PION FORM FACTORS FROM SEPARABLE KERNEL

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Outlook

In the report we discuss the interaction of the two-quarks system with the photon: construct the electromagnetic current of the system in general form including the generalized $\gamma q\bar{q}$-vertex and interaction current.
The influence of the interaction current and anomalous magnetic moment of the quark in the phenomenological separable model for pion is discussed.

Bethe-Salpeter equation for the fermion-antifermion T matrix

$$T(k',k;p) = V(k',k;p) + \frac{i}{(2\pi)^4}\int d^4k''\, V(k',k'';p)\, S_2(k'';p)\, T(k'',k;p)$$

k'', k', p - the relative four-momenta, p - the total four-momentum

$V(k',k;p)$ - the interaction kernel

$S_2^{-1}(k;p) = \left(k\cdot\gamma + \frac{1}{2}\,p\cdot\gamma - m\right)^{(1)}\left(k\cdot\gamma - \frac{1}{2}\,p\cdot\gamma - m\right)^{(2)}$
free two-particles Green function

Bethe-Salpeter equation for the fermion-antifermion vertex function

The bound state in the channel gives the simple pole at the mass M_B in the T matrix

$$T(k',k;p) = \frac{\Gamma(k';p)\bar{\Gamma}(k;p)}{p^2 - M_B^2} + Reg(k',k;p)$$

BS equation for the vertex function

$$\Gamma(k;p) = \frac{i}{(2\pi)^4} \int d^4k''\, V(k',k'';p)\, S_2(k'';p)\, \Gamma(k'';p)$$

BS amplitude

$$\Psi(k;p) = S^{(1)}(k+p/2)\Gamma(k;p)S^{(2)}(k-p/2)$$

Separable kernel of the interaction

The separable kernels of the interaction are widely used in the calculations.

One-rank separable ansatz for the kernel

$$V_{\alpha_1\alpha_2,\beta_1\beta_2}(k',k;p) = \lambda\, g(k'^2)\, g(k^2)\, \Omega_{\alpha_1\alpha_2}\, \bar{\Omega}_{\beta_1\beta_2}$$

$\Omega_{\alpha_1\alpha_2}$ - constant matrix, does not depend on momenta.

Solution of the BS equation

$$\Gamma(k;p) = \mathcal{N} g(k^2)\, \Omega$$

with normalization constant $\mathcal{N}$. Noting that

$$V(k',k;p) = \frac{\lambda}{\mathcal{N}^2}\Gamma(k')\bar{\Gamma}(k)$$

the eigenvalue equation becomes

$$1 = i\frac{\lambda}{\mathcal{N}^2}\int \frac{d^4k}{(2\pi)^4}\{\bar{\Gamma}(k)S(k+p/2)\Gamma(k)S(k-p/2)\}.$$

Equation connects constant λ, pion m_π and quark m_q masses, parameters of g-function.

Model for pion

$$\Omega_{\alpha\beta} = \gamma^5_{\alpha\beta}$$

$$g(k^2) = 1/D(k^2), \qquad D(k^2) = k^2 - \Lambda^2$$

Λ is parameter of the model.
The weak pion decay constant is

$$f_\pi = (-i)\frac{m_q}{2\sqrt{2}}\sqrt{n_c}\mathcal{N}\int\frac{d^4k}{(2\pi)^4}\frac{1}{D(k^2)[(k+p/2)^2-m_q^2][(k-p/2)^2-m_q^2]}$$

The pion charge radius is

$$< r_\pi^2 >= -6\frac{d}{dq^2}F_\pi(q^2)|_{q^2=0}$$

Parameters of the model and static observables

	m_q (GeV)	Λ (GeV)	r_π (fm)	f_π (GeV)
Set I	0.3	0.5	0.64	0.108
Set II	0.3	0.75	0.55	0.123
Set III	0.2	0.5	0.84	0.0819
Experiment			0.663 $\pm$ 0.006	0.13041 $\pm$ 0.00020

Introduction of the electromagnetic interaction

$$\partial_i^\mu \to \partial_i^\mu + ie_i \int d^4\xi \, f_1^{(i)}(x_i - \xi)\, A^\mu(\xi) - ie_i \,\frac{i\sigma_{\alpha\beta}}{8m_q}\gamma^\mu \int d^4\xi \, f_2^{(i)}(x_i - \xi)\, \partial_i^\beta A^\alpha(\xi)$$

term in red is minimal substitution, term in blue is minimal substitution, e_i are charge of the quarks, A is electromagnetic field.
Functions $F_{1,2}^{(i)}(q^2) = \int d\xi \, f_{1,2}^{(i)}(\xi)\, \exp(-iq\xi)$ are the Dirac and Pauli form factors of quarks

$$F_1(0) = 1$$

$$F_2(0) = \kappa \, m_q / m_N$$

κ is quark anomalous magnetic moment in nucleon magnetons $e\hbar/2m_N$.

