(0)}\mathcal{O}\right)\rangle_{F.T.}$$

Ohm's Laws

$$\begin{pmatrix} J_i \\ T_{ti} \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \alpha T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E_i \\ -(\nabla_i T)/T \end{pmatrix},$$

where

$$E_j = F_{jt} = -\partial_t(\delta A_j^{(0)} e^{-i\omega t}) = i\omega\delta A_j^{(0)}$$

Diffeomorphism transformation

$$\delta g^{(0)}_{ab} = \partial_a \xi_b + \partial_b \xi_a \,, \qquad \delta A^{(0)}_a = A^{(0)}_b \partial_a \xi^b + \xi^b \partial_b A^{(0)}_a \,,\ \xi_t = \frac{i x^j \nabla_j T}{\omega T} \,,\ \xi_{x^j} = 0$$

$$\delta g^{(0)}_{tt} = 0 \,, \quad \delta g^{(0)}_{tj} = \frac{i \nabla_j T}{\omega T} \,, \quad \delta A^{(0)}_j = A^{(0)}_t \partial_j (-\xi_t) = \frac{-i\mu \nabla_j T}{\omega T} \,,$$

$$\begin{aligned} \delta S &= \int d^d x \sqrt{-g^{(0)}} \left[T^{tj} \delta g^{(0)}_{tj} + J^j \left(\delta A^{(0)}_j - \frac{i\mu \nabla_j T}{\omega T} \right) \right] , \\ &= \int d^d x \sqrt{-g^{(0)}} \left[J^j \frac{E_j}{i\omega} + \left(T^{tj} - \mu J^j \right) \frac{i \nabla_j T}{T\omega} \right] . \end{aligned}$$

Sources, Operator for metric and vector field fluctuation

$$\begin{pmatrix} J_j \\ Q_j \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \alpha T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} i\omega (\delta A^{(0)}_j + \mu \delta g^{(0)}_{tj}) \\ i\omega \delta g^{(0)}_{tj} \end{pmatrix} , \ Q_j = T^{tj} - \mu J^j \,.$$

the Einstein dilaton gravity theory

S. Kulkarni, B-HL C. Park, R. Roychowdhury JHEP (2012)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - 2(\partial\phi)^2 - V(\phi)\right], \qquad V(\phi) = 2\Lambda e^{\eta\phi}, \qquad \Lambda < 0$$

Liouville-type scalar potential

Note : If phi = 0, the geometry is
the Anti de-Sitter (AdS) space or Schwarzschild AdS (SAdS) black hole (or brane).

Equations of motion :

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{1}{2}g_{\mu\nu}V(\phi) = 2\partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}(\partial\phi)^2,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) = \frac{1}{4}\frac{\partial V(\phi)}{\partial\phi}.$$

Ansatz (Note : generically hyperscaling violating solutions)

$$ds^2 = -a(r)^2dt^2 + \frac{dr^2}{a(r)^2} + b(r)^2(dx^2 + dy^2), \quad \text{with} \quad a(r) = a_0 r^{a_1}, \quad b(r) = b_0 r^{b_1},$$

$$\phi(r) = -k_0 \log r$$

Note :

1) Can choose $b_0 = 1$, by rescaling the x & y coordinates.
2) The general solution for phi includes constant term, which can be put to zero by rescaling the cosmological constant.
3) Isometry : ISO(2,1). 2+1 dim. Poincare symmetry
→ relativistic nonconformal matter

The non-black hole solution

$$a_0 = \frac{(4+\eta^2)\sqrt{-\Lambda}}{2\sqrt{12-\eta^2}}, \qquad k_0 = \frac{2\eta}{4+\eta^2}, \qquad a_1 = b_1 = \frac{4}{4+\eta^2},$$

Note : For eta = 0 and lambda = −3, the metric reduces to the usual AdS metric, with the isometry group is enhanced to SO(2,3) the conformal group of the dual gauge theory.

By introducing new coordinates, the metric solution can be rewritten in the form of an warped geometry

$$ds^2 = \tilde{r}^{2a_1/(1-a_1)}\left[-d\tilde{t}^2 + d\tilde{x}^2 + d\tilde{y}^2\right] + d\tilde{r}^2.$$

The black hole (brane solution),

$$ds^2 = -a(r)^2 f(r) dt^2 + \frac{dr^2}{a(r)^2 f(r)} + b(r)^2 (dx^2 + dy^2),$$

with the following black hole factor $\quad f(r) = 1 - \delta\ m\ r^{-c},$

$$\delta = \frac{8\pi}{V_2(-\Lambda)}\frac{12-\eta^2}{4+\eta^2}. \qquad V_2 = \int_0^L dx dy$$

m : the black hole mass

Then, the solution is described by the same a(r) and b(r), along with $\quad c = \dfrac{12-\eta^2}{4+\eta^2}.$

Note :
In the asymptotic region r → ∞, the metric agrees with the previous non-black hole one.

the horizon r_h satisfying $f(r_h) = 0$

$$m = \frac{r_h^{(12-\eta^2)/(4+\eta^2)}}{\delta}.$$

The Hawking temperature T_H defined by the surface gravity at the horizon, is given by

$$T_{\rm H} \equiv \frac{1}{4\pi}\frac{\partial}{\partial r}\{a(r)^2 f(r) \quad = \frac{(-\Lambda)(4+\eta^2)}{16\pi}\, r_h^{(4-\eta^2)/(4+\eta^2)}.$$

The Bekenstein–Hawking entropy S_{BH} is

$$S_{\rm BH} \equiv \frac{A(r_h)}{4} = \frac{V_2}{4}\, r_h^{8/(4+\eta^2)},$$ $A(r_h)$:the area at the black hole horizon.

The black hole thermodynamics

$$0 = dE - T_{\rm H} dS_{\rm BH}.$$

the energy is is obtained by integrating :

$$E = \frac{(-\Lambda)V_2}{8\pi}\frac{4+\eta^2}{12-\eta^2}\, r_h^{(12-\eta^2)/(4+\eta^2)},$$

and the free energy of this black hole is given by

$$F \equiv E - TS = -\frac{(-\Lambda)V_2}{64\pi}\frac{16-\eta^4}{12-\eta^2}\, r_h^{(12-\eta^2)/(4+\eta^2)}.$$

the pressure of the system is given by

$$P = -\partial F/\partial V_{2;}$$

the eq of state parameter w can be read from the relation $P = wE/V_2$

$$w = \frac{1}{8}(4 - \eta^2).$$

$\boxed{\text{Note : } 0 < \eta^2 < 12 \quad \leftrightarrow \quad -1 < w < 1/2}$

The gauge theory interpretation of the BH thermodynamic quantities:

For eta = 0 (or w = $\frac{1}{2}$),
the gravity is given by the AdS black hole and corresponds to the gauge theory with the conformal matter whose energy-momentum tensor is traceless.
For 0 <eta-squared < 12, (-1< w < $\frac{1}{2}$),
the gauge theory is with the non-conformal matter.

The thermodynamic stability of the dual gauge theory

The specific heat of the black hole

$$
\begin{aligned}
C_{uh} &\equiv \frac{dE}{dT_{\rm H}} \\
&= \frac{(-\Lambda)V_2}{8\pi}\frac{4+\eta^2}{4-\eta^2}\left(\frac{16\pi}{(-\Lambda)(4+\eta^2)}\right)^{(12-\eta^2)/(4-\eta^2)} T_{\rm H}^{8/(4-\eta^2)}.
\end{aligned}
$$

Note : The specific heat of the SAdS black hole (obtained by setting eta = 0) is always positive, which implies that the SAdS black hole is thermodynamically stable.

There exists another critical point called the crossover point at eta-squared = 4.

For eta-squared < 4, the heat capacity is positive.
For eta-squared = 4, the heat capacity is singular.
For 4 < eta-squared < 12, it is negative.

Note :
the black hole is stable only for 0 ≤ eta-squared < 4,

For 0 < w <1/2 the non-conformal matter provides the thermodynamically stable system.
For −1 < w < 0, the dual gauge theory is thermodynamically unstable.

