Symmetries in Nuclear Structure physics

R.V.Jolos, JINR, Dubna

St.Petersburg, 24.07.2017

Content

- Pseudospin and Supersymmetry
- Octupole excitations – application of the Supersymmetric Quantum Mechanics
- Interacting Boson Model – Dynamical symmetries

Introduction

The mean field theory of the nucleonic interaction plays a role of a microscopic reference theory. It, is therefore , of fundamental importance for the whole field of nuclear structure physics to discover, examine and use the consequences of the underlying symmetries of the mean field, even if they are approximate.

Introduction

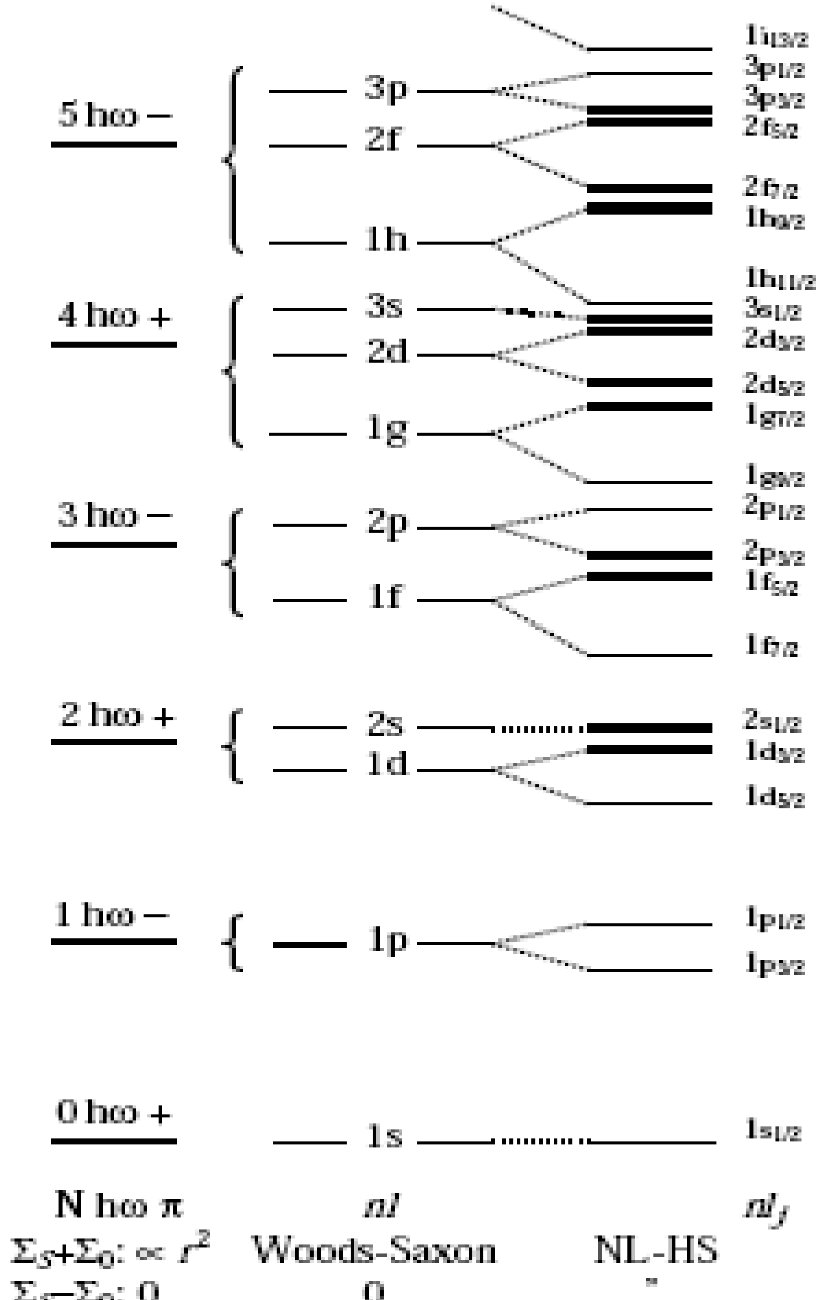
Symmetries imply existence of the characteristic multiplet structures. From the physics point of view, however, the fact that nuclear excited state multiplets exist in the realistic spectra is an intriguing result.

Indeed, let us recall that the nuclear mean field is a potential corresponding to averaging of the nucleon-nucleon interaction over many occupied single-nucleonic configurations. And if at the end it resembles any simple-looking function it is either incidental or the result of a symmetry.

Pseudospin

- 48 years ago a quasidegeneracy was observed, at first, for spherical nuclei (Arima, Harvey,… and Hecht, Adler – 1969):
  Single-particle states with J=l+1/2
  and J=(l+2)-1/2 lie very close in energy.

It is convenient to label them as pseudospin doublets.