Introduction of the electromagnetic interaction

BS equation in operator form

$$(i\partial_1 \cdot \gamma - m_q)(i\partial_2 \cdot \gamma - m_q)\Psi - V(\partial_1, \partial_2)\Psi = 0$$

First term gives one-body current (relativistic impulse approximation, RIC),
second term - two-body current (interaction current, INT),

Relativistic impulse approximation

$$J^{\mu}_{\mathrm{RIA}}(k',k;p',p) =$$

$$-ie_1(2\pi)^4\delta^4(k'-k-q/2)\left(\gamma^\mu F_1^{(1)}(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_q}F_2^{(1)}(q^2)\right)^{(1)} S^{(2)\,-1}(k-p/2)$$

$$-ie_2(2\pi)^4\delta^4(k'-k+q/2)\left(\gamma^\mu F_1^{(2)}(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_q}F_2^{(2)}(q^2)\right)^{(2)} S^{(1)\,-1}(k+p/2)$$

Interaction current

If Ω is constant and $g = g(k^2)$ then

$$J^{\mu}_{\mathrm{INT}}(k', k; p', p) =$$

$$-e_1 \left({l_+}^{\mu} F_1^{(1)}(q^2) - \frac{i\sigma^{\mu\alpha} q_\alpha}{8m_q} l_+ \cdot \gamma F_2^{(1)}(q^2) \right) / (l_+ q) \left[V(k', k_+; p') - V(k', k; p) \right]$$

$$+e_1 \left({l'_-}^{\mu} F_1^{(1)}(q^2) - \frac{i\sigma^{\mu\alpha} q_\alpha}{8m_q} l'_- \cdot \gamma F_2^{(1)}(q^2) \right) / (l'_- q) \left[V(k'_-, k; p) - V(k', k; p) \right]$$

$$((1) \longleftrightarrow (2))$$

$l_{\pm}^{(')} = k^{(')} \pm q/4$, $k_{\pm}^{(')} = k^{(')} \pm q/2$, $p' = p + q$.
The techniques is based on the paper by *Hiroshi Ito, W.W. Buck, Franz Gross* Phys.Rev. C45 (1992) 1918-1934.

Gauge invariance

Using identity

$$q_\mu \gamma^\mu = S^{-1}(k + p/2 + q) - S^{-1}(k + p/2)$$

and Bethe-Salpeter equation one has for RIA

$$q_\mu \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \{\bar{\Psi}(k',p') J^\mu_{\rm RIA}(k',k;p',p) \Psi(k,p)\} =$$

$$+e_1\, F_1^{(1)}(q^2) \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \bar{\Psi}(k',p')[V(k',k_+;p') - V(k',k;p)]\Psi(k,p)$$

$$-e_2\, F_1^{(2)}(q^2) \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \bar{\Psi}(k',p')[V(k'_-,k;p) - V(k',k;p)]\Psi(k,p)$$

Gauge invariance

From another hand one has for INT

$$q_\mu \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \{\bar{\Psi}(k',p') J^\mu_{\rm INT}(k',k;p',p)\Psi(k,p)\} =$$

$$-e_1\, F_1^{(1)}(q^2) \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \bar{\Psi}(k',p')[V(k',k_+;p') - V(k',k;p)]\Psi(k,p)$$

$$+e_2\, F_1^{(2)}(q^2) \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \bar{\Psi}(k',p')[V(k'_-,k;p) - V(k',k;p)]\Psi(k,p)$$

So in sum

$$q_\mu \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \{\bar{\Psi}(k',p')(J^\mu_{\rm RIA}(k',k;p',p) + J^\mu_{\rm INT}(k',k;p',p))\Psi(k,p)\} = 0$$

