Compositeness for the N^* and Δ^* resonances from the πN scattering amplitude

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1. Introduction
2. Two-body wave functions from scattering amplitudes
3. The N^* compositeness program
4. Summary

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[1] T. S. , *Phys. Rev.* C95 (2017) 025206.

[2] T. S. , in preparation.

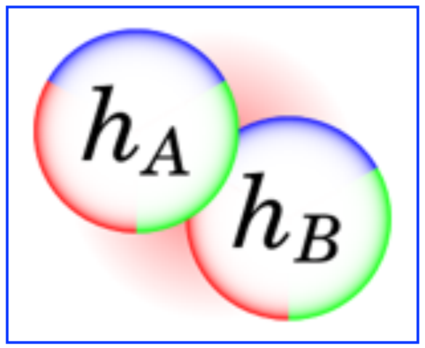
[3] T. S. , T. Hyodo and D. Jido, *PTEP* 2015 063D04.

[4] T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, *Phys. Rev.* C93 (2016) 035204.

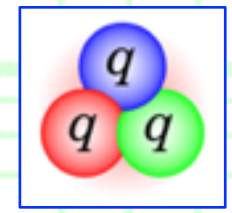
1. Introduction

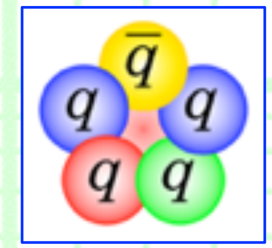
++ What is the compositeness ? ++

- Compositeness (X) is a quantity to "measure" the hadronic molecular component inside an excited hadron of interest.



Hadronic molecules





Compact states

$$\langle \tilde{\Psi} | \Psi \rangle = X + Z = 1$$

- Compositeness is defined as the norm of the two-body part of the bound-state wave function:

$$X = \int \frac{d^3 q}{(2\pi)^3} \langle \tilde{\Psi} | \mathbf{q} \rangle \langle \mathbf{q} | \Psi \rangle = \int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}(\mathbf{q}) \right]^2$$

--- To obtain the bound-state wave function, it is better to solve the Lippmann-Schwinger Eq. rather than Schrödinger Eq.

- In general compositeness is a model dependent quantity, but becomes model independent if the pole exists near the threshold.

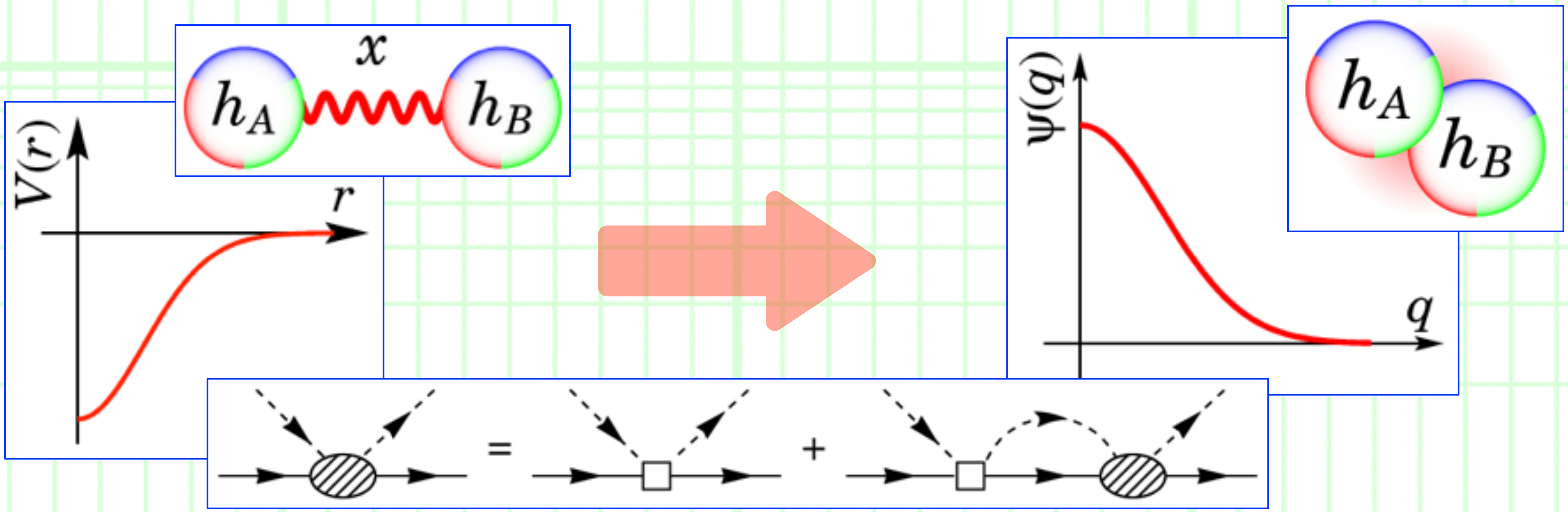
--- Weinberg's compositeness condition ($B_E << E_{\text{typical}}$). Weinberg (1965).

1. Introduction

++ Motivation ++

- We evaluate **the wave function of hadron-hadron composite part**.

--- *cf.* Wave function for relative motion of two nucleons in deuteron.



- For a given interaction (potential) which generates a bound state, we solve **the Lippmann-Schwinger Eq.**

$$T(E;\, \mathbf{q}', \mathbf{q}) = \langle \mathbf{q}' | \hat{T}(E) | \mathbf{q} \rangle \approx \frac{\gamma(q')\gamma(q)}{E - E_{\rm pole}}$$

$$\langle \mathbf{q} | \Psi \rangle = \tilde{\psi}(q) = \frac{\gamma(q)}{E_{\rm pole} - \mathcal{E}(q)}$$

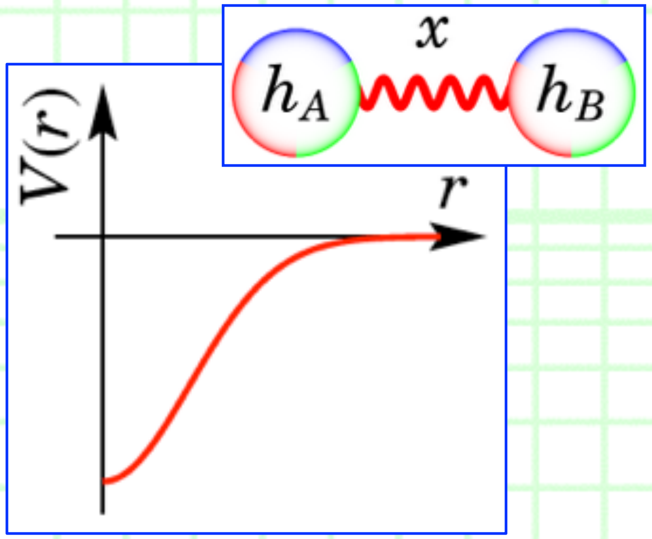
$$X \equiv \int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}(\mathbf{q}) \right]^2$$

--- The WF and compositeness (= norm) are **automatically scaled**.

2. Wave functions from amplitudes

++ Setup of our model ++

- We consider the following system in quantum mechanics.
  - □ **Full Hamiltonian**: $\hat{H} = \hat{H}_0 + \hat{V}(E)$



--- Composed of free part H_0 and interaction V.
--- The interaction V, determined in some theory, may depend on the energy of the system E.

□ **The free Hamiltonian** has eigenstates of free two-body state:

$\hat{H}_0|\mathbf{q}\rangle = \mathcal{E}(q)|\mathbf{q}\rangle$ where $\mathcal{E}(q) = \sqrt{m_1^2 + q^2} + \sqrt{m_2^2 + q^2}$ or $\mathcal{E}(q) = m_1 + m_2 + \dfrac{q^2}{2\mu}$

--- The two-body state with relative momentum q.

□ **The full Hamiltonian** has an eigenstate of a bound state:

$$\hat{H}|\Psi\rangle = (\hat{H}_0 + \hat{V})|\Psi\rangle = E_{\text{pole}}|\Psi\rangle$$

--- The eigenvalue E_{pole} is real (stable bound state) or complex (resonance).

2. Wave functions from amplitudes

++ Compositeness as a norm ++

- **Definition:** **Compositeness is defined as the norm of the two-body part of the bound-state wave function.**

  - Two-body bound-state wave function in momentum space:

$$\langle \mathbf{q} | \Psi \rangle = \tilde{\psi}(\mathbf{q})$$

  - The norm of the two-body wave function is:

$$X \equiv \int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}(\mathbf{q}) \right]^2$$

$$\langle \mathbf{q}' | \mathbf{q} \rangle = (2\pi)^3 \delta(\mathbf{q}' - \mathbf{q})$$

$$\mathbb{1}_{\text{two-body}} = \int \frac{d^3 q}{(2\pi)^3} | \mathbf{q} \rangle \langle \mathbf{q} |$$

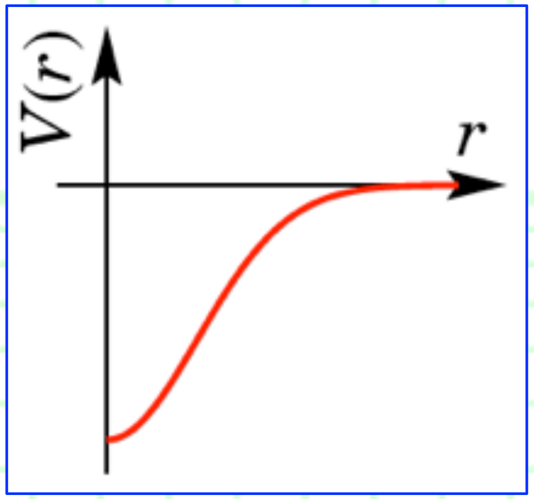
  - However, for the moment **we have not normalized the bound-state wave function**.

--> To interpret the compositeness, we have to normalize it.

2. Wave functions from amplitudes

++ Wave function from Lippmann-Schwinger Eq. ++

- To obtain the correct normalization of bound-state wave function it is better to solve the Lippmann-Schwinger Eq. than the Schrödinger Eq. !



□ Schrödinger Eq. in momentum space:

$$\mathcal{E}(q)\tilde{\psi}(\mathbf{q}) + \int \frac{d^3q'}{(2\pi)^3} V(\mathbf{q}, \mathbf{q}')\tilde{\psi}(\mathbf{q}') = E_{\rm pole}\tilde{\psi}(\mathbf{q})$$

--- Homogeneous integral Eq., so we have to normalize it by hand !

□ Lippmann-Schwinger Eq. in momentum space:

$$T(E; \mathbf{q}', \mathbf{q}) = V(\mathbf{q}', \mathbf{q}) + \int \frac{d^3k}{(2\pi)^3} \frac{V(\mathbf{q}', \mathbf{k})T(E; \mathbf{k}, \mathbf{q})}{E - \mathcal{E}(k)}$$

--- Inhomogeneous integral Eq., so we need not take care of the normalization of the scattering amplitude !

□ Where is the wave function in Lippmann-Schwinger Eq. ?