5 hω −
4 hω +
3 hω −
2 hω +
1 hω −
0 hω +
3p
2f
1h
3s
2d
1g
2p
1f
2s
1d
1p
1s
1i13/2
3p1/2
3p3/2
2f5/2
2f7/2
1h9/2
1h11/2
3s1/2
2d3/2
2d5/2
1g7/2
1g9/2
2p1/2
2p3/2
1f5/2
1f7/2
2s1/2
1d3/2
1d5/2
1p1/2
1p3/2
1s1/2
N hω π
nl
nlj
ΣS+Σ0: ∝ r²
Woods-Saxon
NL-HS
ΣS−Σ0: 0
0

At small deformations the geometrical characteristics such as nucleonic probability distribution in space are very different from those at moderate and large deformations. However, the small energy spread of the pseudospin multiplets is nearly independent of deformation.

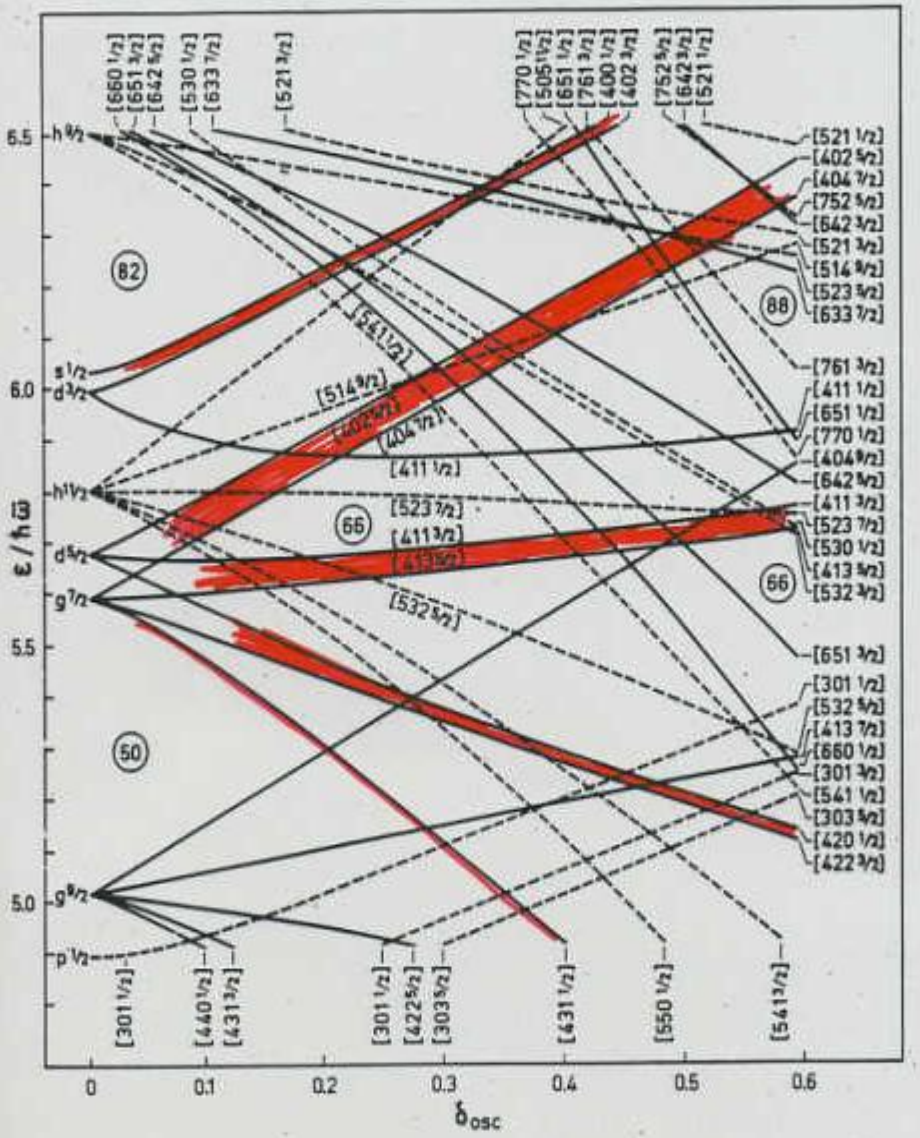


Figure 5-2 Proton orbits in prolate potential ($50 < Z < 82$). The spectra in this and the following figures (Figs. 5-2 to 5-5) are taken from C. Gustafson, I. L. Lamm, B. Nilsson, and S. G. Nilsson, *Arkiv Fysik* **36**, 613 (1967). The orbits are labeled by the asymptotic quantum numbers [$Nn_z\Lambda\Omega$]. Levels with even and odd parity are drawn with solid and dashed lines, respectively. (Erratum: The orbit [301 3/2] is incorrectly labeled [301 1/2] at bottom of figure.)

