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Modern problems in nuclear and elementary particle physics
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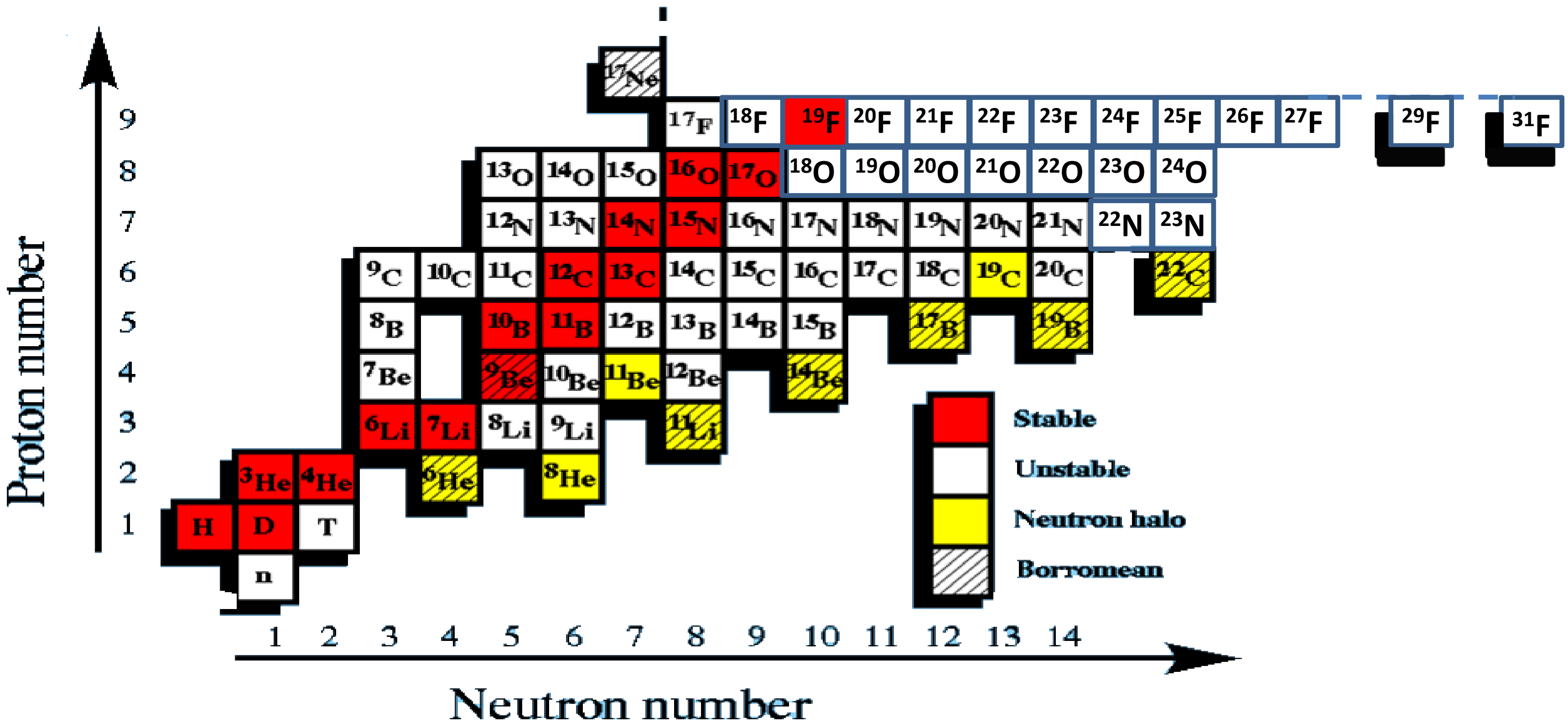
Cluster structure of light nuclei : the ^{22}C example


Proton number
Neutron number
Stable
Unstable
Neutron halo
Borromean
n
H
D
T
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4He
6He
8He
6Li
7Li
8Li
9Li
11Li
7Be
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10Be
11Be
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^{22}C was first observed to be bound in 1986

β-decay half-lives $t_{1/2} = 6.1^{+1.4}_{-1.2}$ ms in 2003

(evaluation, 2003)

