Cluster approach to the structure of heavy nuclei

T.M. Shneidman[1],

G.G. Adamian[1], N.V. Antonenko[1], R.V. Jolos[1], H. Hua[2], Shan-Gui Zhou[3]

[1] *Joint Institute for Nuclear Research, Dubna, R ussia*
[2] *State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing. China*
[3] *Institute of Theoretical Physics, CAS, Beijing, China*

Content:

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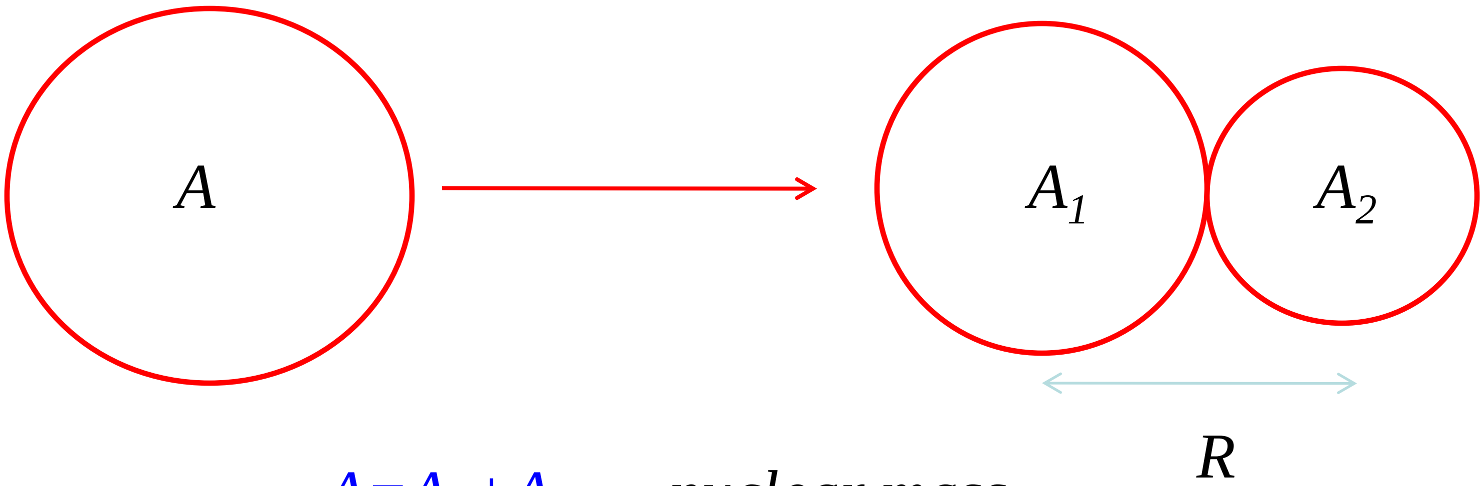
- *Parity splitting and dipole transitions in actinides and rare-earth nuclei*

- *Multiple reflection-asymmetric type bands structure*

- *Excitation spectra of fission isomers*

Conclusion

Clusters in nuclei



$A=A_1+A_2$ -- *nuclear mass*

R -- *relative distance*

$\xi=2A_2/A$ – *mass asymmetry*

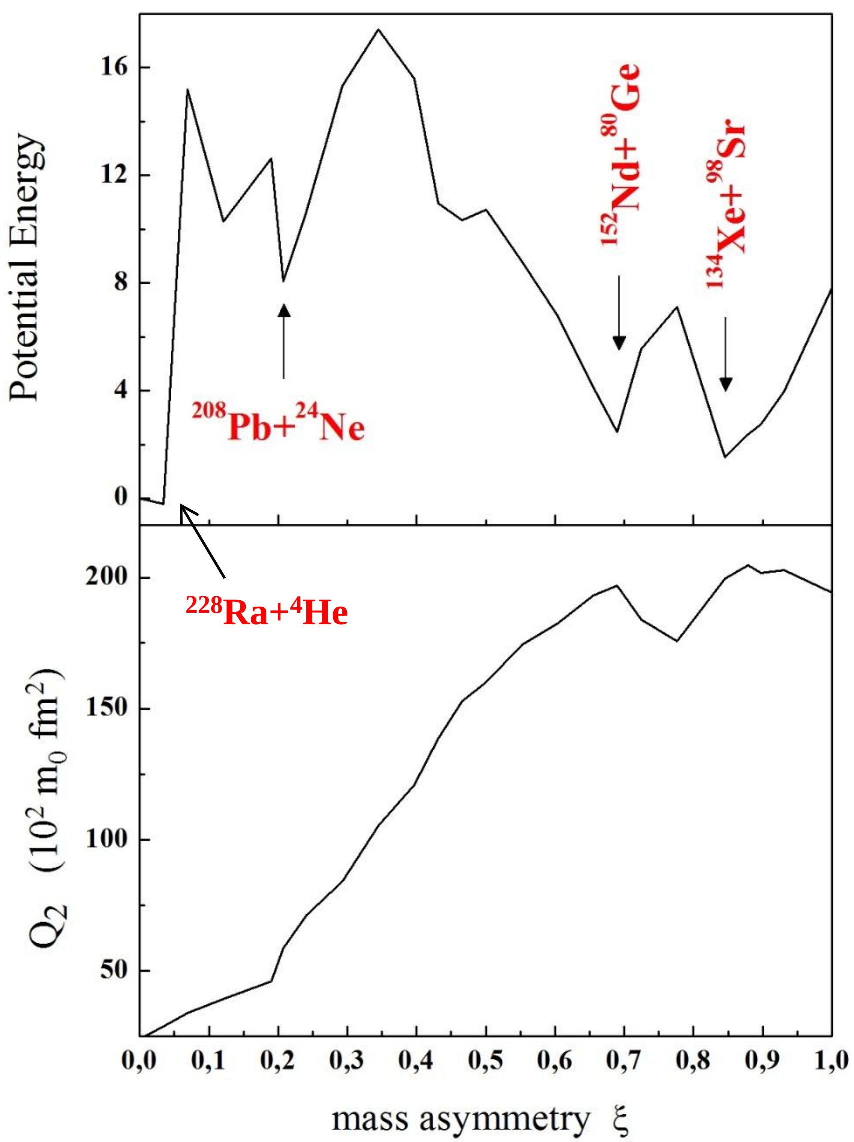
Light nuclei : ξ is fixed, dynamics in R

$$\psi_{ijk}\left(\vec{r}_1,\ldots,\vec{r}_{A_0},\vec{R}\right)=\hat{A}\left[\phi_i\left(A_1\right)\phi_j\left(A_2\right)\chi_k\left(\vec{R}\right)\right]$$

Heavy nuclei: R is fixed in touching, dynamics in ξ

$$\Psi\left(\vec{r}_1,\ldots,\vec{r}_{A_0}\right)=\sum_h\sum_{ijk}a^h_{ijk}\psi^h_{ijk}\left(\vec{r}_1,\ldots,\vec{r}_{A_0},\vec{R}_{touch}\right)$$

Driving potential for ^{232}U



The potential energy of the DNS

$$V(\xi) = E_1(\xi) + E_2(\xi) + V_N(R,\xi) + V_C(R,\xi)$$

Mass quadrupole moments of the DNS

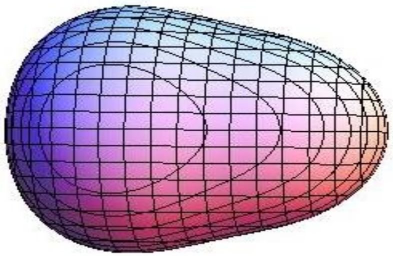
$$Q_2(\xi, R) = 2m_0 \frac{A_1 A_2}{A_1 + A_2} R^2 + Q_2(A_1) + Q_2(A_2)$$

(nuclear deformations from ***S.Raman et al., At. Data and Nuclear Data tables, Vol. 78, 2001***)

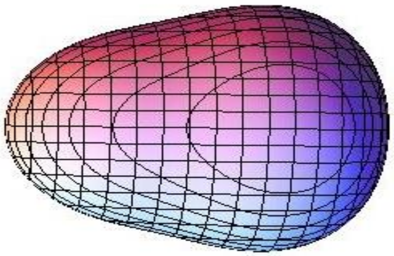
Reflection Asymmetric Deformation

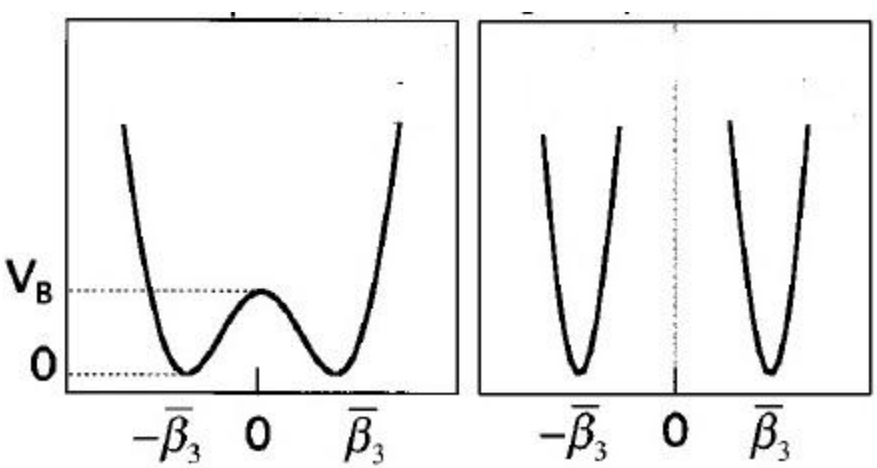
Intrinsic states $\Psi(\beta_{30})$ and $\Psi(-\beta_{30})$ are physically equivalent.