Interrelation between the pseudospin symmetry and microscopic mean field approach

- J.Meng, H.Toki, S.G.Zhou, S.Q.Zhang, W.H.Long, L.S.Geng, *Prog.Part.Nucl.Phys.*, **57**, 470 (2006).
- J.Meng, J.Y.Guo, J.Li, Z.P.Li, *et al.*, *Prog.Phys*. **31**, 199 (2011).
- J.Meng, S.G.Zhou, *J.Phys.* G, **42**, 093101 (2015).

Supersymmetry

- The idea of supersymmetry was invented in particle physics. However, actual examples of supersymmetry were found in the spectra of nuclei.

SUPERSYMMETRY IN SOME NUCLEAR-STRUCTURE MODELS 1129

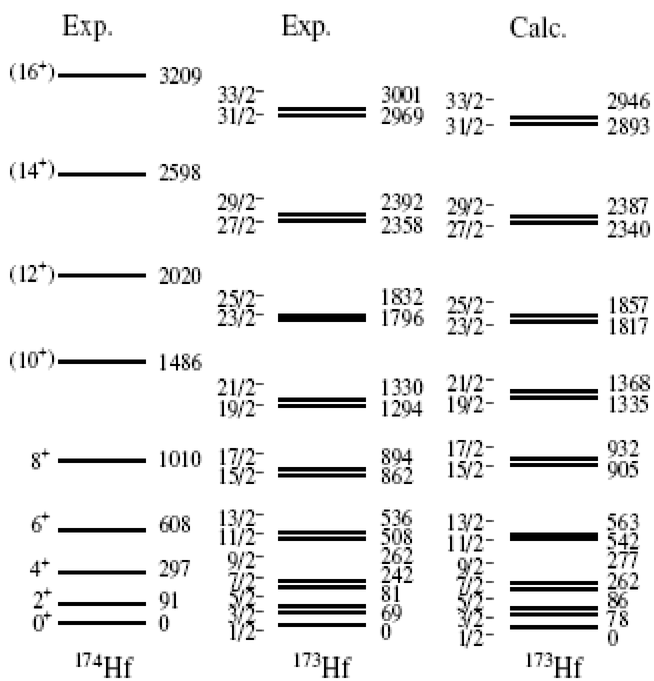


Fig. 3. Observed levels of the ground-state band of ^{174}Hf and experimental and calculated states of ^{173}Hf that form supermultiplets. The energies are given in keV. Experimental data were taken from [17−19].

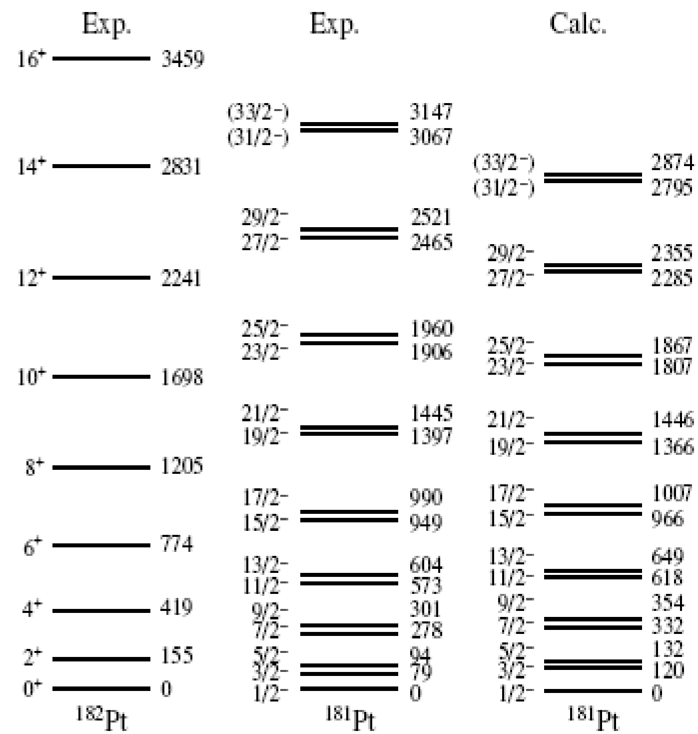


Fig. 4. As in Fig. 3, but for ^{181,182}Pt. Experimental data were taken from [17, 20, 21].

These figures display rotational bands belonging to odd deformed or nearly deformed nuclei. In both cases the bandheads are $1/2^-$ states. These bands consist of weakly splited doublets whose centers of gravity are close to the energies of the corresponding rotational states of the neighboring even-even nuclei. Doublets are: ($3/2^-$, $5/2^-$), ($7/2^-$, $9/2^-$), and so on. This means that the angular momenta of the states of the odd nuclei belonging to the multiplets can be treated as the result of the vector coupling of the orbital momenta L=0, 2, 4,… of the even-even core and the fermion momentum j=1/2. The fermion momentum is decoupled from the interaction. It is assumed that it is pseudospin. Because of the approximate pseudospin symmetry, a pseudospin-orbit interaction is weak.