$S_{2n} = 420 \pm 940$ keV

Reaction cross sections (σ_R) for

^{19,20,22}C + p at E = 40A MeV

A	19	20	22
σ_R (mb)	754 ± 22	791 ± 34	1338 ± 274

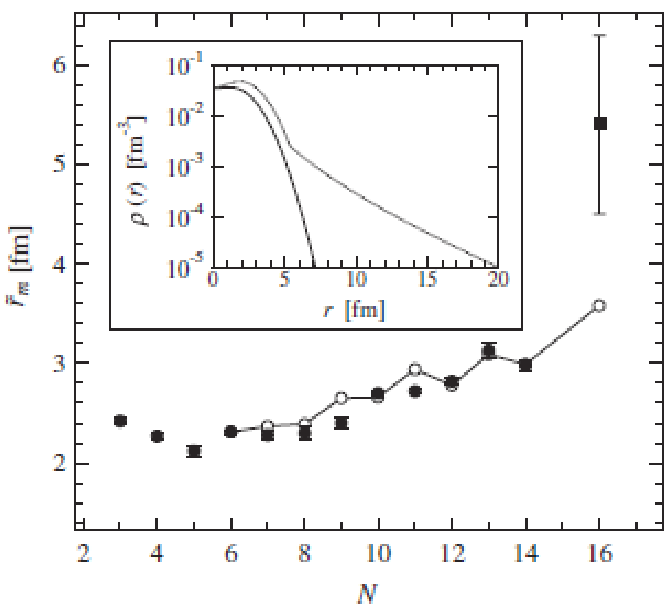
(direct time-of-flight measurement, 2012)

$S_{2n} = 0^{+320}_{-0}$ keV

Interaction cross sections (σ_I) for

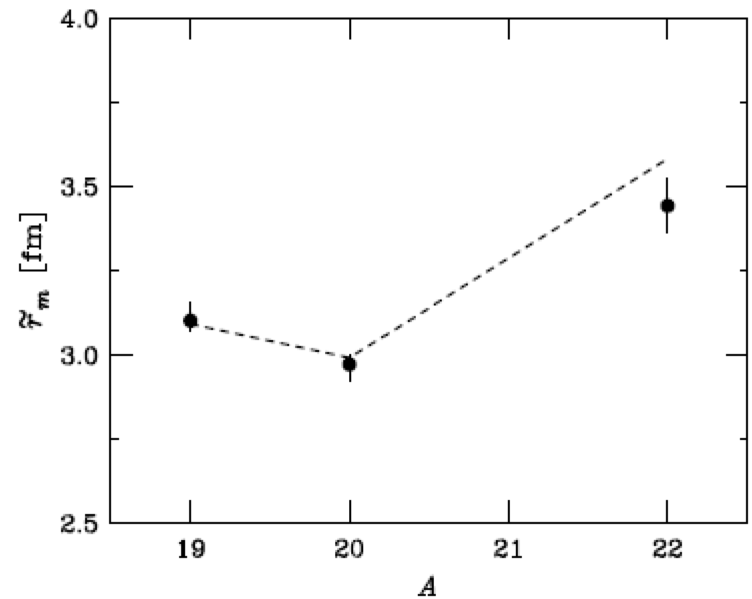
^{19,20,22}C + C at E = (235-307)A MeV

A	19	20	22
σ_I (mb)	1125 ± 25	1111 ± 8	1280 ± 22



$\langle r_m^2 \rangle^{1/2} = 5.4 \pm 0.9$ fm

K. Tanaka et al., PRL 104, 062701 (2010)



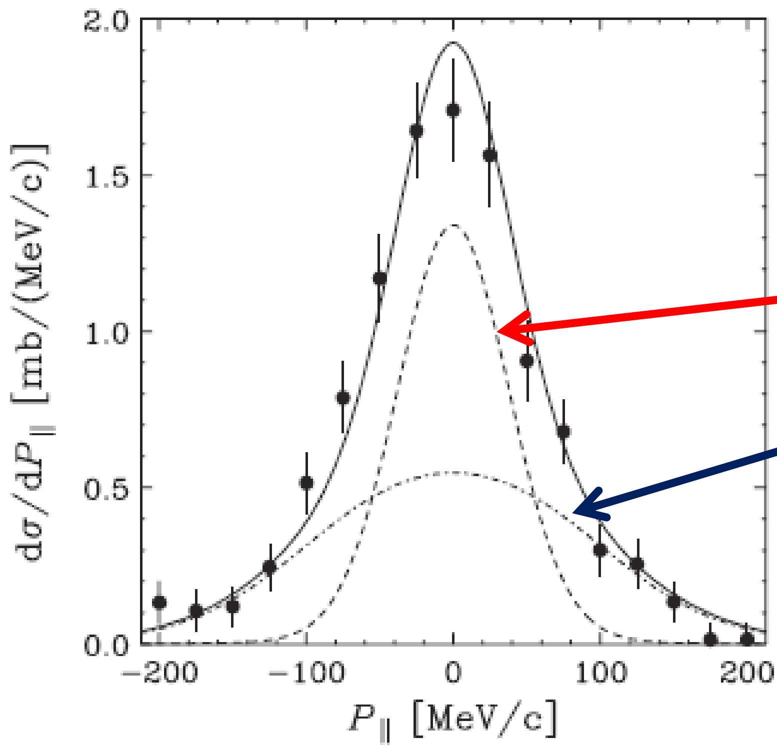
$\langle r_m^2 \rangle^{1/2} = 3.44 \pm 0.08$ fm