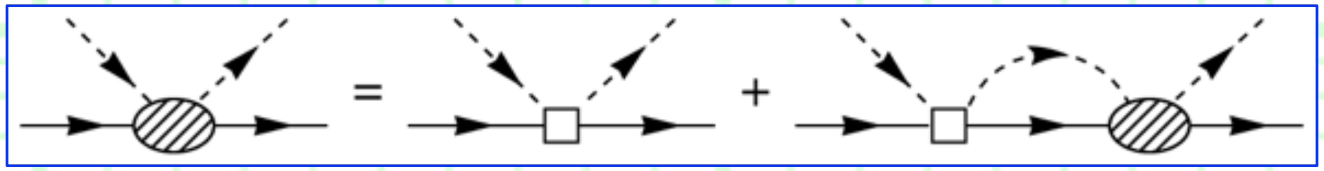
2. Wave functions from amplitudes

++ Wave function from Lippmann-Schwinger Eq. ++

- Solve the Lippmann-Schwinger equation at the pole position of the bound state.

$$\hat{T}(E) = \hat{V} + \hat{V}\frac{1}{E-\hat{H}_0}\hat{T} = \hat{V} + \hat{V}\frac{1}{E-\hat{H}}\hat{V}$$

$$T(E;\mathbf{q}',\mathbf{q}) = \langle \mathbf{q}'|\hat{T}(E)|\mathbf{q}\rangle$$



--- Near the resonance pole position $E_{\rm pole}$, amplitude is dominated by the pole term in the expansion by the eigenstates of H as

$$\langle \mathbf{q}'|\hat{T}(E)|\mathbf{q}\rangle \approx \langle \mathbf{q}'|\hat{V}|\Psi\rangle \frac{1}{E-E_{\rm pole}} \langle \tilde{\Psi}|\hat{V}|\mathbf{q}\rangle$$

$|\Psi\rangle$, $|\mathbf{q}_{\rm full}\rangle$, ... $\langle\tilde{\Psi}|$, $\langle\mathbf{q}_{\rm full}|$, ...

$$\mathbb{1} = |\Psi\rangle\langle\tilde{\Psi}| + \cdots$$

--- The residue of the amplitude at the pole position has information on the wave function !

$$\langle \mathbf{q}|\hat{V}|\Psi\rangle = \langle \mathbf{q}|(\hat{H}-\hat{H}_0)|\Psi\rangle = [E_{\rm pole} - \mathcal{E}(q)]\tilde{\psi}(\mathbf{q})$$

$$\langle \tilde{\Psi}|\hat{V}|\mathbf{q}\rangle = [E_{\rm pole} - \mathcal{E}(q)]\tilde{\psi}(\mathbf{q})$$

$$\mathcal{E}(q) = \sqrt{m_1^2+q^2} + \sqrt{m_2^2+q^2}$$

or

$$\mathcal{E}(q) = m_1 + m_2 + \frac{q^2}{2\mu}$$

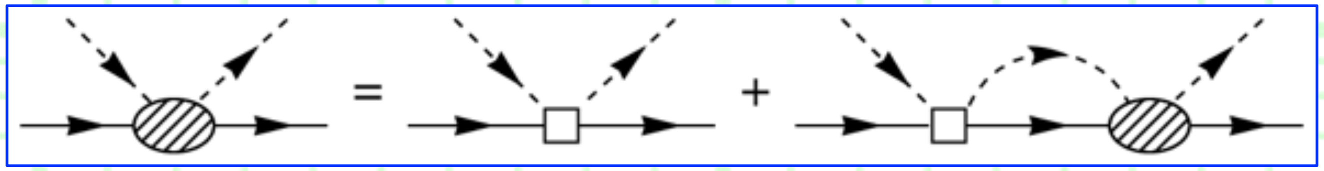
2. Wave functions from amplitudes

++ Wave function from Lippmann-Schwinger Eq. ++

- Solve the Lippmann-Schwinger equation at the pole position of the bound state.

$$T(E; \mathbf{q}', \mathbf{q}) = \langle \mathbf{q}' | \hat{T}(E) | \mathbf{q} \rangle$$

$$\hat{T}(E) = \hat{V} + \hat{V} \frac{1}{E - \hat{H}_0} \hat{T} = \hat{V} + \hat{V} \frac{1}{E - \hat{H}} \hat{V}$$



--- The wave function can be extracted from the residue of the amplitude at the pole position:

$$T(E; \mathbf{q}', \mathbf{q}) = \langle \mathbf{q}' | \hat{T}(E) | \mathbf{q} \rangle \approx \frac{\gamma(q')\gamma(q)}{E - E_{\text{pole}}}$$

<-- Off-shell Amp. !

$$\gamma(q) \equiv \langle \mathbf{q} | \hat{V} | \Psi \rangle = [E_{\text{pole}} - \mathcal{E}(q)] \tilde{\psi}(q)$$

--> Because the scattering amplitude cannot be freely scaled (Lippmann-Schwinger Eq. is inhomogeneous !), the WF from the residue of the amplitude is automatically scaled as well !

If purely molecule -->

$$\int \frac{d^3 q}{(2\pi)^3} \left[\frac{\gamma(q)}{E_{\text{pole}} - \mathcal{E}(q)} \right]^2 = 1$$

<-- We obtain !

E. Hernandez and A. Mondragon, *Phys. Rev.* C29 (1984) 722.

2. Wave functions from amplitudes

++ Example: Stable bound state ++

- A Λ hyperon in $A \sim 40$ nucleus.

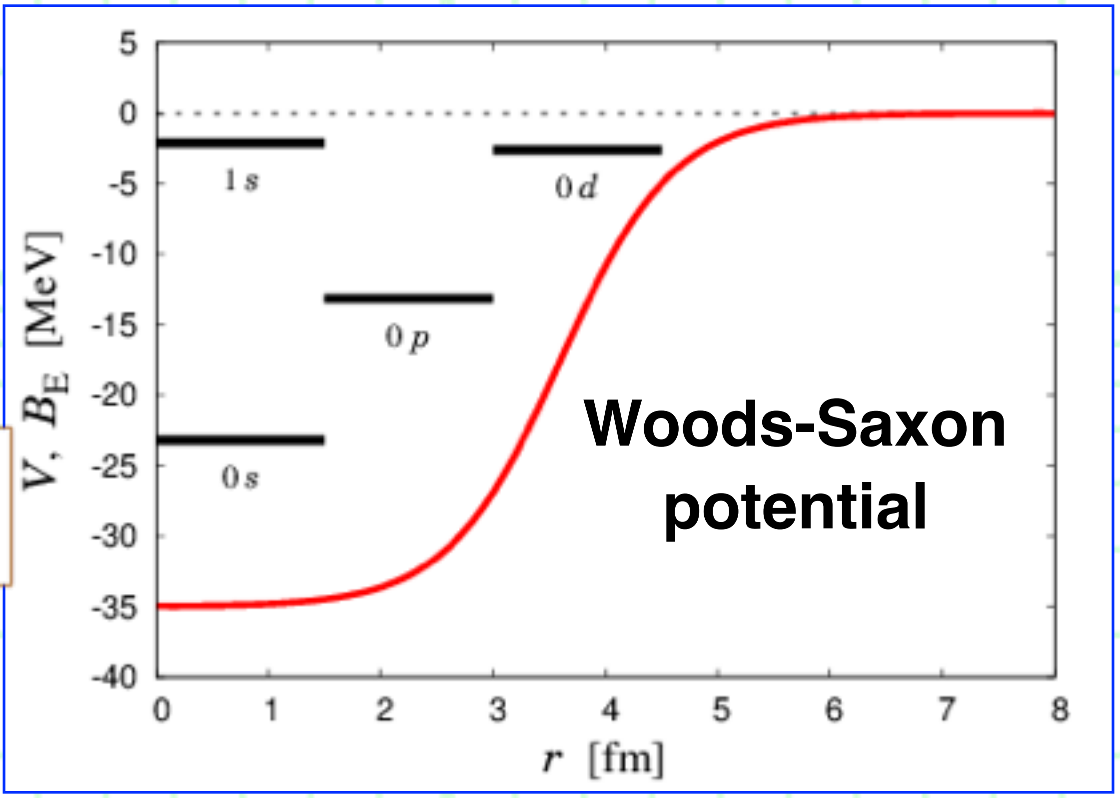
--> Calculate wave functions in 2 ways.

1. Solve Schrödinger equation:

$$\mathcal{E}(q)\tilde{\psi}(q) + \int \frac{d^3 q}{(2\pi)^3} \tilde{V}(\mathbf{q}, \mathbf{q}')\tilde{\psi}(q') = E_{\rm pole}\tilde{\psi}(q)$$

--> Normalize ψ by hand !

$$\int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}(q)\right]^2 = 1$$



2. Solve Lippmann-Schwinger equation:

$$T(E; \mathbf{q}', \mathbf{q}) = V(\mathbf{q}', \mathbf{q}) + \int \frac{d^3 k}{(2\pi)^3} \frac{\tilde{V}(\mathbf{q}', \mathbf{k})T(E; \mathbf{k}, \mathbf{q})}{E - \mathcal{E}(k)}$$

--> Extract WF from the residue:

$$T(E; \mathbf{q}', \mathbf{q}) \approx \frac{\gamma(q')\gamma(q)}{E - E_{\rm pole}}$$ --> $$\tilde{\psi}(q) = \frac{\gamma(q)}{E_{\rm pole} - \mathcal{E}(q)}$$

--- Without normalizing by hand !

2. Wave functions from amplitudes

++ Example: Stable boun

$$\mathrm{P}(q) = \frac{4\pi q^2}{(2\pi)^3}\left[\tilde{\psi}(\mathbf{q})\right]^2$$

- A Λ hyperon in $A \sim 40$ nucleus.

--> Calculate wave functions in 2

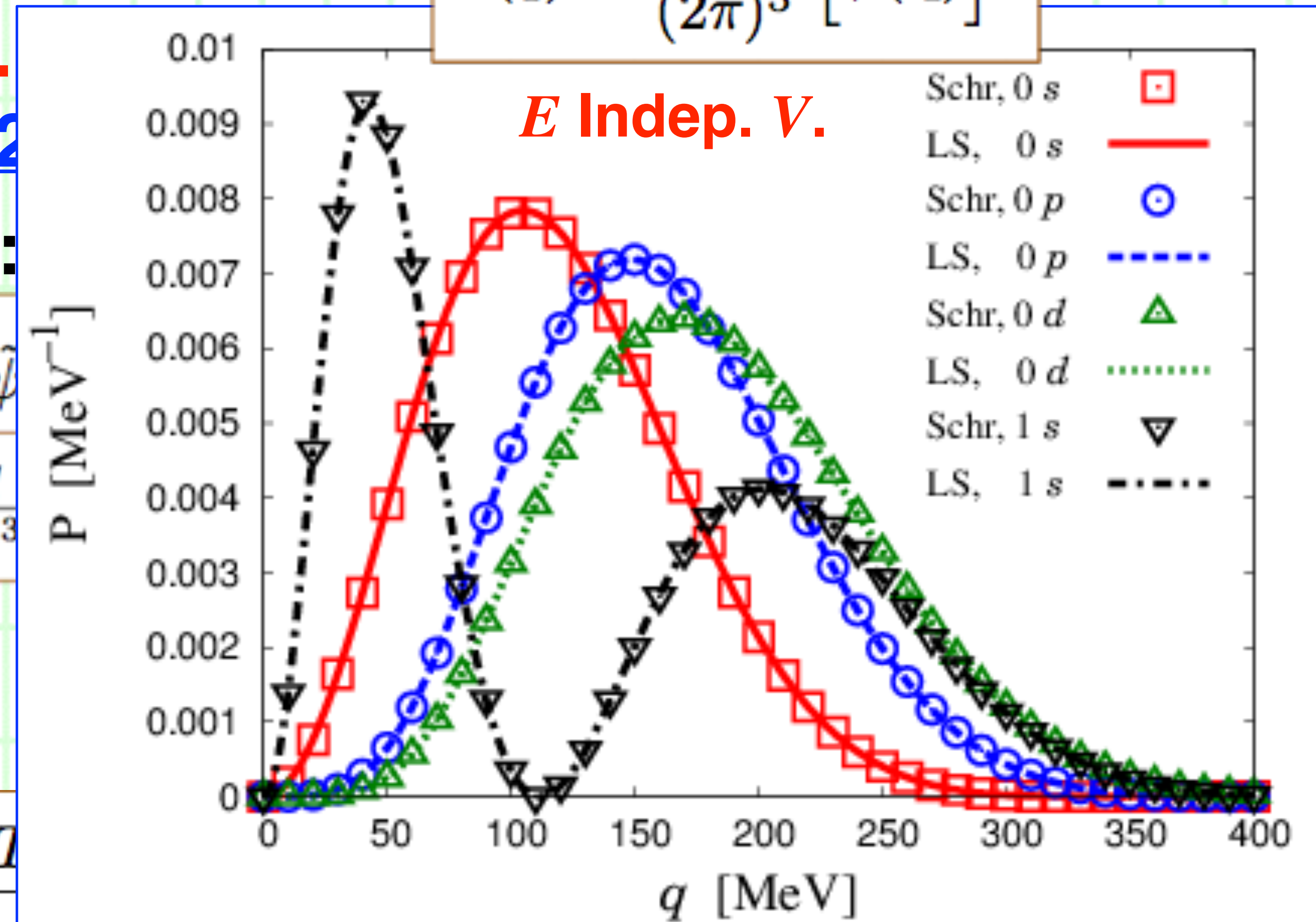
1. Solve Schrödinger equation:

$$\mathcal{E}(q)\tilde{\psi}(q) + \int \frac{d^3q}{(2\pi)^3}\tilde{V}(\mathbf{q}, \mathbf{q}')\tilde{\psi}(q') = E_{\mathrm{pole}}\tilde{\psi}$$

--> Normalize ψ by hand ! $\int \frac{d^3q}{(2\pi)^3}$

2. Solve Lippmann-Schwinger equation:

$$T(E; \mathbf{q}', \mathbf{q}) = V(\mathbf{q}', \mathbf{q}) + \int \frac{d^3k}{(2\pi)^3}\frac{\tilde{V}(\mathbf{q}', \mathbf{k})T}{E - \mathcal{E}(k)}$$



--> Extract WF from the residue:

$$T(E; \mathbf{q}', \mathbf{q}) \approx \frac{\gamma(q')\gamma(q)}{E - E_{\mathrm{pole}}} \quad \text{-->} \quad \tilde{\psi}(q) = \frac{\gamma(q)}{E_{\mathrm{pole}} - \mathcal{E}(q)}$$

--- Without normalizing by hand !