Superdeformed bands

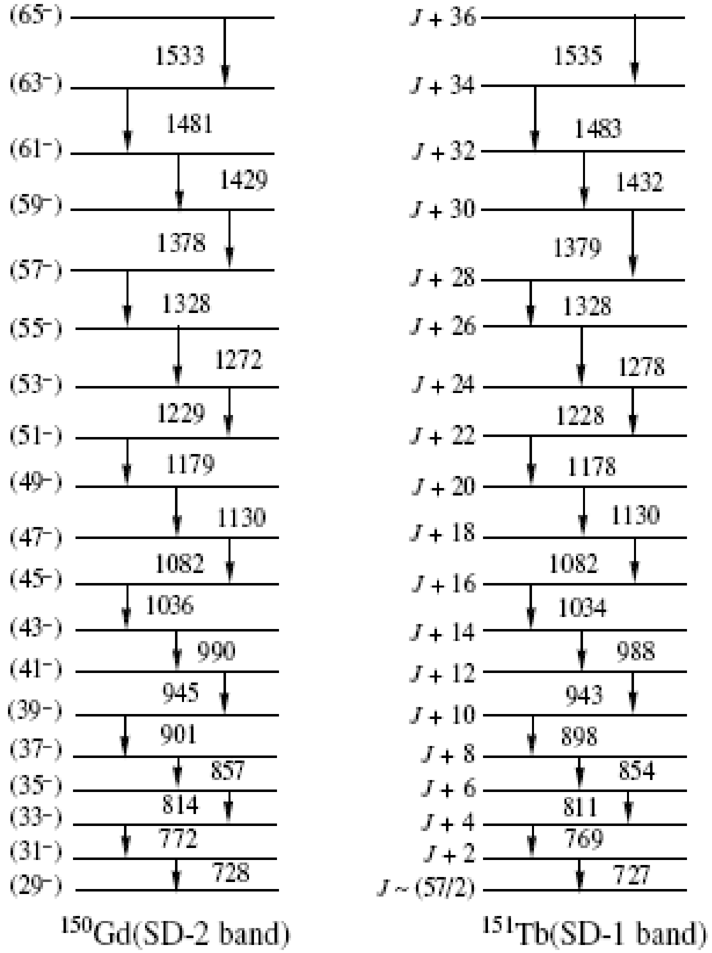


Fig. 5. Observed states of the identical superdeformed bands of ^{150}Gd and ^{151}Tb. The energies are given in keV. Experimental data were taken from [22].

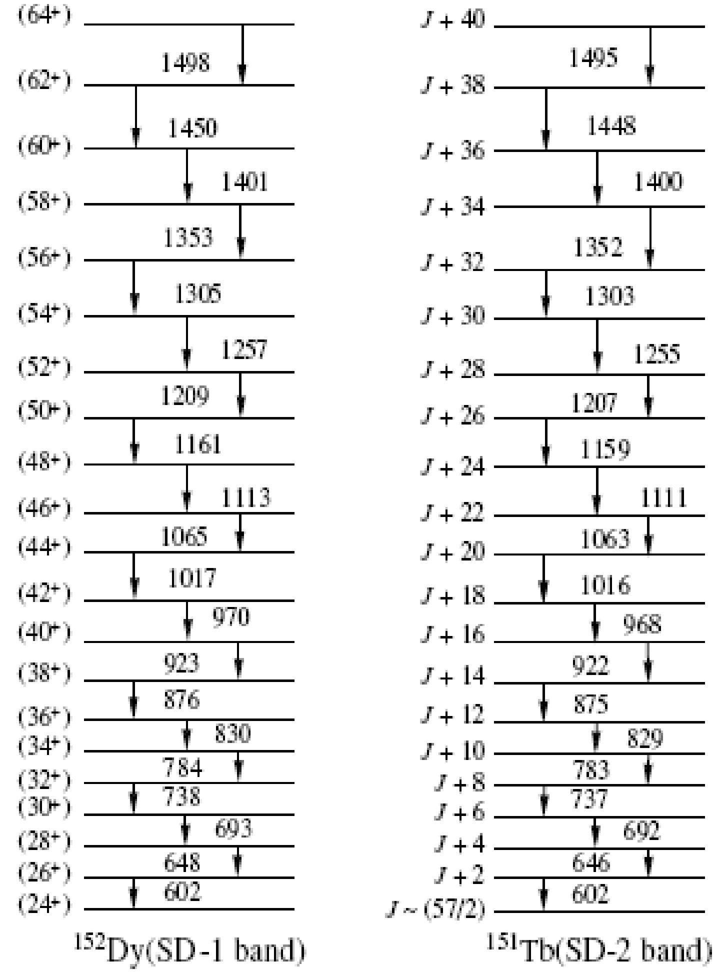


Fig. 6. Observed states of the identical superdeformed bands of ^{152}Dy and ^{151}Tb. The energies are given in keV. Experimental data were taken from [22].