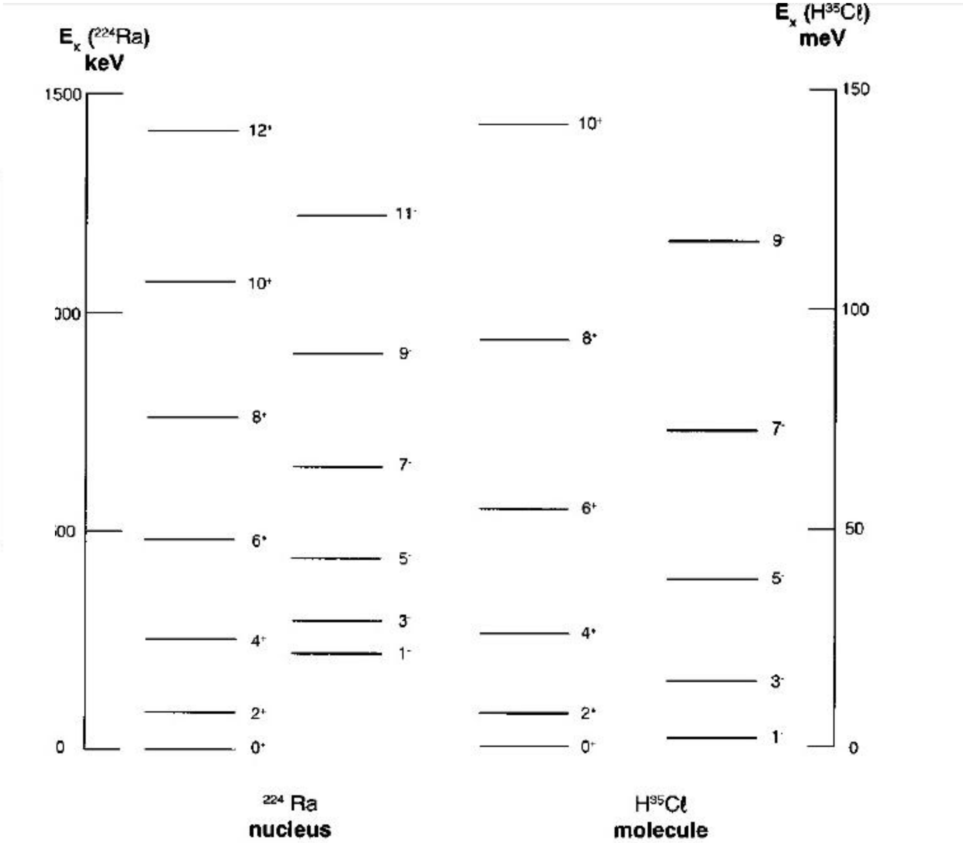
β_{20}=0.6, β_{30}=-0.5



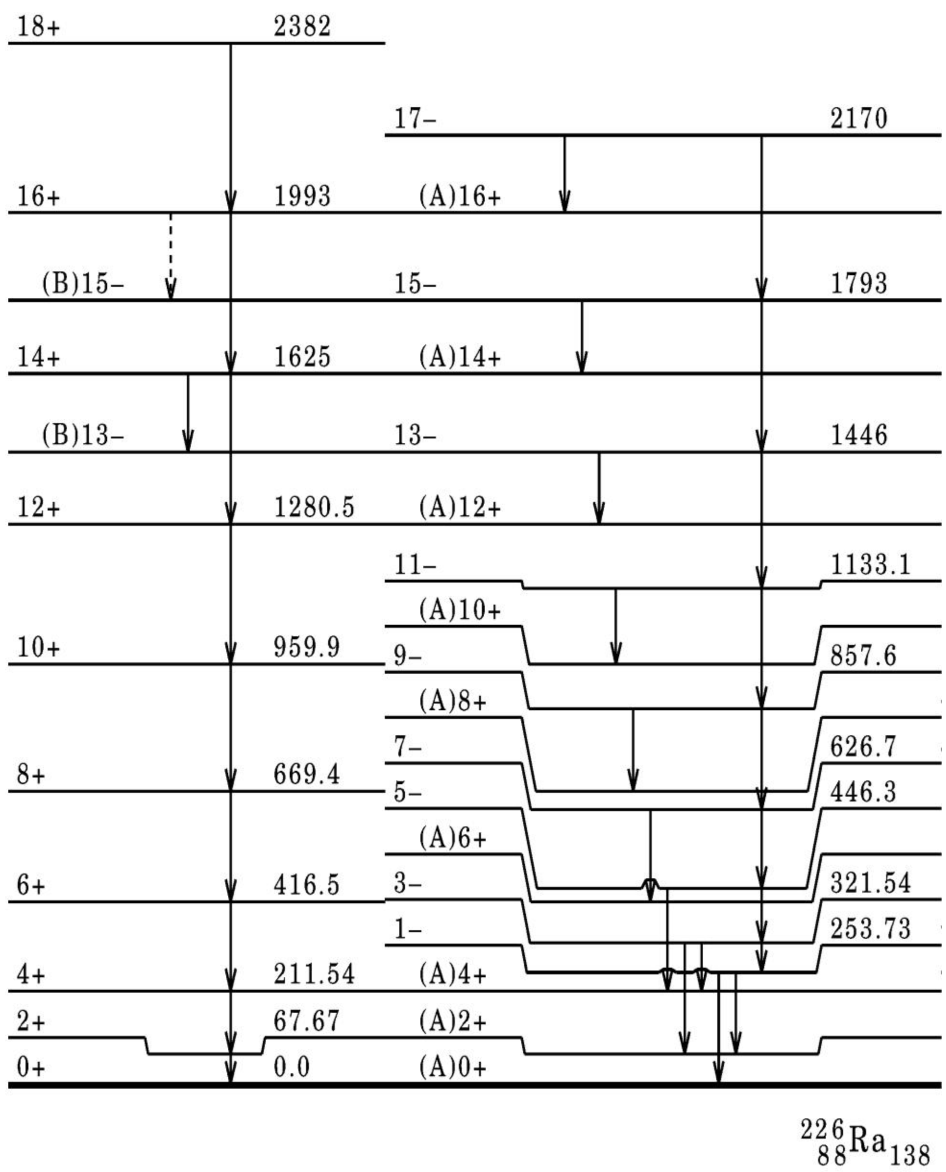
β_{20}=0.6, β_{30}=0.5

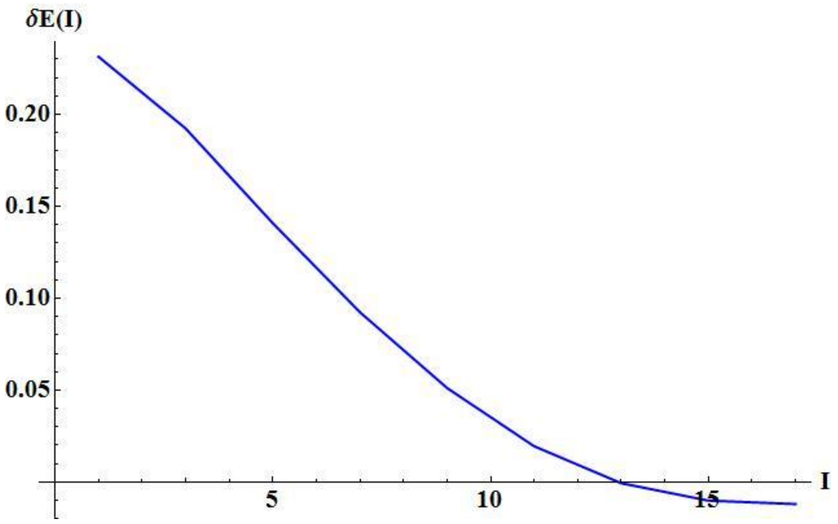


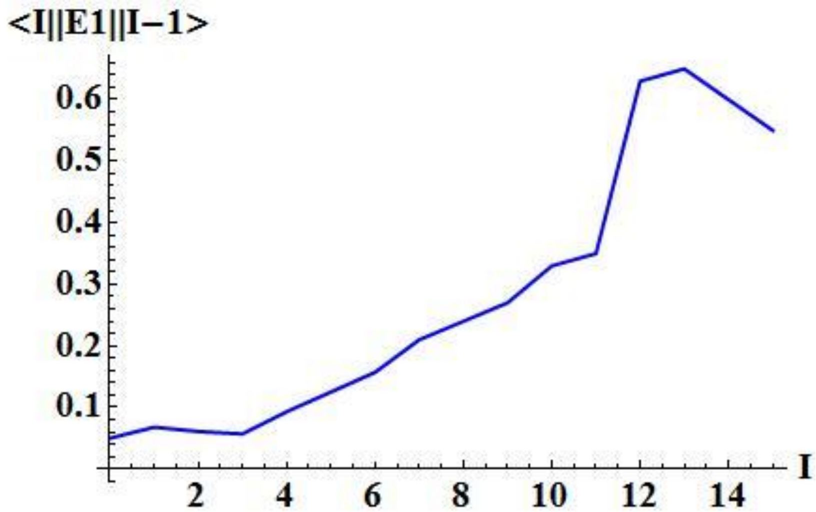




Excitation spectrum of nucleus with R.-A. deformation







Dinuclear system model and motion in mass asymmetry

$$\Psi_{p,IMK} = \sqrt{\frac{2I+1}{16\pi^2}}\left(\Phi_{n,K}(\xi)D^I_{MK} + p(-1)^{I+K}\Phi_{n,\overline{K}}(\xi)D^I_{M,-K}\right)$$

Wave function in ξ defined by the equation:

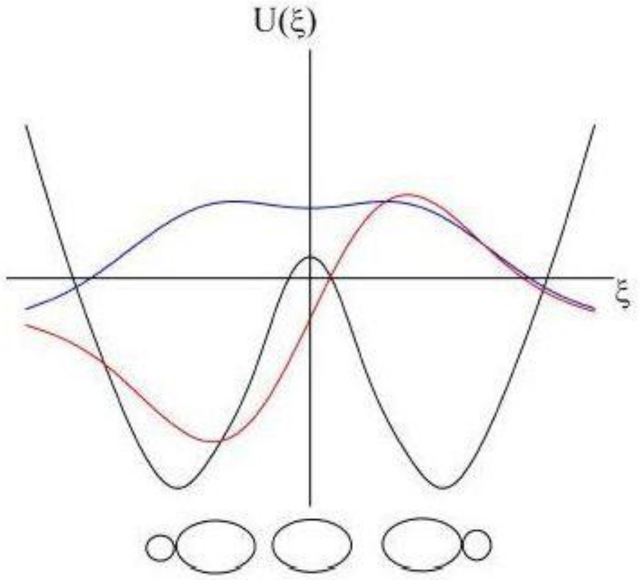
$$\left(-\frac{\hbar^2}{2B_\xi}\frac{d^2}{d\xi^2} + U(\xi) + \frac{\hbar^2}{2\Im(\xi)}I(I+1)\right)\Psi_{n,K}(\xi) = E_{n,K}\Psi_{n,K}(\xi),$$