Properties of the non-conformal dual gauge theory

the linear response of the vector fluctuation in the hydrodynamic limit with small frequency and momentum.

In the gauge/gravity duality, the nontrivial dilaton profile can be identified with the running coupling constant of the dual gauge theory.

Vector fluctuation without the dilaton coupling

Maxwell field action as a fluctuation over the background geometry

$$S_{\rm M} = -\frac{1}{4g_4^2}\int d^4x\sqrt{-g}F^{\mu\nu}F_{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

g is a constant coupling of the bulk gauge field.

In the Ar = 0 gauge, we take Ai (i = t, x, y) in the Fourier space as

$$A_i(t,\mathbf{x},r) = \int \frac{d\omega d^2q}{(2\pi)^3} e^{-i(\omega t - \mathbf{q}\cdot\mathbf{x})} A_i(\omega,\mathbf{q},r),$$

The equations of motion for the gauge fluctuations

$$\partial_\nu \left[\sqrt{-g} g^{\nu\rho} g^{\mu\sigma} F_{\sigma\rho}\right] = 0,$$

can be reduced to two parts, longitudinal and transverse.

The longitudinal part is given by the coupled equations for A_t and A_y

$$\begin{aligned}
0 &= b^2 \omega A_t' + g q A_y' \\
0 &= b^2 A_t'' + 2bb' A_t' - \frac{1}{g}(q\omega A_y + q^2 A_t) \\
0 &= g A_y'' + g' A_y' + \frac{1}{g}(\omega q A_t + \omega^2 A_y),
\end{aligned}$$

where the prime denotes the derivative with respect to the radial coordinate r and new function g(r) is introduced for later convenience

$$g(r) = a(r)^2 f(r).$$

The transverse mode governed by A_x satisfies the following equation

$$0 = A_x'' + \frac{g'}{g} A_x' + \frac{1}{g^2}\left[\omega^2 - q^2 \frac{g}{b^2}\right] A_x.$$

Charge diffusion constant and conductivity

Calculate the retarded Green's functions on the boundary:
According to the gauge/gravity duality, the on-shell gravity action corresponding to the boundary term can be regarded as a generating functional of the dual gauge theory.
Thus, the generating functional of the dual gauge theory can be described by the boundary action of the bulk gauge fluctuation

$$\boxed{S_B = \frac{1}{2g_4^2} \int dt dx dy \sqrt{-g} g^{zz}(z) g^{ij}(z)\ A_i(z) \partial_z A_j \Big|_{z=0},}$$

where i, j = t, x, y.

Since, the boundary value of the gauge fluctuation plays the role of the source A_{0i} for an operator in the dual gauge theory, the retarded Green function of the dual operator can be derived by the following relation

$$\boxed{\mathcal{G}_{ij} = \lim_{z \to 0} \frac{\delta^2 S_B(z)}{\delta A_i^0 \delta A_j^0}.}$$

Near the boundary, we get

$$A_t(z)A_t'(z) \approx A_t^0 \frac{(\tilde{\omega}\tilde{q}A_y^0 + \tilde{q}^2 A_t^0)}{\left(i\frac{\tilde{\omega}}{\tilde{\Lambda}} - \tilde{q}^2\right)} + \cdots ,$$

$$A_y(z)A_y'(z) \approx A_y^0 \frac{(\tilde{\omega}^2 A_y^0 + \tilde{q}\tilde{\omega}A_t^0)}{a_0^2\left(i\frac{\tilde{\omega}}{\tilde{\Lambda}} - \tilde{q}^2\right)} + \cdots ,$$

$$A_x(z)A_x'(z) \approx \left(\frac{\tilde{\Lambda}}{a_0}\right)^2 (A_x^0)^2 \left(i\frac{\tilde{\omega}}{\tilde{\Lambda}} - \tilde{q}^2\right) + \cdots ,$$

The boundary action reduces to

$$S_B = -\frac{T_{\rm H}}{2g_4^2}\left\{\frac{A_t^0(\tilde{q}^2 A_t^0 + \tilde{\omega}\tilde{q}A_y^0) + A_y^0(\tilde{\omega}^2 A_y^0 + \tilde{\omega}\tilde{q})A_t^0}{i\tilde{\omega} - \tilde{\Lambda}\tilde{q}^2} - (A_x^0)^2(i\tilde{\omega} - \tilde{\Lambda}\tilde{q}^2)\right\} + \cdots .$$

Thus, the retarded Green's functions of the longitudinal modes are given by

$$\mathcal{G}_{tt} = -\frac{1}{g_4^2}\left[\frac{q^2}{i\omega - \left(\frac{\tilde{\Lambda}}{T_{\rm H}}\right)q^2}\right],$$

$$\mathcal{G}_{yy} = -\frac{1}{g_4^2}\left[\frac{\omega^2}{i\omega - \left(\frac{\tilde{\Lambda}}{T_{\rm H}}\right)q^2}\right],$$

$$\mathcal{G}_{ty} = \mathcal{G}_{yt} = -\frac{1}{g_4^2}\left[\frac{\omega q}{i\omega - \left(\frac{\tilde{\Lambda}}{T_{\rm H}}\right)q^2}\right],$$

Note : the longitudinal modes have a quasi normal pole as we expected

The charge diffusion constant D of this quasi normal mode is

$$D = \frac{\tilde{\Lambda}}{T_{\rm H}} = \frac{(-\Lambda)}{16\pi T_{\rm H}} \frac{(4+\eta^2)^2}{4-\eta^2}.$$

For the AdS black brane (eta = 0 and lambda = −3), the charge diffusion constant is given by

$$D = \frac{3}{4\pi T_{\rm H}},$$

which shows the pole structure of the quasi normal mode in the dual conformal gauge theory.

Note : The quasi normal mode decays with a half-life time $t_{1/2}$ = 1/D q2 .

Note :the dispersion relation $\omega = -iDq^2$

For conformal gauge theory,
a quasi normal mode with a larger momentum will decay rapidly.
Alternatively, the quasi normal mode can sustain longer in high temperature.

On the other hand, in the non-conformal dual gauge theory,
the diffusion constant depends on the temperature as well as on the parameter eta and increases with eta.
Thus, we infer that when the system deviates from the conformality, the quasi normal mode decays more rapidly.

The Green function of the transverse mode is

$$\mathcal{G}_{xx} = \frac{1}{g_4^2}\left[i\omega - \left(\frac{\tilde{\Lambda}}{T_{\rm H}}\right) q^2\right],$$

which has no pole as we expected.

Note : the Green function of the transverse mode turned out to be the inverse of the longitudinal one up to a multiplication factor.

The real DC conductivity of this system can be easily read off

$$\sigma = \lim_{\omega\to 0} \mathrm{Re}\left(\frac{\mathcal{G}_{xx}}{i\omega}\right) = \frac{1}{g_4^2}.$$

Note that the non-conformality does not influence the DC conductivity.