Y. Togano et al., PLB 761, 412 (2016)

S_{2n} of ^{22}C has large uncertainty
properties of unbound ^{21}C and ^{23}C are not well known

parallel momentum distribution of ^{20}C following two-neutron neutron removal reactions from ^{22}C

E/A $\sim$ 240 MeV



The individual contributions from the two dominant shell-model final states of ^{21}C

$J^\pi = 1/2^+$, $C^2S = 1.4$

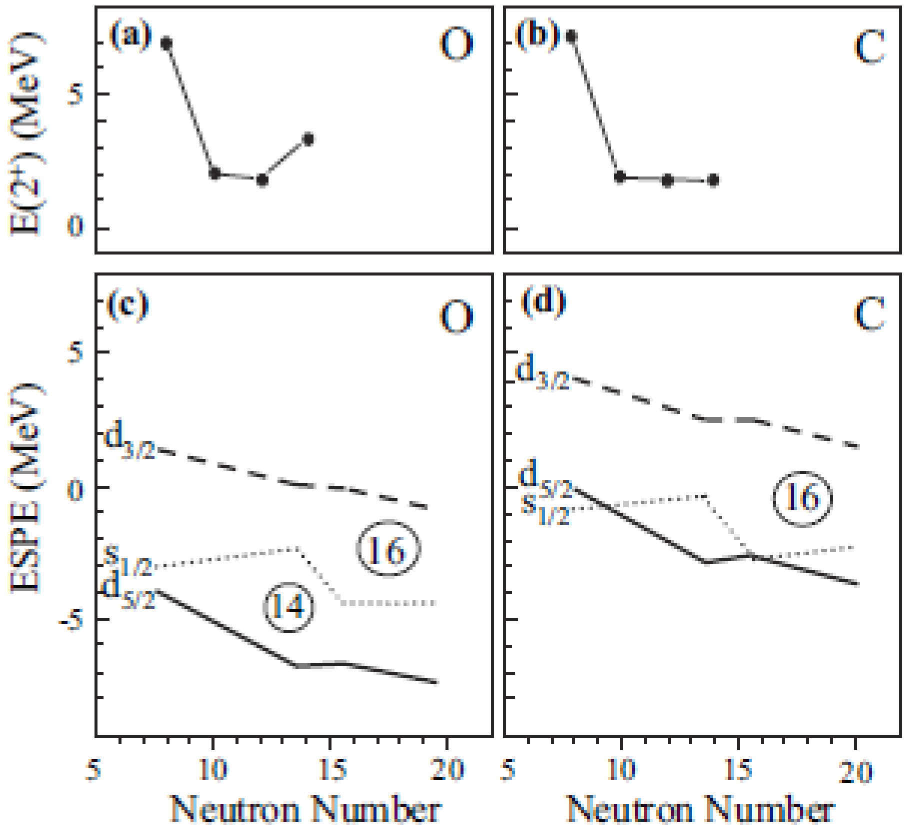
$J^\pi = 5/2^+$, $C^2S = 4.2$

the relative insensitivity of momentum distributions to the experimental uncertainty in $S_{2n}(^{22}\mathrm{C})$

N. Kobayashi et al., PRC 86, 054604 (2012)

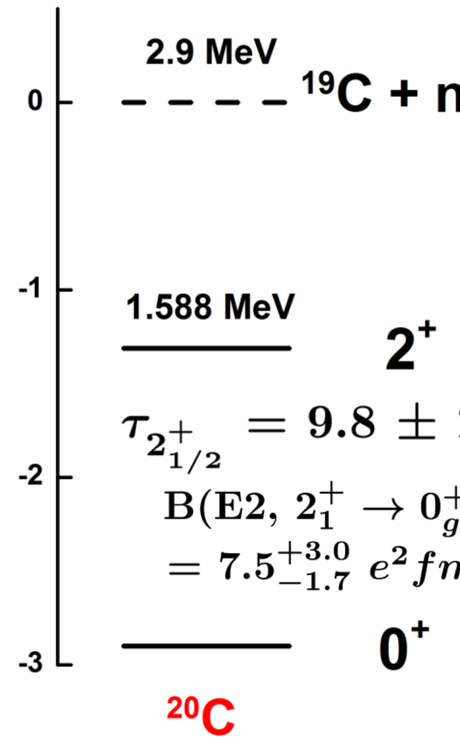
the spectroscopy of the neutron-rich C isotopes till ^{20}C