- In 1st way: Points.
  2nd way: Lines.
- Exact coincidence !

--- We obtain auto-matically normalized WF from the Amp. !

2. Wave functions from amplitudes

++ Example: Stable bound state ++

- We define **the compositeness X as the norm of the wave function**:

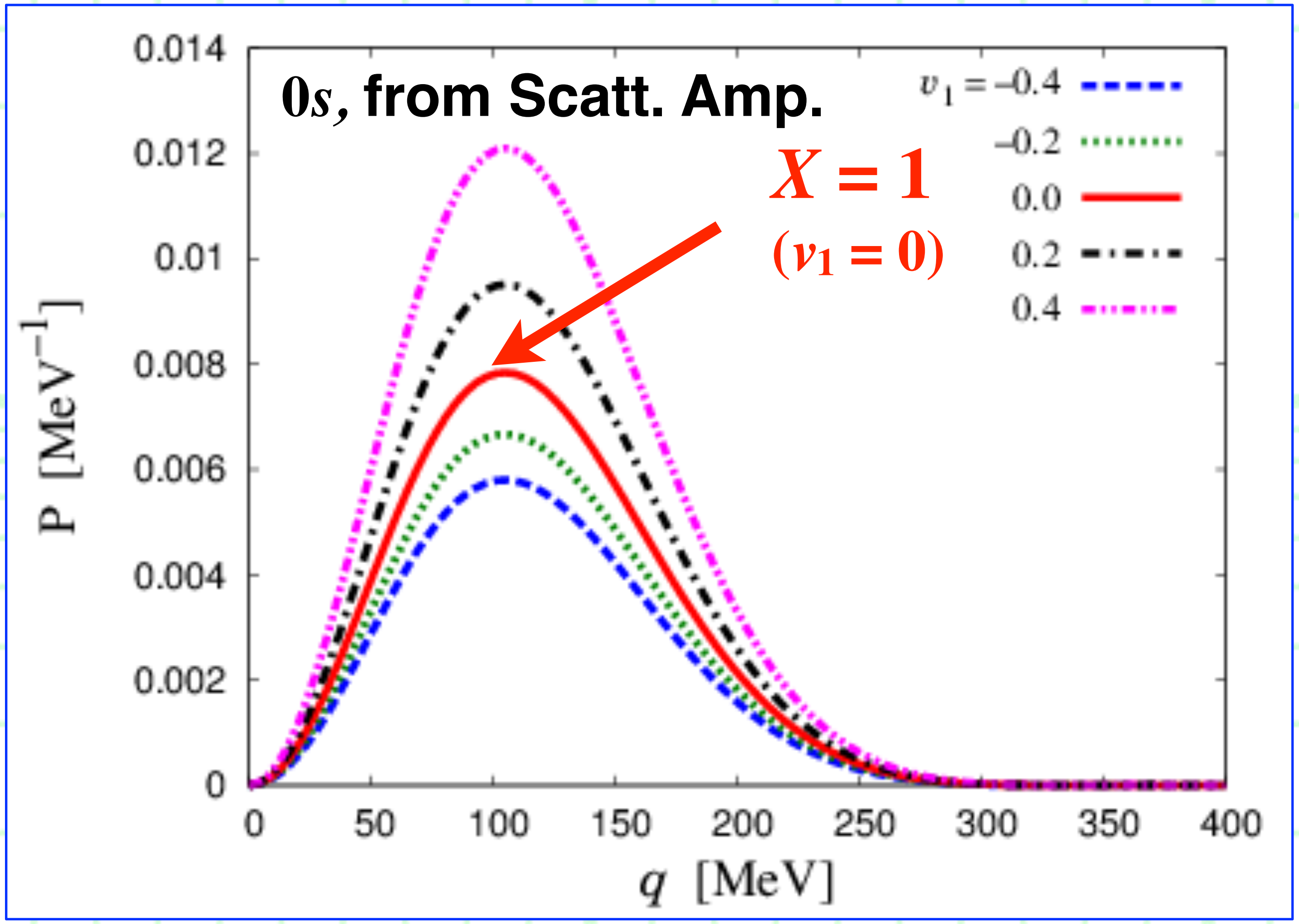
$$X \equiv \int \frac{d^3 q}{(2\pi)^3} \langle \tilde{\Psi} | \mathbf{q} \rangle \langle \mathbf{q} | \Psi \rangle = \int_0^\infty dq \, \mathrm{P}(q)$$

$$\mathrm{P}(q) = \frac{4\pi q^2}{(2\pi)^3} \left[\frac{\gamma(q)}{E_{\mathrm{pole}} - \mathcal{E}(q)} \right]^2$$

--- In the following, we calculate X from the scattering amplitude.

□ The compositeness is **unity for energy independent interaction**.

Hernandez and Mondragon (1984).



□ An interesting thing happens when **we introduce the energy dependence for V**.

□ If the interaction depends on the energy, the compositeness from the scattering amplitude **deviates from unity**.

$$V(r;\, E) \propto [v_0 + v_1 (E - E_{\mathrm{pole}})]$$

2. Wave functions from amplitudes

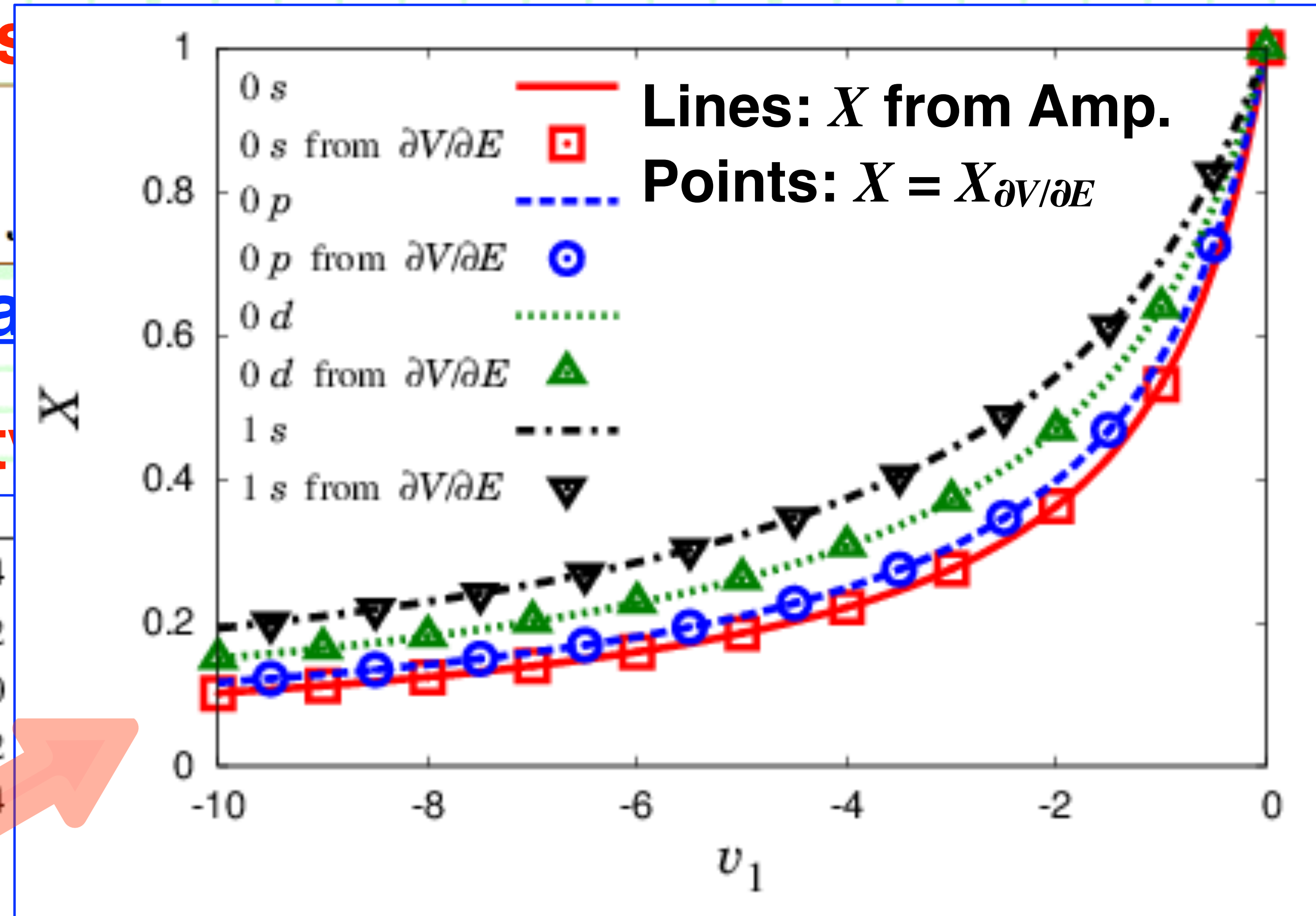
++ Example: Stable bound state ++

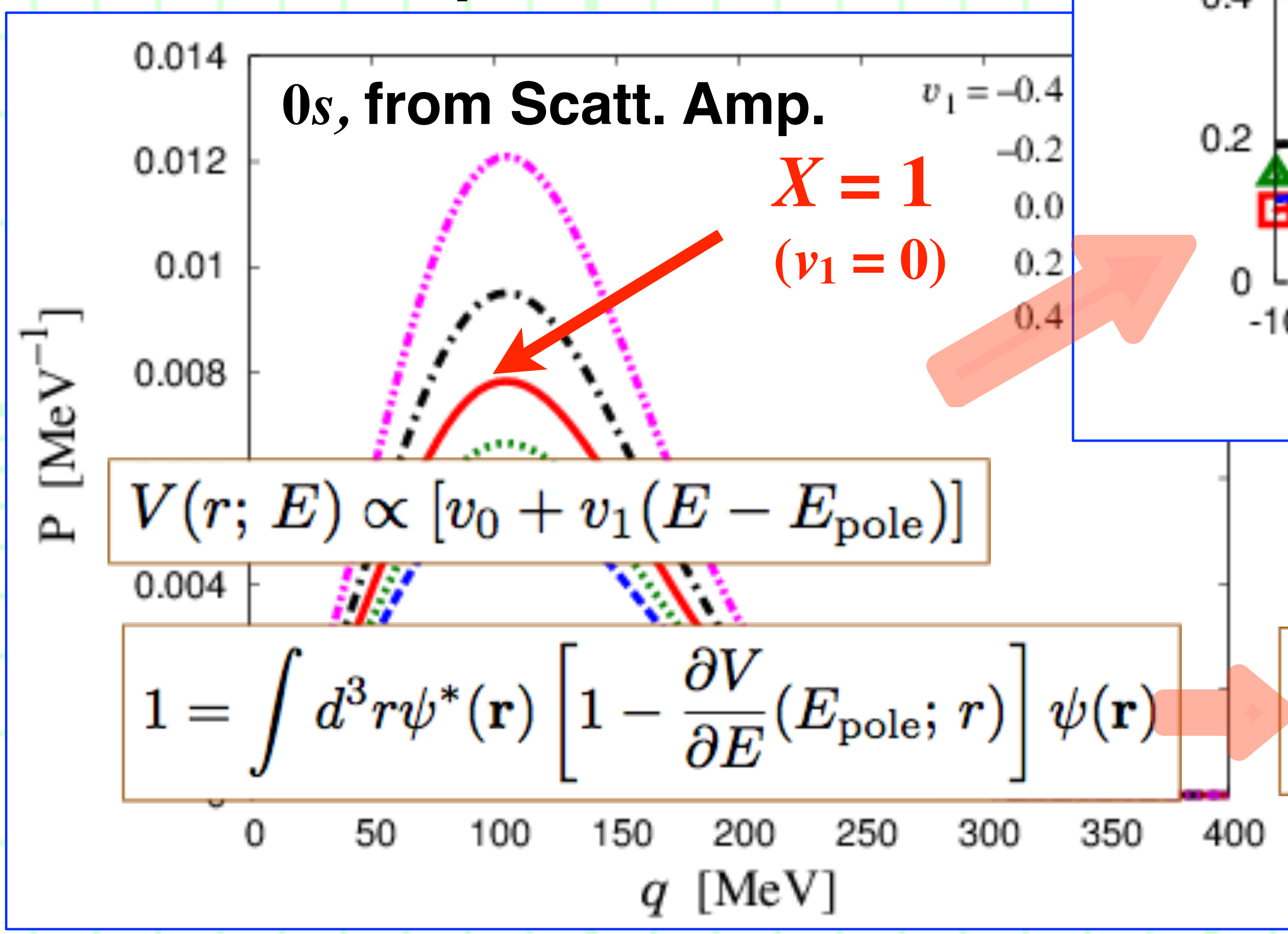
- We define **the compositenes**

$$X \equiv \int \frac{d^3q}{(2\pi)^3} \langle \tilde{\Psi} | \mathbf{q} \rangle \langle \mathbf{q} | \Psi \rangle =$$