where

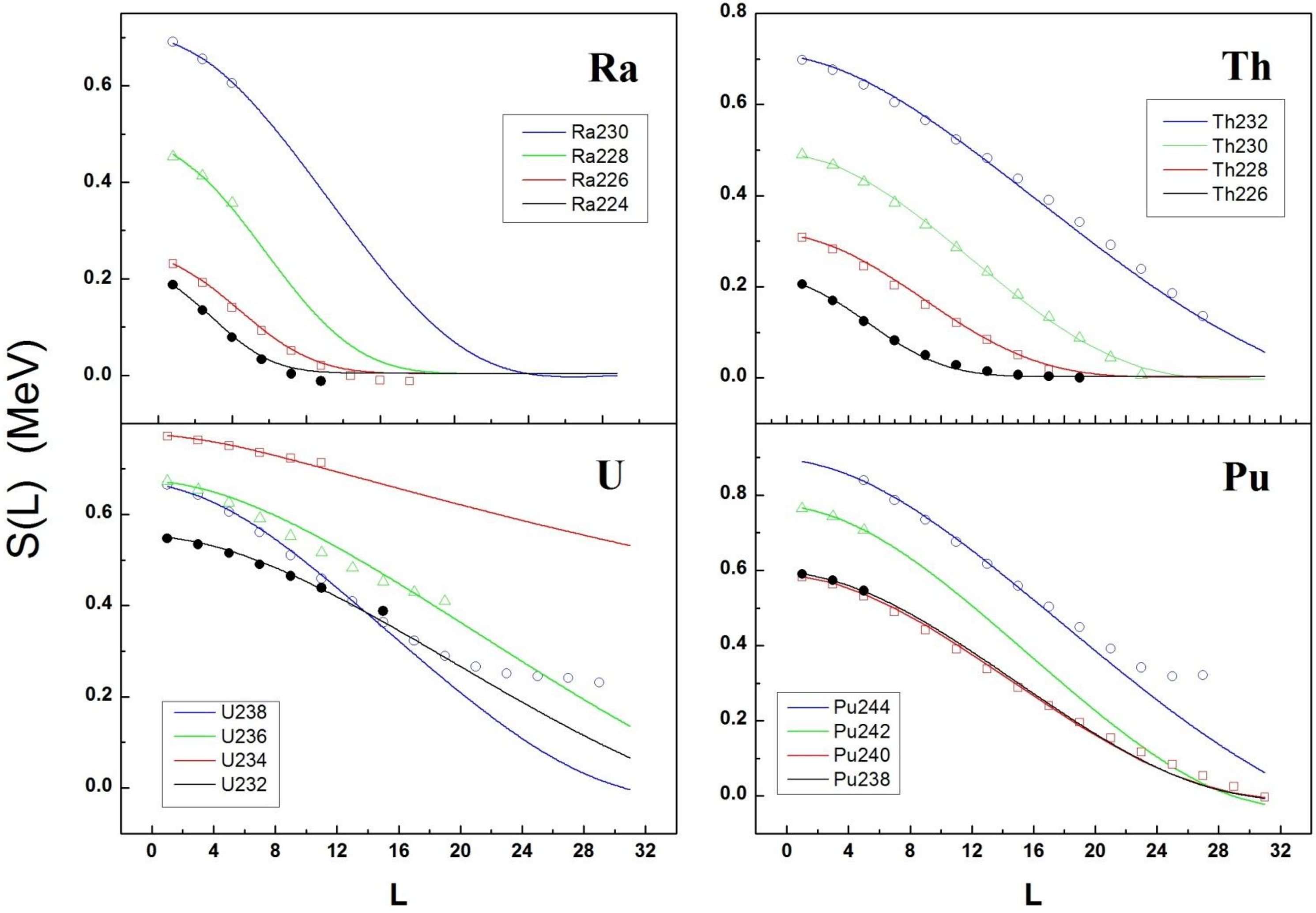
$$\Im(\xi) = 0.85(\Im_1^r + \Im_2^r + m_0\frac{A_1A_2}{A}R^2)$$

Exitation spectra:

$$I^p(\text{ for } K=0) = 0^+, 1^-, 2^+ ...$$
$$I^p(\text{ for } K\neq 0) = K^\pm, (K+1)^\pm ...$$



Parity splitting in alternating parity bands



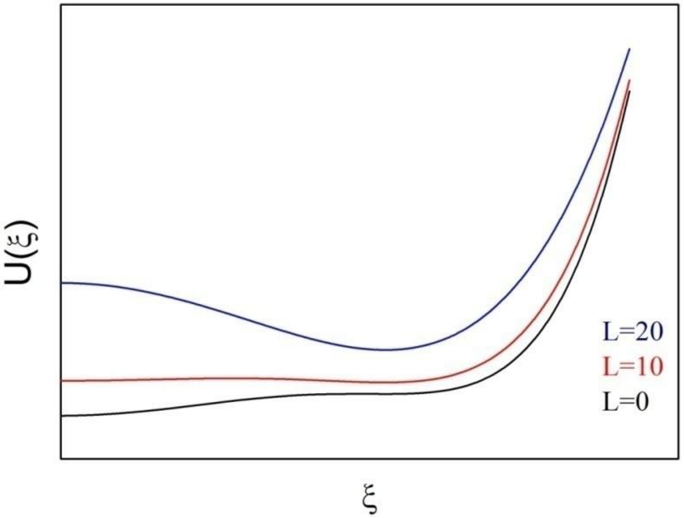
$$S(I^-) = E(I^-) - \frac{(I+1)E^+_{(I-1)} + IE^+_{(I+1)}}{2I+1}$$

EPJ WC 107, 03009, (2016)

Angular momentum dependence of the parity splitting

Hamiltonian in mass asymmetry

$$H(\xi,L) = -\frac{\hbar^2}{2B}\frac{1}{\xi^{3/2}}\frac{\partial}{\partial\xi}\xi^{3/2}\frac{\partial}{\partial\xi} + U_0(\xi) + \frac{\hbar^2 L(L+1)}{2J(\xi)}$$



$$\xi = 0;$$

$$U(\xi,L) = U(\xi,L=0) + \frac{\hbar^2}{2}\frac{L(L+1)}{J_h}$$

$$\xi = 1;$$

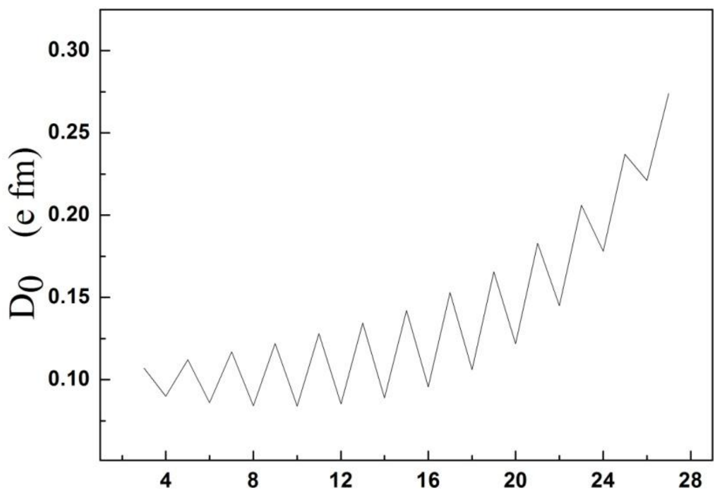
$$U(\xi,L) = U(\xi,L=0) + \frac{\hbar^2}{2}\frac{L(L+1)}{J_{tot}}$$

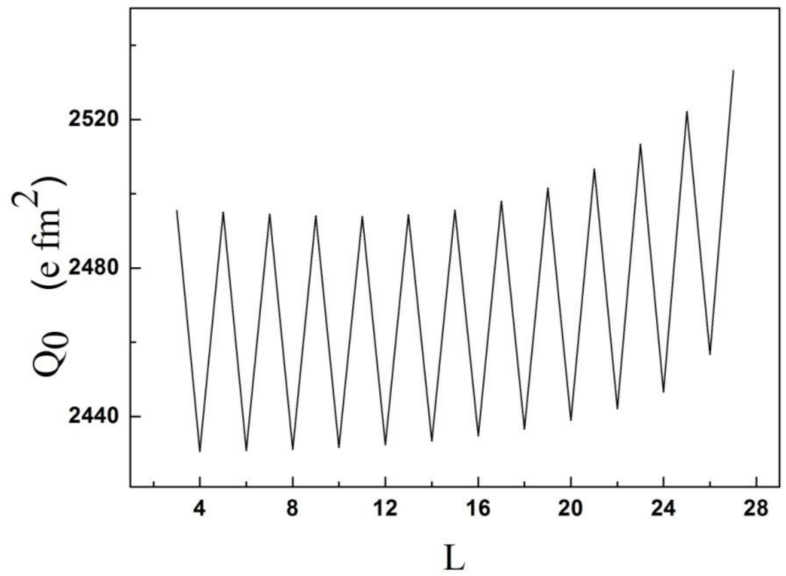
$$J_{tot} > J_h$$

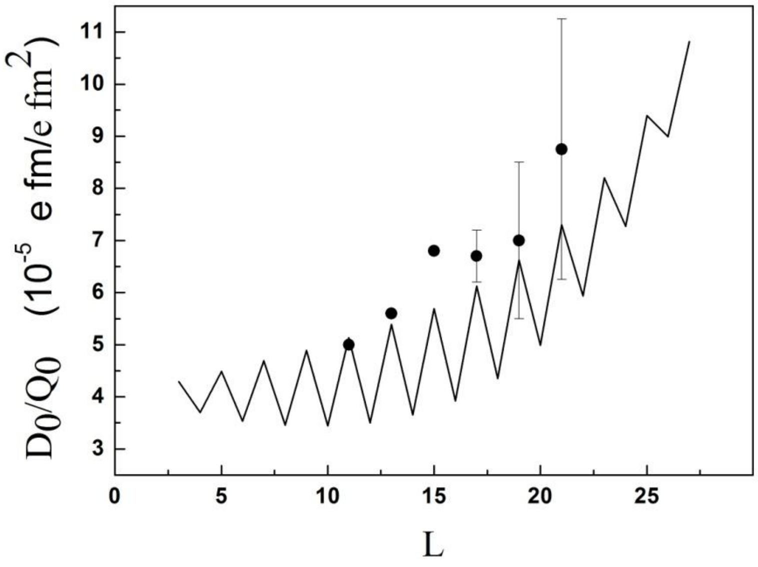
As a result the parity splitting decreases with angular momentum.