Evolution of the 2^+ energies and effective single-particle energies



N = 14 subshell gap is no longer present in the C isotopic chain

M. Stanoiu et al., PRC 78, 034315 (2008)



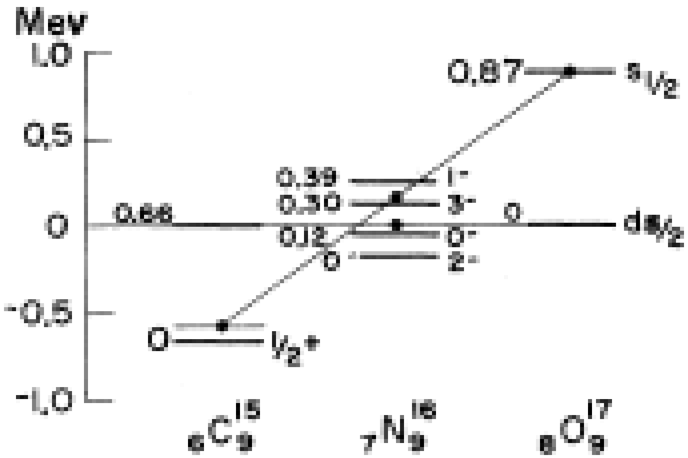
$\tau_{2^+_{1/2}} = 9.8 \pm 2.8$ ps

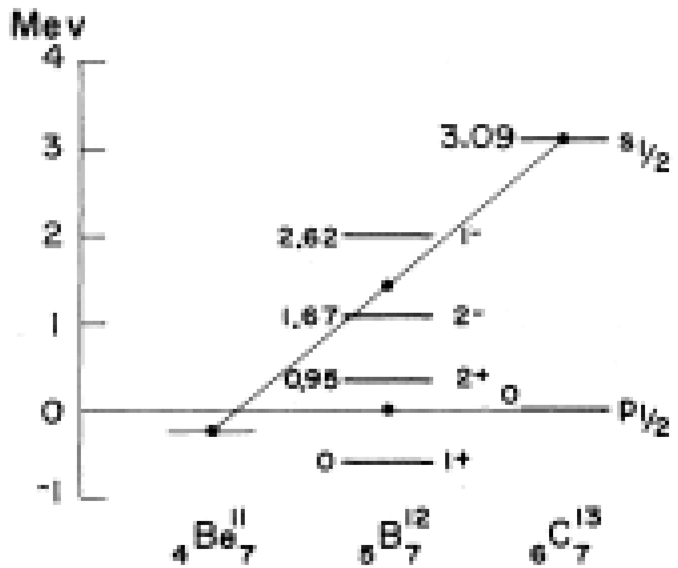
$\mathrm{B(E2,}\ 2_1^+ \rightarrow 0^+_{g.s.}) = 7.5^{+3.0}_{-1.7}\ e^2 fm^4$

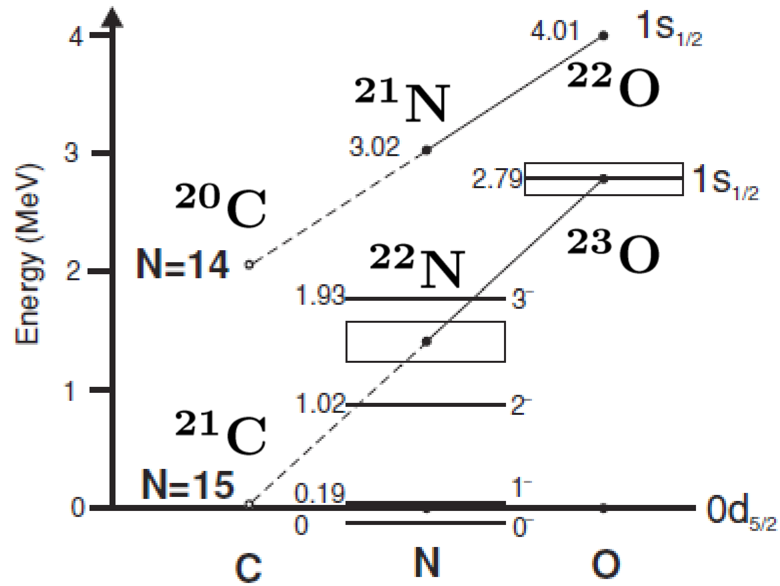
configuration mixing in C nuclei is large
the ground state as well as the 2_1^+ state in ^{16,18,20}C are dominated by $(d_{5/2},\ s_{1/2})^{2,4,6}$ configurations

close spacing between $s_{1/2}$ and $d_{5/2}$:
possibility of many halo configurations in C isotopes related to the loosely bound $s_{1/2}$ orbital

evolution of a shell gap with neutron number N can be extracted from the single particle or single hole levels in the N - 1 or N + 1 nuclei