--- In the following, we calcula

□ The compositeness is unit





$$V(r;\, E) \propto [v_0 + v_1(E - E_{\rm pole})]$$

$$1 = \int d^3r \psi^*(\mathbf{r}) \left[1 - \frac{\partial V}{\partial E}(E_{\rm pole};\, r) \right] \psi(\mathbf{r})$$

- **Consistent with the norm with energy-dep. interaction.**

$$X_{\partial V/\partial E} = 1 + \int d^3r \psi^*(\mathbf{r}) \frac{\partial V}{\partial E}(E_{\rm pole};\, r) \psi(\mathbf{r})$$

Formanek, Lombard and Mares (2004);
Miyahara and Hyodo (2016).

2. Wave functions from amplitudes

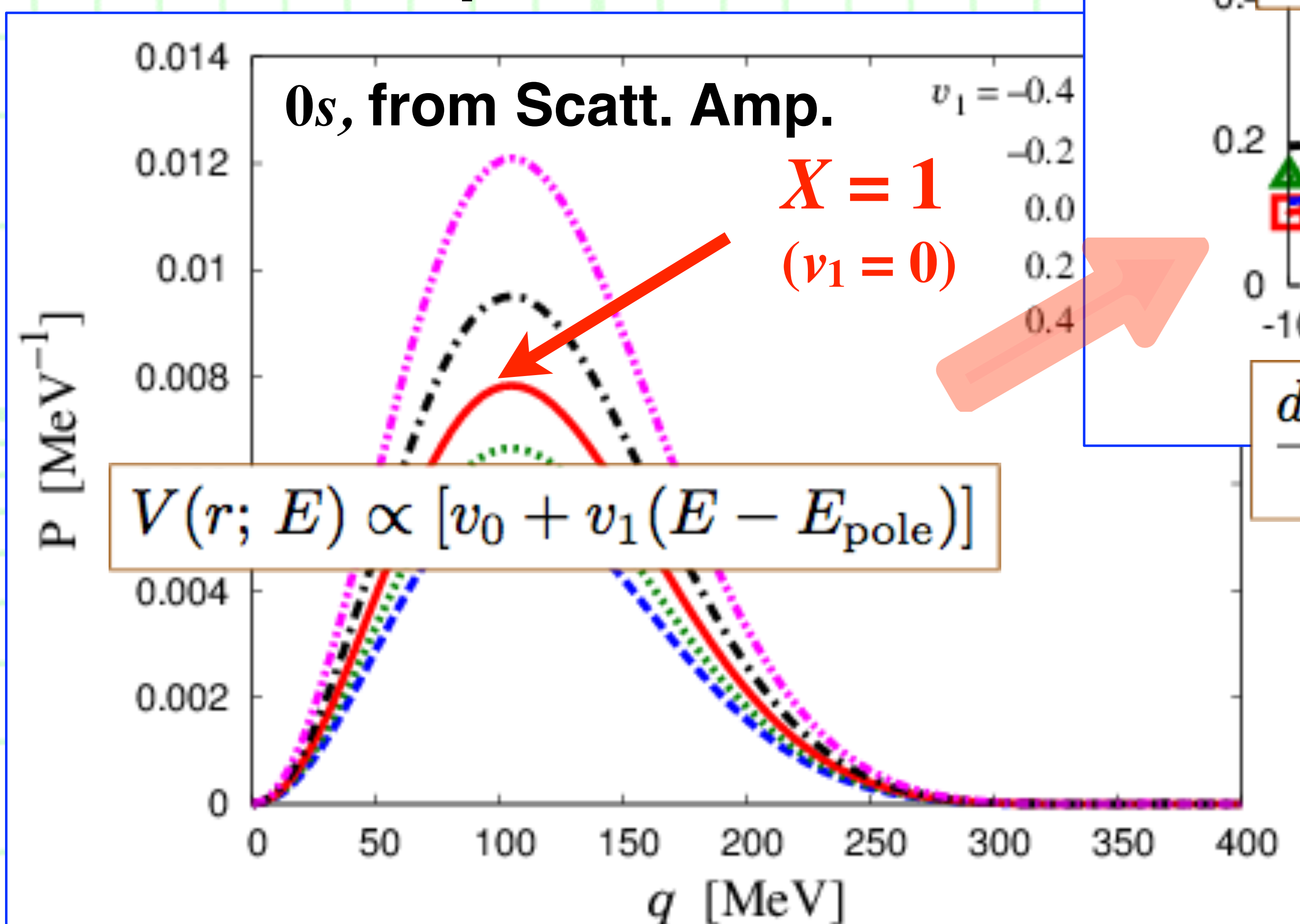
++ Example: Stable bound state ++

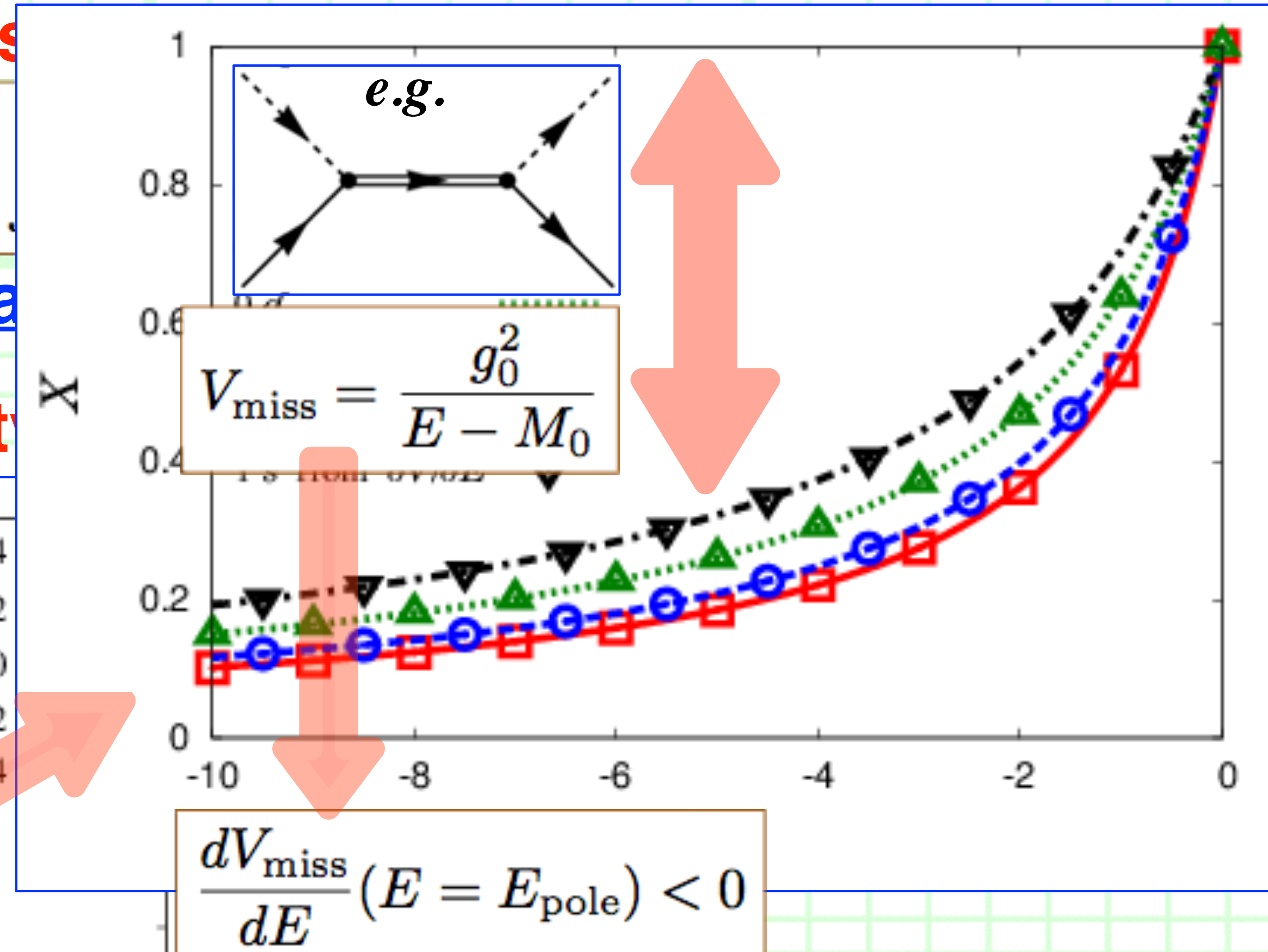
- We define **the compositenes**

$$X \equiv \int \frac{d^3 q}{(2\pi)^3} \langle \tilde{\Psi} | \mathbf{q} \rangle \langle \mathbf{q} | \Psi \rangle =$$

--- In the following, we calcula

□ The compositeness is unit





- **Deviation of compositeness from unity** can be interpreted **as a missing-channel part.**

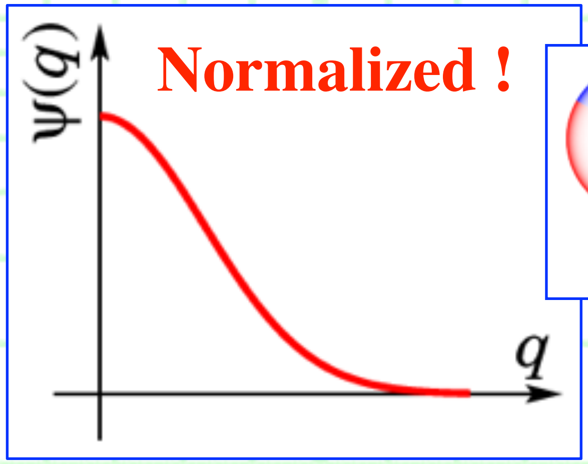
T. S. , Hyodo and Jido, *PTEP* 2015 063D04.

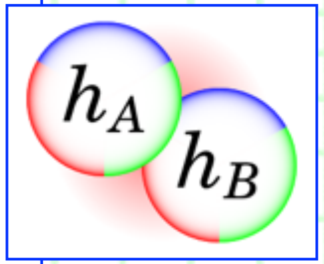
3. The N^* compositeness program

++ What I want to do is ++

- For a given interaction, we can calculate two-body wave functions from the scattering amplitude.
  - In particular, compositeness (= the norm of the wave function) is automatically normalized !

$$T(E; \mathbf{q}', \mathbf{q}) \approx \frac{\gamma(q')\gamma(q)}{E - E_{\text{pole}}}$$





  - For an energy dependent interaction, the compositeness deviates from unity, interpreted as a missing channel contribution.

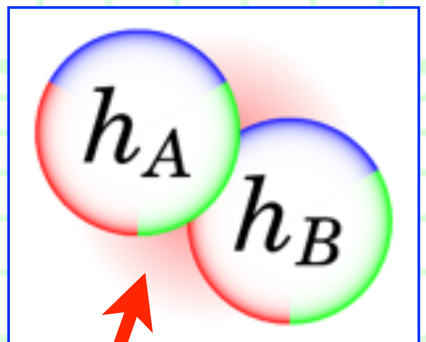
- Therefore, we can investigate:
  - Compositeness for "interesting" resonances from amplitudes.
  - Experimental information on the scattering amplitudes available.
  - Construction of detailed interactions possible.