Example 1.

The model describing a system of *s* and *d* bosons and the fermions occupying single particle states with angular momenta j=1/2,3/2,5/2.

This model is based on the U(6/12) graded algebra.

$$p_{1/2}, (p_{3/2}, f_{5/2}) \to \tilde{l} = 0, 2, \quad \tilde{s} = \frac{1}{2},$$

$$a^+_{jm} = \sum_{\mu,\sigma} C^{jm}_{\tilde{l}\mu 1/2\sigma} a^+_{\tilde{l}\mu, 1/2\sigma},$$

$$H = \epsilon_0 \bar{N}(0) + \epsilon_2 \bar{N}(2) - \kappa Q_2 \cdot Q_2,$$

$$\bar{N}(0) = \bar{N}^B_s + \bar{N}^F(\tilde{l} = 0),$$

$$\bar{N}(2) = \bar{N}^B_d + \bar{N}^F(\tilde{l} = 2),$$

$$\bar{N}^B_s = S^+ s, \quad \bar{N}^F(\tilde{l} = 0) = \sum_{\sigma} a^+_{00,1/2\sigma} a_{00,1/2\sigma},$$

$$\bar{N}^B_d = \sum_{\mu} d^+_\mu d_\mu, \quad \bar{N}^F(\tilde{l} = 2) = \sum_{\mu,\sigma} a^+_{2\mu,1/2\sigma} a_{2\mu,1/2\sigma},$$

$$Q_{2\mu} = Q^B_{2\mu} + Q^F_{2\mu},$$

$$Q^B_{2\mu} = d^+_\mu s + s^+(-1)^\mu d_{-\mu} + \chi \sum_{\eta,\eta'} C^{2\mu}_{2\eta 2\eta'} d^+_\eta (-1)^{\eta'} d_{-\eta'},$$

$$Q^F_{2\mu} = \sum_\sigma \Big(a^+_{2\mu,1/2\sigma} a_{00,1/2\sigma} + a^+_{00,1/2\sigma}(-1)^\mu a_{2-\mu,1/2\sigma}$$

$$+\chi \sum_{\eta,\eta'} C^{2\mu}_{2\eta 2\eta'} a^+_{2\eta,1/2\sigma}(-1)^{\eta'} a_{2-\eta',1/2\sigma} \Big)$$

The superoperator commuting with the Hamiltonian is:

$$P_{1/2\sigma} = a^{+}_{00,1/2\sigma} s + \sum_{\mu} a^{+}_{2\mu,1/2\sigma} d_{\mu},$$

$$|JM N_F = 1, N_B = N-1, L\rangle = -\frac{1}{\sqrt{N}} \sum_{\mu,\sigma} C^{JM}_{1/2\sigma L\mu} P_{1/2\sigma} |L\mu, N_F = 0, N_B = N\rangle.$$

Together with the corresponding eigen states of the even-even nucleus, these states form supersymmetric multiplets. Since the operator $P_{1/2\sigma}$ has the angular momentum j=1/2, we obtain doublets of the states with angular momenta $J = L \pm 1/2$ in the odd nucleus.

The superoperator $P_{1/2\ m}$ together with its Hermitian conjugate and the operators:

$N_B = N^B{}_s + N^B{}_d$, $N_F = N^F{}_{(l=0)} + N^F{}_{(l=2)}$

$S_{1\eta} = \sqrt{3/2} \sum C^{1/2m}{}_{1/2m'\ 1\eta} a^+{}_{l\mu\ \ 1/2m}\ a_{l\mu\ \ 1/2m'}$

form a graded Lie algebra U(1/2)

Example 2

$$H = \sum_{\tilde{l},\mu,\sigma} E_{\tilde{l}} a^{+}_{\tilde{l}\mu,1/2\sigma} a_{\tilde{l}\mu,1/2\sigma} + \frac{\hbar^2}{2\Im}\vec{L}^2$$

$$-\hbar\omega_0 \sum_{\mu} (-1)^{\mu} q_{2\mu} Q^{c}_{2-\mu},$$

$$q_{2\mu} =< r^2 > \sum_{\tilde{l},\tilde{l}',\nu,\nu',\sigma} \frac{1}{\sqrt{5}} < \tilde{l}'||Y_2||\tilde{l} >$$

$$C^{2\mu}_{\tilde{l}'\nu'\tilde{l},\nu,} a^{+}_{\tilde{l}'\nu',1/2\sigma} a_{\tilde{l}\nu,1/2\sigma}.$$