Electromagnetic transition in ^{240}Pu

(*I. Wiedenhöver et al., Phys. Rev. Lett. 83, Number 11, (1999)*)



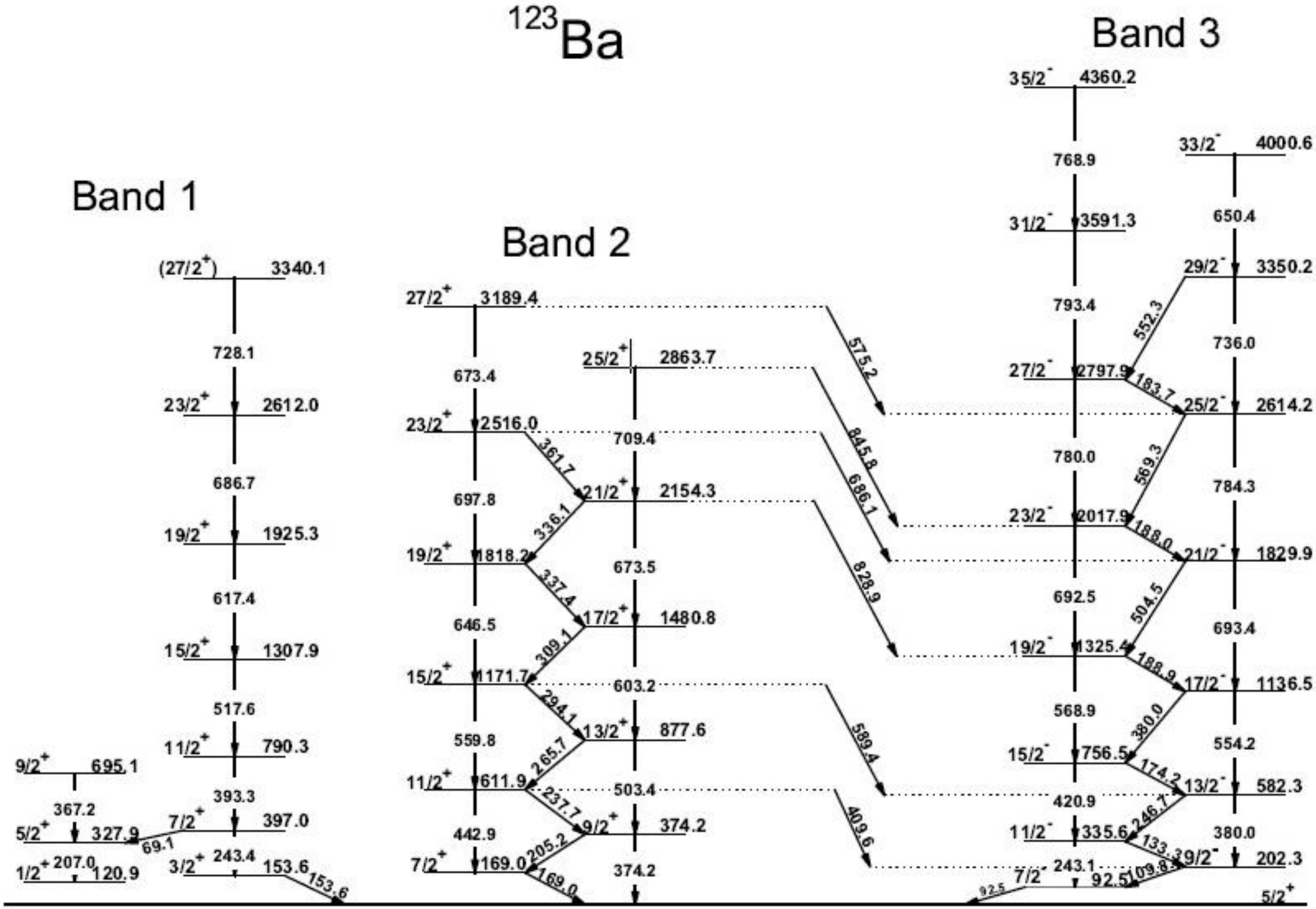




Ratio of transition dipole and quadrupole moments extracted from the *E1* and *E2* branchings $E1(I^- \longrightarrow (I\text{-}1)^+)/E2(I^- \longrightarrow (I\text{-}2)^-)$ as a function of the initial spin *I*.

PRC92, 034302 (2015)

Reflection-asymmetric correlations in ^{123}Ba



PRC94, 021301(R) (2016)

Odd-Mass Nuclei: illustrative Example

(D.M. Brink *et al.*, J. Phys. G: Nucl. Phys. 13 (1987))

Assumptions:

- Coriolis and recoil terms are neglected

- only two single-particle states with positive and negative parity:

$\chi_{+K}(\vec{r}), \chi_{-K}(\vec{r})$ corresponding energies: $\varepsilon_{+K}, \varepsilon_{-K}$

- only two core states with positive and negative parity:

$\varphi_{+K}(\xi), \varphi_{-K}(\xi)$ corresponding energies: $0, \delta E(I)$

Simplified Hamiltonian

$$H = H_{core} + \frac{\hbar^2}{2J(\xi)}(I^2 - I_3^2)$$

$$+ \varepsilon_{+K} a^+_{+K} a_{+K} + \varepsilon_{-K} a^+_{-K} a_{-K} + g(\xi)(a^+_{+K} a_{-K} + a^+_{-K} a_{+K})$$

Odd-Mass Nuclei: illustrative Example

(D.M. Brink *et al.*, J. Phys. G: Nucl. Phys. 13 (1987))

Smallest eigenvalues of positive and negative parities
(without rotational energy):

$$\tilde{\varepsilon}_{+K}(I) = \frac{1}{2}(\delta E(I) + \varepsilon_{+K} + \varepsilon_{-K}) - \frac{1}{2}\sqrt{(\delta E(I) + (\varepsilon_{+K} - \varepsilon_{-K}))^2 + 4g^2}$$

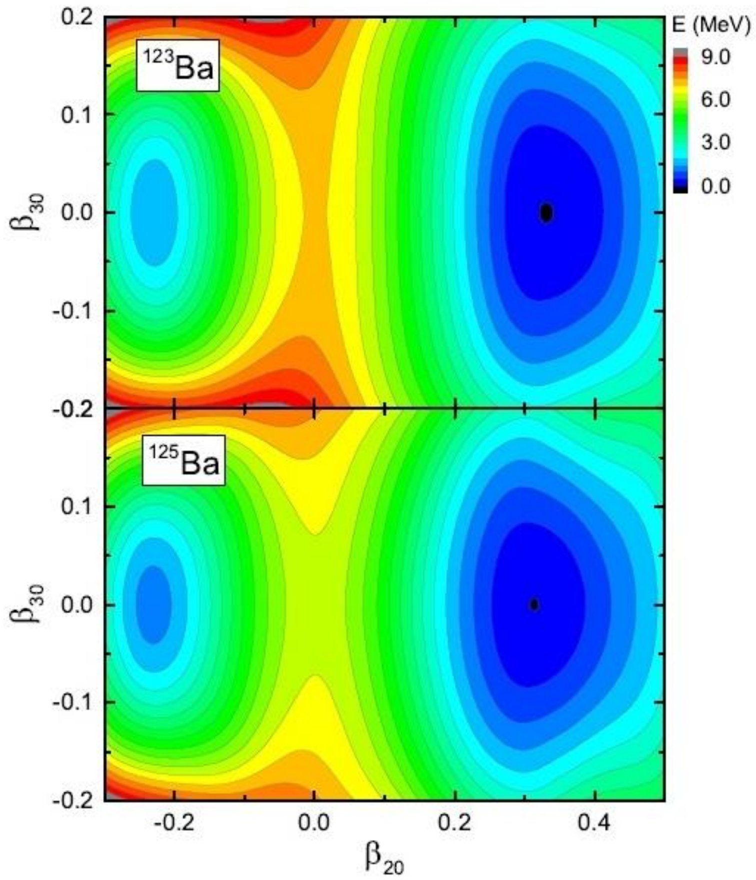
$$\tilde{\varepsilon}_{-K}(I) = \frac{1}{2}(\delta E(I) + \varepsilon_{+K} + \varepsilon_{-K}) - \frac{1}{2}\sqrt{(\delta E(I) - (\varepsilon_{+K} - \varepsilon_{-K}))^2 + 4g^2}$$

Parity splitting at the limits:

$$\varepsilon_{-K} - \varepsilon_{+K} << \delta E(I), g << 1 \quad S(I) = \tilde{\varepsilon}_{-K}(I) - \tilde{\varepsilon}_{+K}(I) \approx \varepsilon_{-K} - \varepsilon_{+K}$$

$$\delta E(I) << \varepsilon_{-K} - \varepsilon_{+K}, g << 1 \quad S(I) = \tilde{\varepsilon}_{-K}(I) - \tilde{\varepsilon}_{+K}(I) \approx \delta E(I)$$

PES for ^{123,125}Ba



Calculations have been performed in the frame of MDC-RMF model.

Although the minimum of the nuclear potential energy corresponds to the reflection-symmetric shape, PES for ^{123,135}Ba are very soft with respect to the reflection-asymmetric deformation.

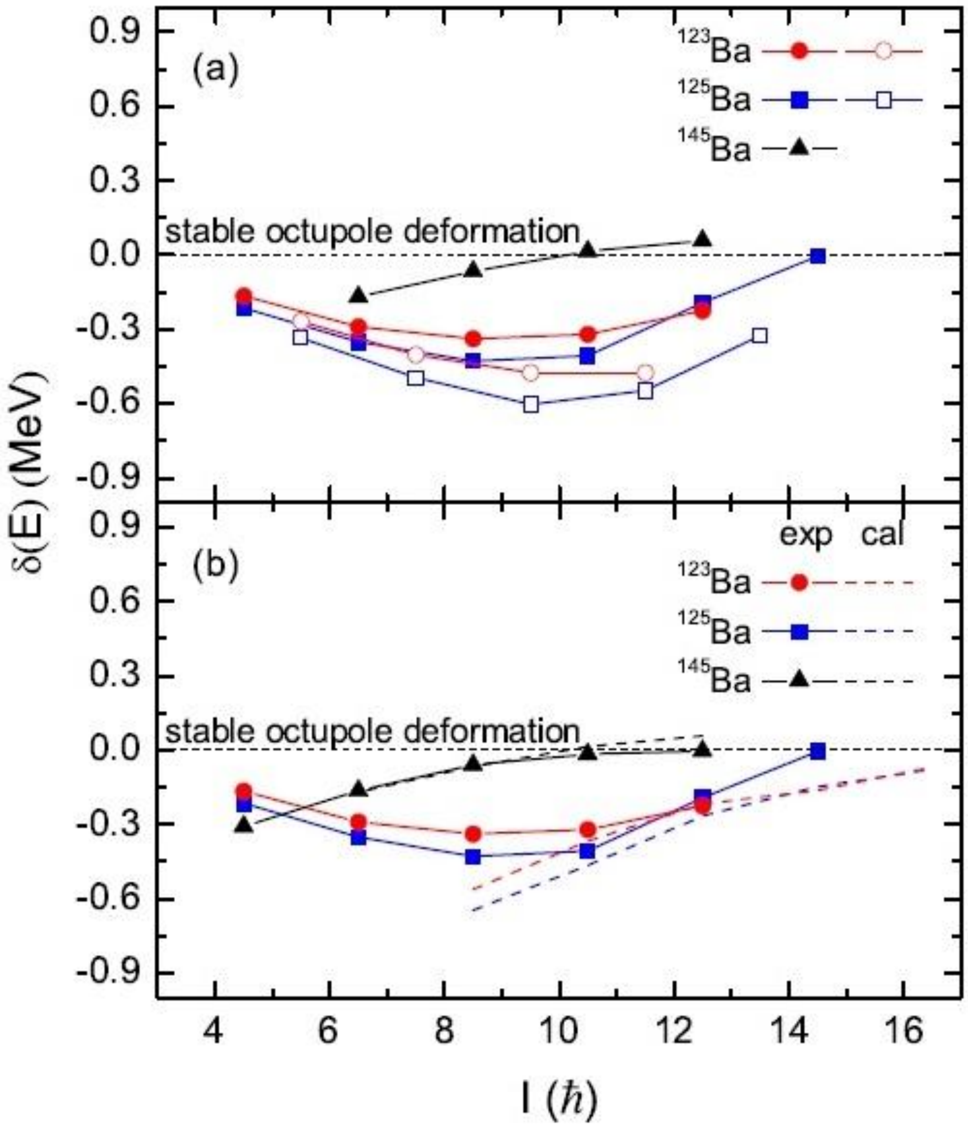
Using the DNS model one can estimate the critical value of angular momentum at which the stable reflection-asymmetric is developed.