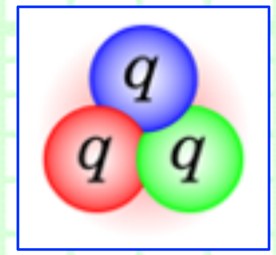
3. The N^* compositeness program

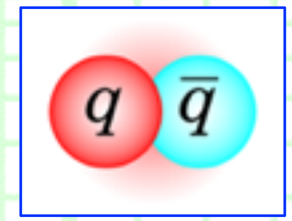
++ Wave functions for hadrons ++

- By using **the two-body wave function and compositeness** (norm), we can distinguish a certain configuration of hadrons in a model.

Hadronic molecules as a bound state of hadrons (*cf.* deuteron)



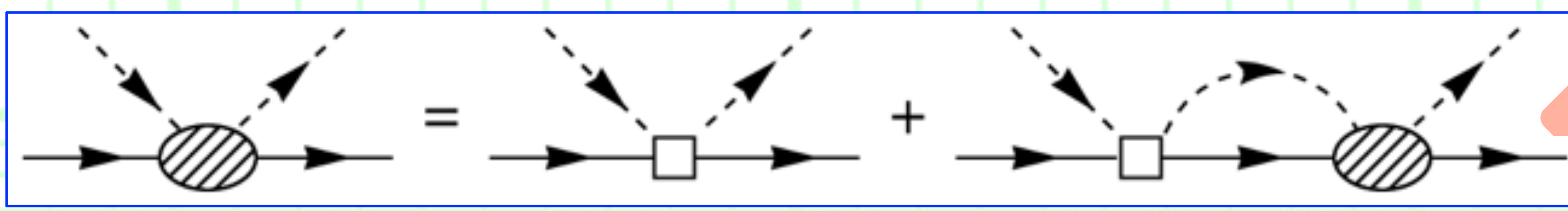




Ordinary hadrons

$$\langle \tilde{\Psi} | \Psi \rangle = X + Z = 1$$

$$X = \int \frac{d^3 q}{(2\pi)^3} \langle \tilde{\Psi} | \mathbf{q} \rangle \langle \mathbf{q} | \Psi \rangle = \int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}(\mathbf{q}) \right]^2$$



- In the previous studies, we have investigated:
  - □ $\Lambda(1405)$. □ $\Xi(1690)$. □ $N(1535)$ & $N(1650)$. □ ...

T. S. , Hyodo and Jido, *PTEP* 2015, 063D04; T. S. , *PTEP* 2015 091D01;
T. S. T. Arai, J. Yamagata-Sekihara and S. Yasui, *Phys. Rev.* C93 (2016) 035204..

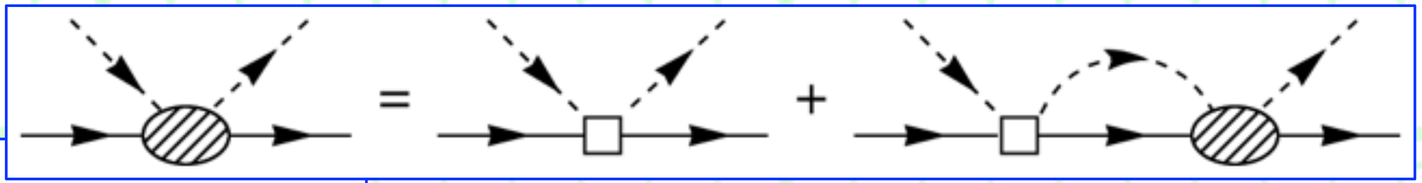
--- Evaluated X for these "dynamically generated resonances".

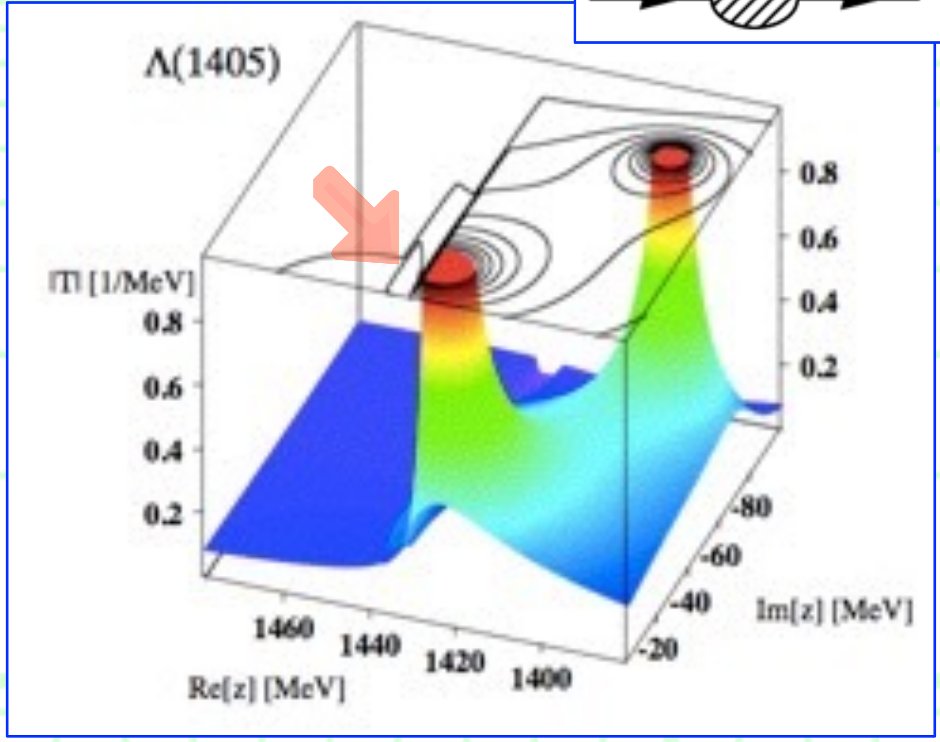
3. The N^* compositeness program

++ Example: compositeness for Λ(1405) ++

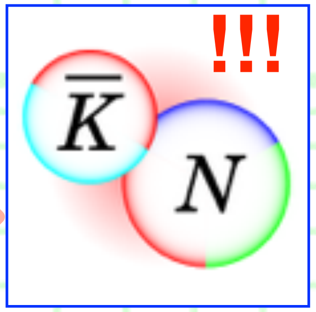
- **Compositeness X for Λ(1405) in the chiral unitary approach.**

Amplitude taken from: Ikeda, Hyodo and Weise, *Phys. Lett.* B706, (2011) 63; *Nucl. Phys.* A881 (2012) 98.





Hyodo and Jido ('12).



	Λ(1405), higher pole	Λ(1405), lower pole
$\sqrt{s_{\rm pole}}$	$1424 - 26i$ MeV	$1381 - 81i$ MeV
$X_{\bar{K}N}$	$1.14 + 0.01i$	$-0.39 - 0.07i$
$X_{\pi\Sigma}$	$-0.19 - 0.22i$	$0.66 + 0.52i$
$X_{\eta\Lambda}$	$0.13 + 0.02i$	$-0.04 + 0.01i$
$X_{K\Xi}$	$0.00 + 0.00i$	$-0.00 + 0.00i$
Z	$-0.08 + 0.19i$	$0.77 - 0.46i$

T. S. , Hyodo and Jido, *PTEP* 2015, 063D04.

--- **Large $\bar{K}N$ component for (higher pole) Λ(1405), since $X_{\bar{K}N}$ is almost unity with small imaginary parts.**

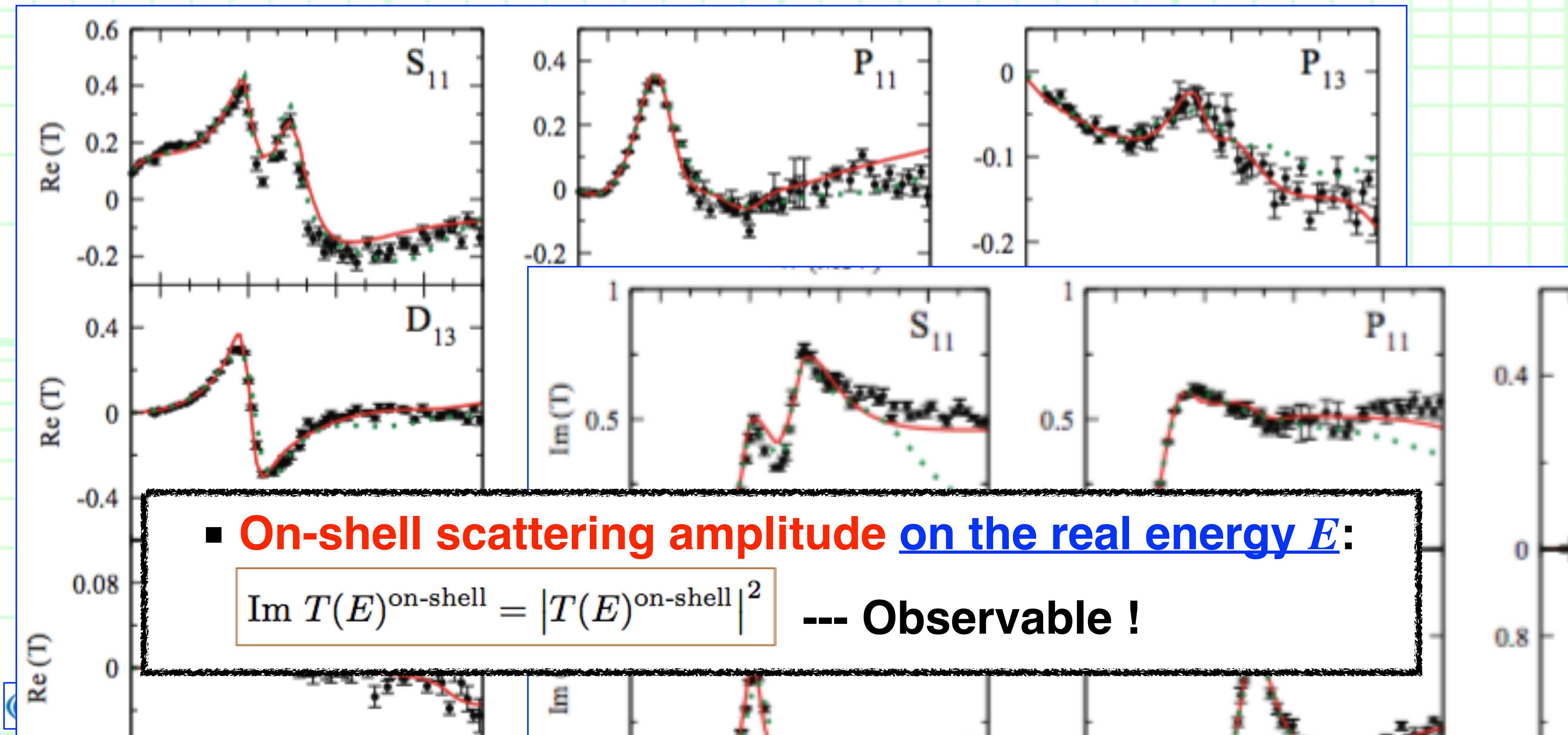
3. The N^* compositeness program

++ The N^* compositeness from πN amplitude ++

- Next target: Comprehensive analysis of the N^* and Δ^* resonances from the precise on-shell πN amplitude !

--- The precise on-shell πN scattering amplitude is available.

Kamano *et al.*, *Phys. Rev.* C88 (2014) 035209.