Pion form factor

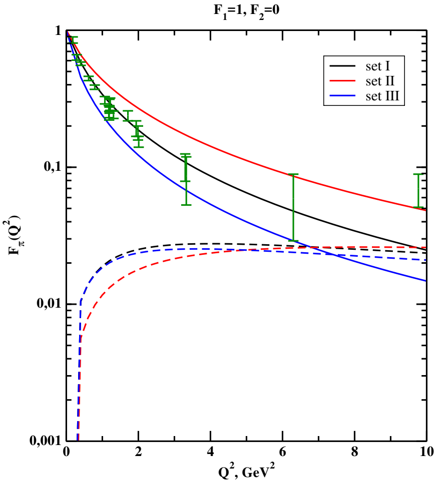
$$F_\pi(q^2)(p'+p)^\mu = \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \{\Psi(k',p') J^\mu(k',k;p',p) \Psi(k,p)\}$$

$$J^\mu(k',k;p',p) = J^\mu_{\mathrm{RIA}}(k',k;p',p) + J^\mu_{\mathrm{INT}}(k',k;p',p)$$

Calculations are performed by using the Cauchy theorem and numerical integration in the Briet frame

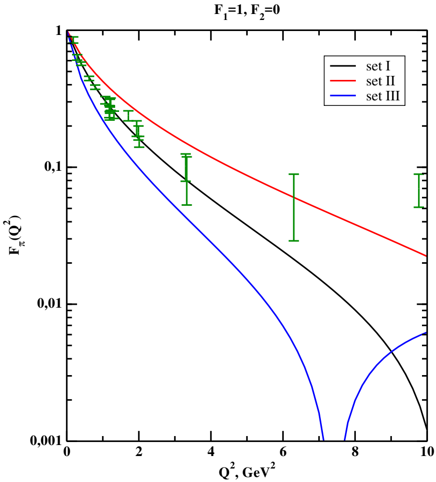
$$q=(0,\mathbf{q}) \quad p=(E_q,-\mathbf{q}/2) \quad p'=(E_q,\mathbf{q}/2) \quad E_q=\sqrt{\mathbf{q}^2+m_\pi^2}$$

Pion form factor $(F_1 = 1, F_2 = 0)$



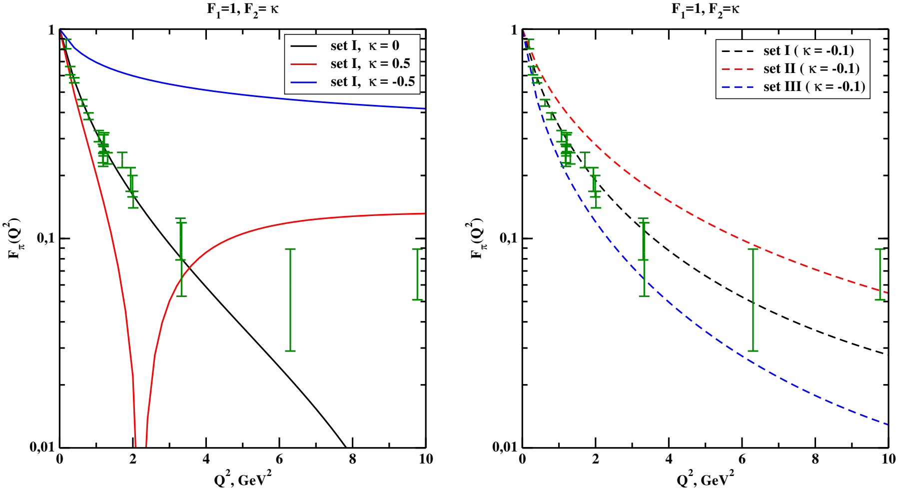
RIA - solid line (positive)

INT - dashed line (negative)



RIA + INT

Pion form factor (RIA+INT) $(F_1 = 1, F_2 = \kappa)$



Conclusion

- the pion charge form factor is calculated in the Bethe-Salpeter approach with separable kernel of quark-quark interaction
- the interaction current contribution is found to be negative and large for the high momentum transfer ($Q^2 > 6 - 8$ GeV^2) for point-like quarks
- the quark anomalous magnetic moment is introduced and estimated to be rather small (in nucleon magnetons $e\hbar/2m_N$) and negative (about -0.1). Important! The quark form factors will change the behaviour of the pion form factor.

Nearest plans

- estimation of the quark form factors contribution