$$I_{crit} \approx 13\hbar \quad \text{- for } {}^{123}\text{Ba},$$

$$I_{crit} \approx 12\hbar \quad \text{- for } {}^{125}\text{Ba}.$$

PRC94, 021301(R) (2016)

Parity splitting of ^{123,125,145}Ba



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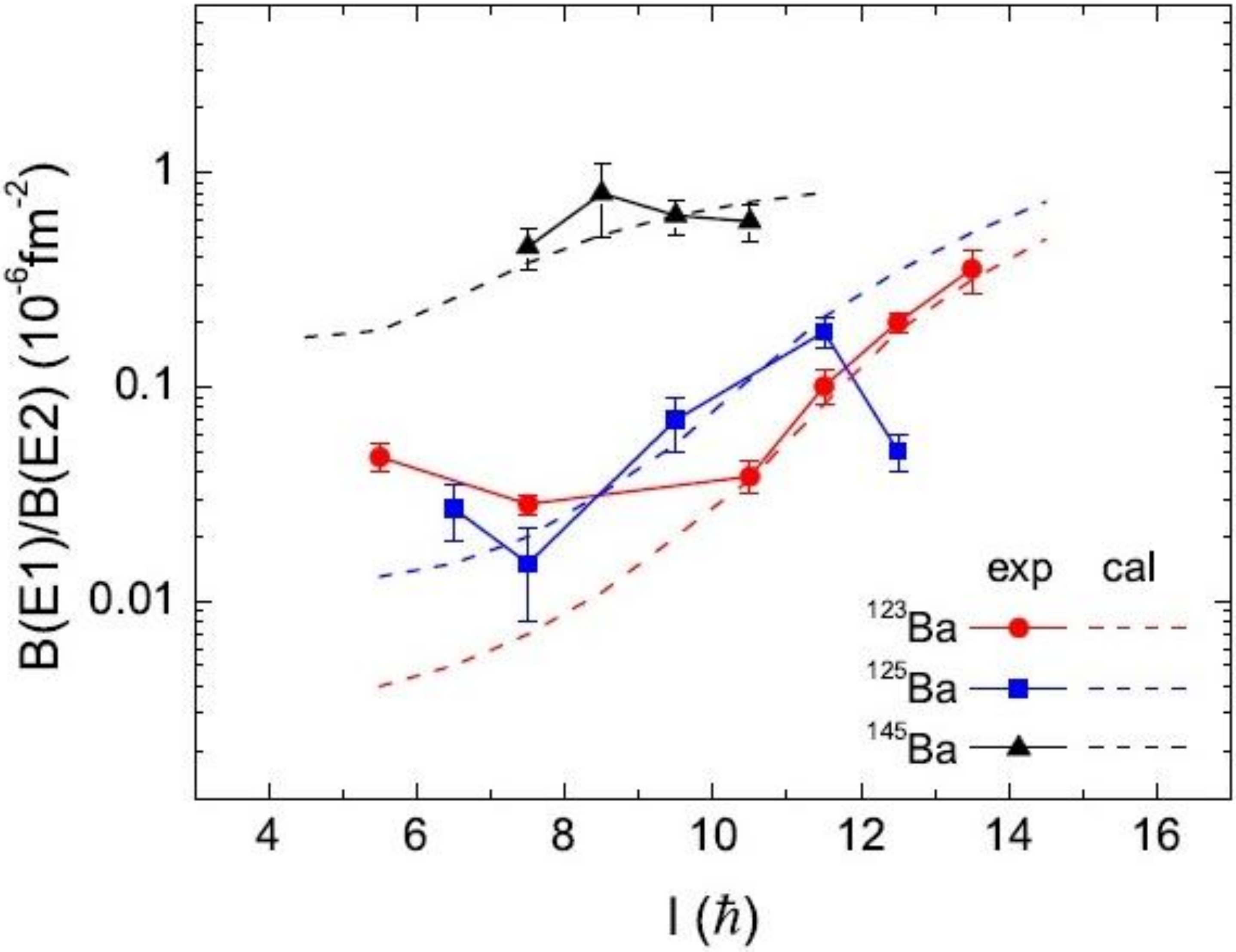
B(E1)/B(E2)-values for ^{123,125,145}Ba



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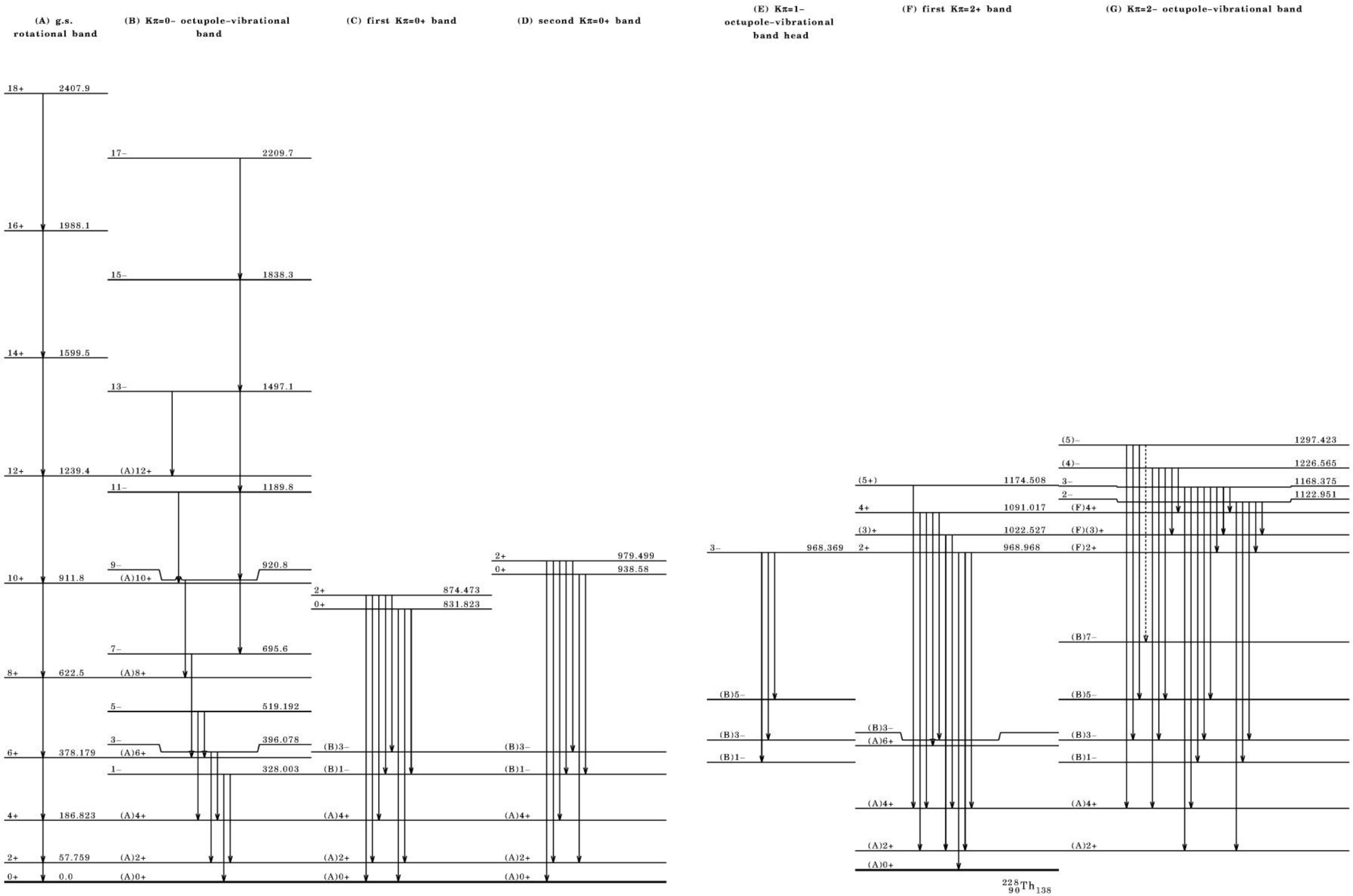
Experimental example: ^{228}Th



(from www.nndc.bnl.gov/ensdf)

Degrees of freedom of dinuclear system model

The dinuclear system (A,Z) consists of a configuration of two touching nuclei (clusters) (A_1,Z_1) and (A_2,Z_2) with $A = A_1 + A_2$ and $Z = Z_1 + Z_2$, which keep their individuality.

DNS has totally 15 collective degrees of freedom which govern its dynamics.