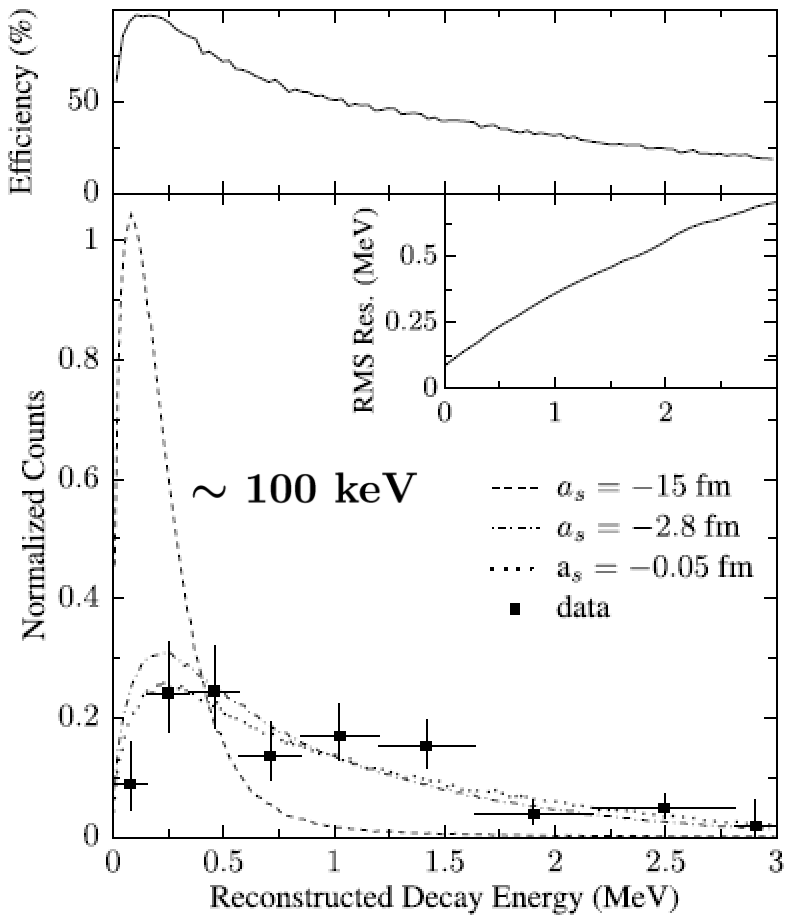
disappearance of the gap $\nu 1s_{1/2}$ - $\nu 0d_{5/2}$ and even a level inversion of the $(\nu 1s_{1/2}, \nu 0d_{5/2})$ levels in the nucleus ^{21}C seems likely

I. Talmi, I. Unna, PRL 4, 469 (1960)

M. J. Strongman et al., PRC 80, 021302(R) (2009)

A search for the neutron-unbound nucleus ^{21}C

^{21}C had been shown to be unbound in 1985



S. Mosby et al., NPA 909, 69 (2013)

the single-proton removal reaction from a beam of ^{22}N at 68 MeV/u

No evidence for a low-lying state

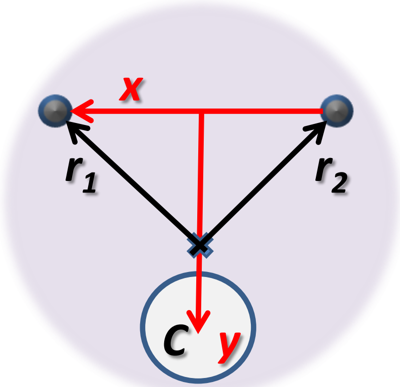
the ^{20}C + n decay-energy spectrum could be described with an *s*-wave line shape with a scattering length limit of -2.8 fm $< a_s <$ 0 fm

the $l = 2$ states are not populated by the reaction mechanism

the search for ^{21}C should concentrate on populating the *d* wave state

the first case:
the two-body subsystem of a Borromean two-neutron halo nucleus does not have a low-lying virtual state or resonance below 1 MeV

FEW-BODY CLUSTER MODELS



The Schrodinger equation

$$H_A\,\Psi(\mathbf{r}_1,\cdots,\mathbf{r}_A) = E\,\Psi(\mathbf{r}_1,\cdots,\mathbf{r}_A)$$

Total hamiltonian of the three-body cluster models $(A = A_C + 2)$

$$H_A = H_{A_C} + T_{x,y} + V(r_1, r_2) + \sum_{i=1}^{A_C} V(r_1, r_i) + \sum_{i=1}^{A_C} V(r_2, r_i)$$

wave function is factorized into a sum of products from two parts

$$\Psi(\mathbf{r}_1,\cdots,\mathbf{r}_A) = \sum_i \phi_i(\mathbf{r}_1,\cdots,\underline{\mathbf{r}}_{A_C})\,\psi_i(\mathbf{x},\mathbf{y})$$