Vector fluctuation without the dilaton coupling

Maxwell field fluctuations coupled to a dilaton $S_{\rm MD} = -\frac{1}{4g_4^2}\int d^4x\sqrt{-g}e^{\alpha\phi}F^{\mu\nu}F_{\mu\nu},$

The equations of motion for gauge fluctuations $0 = \partial_\nu\left[\sqrt{-g}g^{\nu\rho}g^{\mu\sigma}e^{\alpha\phi}F_{\sigma\rho}\right].$

Then the coupled equations for the longitudinal modes are rewritten as

$$\begin{aligned}
0 &= \tilde\omega A_t' + F(u)\tilde q A_y', \\
0 &= A_t'' - \frac{\tilde\Lambda_{\rm eff}^2}{H(u)}\left[\tilde q\tilde\omega A_y + \tilde q^2 A_t\right], \\
0 &= A_y'' + \frac{F'(u)}{F(u)}A_y' + \frac{\tilde\Lambda_{\rm eff}^2}{F(u)H(u)}\left[\tilde\omega\tilde q A_t + \tilde\omega^2 A_y\right],
\end{aligned}$$

the equation of the decoupled transverse mode A_x is given by

$$0 = A_x'' + \frac{F'(u)}{F(u)}A_x' + \frac{\tilde\Lambda_{\rm eff}^2}{F(u)H(u)}\left[\tilde\omega^2 - F(u)\tilde q^2\right],$$

Near the asymptotic boundary u = 0,

$$A_t' = \left(\frac{\tilde\Lambda_{\rm eff}^2}{a_0^2}\frac{(\tilde\omega\tilde q A_y^0 + \tilde q^2 A_y^0)}{\left[(2a_1+\delta-1)\left(\frac{\tilde\Lambda_{\rm eff}}{T_{\rm H}}\right)\right]^\gamma}\right)\frac{u^{1-\gamma}}{1-\gamma} + \frac{\tilde\omega\tilde q A_y^0 + \tilde q^2 A_t^0}{\left(\frac{i\tilde\omega}{\tilde\Lambda_{\rm eff}}\left[(2a_1+\delta-1)\left(\frac{\tilde\Lambda_{\rm eff}}{T_{\rm H}}\right)\right]^{\gamma/2} - \tilde q^2\right)},$$

$$A_y' = -\left(\frac{\tilde\Lambda_{\rm eff}^2}{a_0^2}\frac{(\tilde\omega\tilde q A_t^0 + \tilde\omega^2 A_t^0)}{\left[(2a_1+\delta-1)\left(\frac{\tilde\Lambda_{\rm eff}}{T_{\rm H}}\right)\right]^\gamma}\right)\frac{u^{1-\gamma}}{1-\gamma} - \frac{\tilde\omega\tilde q A_t^0 + \tilde\omega^2 A_y^0}{\left(\frac{i\tilde\omega}{\tilde\Lambda_{\rm eff}}\left[(2a_1+\delta-1)\left(\frac{\tilde\Lambda_{\rm eff}}{T_{\rm H}}\right)\right]^{\gamma/2} - \tilde q^2\right)}.$$

Similarly, the transverse mode A'_x is related to the boundary value of A_x

The non-vanishing components of the Green's function to be

$$\mathcal{G}_{tt} = -\frac{1}{g_4^2}\frac{q^2}{\left(i\omega\left[(2a_1+\delta-1)\left(\frac{\tilde{\Lambda}_{\rm eff}}{T_{\rm H}}\right)\right]^{\gamma/2} - \left(\frac{\tilde{\Lambda}_{\rm eff}}{T_{\rm H}}\right)q^2\right)},$$

$$\mathcal{G}_{yy} = -\frac{1}{g_4^2}\frac{\omega^2}{\left(i\omega\left[(2a_1+\delta-1)\left(\frac{\tilde{\Lambda}_{\rm eff}}{T_{\rm H}}\right)\right]^{\gamma/2} - \left(\frac{\tilde{\Lambda}_{\rm eff}}{T_{\rm H}}\right)q^2\right)},$$

$$\mathcal{G}_{ty} = \mathcal{G}_{yt} = -\frac{1}{g_4^2}\frac{\omega q}{\left(i\omega\left[(2a_1+\delta-1)\left(\frac{\tilde{\Lambda}_{\rm eff}}{T_{\rm H}}\right)\right]^{\gamma/2} - \left(\frac{\tilde{\Lambda}_{\rm eff}}{T_{\rm H}}\right)q^2\right)},$$

$$\mathcal{G}_{xx} = \frac{1}{g_4^2}\left[\frac{i\omega}{\left[(2a_1+\delta-1)\left(\frac{\tilde{\Lambda}_{\rm eff}}{T_{\rm H}}\right)\right]^{\gamma/2}} - \frac{q^2}{(1-\gamma)\left[(2a_1+\delta-1)\left(\frac{\tilde{\Lambda}_{\rm eff}}{T_{\rm H}}\right)\right]^{\gamma}}\left(\frac{\tilde{\Lambda}_{\rm eff}}{T_{\rm H}}\right)\right].$$

Note : the Green function of the transverse mode is not exactly the inverse of that of the longitudinal one.

Charge diffusion constant and conductivity

We can easily read off the diffusion constant D, to be

$$D = \frac{(-\Lambda)}{16\pi T_{\rm H}}\frac{(4+\eta^2)^2}{4}.$$

The quasi normal modes possess similar qualitative features.

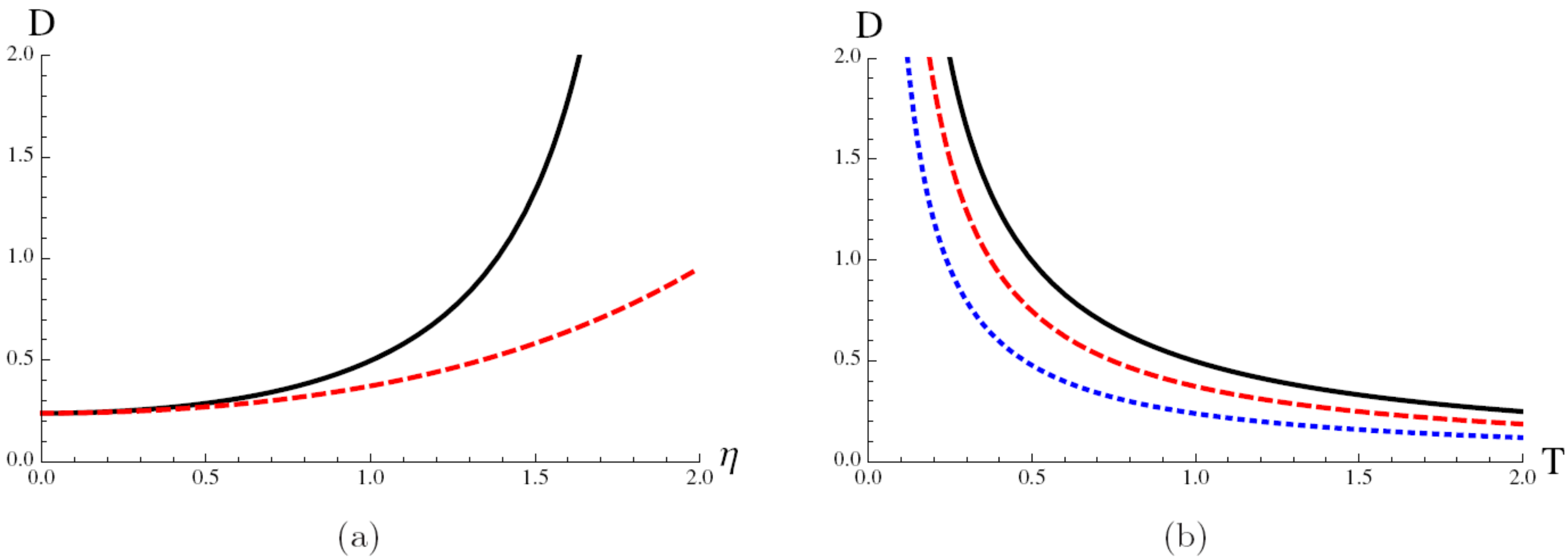


Figure 1. Charge diffusion constants:
(a) eta dependence and (b) temperature dependence.
The dashed and solid curve implies with or without a non-trivial dilaton coupling, respectively
(The AdS_4 result is represented as a dotted curve).

From the retarded Green function of the transverse mode, the real DC conductivity is given by

$$\sigma = \frac{1}{g_4^2}\left[\frac{16\pi}{(-\Lambda)(4+\eta^2)}\right]^{\frac{\eta^2}{4-\eta^2}} T_{\rm H}^{\frac{\eta^2}{4-\eta^2}},$$

Note 1)a significant change in the behaviour from that of the previous one.
2) The conductivity obeys a power law behaviour.