- *Relative motion of the clusters* $\mathbf{R} = (R, \theta_R, \phi_R)$
- *Rotation of the clusters* $\Omega_1 = (\phi_1, \theta_1, \chi_1)$, $\Omega_2 = (\phi_2, \theta_2, \chi_2)$
- *Intrinsic excitations of the clusters* (β_1, γ_1), (β_2, γ_2)
- *Nucleon transfer between the clusters* ξ, ξ_Z

Mass asymmetry $\xi = \frac{2A_2}{A_1+A_2}$. Charge asymmetry $\xi_Z = \frac{2Z_2}{Z_1+Z_2}$

Hamiltonian of the DNS model

The kinetic energy operator of the DNS then becomes

$$\hat{T} = -\frac{\hbar^2}{2B(\xi_0)}\frac{1}{\mu^{3/2}(\xi)}\frac{\partial}{\partial\xi}\mu^{3/2}(\xi)\frac{\partial}{\partial\xi} - \frac{\hbar^2}{2\mu(\xi)}\frac{1}{R^2}\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R}$$

$$+\frac{\hbar^2}{2\mu(\xi)R^2}\hat{l}_0^2 + \frac{\hbar^2}{2}\sum_{n=1}^{2}\sum_{k=1}^{3}\frac{\hat{l}_{(n)k}^2}{I_k^{(n)}(\beta_n,\gamma_n)} \qquad \left(\equiv \hat{T}_{rot}\right)$$

$$-\frac{\hbar^2}{2}\sum_{n=1}^{2}\frac{1}{D_n(\xi_0)}\left(\frac{1}{\beta_n^4}\frac{\partial}{\partial\beta_n}\beta_n^4\frac{\partial}{\partial\beta_n} + \frac{1}{\beta_n^2}\frac{1}{\sin 3\gamma_n}\frac{\partial}{\partial\gamma_n}\sin 3\gamma_n\frac{\partial}{\partial\gamma_n}\right)$$

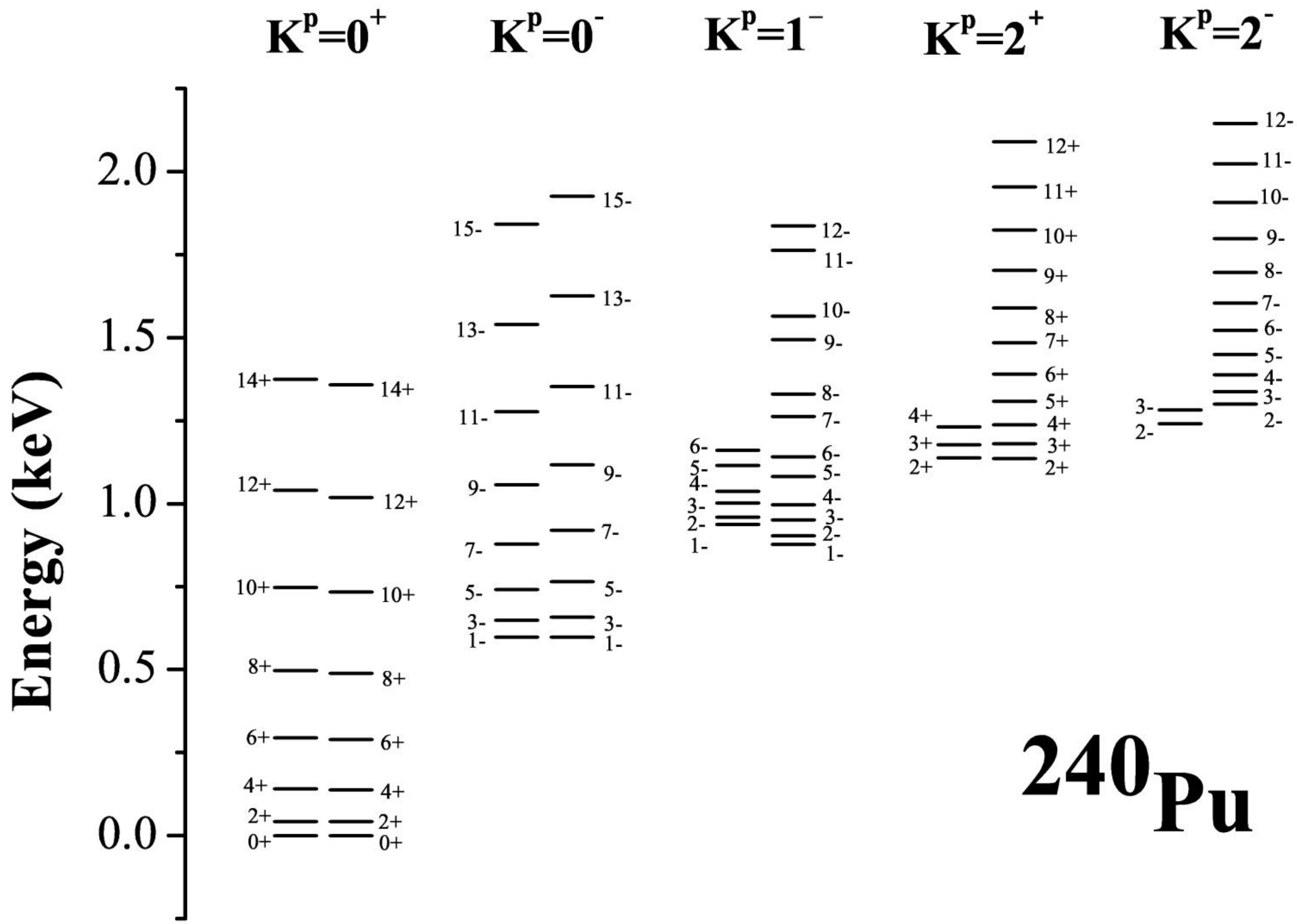
$$\left(\equiv \hat{T}_{\mathrm{int}r}\right)$$

The potential energy of the DNS is

$$V(\xi) = E_1(\xi,\beta_1,\gamma_1) + E_2(\xi,\beta_2,\gamma_2) + V_N(R,\xi,\beta_{\{1,2\}},\gamma_{\{1,2\}},\Omega_{\{1,2\}}) + V_C(R,\xi,\beta_{\{1,2\}},\gamma_{\{1,2\}},\Omega_{\{1,2\}})$$

Ground-State Well (^{240}Pu)

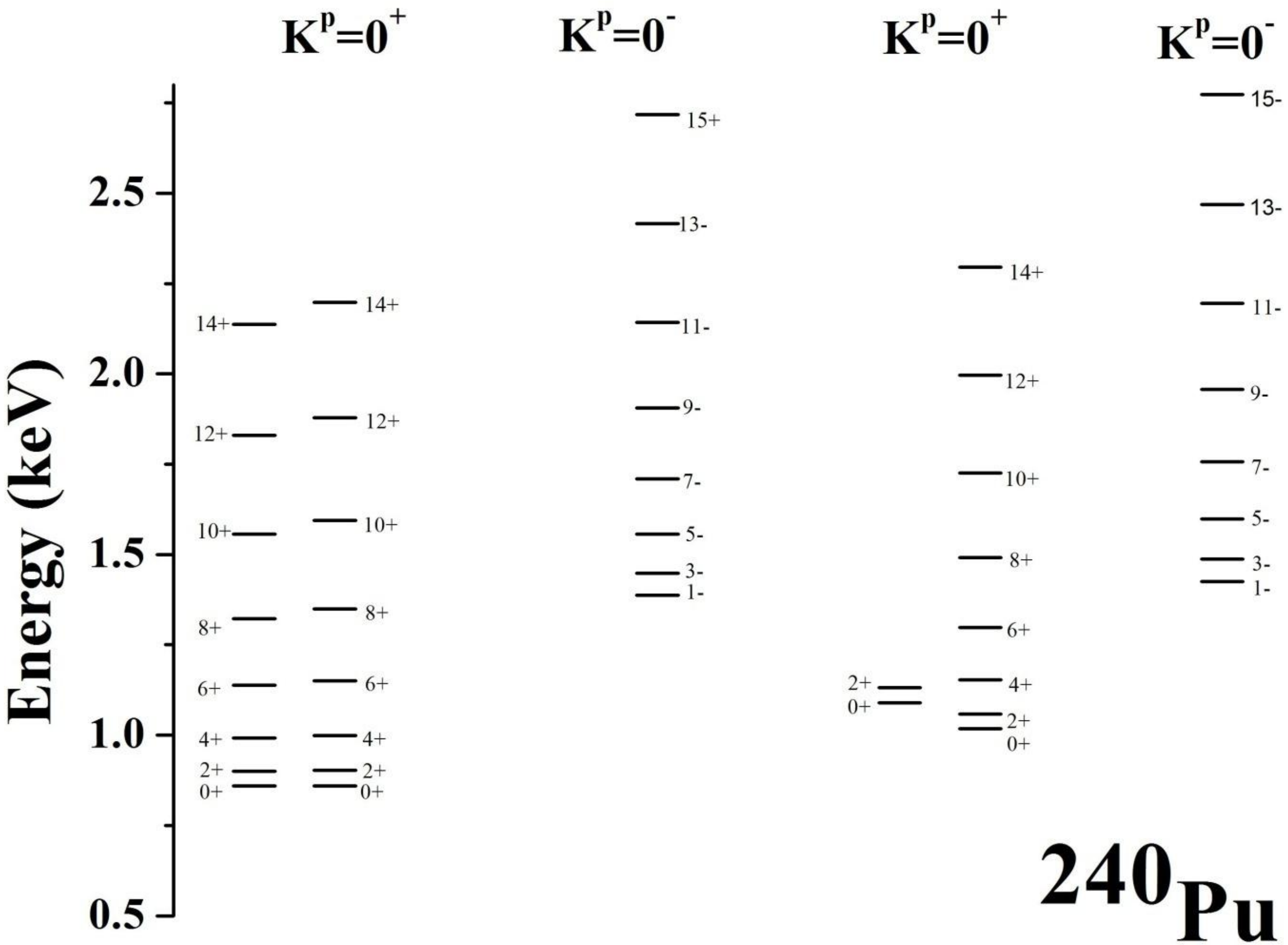
(Exp. data are taken from: http://www.nndc.bnl.gov/ensdf/)



PRC92, 034302 (2015)

Ground-State Well (^{240}Pu) –continued

(Exp. data are taken from: http://www.nndc.bnl.gov/ensdf/)



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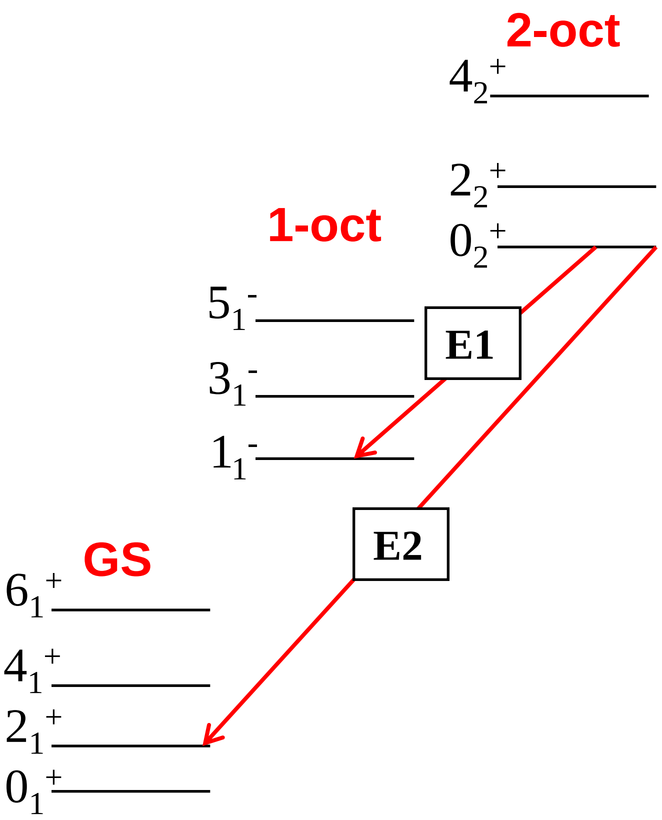
Electromagnetic Transition in ^{240}Pu

(Exp. Data are from *M. Spieker et al., Phys. Rev. C88, 041303(R), (2013)*)



Experimental B(E1)/B(E2) ratios (R_{exp}) are compared to the calculation of our model for the low-spin members of the $K\pi = 0^+{}_2$ rotational band in ^{240}Pu.