The sum may include core excitations

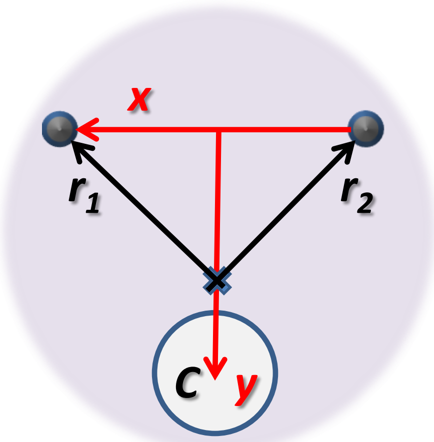
$$H_{A_C}\,\phi_i(\mathbf{r}_1,\cdots,\mathbf{r}_{A_C}) = \epsilon_i\,\phi_i(\mathbf{r}_1,\cdots,\mathbf{r}_{A_C})$$

Calculations of the bound states and continuum wave functions

Borromean nature of halo nuclei

(no bound states between pairs of clusters)

one type of the wave function asymptotic behaviour



$$\rho^2 = \mu_x \bar{\mathrm{x}}^2 + \mu_y \bar{\mathrm{y}}^2$$

$$\alpha_\rho = \arctan\left(\frac{\sqrt{\mu_x}\, x}{\sqrt{\mu_y}\, y}\right)$$

$$\Omega_5^\rho = \{\alpha_\rho, \hat{x}, \hat{y}\}$$

$$\frac{\kappa^2}{2m} = \frac{\bar{\mathrm{k}}_{\mathrm{x}}^2}{2\mu_x} + \frac{\bar{\mathrm{k}}_{\mathrm{y}}^2}{2\mu_y}$$

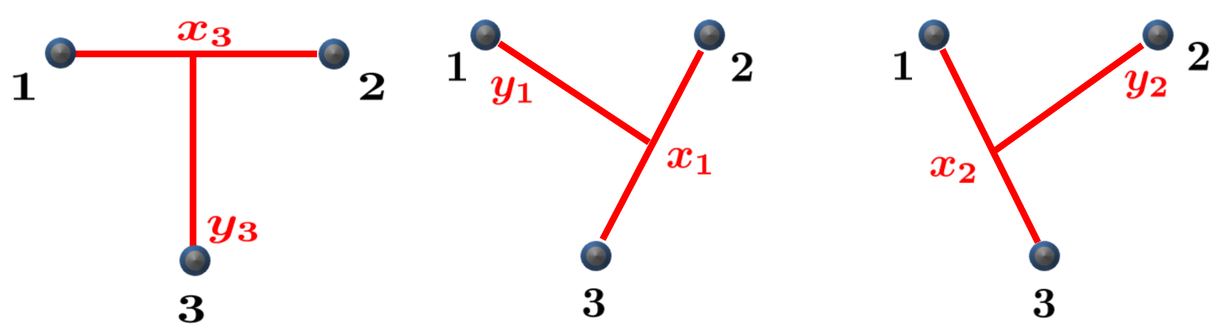
$$\alpha_\kappa = \arctan\left(\frac{\sqrt{\mu_y}\, \mathrm{k_x}}{\sqrt{\mu_x}\, \mathrm{k_y}}\right)$$

$$\Omega_5^\kappa = \{\alpha_\kappa, \hat{k}_x, \hat{k}_y\}$$

$$\{\bar{\mathrm{x}}, \bar{\mathrm{y}}\} \Rightarrow \{\rho\,, \Omega_5^\rho\}$$

$$\{\bar{\mathrm{k}}_x, \bar{\mathrm{k}}_y\} \Rightarrow \{\kappa\,, \Omega_5^\kappa\}$$

$\{\rho, \kappa\}$ are independent of the Jacobi system



The bound state wave function ($\gamma = \{K, l_x, l_y, L, S, I, j\}$)

$$\Psi_{JM}(\mathbf{r}_1, \cdots, \mathbf{r}_A) = \frac{1}{\rho^{5/2}} \sum_{\gamma} \chi^J_\gamma(\rho) \left[\Upsilon^{l_x l_y}_{KL}(\Omega^\rho_5) \otimes [\chi_s \otimes \phi_{nI}]_j\right]_{JM}$$