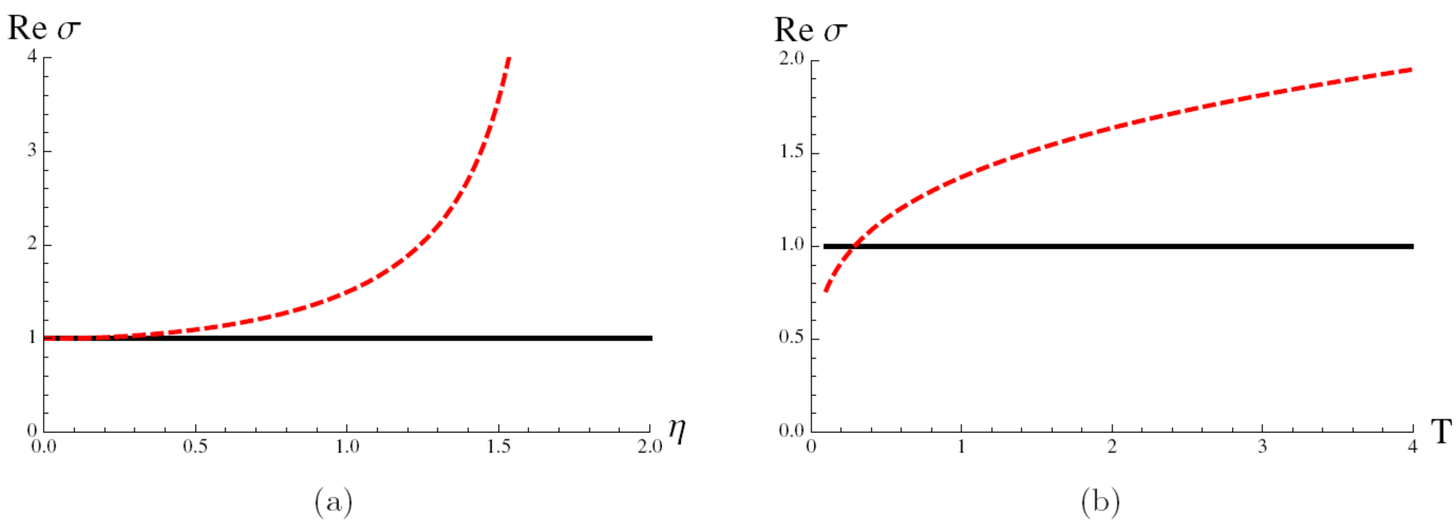


Figure 2. Real conductivities:
(a) eta dependence and (b) temperature dependence.
The dashed and solid curve implies with or without a dilaton coupling respectively.

$$S = \int d^4x \sqrt{-g} \left(R + \frac{6}{L^2} - 2(\nabla\phi)^2 - \frac{1}{2} e^{4\phi} \sum_{i=1}^{2} (\nabla \tilde{a}_i)^2 - e^{-2\phi} F^2 \right) .$$

Einstein and Field Equations

$$R_{\mu\nu} = -\frac{3}{L^2} g_{\mu\nu} + 2\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{2} e^{4\phi} \nabla_\mu \tilde{a} \nabla_\nu \tilde{a} + 2e^{-2\phi} F_{\mu\rho} F_\nu{}^\rho - \frac{1}{2} g_{\mu\nu} e^{-2\phi} F^2 ,$$

$$\Box\phi - \frac{1}{2} e^{4\phi} \sum_{i=1}^{2} (\nabla \tilde{a}_i)^2 + \frac{1}{2} e^{-2\phi} F^2 = 0 ,$$

$$\Box\tilde{a}_i + 4\nabla_\mu\phi\nabla^\mu\tilde{a}_i = 0 ,$$

$$\nabla_\mu(e^{-2\phi} F^{\mu\nu}) = 0 .$$

Linear Axion Fields

$$ds^2 = \frac{L^2}{z^2}\left(-g(z)dt^2 + g(z)^{-1}dz^2 + e^{A(z)+B(z)}dx^2 + e^{A(z)-B(z)}dy^2\right),$$
$$\phi = \phi(z),\ A_\mu dx^\mu = A_t(z)dt,\ \tilde{a}_1 = \alpha_1 x,\ \tilde{a}_2 = \alpha_2 y\,,$$
$$F_{zt} = A_t' = \rho_z L e^{-A+2\phi}\,.$$

EOM

$$2A'' + \left(A'\right)^2 + \left(B'\right)^2 + 4\left(\phi'\right)^2 = 0\,,$$
$$gzB'' + \left(g\left(zA' - 2\right) + zg'\right)B' + \frac{1}{2}ze^{-A-B+4\phi}\left(\alpha_1^2 - \alpha_2^2 e^{2B}\right) = 0\,,$$
$$\left(4z - 2z^2A'\right)g' + \left(-z^2\left(A'\right)^2 + 8zA' + z^2\left(B'\right)^2 + 4z^2\left(\phi'\right)^2 - 12\right)g$$
$$-\alpha_1^2 z^2 e^{-A-B+4\phi} - \alpha_2^2 z^2 e^{-A+B+4\phi} - 4\rho_z^2 z^4 e^{2\phi-2A} + 12 = 0\,,$$
$$e^A g\phi'' + \left(e^A gA' + e^A g' - \frac{2e^A g}{z}\right)\phi' - \rho_z^2 z^2 e^{2\phi-A} - \frac{1}{2}\alpha_1^2 e^{4\phi-B} - \frac{1}{2}\alpha_2^2 e^{B+4\phi} = 0\,,$$

Evolution of background fields

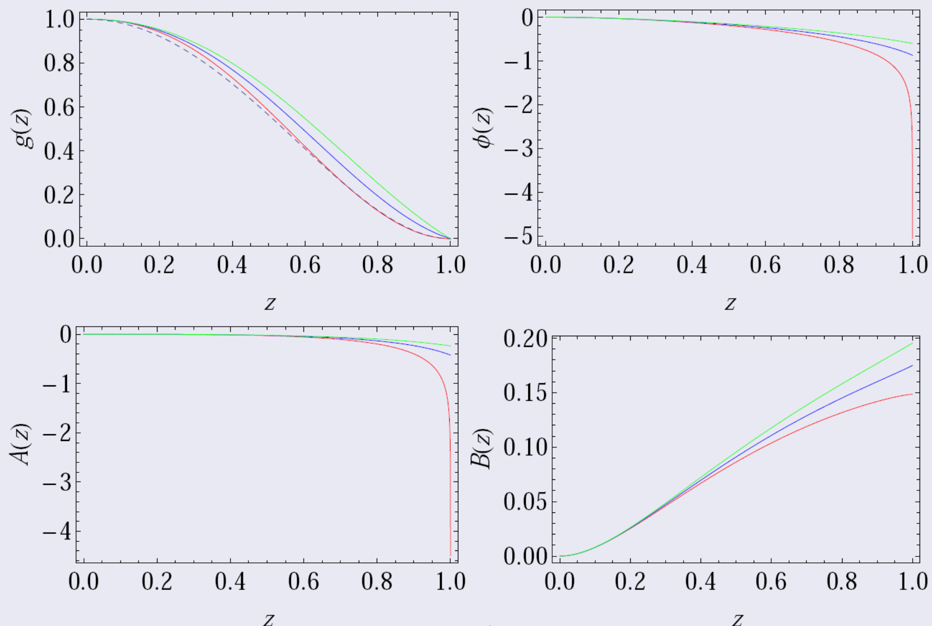


Figure: Fix $\alpha_1 = 2$, $\alpha_2 = 2$, $\kappa = 2.4 \times 10^{-4}$ (Red), 0.5 (Blue), and 1 (Green). Notice that the black-dashed line is
$f(z) = 1 - \frac{\alpha^2}{2} z^2 - \left(1 - \frac{\alpha^2}{2} + \frac{\mu^2}{4L^2}\right) z^3 + \frac{\mu^2}{4L^2} z^4.$