I_i^π	$I_{f,E1}^\pi$	$I_{f,E2}^\pi$	R_{exp} $(10^{-6}\ \mathrm{fm}^{-2})$	R_{DNS} $(10^{-6}\ \mathrm{fm}^{-2})$
0_2^+	1_1^-	2_1^+	13.7(3)	19.17
2_2^+	1_1^-	0_1^+	99(15)	99.95
2_2^+	1_1^-	2_1^+	26(2)	39.15
2_2^+	1_1^-	4_1^+	5.9(3)	8.57
2_2^+	3_1^-	0_1^+	149(22)	165.60
2_2^+	3_1^-	2_1^+	39(2)	64.9
2_2^+	3_1^-	4_1^+	8.9(5)	14.2
4_2^+	3_1^-	6_1^+	4.4(11)	6.9
4_2^+	5_1^-	6_1^+	4.7(13)	10.59

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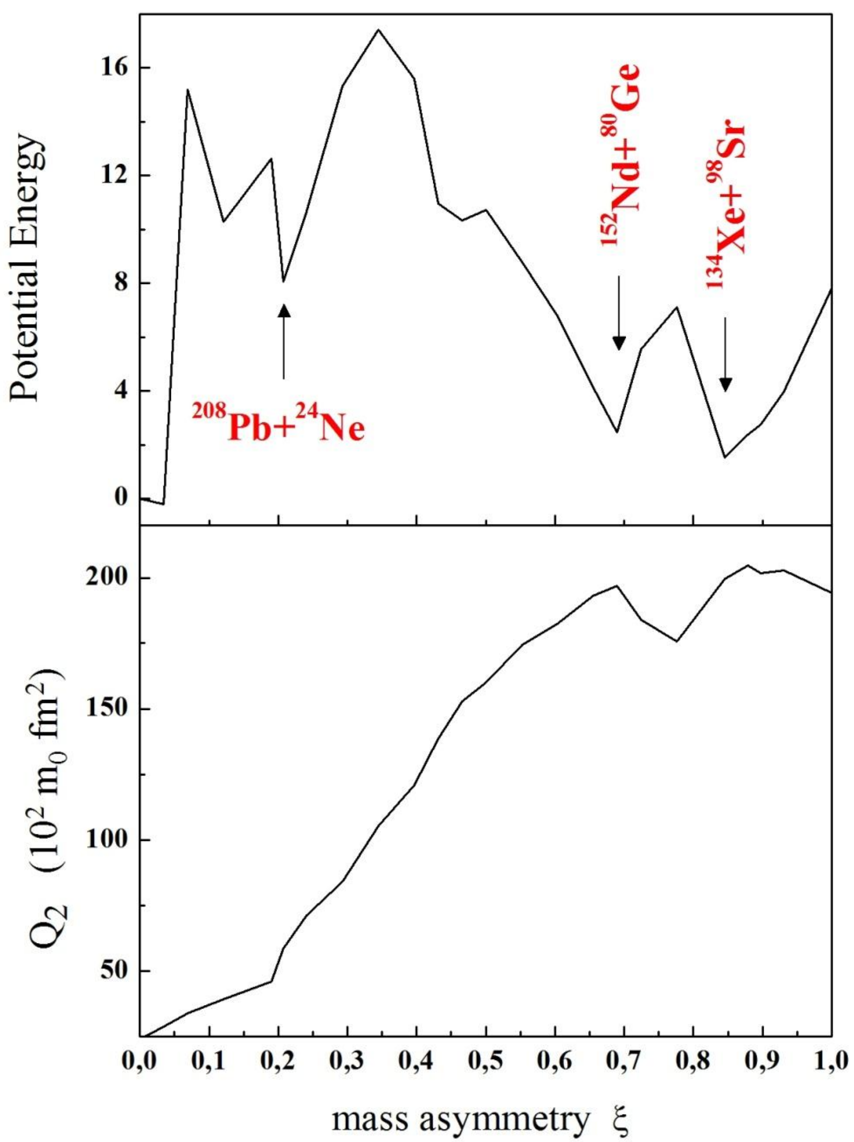
Driving Potential for ^{232}U



The potential energy of the DNS

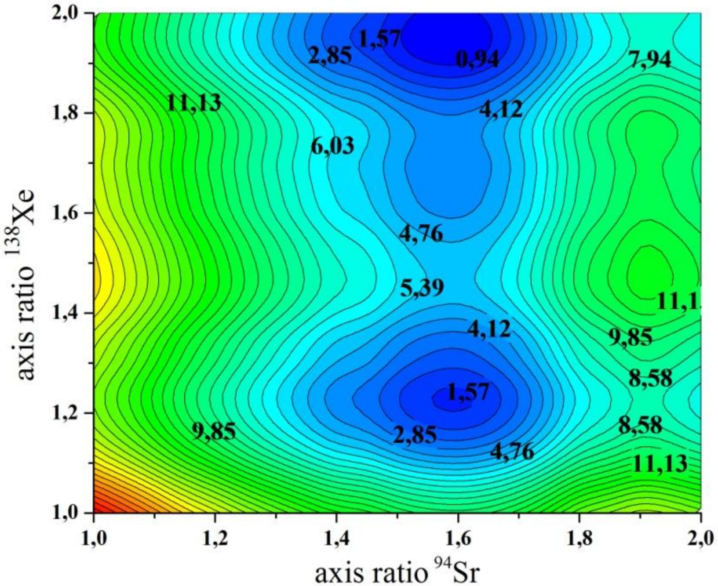
$$V(\xi) = E_1(\xi) + E_2(\xi) + V_N(R,\xi) + V_C(R,\xi)$$

Mass quadrupole moments of the DNS

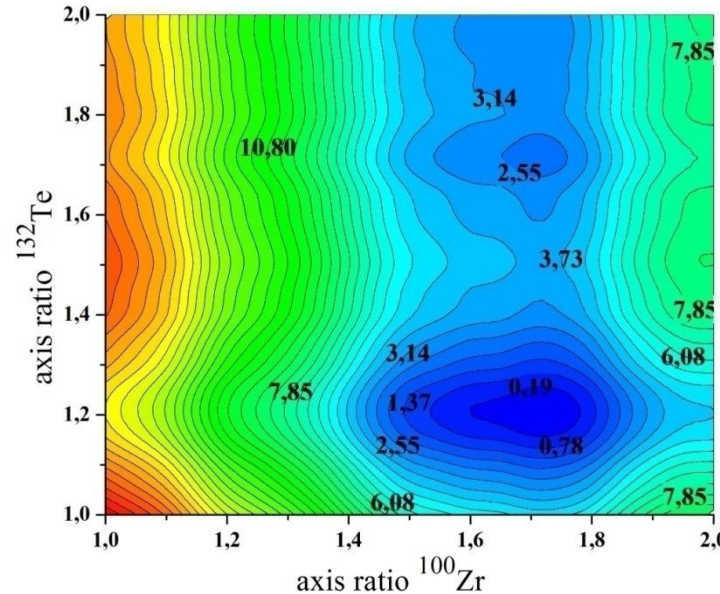
$$Q_2(\xi, R) = 2m_0 \frac{A_1 A_2}{A_1 + A_2} R^2 + Q_2(A_1) + Q_2(A_2)$$

(nuclear deformations from ***S.Raman et al., At. Data and Nuclear Data tables, Vol. 78, 2001***)

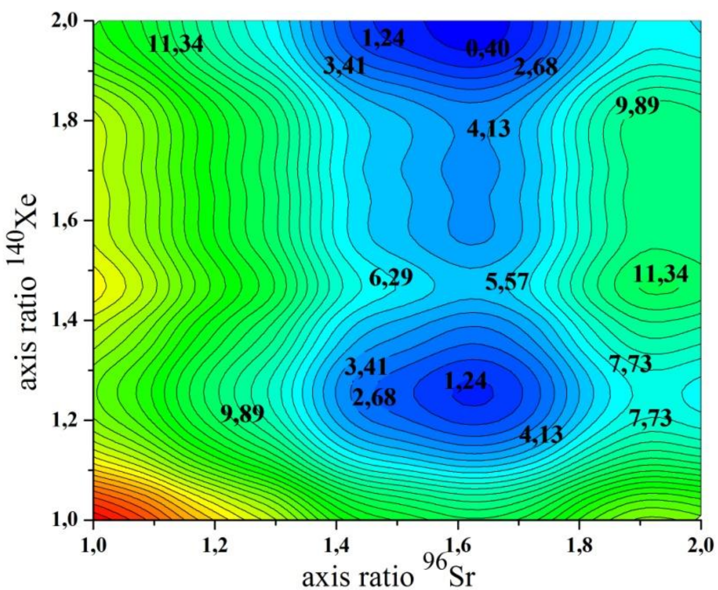
$^{232}U \longrightarrow {}^{94}Sr + {}^{138}Xe$



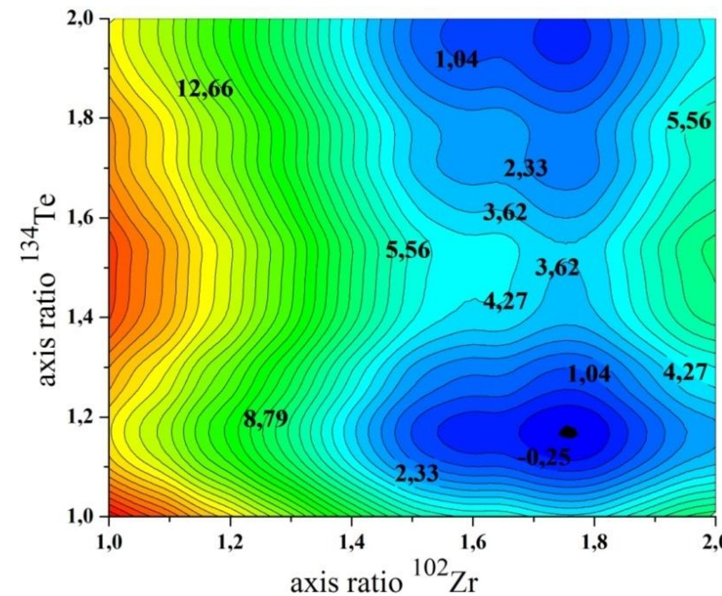
$^{232}U \longrightarrow {}^{100}Zr + {}^{132}Te$