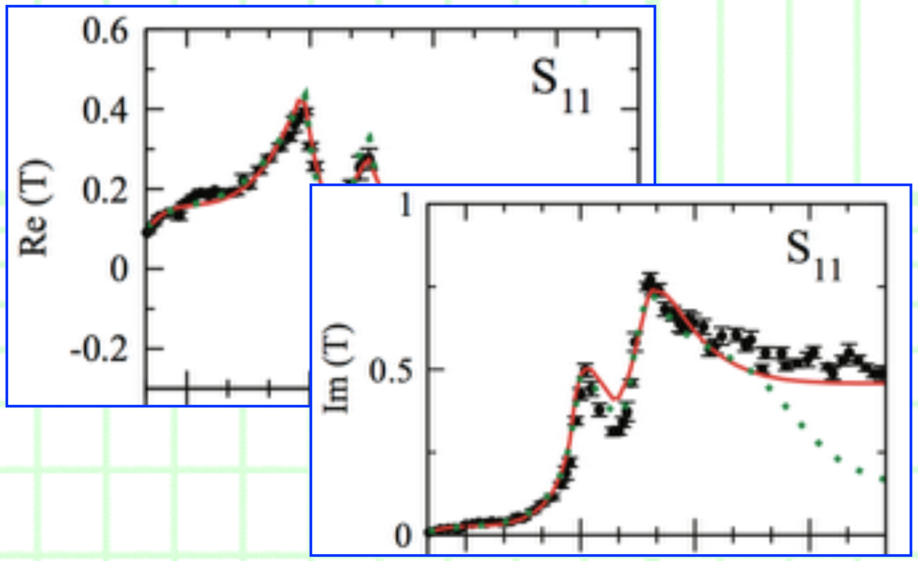
- On-shell scattering amplitude on the real energy E:

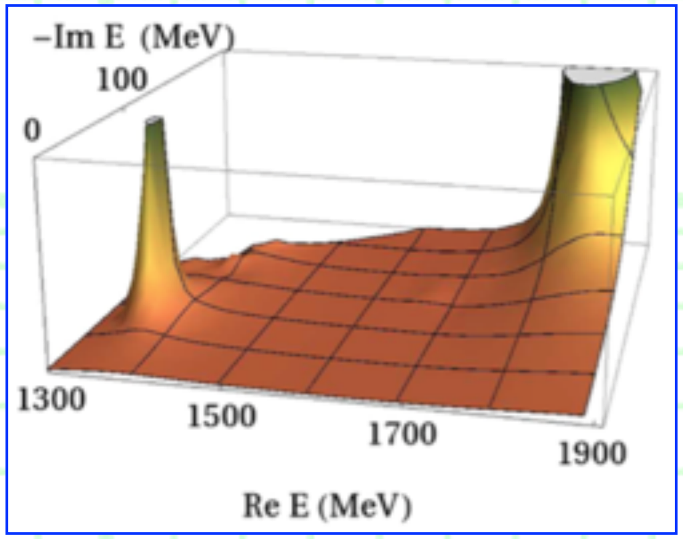
$$\mathrm{Im}\, T(E)^{\text{on-shell}} = \left| T(E)^{\text{on-shell}} \right|^2$$ --- Observable !

3. The *N** compositeness program

++ Many *N** resonances ++

- **Many *N** and Δ* resonances** from the π*N* scattering amplitude.





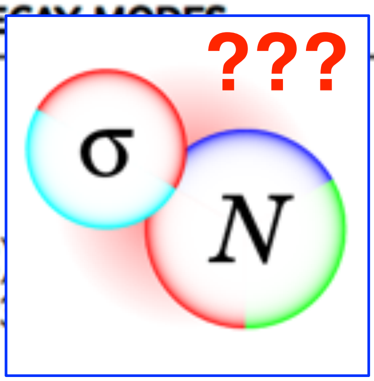
Suzuki *et al.*, *Phys. Rev. Lett.* 104 (2010) 042302.

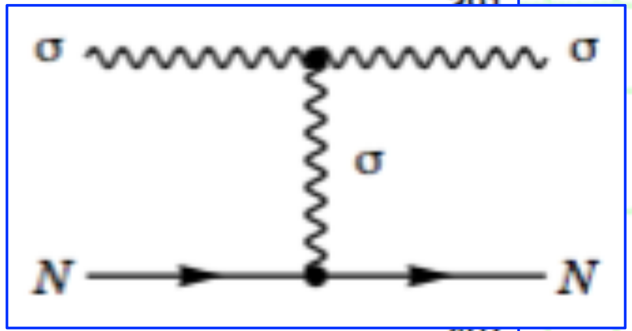
- There are **several "interesting" *N** resonances**, such as:

N(1440) 1/2+ $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ PDG.

Breit-Wigner mass = 1410 to 1450 (≈ 1430) MeV
Breit-Wigner full width = 250 to 450 (≈ 350) MeV

N(1440) DE	Fraction (Γ_i/Γ)	p (MeV/c)
$N\pi$	55–75 %	391
$N\eta$	<1 %	
$N\pi\pi$	25–50 %	
Δ(1232)	20–30 %	
Δ(12	13–27 %	
$N\sigma$	11–23 %	
$p\gamma$, helicity=1/2	0.035–0.048 %	407
$n\gamma$, helicity=1/2	0.02–0.04 %	406





- We can now investigate **their internal structure in terms of the meson-baryon component**.

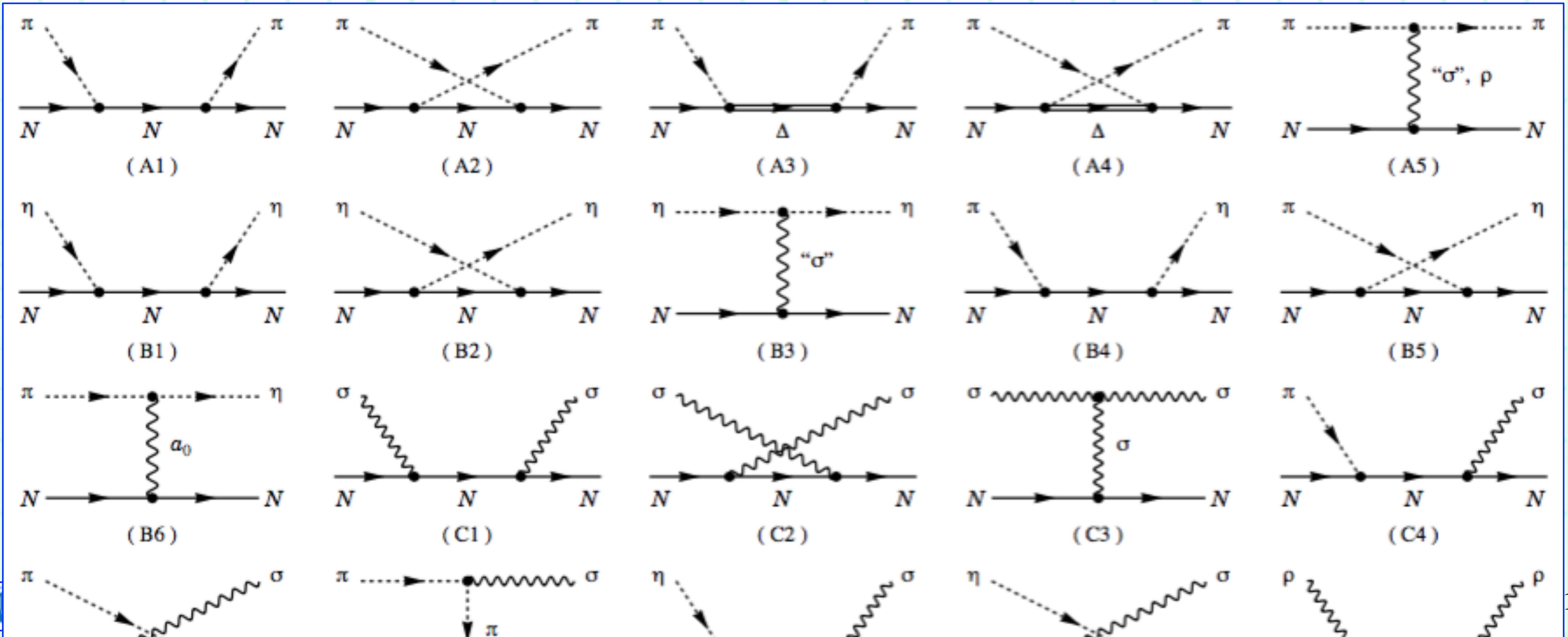
--- ***N*(1440) is a σ*N* bound state ?** *cf.* Jülich group.

Rönchen *et al.* (2013); ...

3. The N^* compositeness program

++ From on-shell to off-shell amplitude ++

- By using the on-shell πN amplitude (<-- observable), I construct the off-shell amplitude, where the N^* wave functions live.
- I take into account bare N^* states and appropriate diagrams for the meson-baryon interaction.
- How much the physical N^* are "dressed" ?



3. The N^* compositeness program

++ Numerical results ++

- Numerical results ...

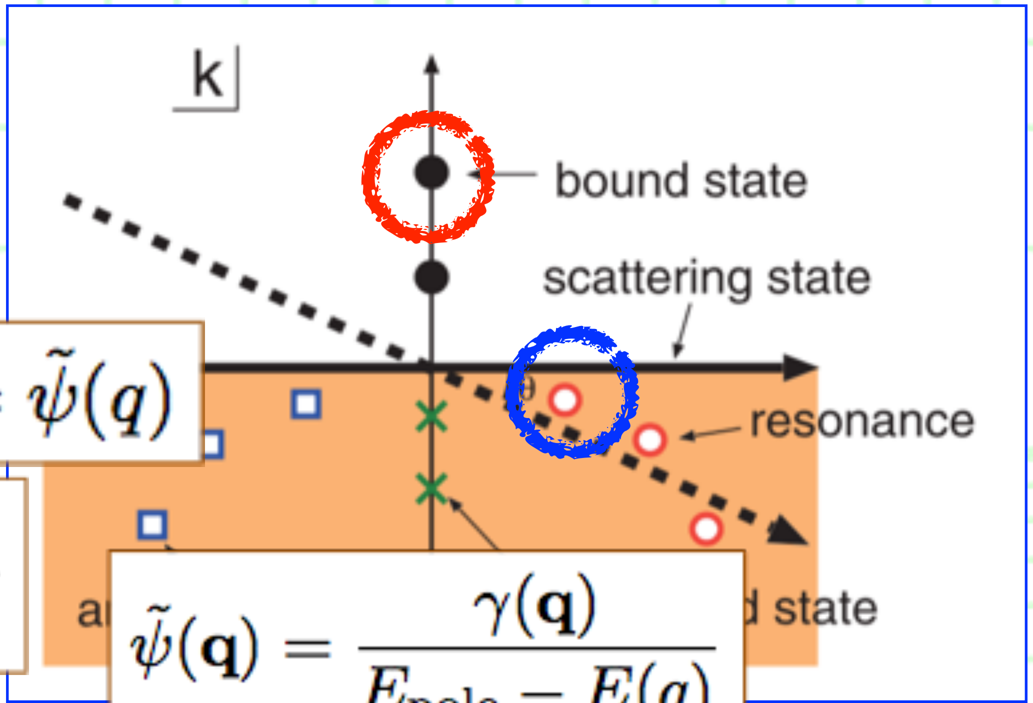
--- Sorry, but now on going !

- If you have your own πN amplitudes as solutions of the Lippmann-Schwinger Eq., you can calculate the N^* compositeness in the manner presented here.

--- Why don't you join me ?

4. Summary

- We can extract the two-body WF from the residue of the scattering amplitude at the pole position, both stable and unstable states.



$$\langle \mathbf{q} | \Psi \rangle = \tilde{\psi}(q)$$

Scattering amplitude: $$T(E;\mathbf{q}',\mathbf{q}) = \langle \mathbf{q}' | \hat{T}(E) | \mathbf{q} \rangle \approx \frac{\gamma(q')\gamma(q)}{E - E_{\mathrm{pole}}}$$

$$\tilde{\psi}(\mathbf{q}) = \frac{\gamma(\mathbf{q})}{E_{\mathrm{pole}} - E(q)}$$

- The WF from the scattering amplitude is automatically scaled.
  - The compositeness (= norm of the two-body WF) is unity for a bound state in an energy independent interaction.
  - For an energy dependent interaction, the compositeness deviates from unity, reflecting a missing channel contribution.

- From the precise πN amplitude with appropriate models, we can evaluate the compositeness for the N^* and Δ^* resonances.
  - In particular, what is the structure of the $N(1440)$ resonance ?

Thank you very much for your kind attention !

Appendix

Appendix

++ Wave function from Lippmann-Schwinger Eq. ++

- Near **the resonance pole position $E_{\rm pole}$**, amplitude is dominated by the pole term in the expansion by the eigenstates of H as

$$\langle \mathbf{q}' | \hat{T}(E) | \mathbf{q} \rangle \approx \langle \mathbf{q}' | \hat{V} | \Psi \rangle \frac{1}{E - E_{\rm pole}} \langle \tilde{\Psi} | \hat{V} | \mathbf{q} \rangle$$

$|\Psi\rangle$, $|\mathbf{q}_{\rm full}\rangle$, ... $\langle \tilde{\Psi}|$, $\langle \mathbf{q}_{\rm full}|$, ...