Horizon Data

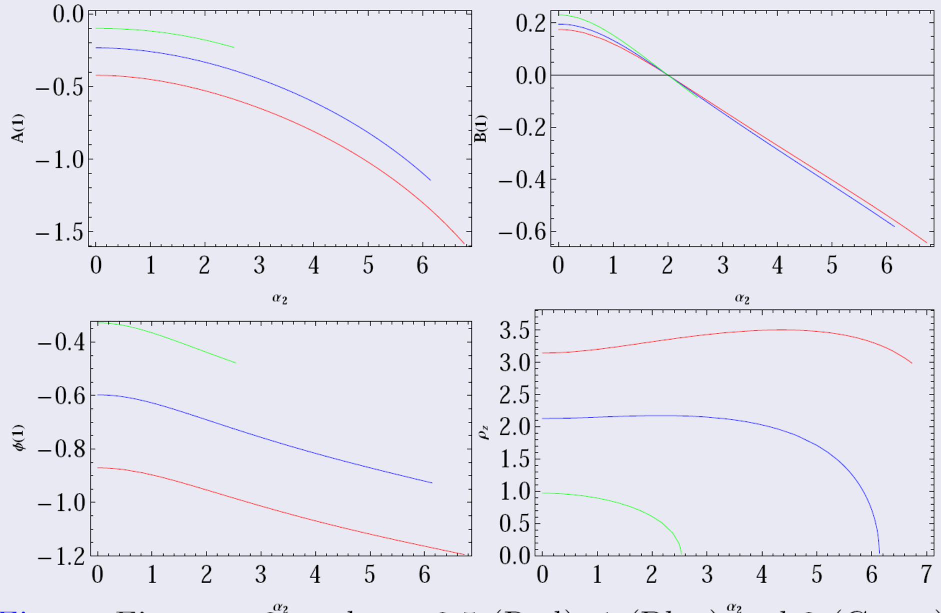


Figure: Fix $\alpha_1 = 2$, and $\kappa = 0.5$ (Red), 1 (Blue) and 2 (Green)

Perturbation

$$A_\mu dx^\mu \to A_t(z)dt + \left[\tilde{A}_x(t,z)dx + \tilde{A}_y(t,z)dy\right] \ ,$$

$$g_{\mu\nu}dx^\mu dx^\nu \to \overline{g}_{\mu\nu}dx^\mu dx^\nu + \frac{2L^2}{z^2}\left[\tilde{g}_{tx}(t,z)dtdx + \tilde{g}_{ty}(t,z)dtdy\right] \ ,$$

Fourier mode decompositions

$$\tilde{A}_i(t,z) = \int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}e^{-i\Omega t}A_i(z), \ \ \tilde{g}_{ti}(t,z) = \int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}e^{-i\Omega t}g_{ti}(z), \ \ \tilde{a}_i \to \alpha_i x^i + i\int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}\Omega e^{-i\Omega t}\chi_i(z),$$

Fluctuation Equations

$$A_i'' + \left(\bar{B_i}' + \frac{g'}{g} - 2\phi'\right)A_i' + \left(\frac{\Omega^2}{g^2} - \frac{4z^2e^{2\phi-2A}\rho_z^2}{g}\right)A_i - \alpha_i L e^{6\phi-A}\rho_z\chi_i' = 0\,,$$

$$\chi_i'' + \left(A' + \frac{g'}{g} - \frac{2}{z} + 4\phi'\right)\chi_i' + \frac{\Omega^2}{g^2}\chi_i - \frac{\alpha_i e^{\bar{B}_i - A}}{g^2}g_{\mathrm{ti}} = 0\,,$$

$$g_{ti}' + \left(\bar{B_i}' - A'\right)g_{ti} - \frac{4e^{-A}z^2\rho_z}{L}A_i - \alpha_i g e^{4\phi}\chi_i' = 0\,.$$

$$\hat{A}_i(z) \equiv g(z)A_i'(z)\,, \quad \hat{\chi}_i(z) \equiv g(z)\chi_i'(z)\,,$$

$$A_i(z) = (1-z)^\lambda a_i(z)\,, \qquad \hat{A}_i(z) = (1-z)^\lambda \hat{a}_i(z)\,, \qquad g_{ti}(z) = (1-z)^\lambda \zeta_{ti}(z)$$

$$\chi_i(z) = (1-z)^\lambda \eta_i(z)\,, \qquad \hat{\chi_i}(z) = (1-z)^\lambda \hat{\eta_i}(z)\,,$$

Fluctuation Equation

$$
\begin{aligned}
\hat{a}_i' + \left(\bar{B}_i{}' - \frac{\lambda}{1-z} - 2\phi'\right)\hat{a}_i + \left(\frac{\Omega^2}{g} - 4z^2 e^{2\phi-2A}\rho_z^2\right)a_i - \theta_i \rho_z L e^{6\phi - A}\hat{\eta}_i &= 0\,,\\
a_i' - \frac{\lambda}{1-z}a_i - \frac{\hat{a}_i}{g} &= 0\,,\\
\hat{\eta}_i' + \left(A' - \frac{\lambda}{1-z} - \frac{2}{z} + 4\phi'\right)\hat{\eta}_i + \frac{\Omega^2}{g}\eta_i - \frac{\theta_i e^{-A+\bar{B}_i}\zeta_{ti}}{g} &= 0\,,\\
\eta_i' - \frac{\lambda}{1-z}\eta_i - \frac{\hat{\eta}_i}{g} &= 0\,,\\
\zeta_{ti}' + \left(-A' + \bar{B}_i{}' - \frac{\lambda}{1-z}\right)\zeta_{ti} - \theta_i e^{4\phi}\hat{\eta}_i - \frac{4\rho_z e^{-A} z^2 a_i}{L} &= 0\,.
\end{aligned}
$$

$$
\begin{pmatrix}
0 & -\frac{\Omega^2}{g'(1)} & 0 & 0 & 0\\
\frac{1}{g'(1)} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\frac{\Omega^2}{g'(1)} & \frac{\alpha_i e^{-A(1)-\bar{B}_i(1)}}{g'(1)}\\
0 & 0 & \frac{1}{g'(1)} & 0 & 0\\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix} \hat{a}_i \\ a_i \\ \hat{\eta}_i \\ \eta_i \\ \zeta_{\mathrm{t}i} \end{pmatrix}
= \lambda
\begin{pmatrix} \hat{a}_i \\ a_i \\ \hat{\eta}_i \\ \eta_i \\ \zeta_{\mathrm{t}i} \end{pmatrix},
$$

$$
\psi_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{\theta_i e^{-A(1)+\bar{B}_i(1)}}{\Omega^2} \\ 1 \end{pmatrix}, \quad
\psi_{1\pm} = \begin{pmatrix} 0 \\ 0 \\ \pm i\Omega \\ 1 \\ 0 \end{pmatrix}, \quad
\psi_{2\pm} = \begin{pmatrix} \pm i\Omega \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.
$$

$$\Phi = \lim_{z\to 0} \sum_{n=0}^{\infty} \Phi^{(n)} z^n \,.$$

$$\begin{aligned}
\chi_i^{(1)} &= g_{ti}^{(1)} = 0\,, \\
\chi_i^{(2)} &= \frac{1}{2}\left(\chi_i^{(0)}\Omega^2 - \alpha_i g_{ti}^{(0)}\right)\,, \qquad g_{ti}^{(2)} = \frac{1}{4}\left(2\alpha_i \chi_i^{(0)}\Omega^2 - \alpha_1^2 g_{ti}^{(0)} - \alpha_2^2 g_{ti}^{(0)}\right)\,, \\
A_i^{(2)} &= \frac{-\Omega}{2} A_i^{(0)}\,, \\
A_i^{(3)} &= \frac{1}{6}\left(-\alpha_j^2 A_i^{(1)} - \Omega^2 A_i^{(1)} - \alpha_i^2 L\rho_z g_{ti}^{(0)} + \alpha_i L\Omega^2 \rho_z \chi_i^{(0)}\right)\,, \\
A_i^{(4)} &= \frac{1}{24}\left((-1)^i(\alpha_1^2 - \alpha_2^2)A_i^{(0)}\Omega^2 + 8A_i^{(0)}\rho_z^2 - 6A_i^{(1)}g^{(3)} + A_i^{(0)}\Omega^4 + 6\alpha_i\rho_z L\chi_i^{(3)}\right)\,,
\end{aligned}$$