The relative weakness of the pseudospin-orbit coupling implies that pseudoorbital angular momenta of the quasiparticles are strongly coupled to the deformation, forming together with the core a rotating system with angular momentum

$$\vec{\tilde{L}} = \vec{R} + \vec{\tilde{l}}$$

The pseudospins are then added to form the total angular momentum

$$\vec{J} = \vec{\tilde{L}} + \vec{\tilde{s}},$$

$$\vec{L} = \vec{R} + \vec{\tilde{l}},$$

$$Q^c_{2\mu} = \beta D^2_{\mu 0}.$$

The operators $a^+_{\tilde{l}\nu,1/2\sigma} a_{\tilde{l}\nu',1/2\sigma'}$, L_μ, D^L_{M0}, $a^+_{\tilde{l}\nu,1/2\sigma} D^l_{m0}$, $a_{\tilde{l}\nu,1/2\sigma} D^l_{m0}$ form a graded Lie algebra.

The superoperator

$$P_{1/2\sigma} \equiv \sum_{\tilde{l},m} \chi_{\tilde{l}}\, C^{00}_{\tilde{l}m\tilde{l}-m} a^+_{\tilde{l}\nu,1/2\sigma} \sqrt{2\tilde{l}+1} D^{\tilde{l}}_{-m0},$$

$$[H, P_{1/2\sigma}] = E_0 P_{1/2\sigma},$$

$$\sum_{\tilde{l}'} h_{\tilde{l}\tilde{l}'} \chi_{\tilde{l}'} = E_0 \chi_{\tilde{l}}$$

$$h_{\tilde{l}\tilde{l}'} = E_{\tilde{l}} \delta_{\tilde{l}\tilde{l}'} - \hbar\omega_0 \beta < r^2 >< \tilde{l}0 | Y_{20} | \tilde{l}'0 > .$$

The superoperator $P_{1/2\sigma}$ acting on the eigenstates of the even-even nucleus produces the eigenstates of the neighboring odd nucleus.
The level energies in the odd nucleus are shifted by a constant E_0 with respect to the energies of the corresponding eigenstates of the even-even nucleus.
Thus the γ-transition energies of the corresponding states in both nuclei are equal.

Dirac equation:

$$\begin{pmatrix} m + V - S & \vec{\sigma}\left(\vec{p} - \vec{V}\right) \\ \vec{\sigma}\left(\vec{p} - \vec{V}\right) & -m + V + S \end{pmatrix} \begin{pmatrix} g \\ f \end{pmatrix}_i = \varepsilon_i \begin{pmatrix} g \\ f \end{pmatrix}_i$$

scalar potential	$S(\mathrm{r})$
vector potential (time-like)	$V(\mathrm{r})$
vector potential (space-like)	$\vec{V}(\mathrm{r})$

Elimination of large components:

$(\varepsilon \longrightarrow m+\varepsilon)$

$$g_i(\mathrm{r}) = \frac{1}{\varepsilon_i - W_-}\vec{\sigma}\vec{\mathrm{p}} f_i(\mathrm{r}) \qquad W_\pm = V \pm S$$

$$\left\{\vec{\sigma}\vec{\mathrm{p}}\frac{1}{\varepsilon_i - W_-}\vec{\sigma}\vec{\mathrm{p}} + W_+ - 2m\right\} f_i(\mathrm{r}) = \varepsilon_i f_i(\mathrm{r})$$

$$\left\{\vec{\mathrm{p}}\frac{1}{\varepsilon_i - W_-}\vec{\mathrm{p}} + \frac{1}{(\varepsilon_i - W_-)^2}\frac{1}{r}\frac{\partial W_-}{\partial r}\vec{l}\vec{s} + W_+ - 2m\right\} f_i(\mathrm{r}) = \varepsilon_i f_i(\mathrm{r})$$