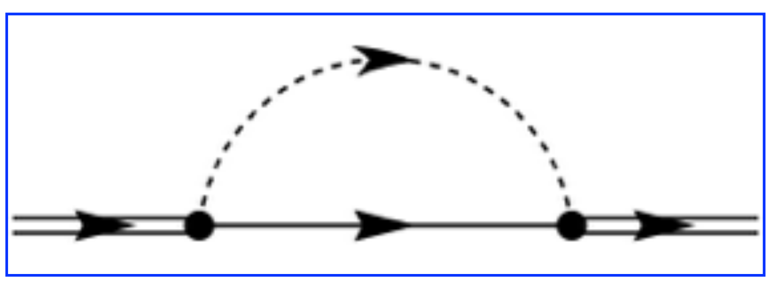
$$\rightarrow \quad \mathbb{1} = |\Psi\rangle \langle \tilde{\Psi}| + \cdots$$

□ **The idea of the renormalization for:**

$$\frac{1}{E - \hat{H}(E)} \approx |\Psi\rangle \frac{1}{E - E_{\rm pole}} \langle \tilde{\Psi}|$$

--- We "(re-)normalize" the total wave function as $\langle \tilde{\Psi} | \Psi \rangle = Z = 1$

cf.



$$\rightarrow \quad \frac{1}{p^2 - m_0^2 - \Sigma(p^2)} \approx \frac{Z}{p^2 - m_{\rm phys}^2}$$

Appendix

++ Model (in)dependence of compositeness ++

- **Compositeness is a model dependent quantity, in general.**
  - Because the wave function and interaction are not observable, they are model dependent quantities.

  --- In the present study, to calculate the residue $\gamma(q)$ and wave function, we need the off-shell amplitude, which is not observable and model dependent quantity.

$$T(E; \mathbf{q}', \mathbf{q}) \approx \frac{\gamma(q')\gamma(q)}{E - E_{\rm pole}} \quad \text{-->} \quad \tilde{\psi}(q) = \frac{\gamma(q)}{E_{\rm pole} - \mathcal{E}(q)}$$

  (E and q are independent as the off-shell Amp.)

  - Furthermore, the compositeness is also not observable and model dependent quantity.

  --- The field renormalization constant is not observable as well.

  cf. Deuteron d-wave probability $P_D \sim 5\%$ is not observable.

Appendix

++ Model (in)dependence of compositeness ++

- Compositeness is a model dependent quantity, in general.

  - Note: We can uniquely determine the compositeness once we fix the interaction (including its energy dependence).

cf. In the pioneering studies, they fixed the interaction first and discussed the compositeness from the scattering amplitude.

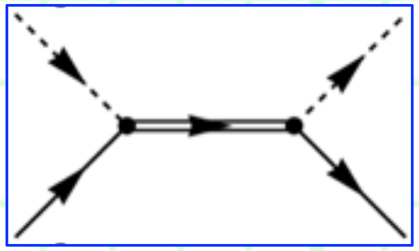
--- Separable interaction.

$$\langle \vec{p}' | V | \vec{p} \rangle = V(\vec{p}, \vec{p}') = v(2l+1)\Theta(\Lambda - p)\Theta(\Lambda - p')P_l(\cos\theta)|\vec{p}|^l|\vec{p}'|^l,$$

Gamermann, Nieves, Oset and Ruiz Arriola, *Phys. Rev.* D81 (2010) 014029;
Yamagata-Sekihara, Nieves and Oset, *Phys. Rev.* D83 (2011) 014003;
Aceti and Oset, *Phys. Rev.* D86 (2012) 014012; ...

--- Interaction with the Yukawa coupling to a bare state.

Hyodo, Jido and Hosaka, *Phys. Rev.* C85 (2012) 015201.



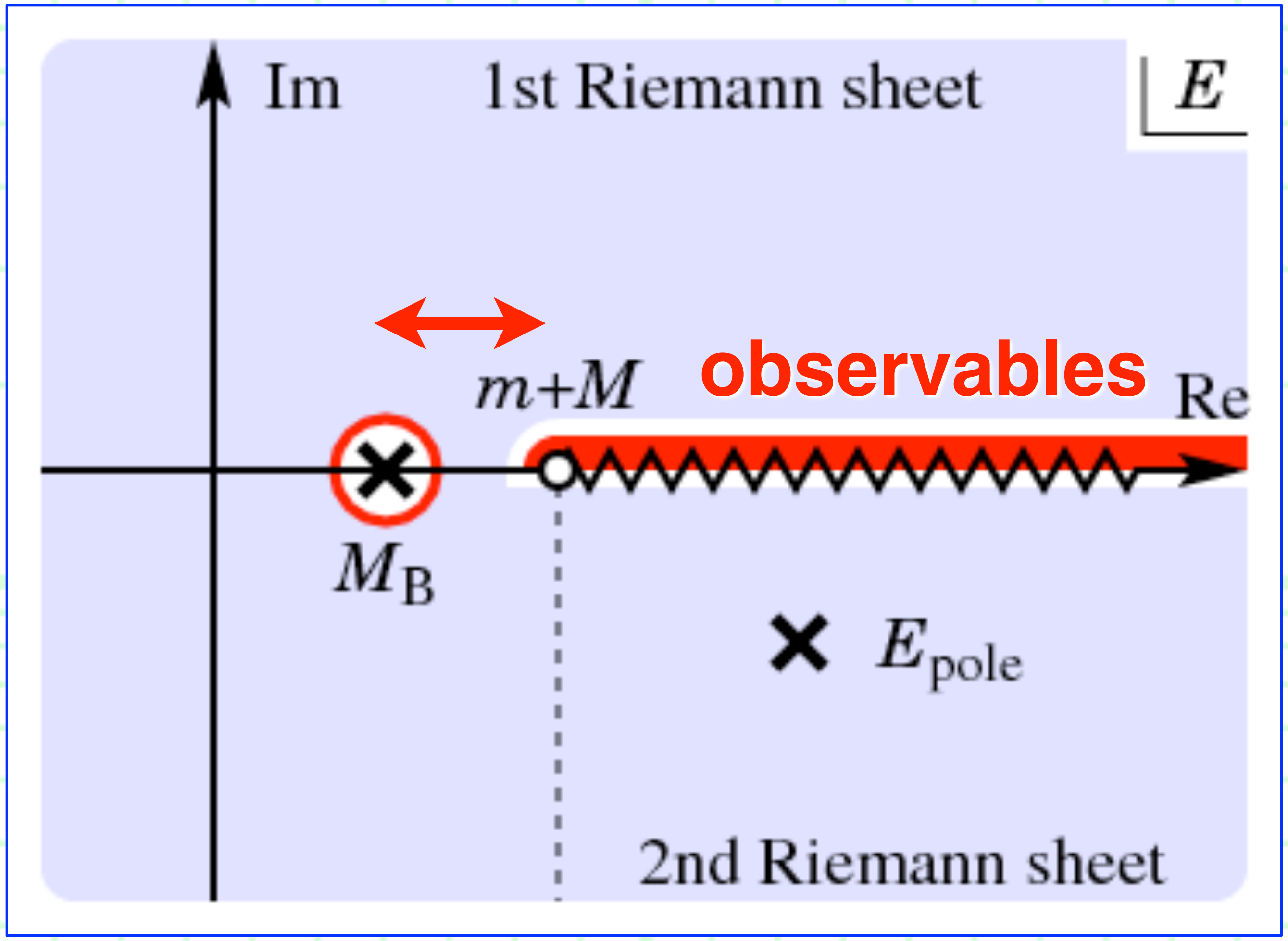
Appendix

++ Model (in)dependence of compositeness ++

- If the pole exists near threshold, compositeness becomes a model independent quantity.

--- Compositeness can be expressed with threshold parameters such as scattering length and effective range, which are observable.

- □ Deuteron.
  Weinberg ('65).
- □ $f_0(980)$ and $a_0(980)$.
  Baru *et al.* ('04),
  Kamiya-Hyodo, *Phys. Rev.* C93 (2016) 035203.
- □ Λ(1405).
  Kamiya-Hyodo, *Phys. Rev.* C93 (2016) 035203.
- □ ...



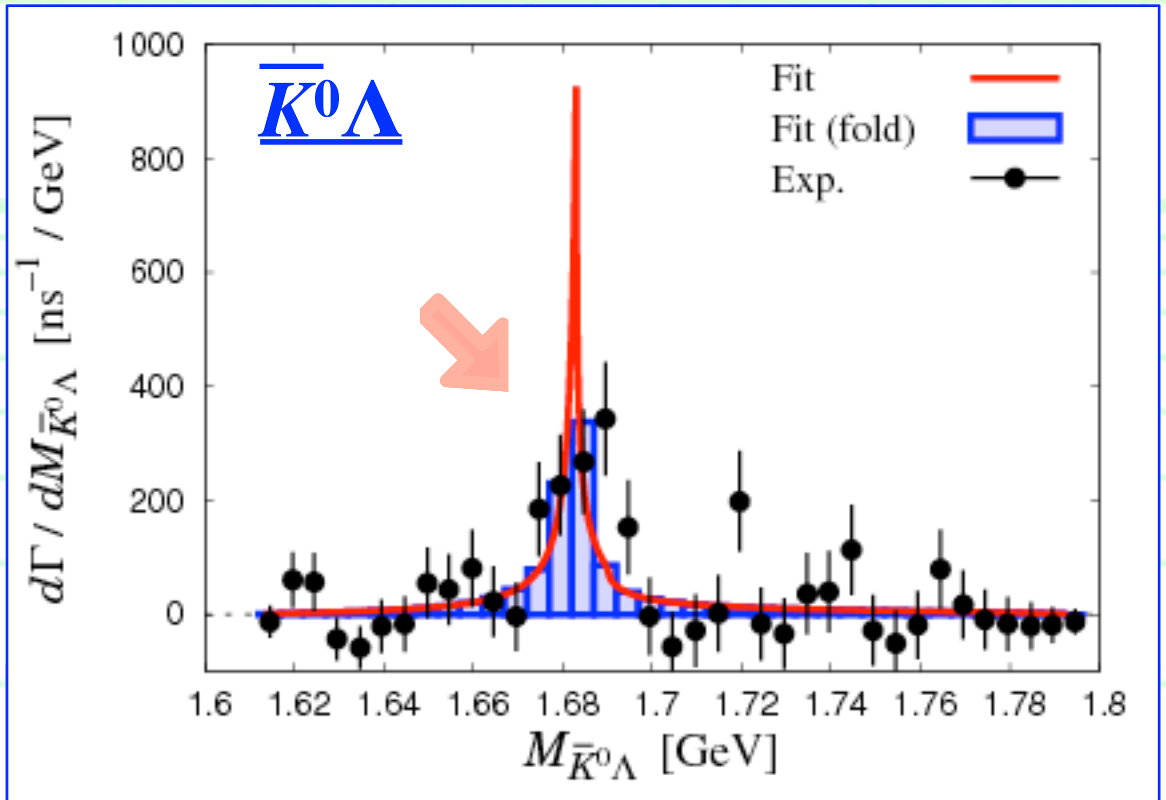
--- Model dependent part is a minor contribution to X !