Quadratic On-shell Action

$$\begin{aligned}
S_{re}^{(2)} &= S_{on} + S_{GH} + S_{ct} \\
&= 2\int d^2x \int \frac{d\Omega}{2\pi}\left[A_i^{(0)}A_i^{(1)} + \frac{1}{2}L^2\left(p_i B^{(3)} + g^{(3)}\right) g_{ti}^{(0)} g_{ti}^{(0)} - 2L\rho_z A_i^{(0)} g_{ti}^{(0)} \right. \\
&\qquad \left. + \frac{3}{4}L^2\Omega^2\chi_i^{(0)}\chi_i^{(3)} - \frac{3}{4}L^2\alpha_i g_{ti}^{(0)}\chi_i^{(3)}\right]\,,
\end{aligned}$$

where p_i is either 1 for $i = x$ or -1 for $i = y$

Multi-interacting fields

$$\Phi_i^{\mathfrak{a}} = \mathbb{S}_i^{\mathfrak{a}} z^{3-\Delta_{\mathfrak{a}}} + \cdots + \mathbb{O}_i^{\mathfrak{a}} z^{\Delta_{\mathfrak{a}}} + \cdots ,$$

where different fields, A_i, g_{ti} and χ_i, are distinguished by an index $\mathfrak{a}$. Here $\Delta_{\mathfrak{a}}$ is a positive value and corresponds to the conformal dimension of the dual operator on-shell gravity action can be written as the following form

$$S_{re} = 2V \int \frac{d\Omega}{2\pi} \left[\bar{\mathbb{S}}_i^{\mathfrak{a}} \mathbb{A}_{\mathfrak{a}\mathfrak{b}}^{ij}(\Omega) \mathbb{S}_j^{\mathfrak{b}} + \bar{\mathbb{S}}_i^{\mathfrak{a}} \mathbb{B}_{\mathfrak{a}\mathfrak{b}}^{ij}(\Omega) \mathbb{O}_j^{\mathfrak{b}} \right] ,$$

where

$$\mathbb{S}_i \equiv \begin{pmatrix} \mathbb{S}_i^1 \\ \mathbb{S}_i^2 \\ \mathbb{S}_i^3 \end{pmatrix} = \begin{pmatrix} A_i^{(0)} \\ g_{ti}^{(0)} \\ \chi_i^{(0)} \end{pmatrix} , \quad \mathbb{O}_i \equiv \begin{pmatrix} \mathbb{O}_i^1 \\ \mathbb{O}_i^2 \\ \mathbb{O}_i^3 \end{pmatrix} = \begin{pmatrix} A_i^{(1)} \\ g_{ti}^{(3)} \\ \chi_i^{(3)} \end{pmatrix} ,$$

$$\mathbb{A}^{ij} = \begin{pmatrix} 0 & -L\rho z & 0 \\ -L\rho z & \frac{1}{4}L^2(g^{(3)} + p_i B^{(3)}) & 0 \\ 0 & 0 & 0 \end{pmatrix} \delta^{ij} , \qquad \mathbb{B}^{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -\frac{3L^2 \alpha_i}{4} \\ 0 & 0 & \frac{3L^2\Omega^2}{4} \end{pmatrix} \delta^{ij} ,$$

Retarded Green Function is

$$G_{\mathfrak{a}\mathfrak{b}}^{ij} \equiv \mathbb{A}_{\mathfrak{a}\mathfrak{b}}^{ij} + \mathbb{B}_{\mathfrak{a}\mathfrak{c}}^{ik} \mathbb{O}_k^{\mathfrak{c}} (\mathbb{S}^{-1})_{\mathfrak{b}}^{j} ,$$

$$\left(\begin{array}{c} J^j \\ T^{tj} \end{array} \right) = \left(\begin{array}{cc} G_{11}^{ij} & G_{12}^{ij} \\ G_{21}^{ij} & G_{22}^{ij} \end{array} \right) \left(\begin{array}{c} A_i^{(0)} \\ g_{ti}^{(0)} \end{array} \right) ,$$

where $J^j = A^{j(1)}$ and $T^{tj} = g^{tj(3)}$. And

$$\left(\begin{array}{c} J^i \\ Q^i \end{array} \right) = \left(\begin{array}{cc} \sigma & \tilde{\alpha} T \\ \bar{\alpha} T & \bar{\kappa} T \end{array} \right) \left(\begin{array}{c} E_i \\ -(\nabla_i T)/T \end{array} \right) ,$$

where $E_i = i\Omega(A_i^{(0)} + \mu g_{ti}^{(0)})$ and $g_{ti}^{(0)} = -\frac{\nabla_i T}{i\Omega T}$. Finally

$$\left(\begin{array}{cc} \sigma_{ii} & \tilde{\alpha}_{ii} T \\ \bar{\alpha}_{ii} T & \bar{\kappa}_{ii} T \end{array} \right) = \left(\begin{array}{cc} -\frac{i G_{11}^{ii}}{\Omega} & \frac{i(\mu G_{11}^{ii} - G_{12}^{ii})}{\Omega} \\ \frac{i(\mu G_{11}^{ii} - G_{21}^{ii})}{\Omega} & -\frac{i\left[G_{22}^{ii} - \tilde{G}_{22}^{ii} - \mu(G_{12}^{ii} + G_{21}^{ii} - \mu G_{11}^{ii})\right]}{\Omega} \end{array} \right) .$$

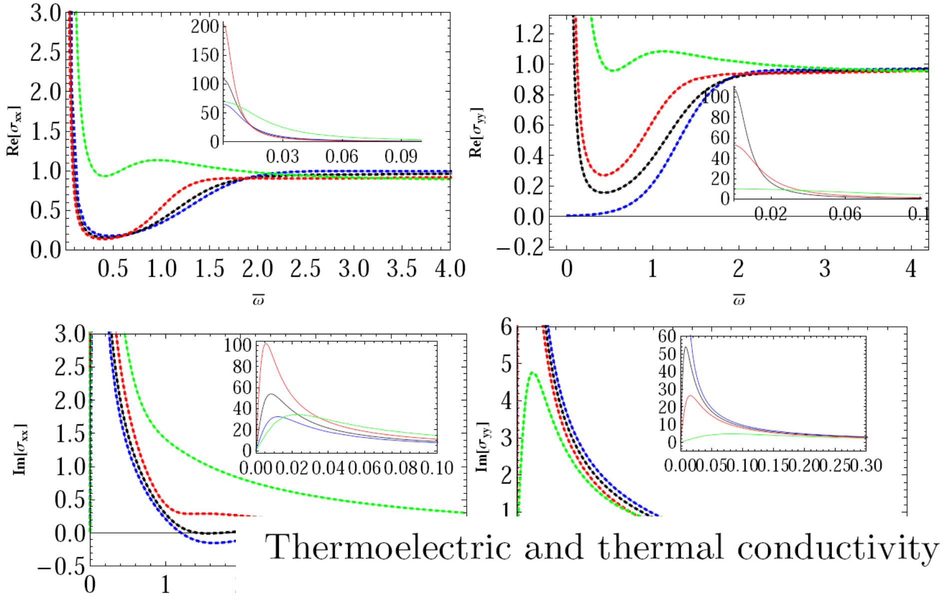


Figure: At a given temper
2 (Black), 4 (Red) and 6 (

Thermoelectric and thermal conductivity

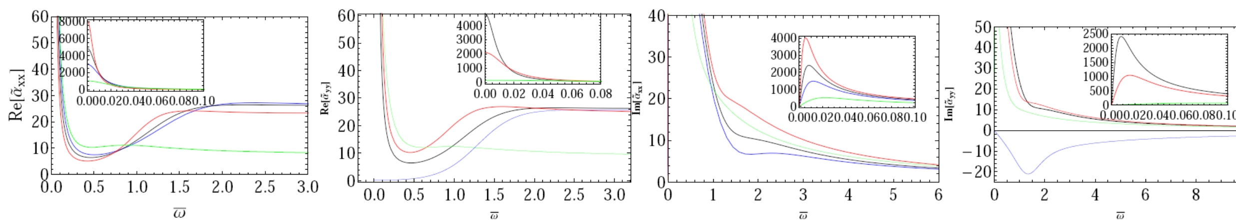


Figure: Fix $\alpha_1 = 2$ with $\kappa = 1$ and $\alpha_2 = 0$ (Blue), 2 (Black), 4 (Red), and 6 (Green).