$^{236}U \longrightarrow {}^{96}Sr + {}^{140}Xe$



$^{236}U \longrightarrow {}^{102}Zr + {}^{134}Te$



Characteristics of HD minima in U isotopes

Exp: L. Csige et al., Journal of Physics: CS**312** (2011) 092022

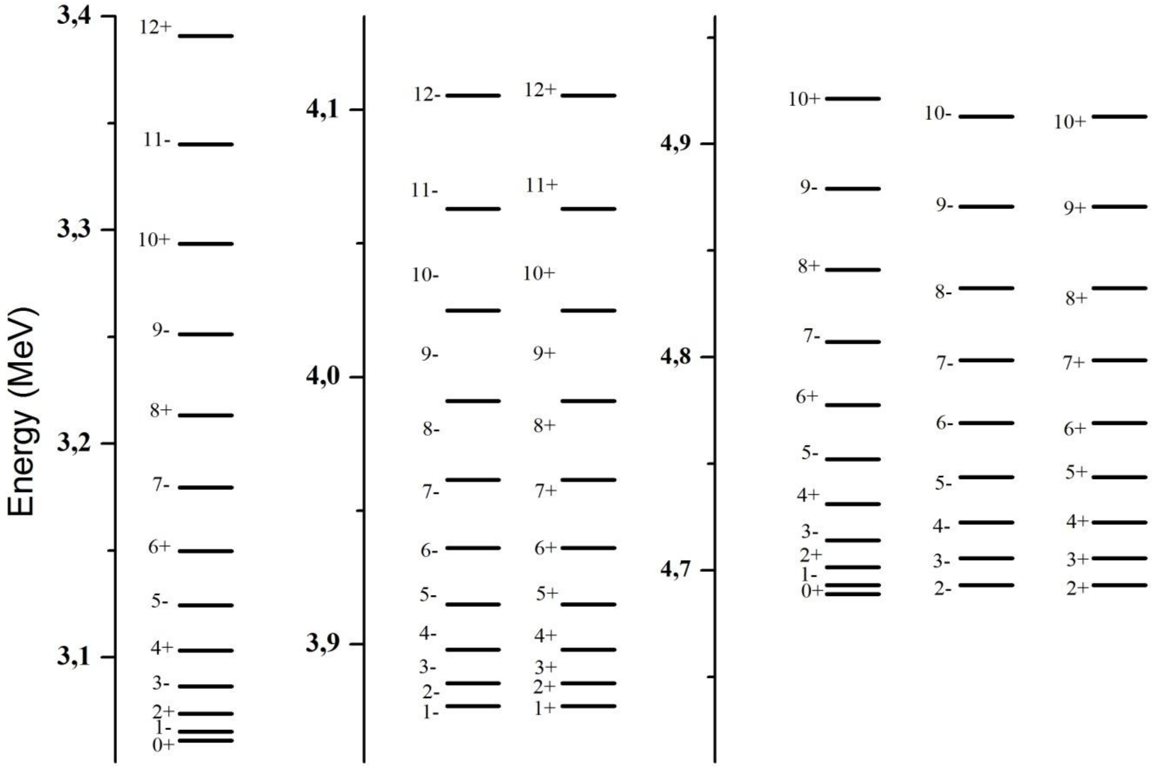
Nucleus	^{232}U	^{234}U	^{236}U	^{238}U
DNS	$^{94}Sr+^{138}Xe$	$^{96}Sr+^{138}Xe$	$^{96}Sr+^{140}Xe$	$^{98}Sr+^{140}Xe$
Energy (MeV)	3.06 (3.2±0.2)	2.6 (3.1±0.4)	2.81 (2.7±0.4)	3.49
Rot. Const. (keV)	1.825 (1.96±0.11)	1.772 (2.1±0.2)	1.751 (2.4±0.4)	1.697
Q_2 (10^2 e fm^2)	92.37	93.021	93.466	96.772
Q_3 (10^3 e fm^3)	29.96	28.48	29.92	27.84

EPJ WC, 38, 07001 (2012)

Conclusion:

- We suggested a cluster interpretation of the multiple negative parity bands in actinides and rare-earth nuclei assuming collective oscillations of nucleus in mass-asymmetry degree of freedom.
- The angular momentum dependence of the parity splitting and electromagnetic transition probabilities *B(E1)* and *B(E2)* are described. The results of calculations are in good agreement with experimental data.
- To take care of non-axially symmetric reflection asymmetric modes, the rotational and vibrational degrees of freedom of the heavy DNS fragment are considered.
- The excited 0^+ bands of reflection-asymmetric nature are explained as a bands built on the first exited state in mass asymmetry degrees of freedom.

Excitation Spectrum of ^{232}U in the HD well



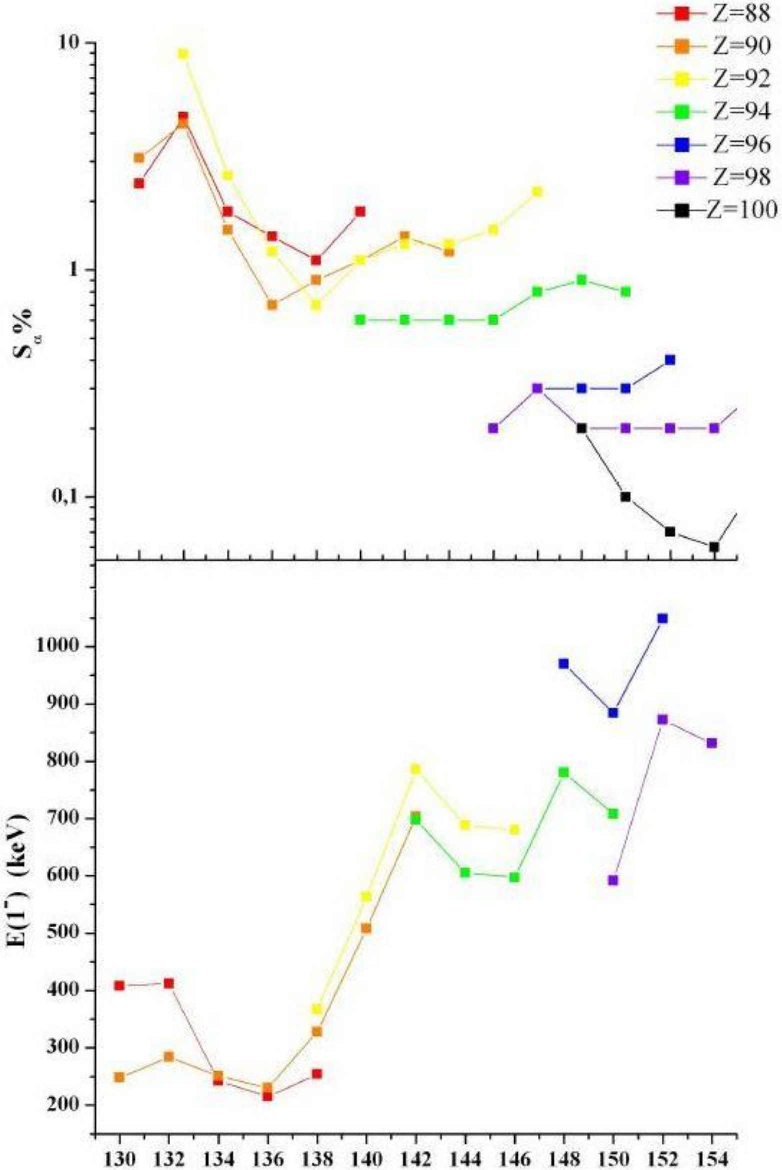
K	π	Energy
0	+	3.06
0	+	4.679
0	+	5.238
1	+	3.921
1	-	3.940
1	+	5.348
1	-	5.385
2	+	4.681
2	-	5.696
2	+	6.169
2	-	6.312

Erot = **1.83 keV** (*experiment*: **1.96±0.11 keV**)

U* = **3.06 MeV** (*experiment*: **3.2 ±0.2 MeV**)

EPJ WC, 38, 07001 (2012)

ΔL= ΔJ=3 or clustering



The value of α-particle preformation factor obtained from the experiment as:

$$S_\alpha^{exp} = T_{1/2}^{exp} / T_{1/2}^{\alpha}$$

$T_{1/2}^{\alpha}$ – *half-life of α-particle dinuclear system.*

Energies of $E(1^-)$*–states as a function of neutron number.*

ΔL= ΔJ=3 or clustering

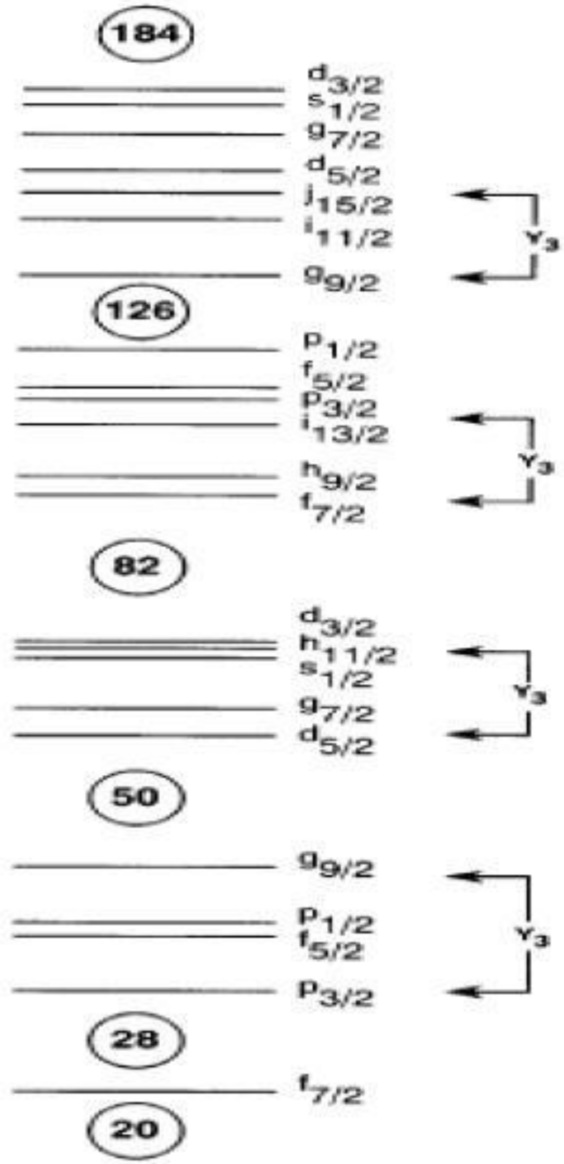


FIG. 4. Nuclear spherical single-particle levels. The most important octupole couplings are indicated.