The continuum wave function at the positive energy

$$\Psi^{(\pm)}_{s\nu IM_I}(\mathbf{k_x}, \mathbf{k_y}; \mathbf{r}_i) = \sum_{\gamma} \imath^K (s\nu IM_I \mid jm_j)(LM_L\, jm_j \mid JM_J)\, \Upsilon^{l_x l_y *}_{KL}(\Omega^\kappa_5) \times$$
$$\times \frac{1}{\rho^{5/2}} \sum_{\gamma'} \chi^J_{\gamma',\gamma}(\kappa, \rho) \left[\Upsilon^{l'_x l'_y}_{K'L'}(\Omega^\rho_5) \otimes [\chi_{s'} \otimes \phi_{n'I'}]_{j'}\right]_{JM}$$

Set of coupled Schrödinger equations for radial wave functions

$$\left(-\frac{\hbar^2}{2\mu}\left[\frac{d^2}{d\rho^2} - \frac{(K+3/2)(K+5/2))}{\rho^2}\right] + \epsilon_\gamma - E\right) \chi^J_{\gamma,\gamma'}(\rho) = -\sum_{\gamma''} V^J_{\gamma,\gamma''}(\rho)\, \chi^J_{\gamma'',\gamma'}(\rho)$$

Hyperspherical harmonics $\Upsilon^{l_x,l_y}_{KLM}(\Omega_5)$ ($K = 2n + l_x + l_y$)

$$\Upsilon^{l_x,l_y}_{KLM}(\Omega^\rho_5) = \psi^{l_x,l_y}_K(\alpha_\rho) \left[Y_{l_x}(\hat{x}) \otimes Y_{l_y}(\hat{y})\right]_{LM}$$

$$\psi^{l_x,l_y}_K(\alpha) = N^{l_x,l_y}_K\, (\sin\alpha)^{l_x}\, (\cos\alpha)^{l_y}\, P^{(l_x+1/2, l_y+1/2)}_n(\cos 2\alpha)$$

$$\Upsilon^{l'_x,l'_y}_{KLM}(\Omega'_5) = \sum_{l_x,l_y} \left\langle l_x, l_y \mid l'_x, l'_y \right\rangle_{KL}\, \Upsilon^{l_x,l_y}_{KLM}(\Omega_5)$$

boundary condition of the radial wave function at the origin

$$\chi^J_\gamma(\rho \to 0) \to 0$$

asymptotic behaviour of the bound state radial wave function

$$\chi^J_\gamma(\rho \to 0) \to \exp(-\kappa_n \rho), \quad \kappa_n = \sqrt{2m \mid E_n - \epsilon_\gamma \mid /\hbar^2}$$

asymptotic behaviour of the continuum radial wave function

$$\chi^J_{\gamma',\gamma}(\kappa, \rho \to \infty) \to \frac{\imath}{\sqrt{2\pi}} \frac{1}{\sqrt{k_\gamma\, k_{\gamma'}}} \left(H^{(-)}_{K+2}(k_\gamma\, \rho)\, \delta_{\gamma,\gamma'} - H^{(+)}_{K'+2}(k_{\gamma'}\, \rho)\, S_{\gamma',\gamma} \right)$$

$$k_\gamma = \sqrt{2m \mid E - \epsilon_\gamma \mid /\hbar^2}$$

In collisions we explore the transition properties of nuclei from ground state to continuum states

$$\langle \Psi^{(-)}(\bar{k}_x, \bar{k}_y) \mid\mid \sum_p \frac{\delta(r - r_p)}{r r_p} \left[Y_L(\hat{r}_p) \times \sigma_p \right]_J \mid\mid \Psi_{gr.st.} \rangle$$

model with a frozen ^{20}C core

only the ground 0^+ core state is taken into account

neutron interaction potential with ^{20}C

$$V(r) = V_c(r) + V_{ls}(r)(l \cdot s) + V^l_{core}(r)$$

potential V(r) has no bound states for d-waves

for s, p waves the repulsive core V^l_{core} (r)
is added to eliminate Pauli forbidden orbits
and fixed by the condition $|a_s| < 2.8$ fm

S_{2n} (keV)	W_Y(l = 0) %	W_Y(l = 2) %	R_m (fm)
66	48	38	3.50
108	40	39	3.43
170	37	41	3.39
224	34	43	3.34