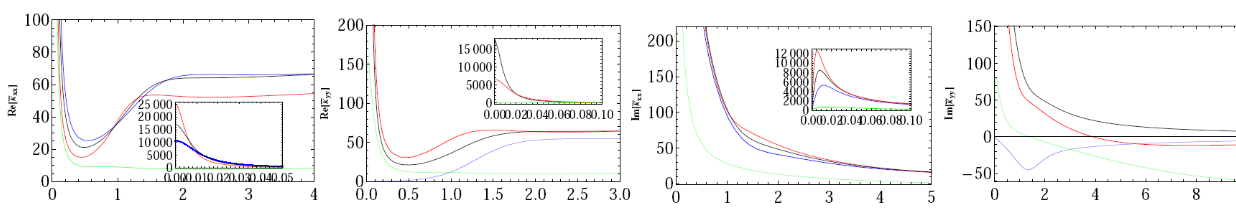


Figure: Fix $\alpha_1 = 2$ with $\kappa = 1$ and $\alpha_2 = 0$ (Blue), 2 (Black), 4 (Red), and 6 (Green).

$$\Gamma = \frac{k\tau}{1 - i\omega\tau},$$

σ_{xx}	γ	b	c	k	τ
$\alpha_2 = 0$	2/3	2.88	-5.65	0.785	82.7
$\alpha_2 = 2$	0.89	1.3	-0.8	0.9	120
$\alpha_2 = 4$	0.626	6.87	-24.5	1.13	180

Table: Parameters of the Drude formula fitting the electric conductivity well.

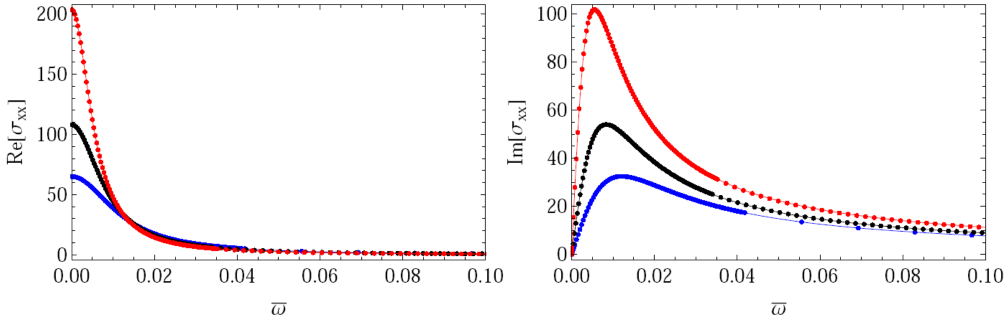


Figure: Drude peak for $\alpha_1 = 2$ and $\kappa = 1$ with $\alpha_2 = 0$ (Blue), 2 (Black) and 4 (Red). Notice that a solid line indicates the analytic result from the Drude formula, while dots represent the numerical results.

Intermediate scaling

$$|\sigma| = \frac{b}{\omega^{\gamma}} + c \tag{2}$$

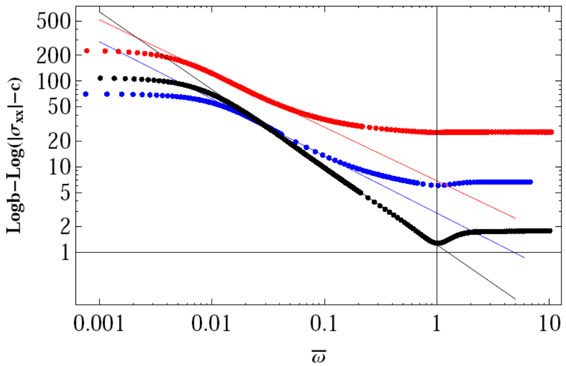


Figure: The magnitude of the electric conductivity for $\alpha_1 = 2$ and $\kappa = 1$ with $\alpha_2 = 0$ (Blue), 2 (Black) and 4 (Red). The slope of the straight line denotes the power law of the electric conductivity.

DC conductivity

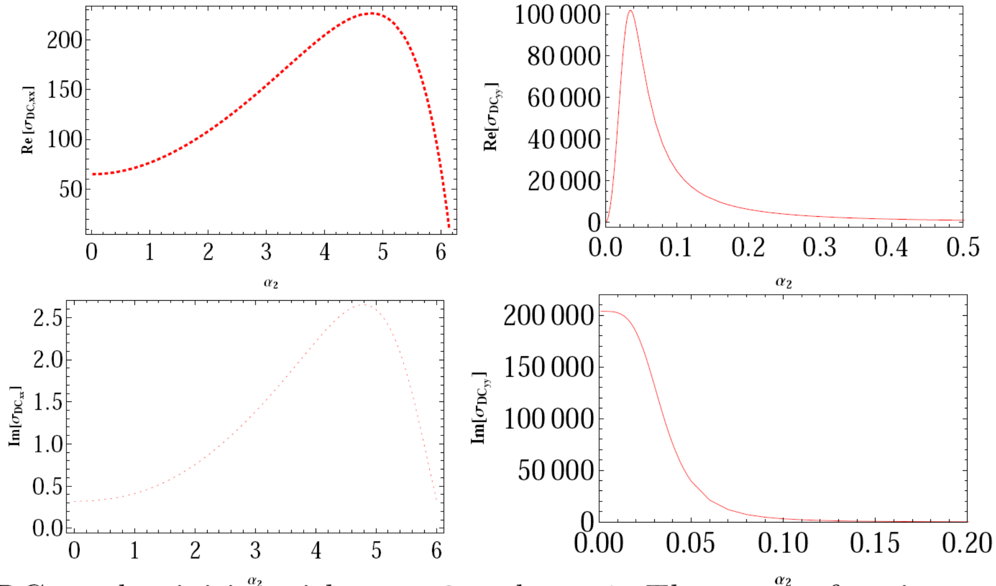


Figure: DC conductivities with $\alpha_1 = 2$ and $\kappa = 1$. The range of α_2 is restricted to $0 < \alpha_2 < 6.1393$.

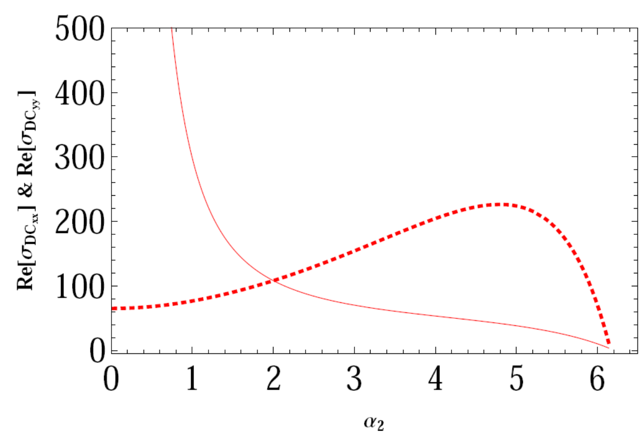
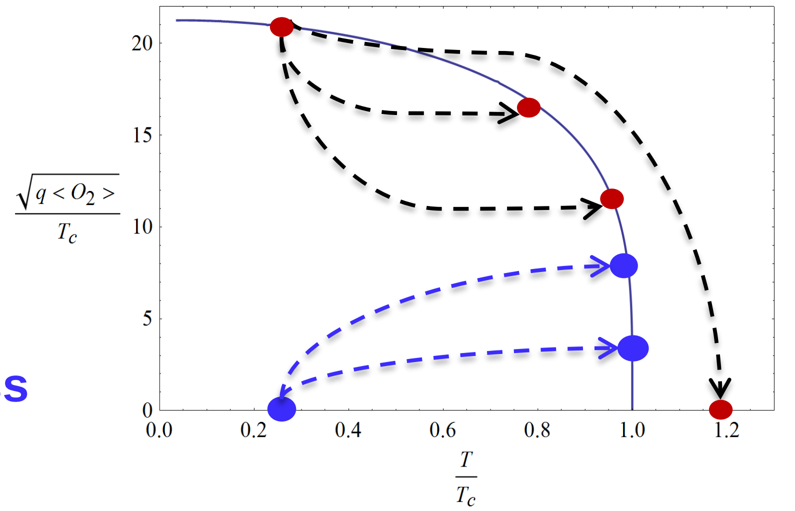


Figure: α_2 dependence of DC conductivities, σ_{xx} (dotted) and σ_{yy} (solid), with $\alpha_1 = 2$ and $\kappa = 1$.