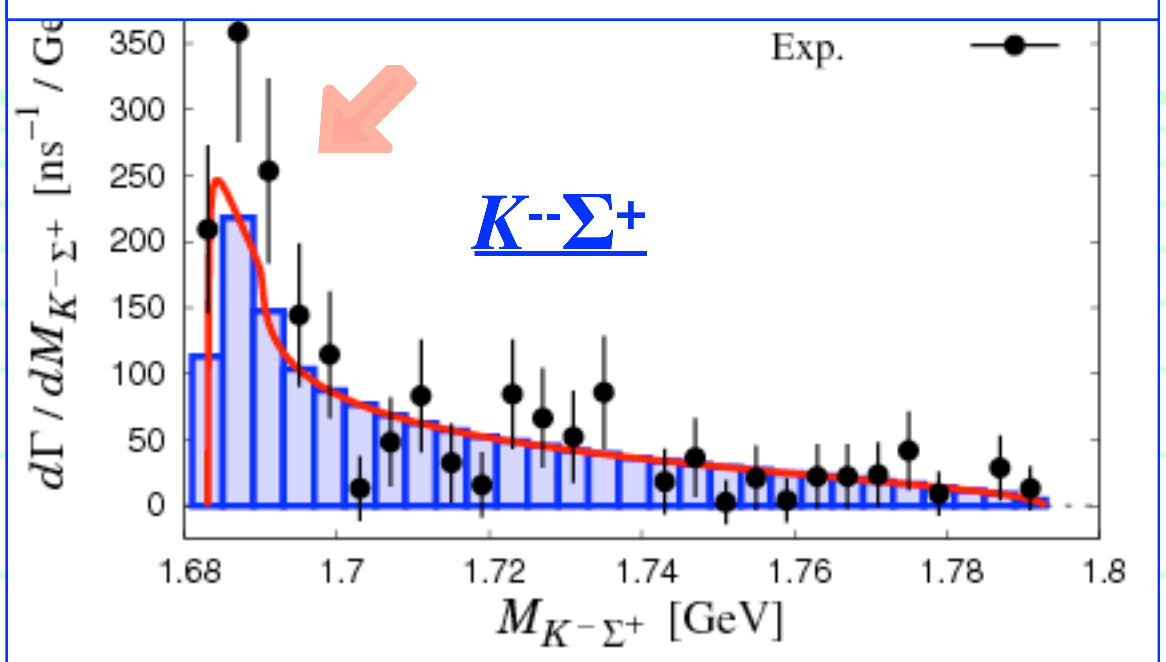
$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(m_\pi^{-1}), \quad r_e = -\frac{Z}{1-Z}R + \mathcal{O}(m_\pi^{-1}), \quad R \equiv \frac{1}{\sqrt{2\mu B}} = 4.318 \text{ fm}$$

Appendix

++ Compositeness for Ξ(1690) ++

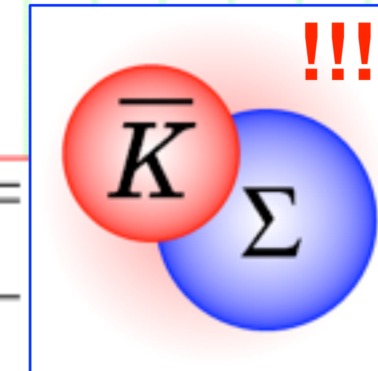
- **Compositeness X for Ξ(1690) in the chiral unitary approach.**





T. S. , *PTEP* 2015 091D01.

	$\Xi(1690)^0$
$\sqrt{s_{\rm pole}}$	$1684.3 - 0.5i$ MeV
$X_{K^-\Sigma^+}$	$0.83 - 0.31i$
$X_{\bar{K}^0\Sigma^0}$	$0.12 + 0.17i$
$X_{\bar{K}^0\Lambda}$	$-0.02 + 0.00i$
$X_{\pi^+\Xi^-}$	$0.00 + 0.00i$
$X_{\pi^0\Xi^0}$	$0.00 + 0.00i$
$X_{\eta\Xi^0}$	$0.01 + 0.02i$
Z	$0.06 + 0.11i$



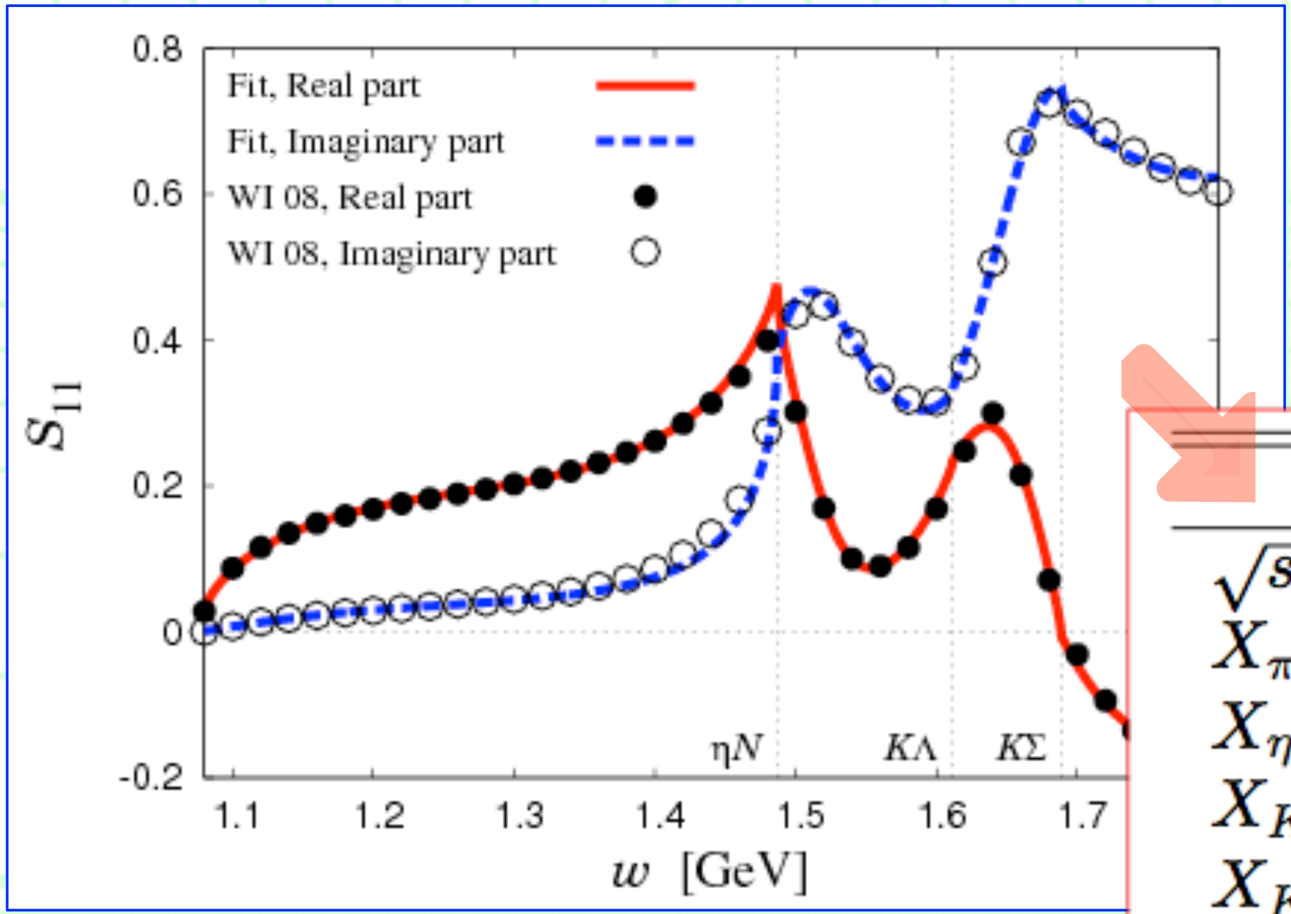
--- **Large $\bar{K}\Sigma$ component for Ξ(1690), since $X_{K\Sigma}$ is almost unity with small imaginary parts.**

Appendix

++ Compositeness for N(1535) and N(1650) ++

- Compositeness X for N(1535) & N(1650) in chiral unitary approach.

T. S. T. Arai, J. Yamagata-Sekihara and S. Yasui,
Phys. Rev. C93 (2016) 035204.



	$N(1535)$	$N(1650)$
$\sqrt{s_{\rm pole}}$ [MeV]	$1496.4-58.7i$	$1660.7-70.0i$
$X_{\pi N}$	$-0.02+0.03i$	$0.00+0.04i$
$X_{\eta N}$	$0.04+0.37i$	$0.00+0.01i$
$X_{K\Lambda}$	$0.14+0.00i$	$0.08+0.05i$
$X_{K\Sigma}$	$0.01-0.02i$	$0.09-0.12i$
Z	$0.84-0.38i$	$0.84+0.01i$

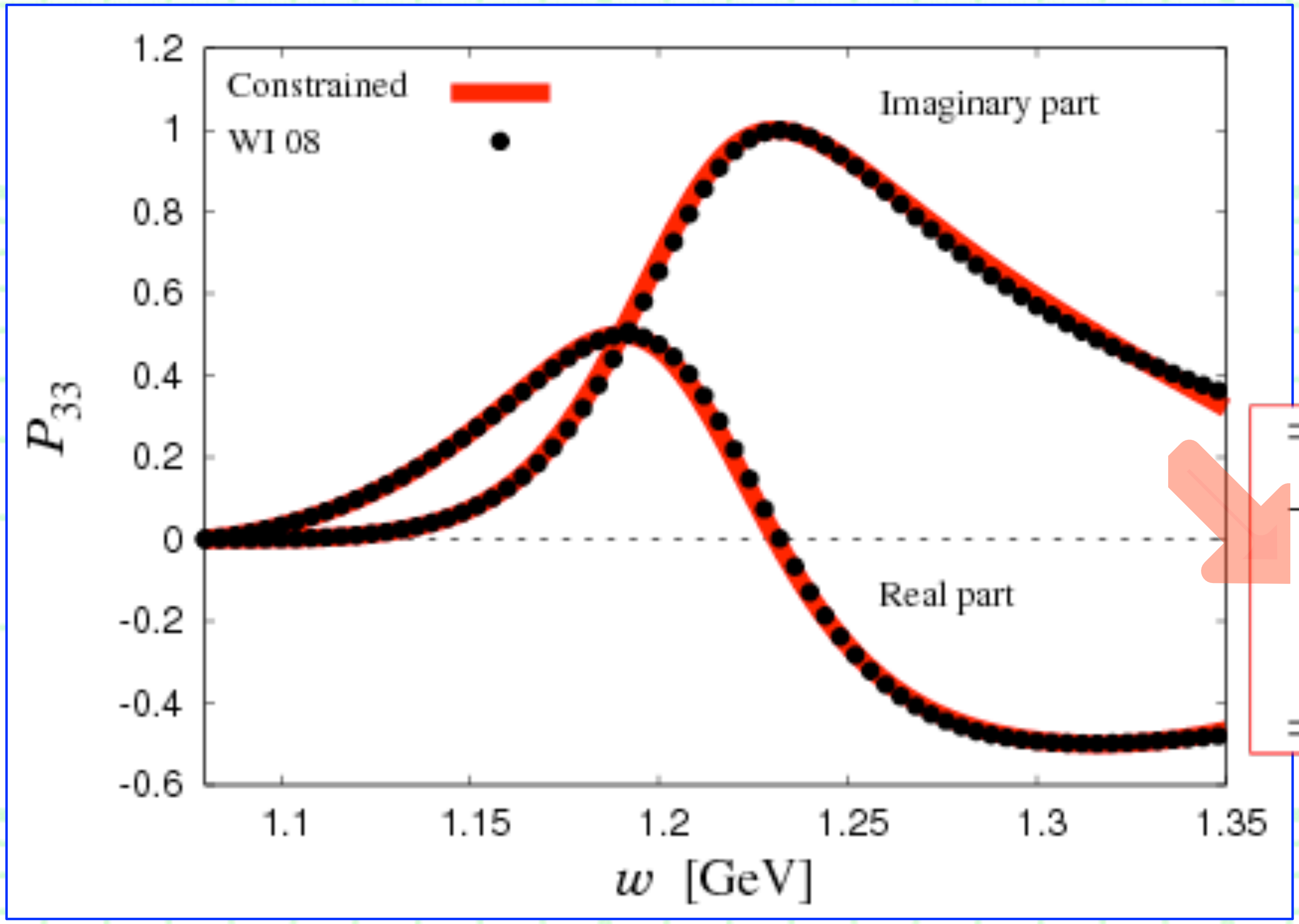
- For both N^* resonances, the missing-channel part Z is dominant.

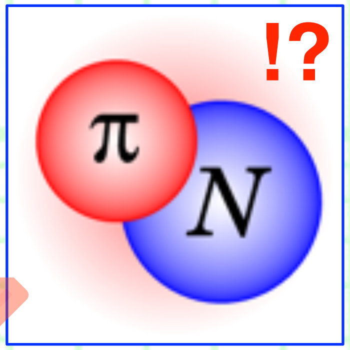
--> N(1535) and N(1650) have large components originating from contributions other than πN, ηN, $K\Lambda$, and $K\Sigma$.

Appendix

++ Compositeness for Δ(1232) ++

- **Compositeness X for Δ(1232) in chiral unitary approach.**





Constrained	Δ(1232)	$N(940)$
$\sqrt{s_{\rm pole}}$ [MeV]	$1206.9 - 49.6i$	938.9
$X_{\pi N}$	$0.87 + 0.35i$	0.00
Z	$0.13 - 0.35i$	1.00

- **The πN compositeness $X_{\pi N}$ takes large real part !** **But non-negligible imaginary part as well.**

--> Large πN component in the Δ(1232) resonance !?