W_Y(l) is weight of the l-component
of wave function in (Cn)-n system

model with an unfrozen ^{20}C core

the ground 0^+ and excited 2^+ (1.588 MeV) core states are taken into account

the 2^+ core state is considered as a vibration excitation with coupling

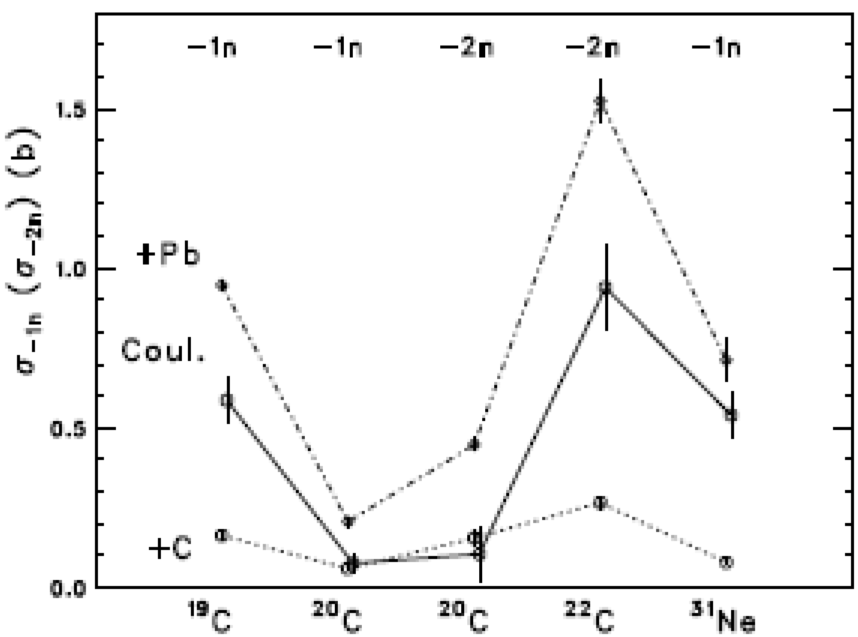
$$V_{\Delta n}(r) = V_I(r)\ (Y_I(\hat{r}) \cdot Y_I(\hat{z}_C))\ (c^{\dagger}_{I0} + c_{I0})$$

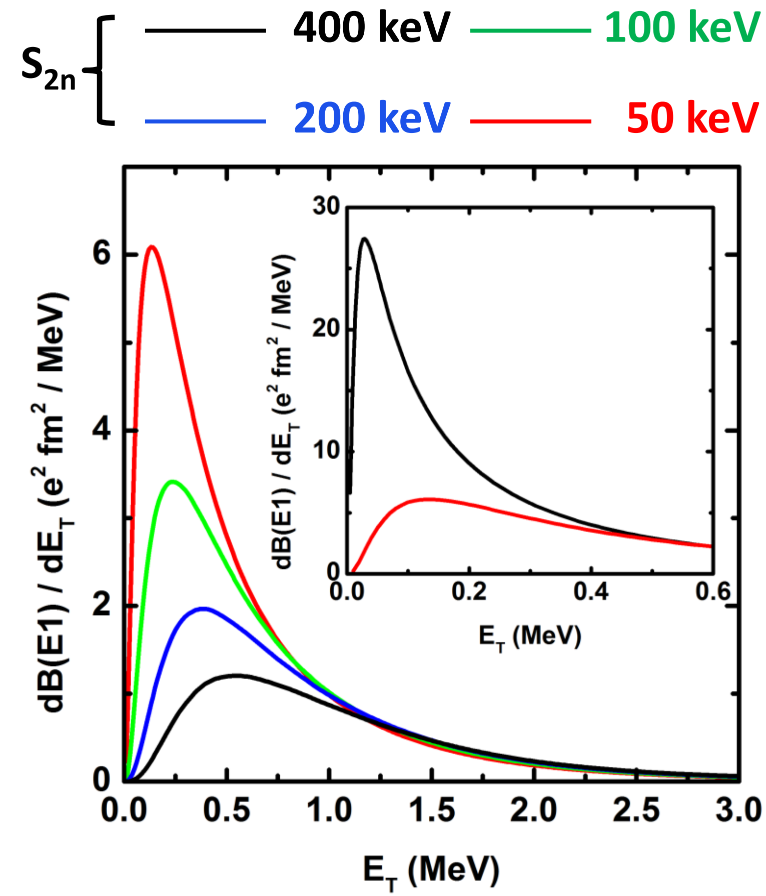
S_{2n} (keV)	$W_Y(l=0)$ %	$W_Y(l=2)$ %	R_m (fm)
60	35	43	3.39
133	32	47	3.32
219	29	49	3.28
319	27	50	3.25

inclusion of core excitation increases the role of quadrupole waves and, respectively, decreases spatial extension

inclusive Coulomb breakup of ^{22}C

E/A ∼ 240 MeV





T. Nakamura, J. Phys. CS 381, 012014 (2012)

the exclusive Coulomb breakup experiment would be desired

Conclusion

The challenge to the microscopic theory
is to understanf and reliably calculate
properties of both bound and continuum states
in nuclei in vicinity of driplines

The few-body cluster models present
a natural and transparent way to describe
specific features of nuclear structure
specified by the cluster degrees of freedom

The quantitative understanding
of the ^{22}C halo nuclear structure
is still open question.
To clarify this open question
the new experimental measurements
are urgently called for.