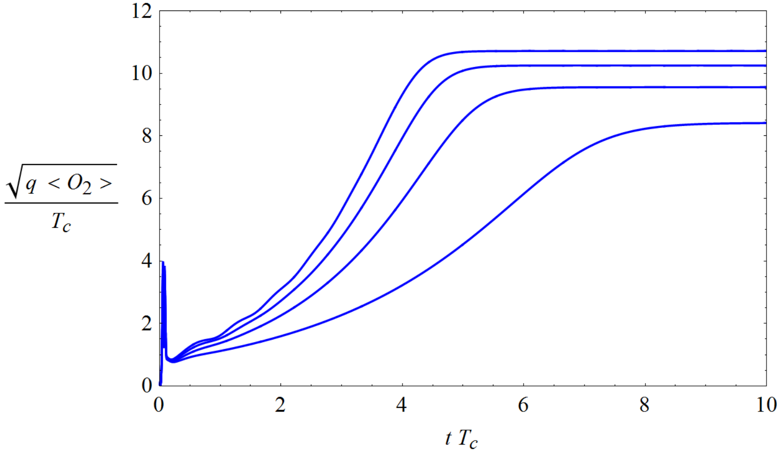
3.Holographic Approach to the nonequilibrium physics

「Two scenarios of
Far-from-equilibrium dynamics



Blue routes : Condensation Process

· Non-equilibrium evolution of an unstable configuration

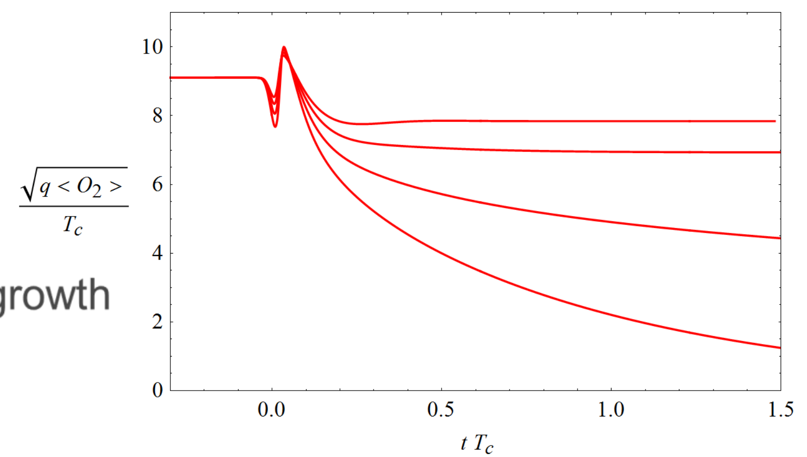


· Condensate undergoes an exponential growth

- Phase transition in “real” time !

Red routes: Quantum Quenching

· Dynamical response to a sudden injection of energy

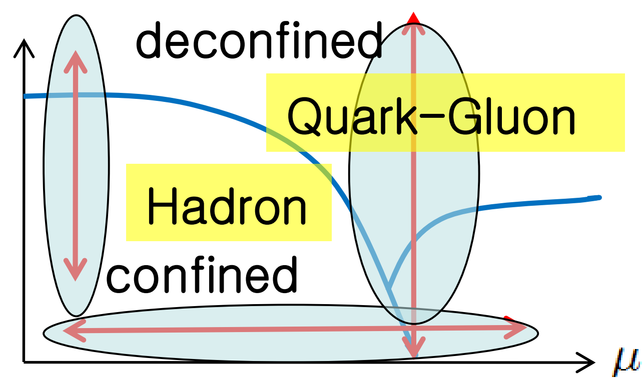


IV. Summary

- Holographic Principles (through the D-brane configuration)

  (d+1 dim.) (classical) gravity $\leftrightarrow$ (d dim.) (quantum) YM theories
- BH geometry $\leftrightarrow$ Finite Temperature field theory
- Constructing the dual geometry :

  Top-down & Bottom-up



- Holographic QCD

– w/o chemical potential –

phase : confined phase $\leftrightarrow$ deconfined phase transition

Geometry : thermal AdS $\leftrightarrow$ AdS BH

Hawking-Page transition

– in dense matter – (U(1) chemical potential → baryon density)

deconfined phase by RNAdS BH $\leftrightarrow$ hadronic phase by tcAdS

Hawking-Page phase transition

IV. Summary – continued

In the hadronic phase, the quark density dependence of the light meson masses has been investigated.

Holographic study on the Baryon Properties in Dense Matter

We have studied the black hole solution of the Einstein-dilaton theory with a Liouville potential. This solution having a non-zero scalar profile modifies the asymptotic geometry from AdS to an warped space.
The warped geometry with one parameter preserves the ISO(1, 2) isometry corresponding to the the dual relativistic nonconformal matter.

We have also shown that the non-conformal gauge theory is thermodynamically stable only for the parameter range 0 ≤ eta-squared < 4.

We have studied the linear response of the vector fluctuations.
Longitudinal modes & the transverse mode propagates independently.

IV. Summary – continued

When the gauge fields couple only to gravity,

the charged diffusion constant and the DC conductivity take the form similar to that of AdS dual gauge theory.

We observed that, the dependence increases the charge diffusion constant compared to its AdS counterpart.

In a nutshell, the quasi normal mode in the non-conformal medium has shorter life-time than that of the conformal case.

The longitudinal modes have a quasi normal pole.

The DC conductivity, computed from the Green's function of the transverse mode is proportional to the bulk gauge coupling.
Since the bulk gauge coupling is constant, there is no significant difference between the DC conductivities of the conformal and non-conformal matter.

IV. Summary – continued

we considered the gauge fluctuations coupled with the dilaton.

This kind of nontrivial dilaton coupling provides a peculiar physical aspect to the dual gauge theory such as the strange metallic behavior.
Here for definiteness, we chose a specific value, = −/2.
With this choice, the charge diffusion constant has a similar form to that of the previous one with some modifications.
For a fixed non-conformality, the diffusion constant is smaller than the one, obtained from the dilaton free gauge fluctuation.
This also leads to the fact that the quasi normal mode with the dilaton coupling survives longer than that of the free one.
We have shown that the effective bulk gauge coupling depending nontrivially on the radial coordinate can change the behaviour of the DC conductivity, dramatically.
The DC conductivity of the system increases with temperature, which is a typical aspect commonly found in electrolytes, and in conductive polymers such as polypyrrole films.
We also would like to explore the paradigm by including the metric fluctuation and thus determine other hydrodynamical quantities like shear viscosity etc.

IV. Summary – continued

- Linear Axion and Massive Gravity theories were discussed motivated by momentum relaxation and anisotropy.
- Transport Coefficients were computed using linear response theory using Green's function and Membrane Paradigm.
- $SL(2, R)$ transformation were used to generate solutions in EM configuration so that Hall conductivity.
- Deeper investigation for resistivity and its temperature dependency would provide more evidence of cupates.

- Holographic principle can also be applied to the strongly interacting condensed matter system as well as nonequilibrium physics
- Holographic Principle may provide a new methodology for the strongly interacting phenomena!

Thank you!!

Спасибо!!