For V=S is $W_-=0$, i.e. pseudo-spin orbit spitting vanishes

J.N.Ginocchio, PRL 78 (1997) 436

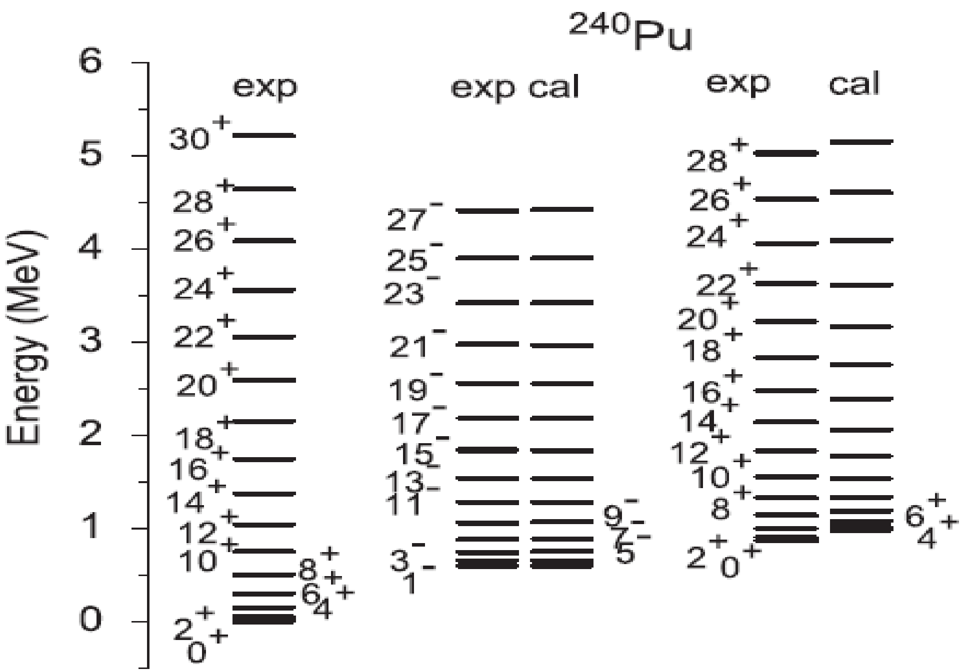
QCD-sum rules: V≈S

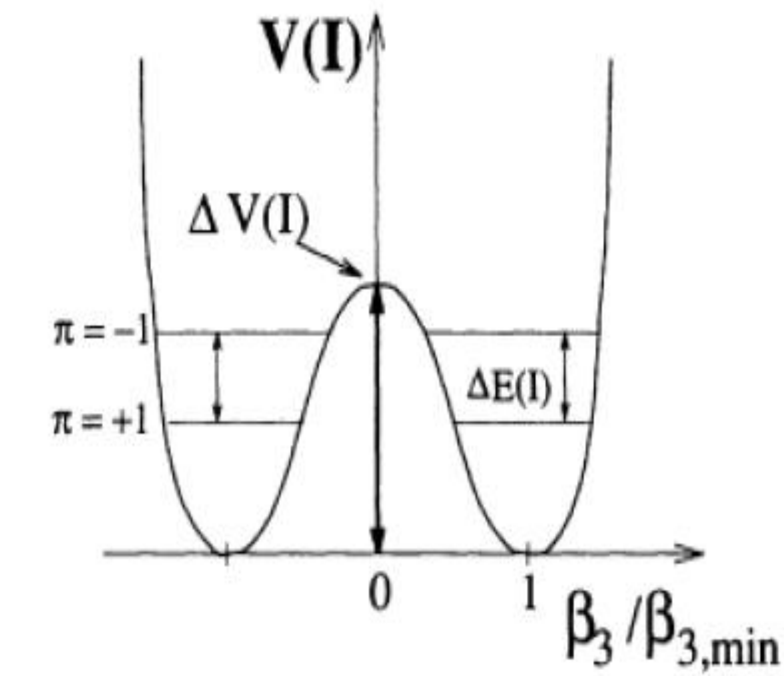
Furnstahl et al, PRC 46 (1992) 1507

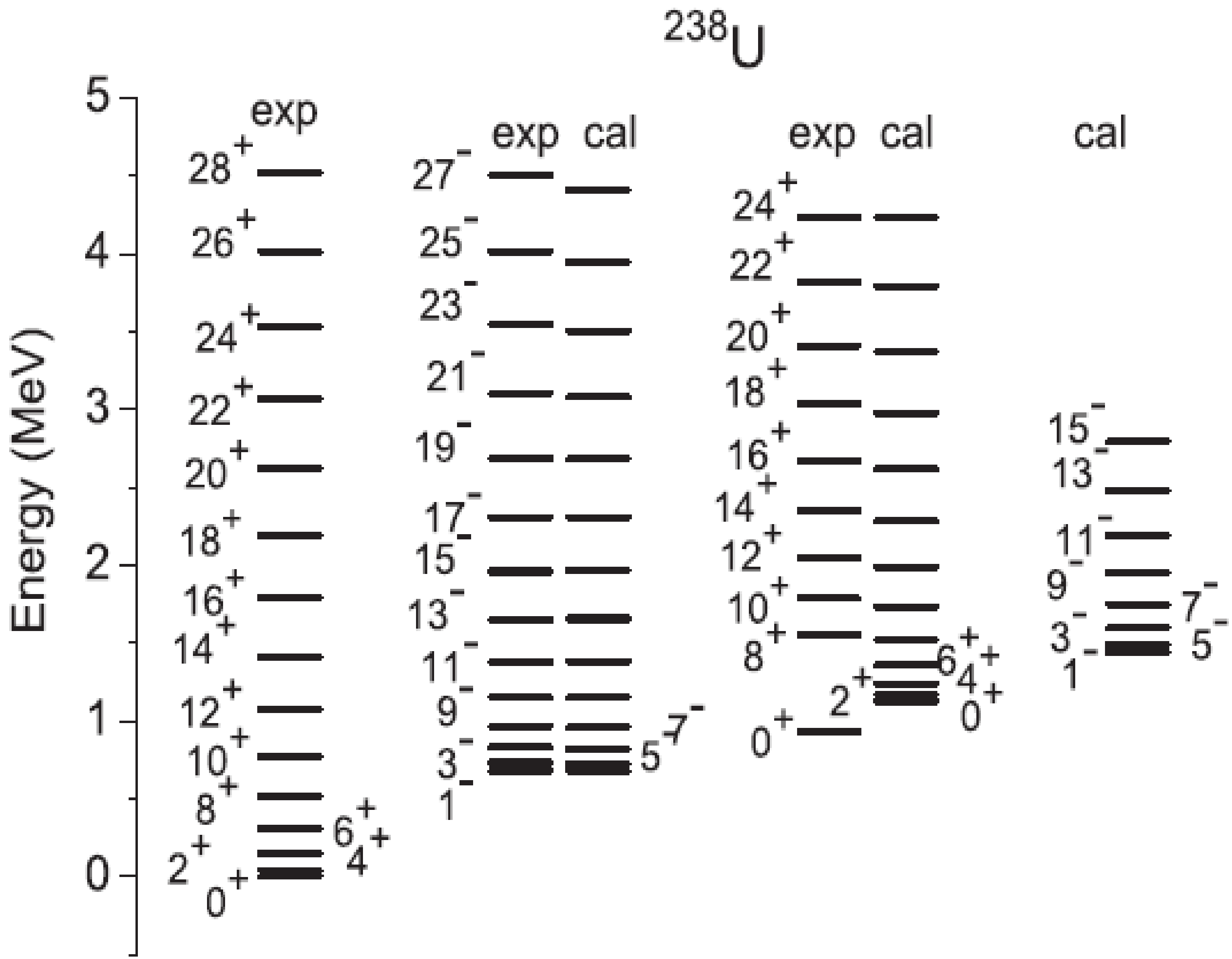
Space-reflection asymmetric modes of nuclear excitation and Supersymmetric Quantum Mechanics

The Hamiltonian of the model used can be presented as

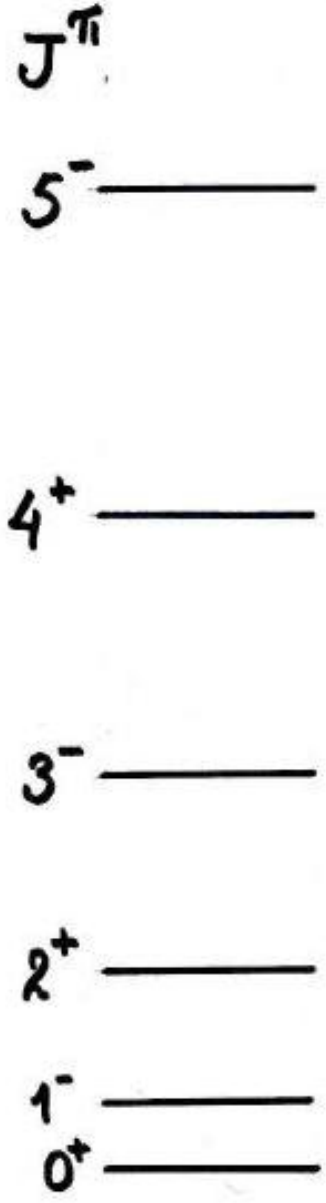
$$H_I = -\frac{\hbar^2}{2B}\frac{d^2}{d\beta_{30}^2} + V_I^{(1)}(\beta_{30}),$$







^{238}U
Energy (MeV)
exp
exp cal
exp cal
cal



CH

$$E_J = B\,J(J+1) - D\,J^2(J+1)^2$$

$$\frac{B}{\hbar c} = 14.19\,\mathrm{cm}^{-1}$$

$$\frac{D}{\hbar c} = 14.4 \cdot 10^{-4}\,\mathrm{cm}^{-1}$$

The numerical solution of the Schrodinger equation with the Hamiltonian shown above with different variants of the two-center potential having very small or very large distances between two minima have shown that the wave function of the positive parity state belonging to the ground state alternating parity band can be approximated to a good accuracy by the sum of two Gaussians:

$$\Psi_I(\beta_{30}) = \left(\frac{B\omega}{\pi\hbar}\right)^{\frac{1}{4}} \{2[1+\exp(-s_3^2(I))]\}^{-1/2} \times \left(\exp\left\{-\frac{1}{2}s_3^2(I)[\beta_{30}/\beta_m(I)-1]^2\right\} + \exp\left\{-\frac{1}{2}s_3^2(I)[\beta_{30}/\beta_m(I)+1]^2\right\}\right),$$

where

$$s_3(I) \equiv \sqrt{\frac{B\omega}{\hbar}}\beta_m(I).$$

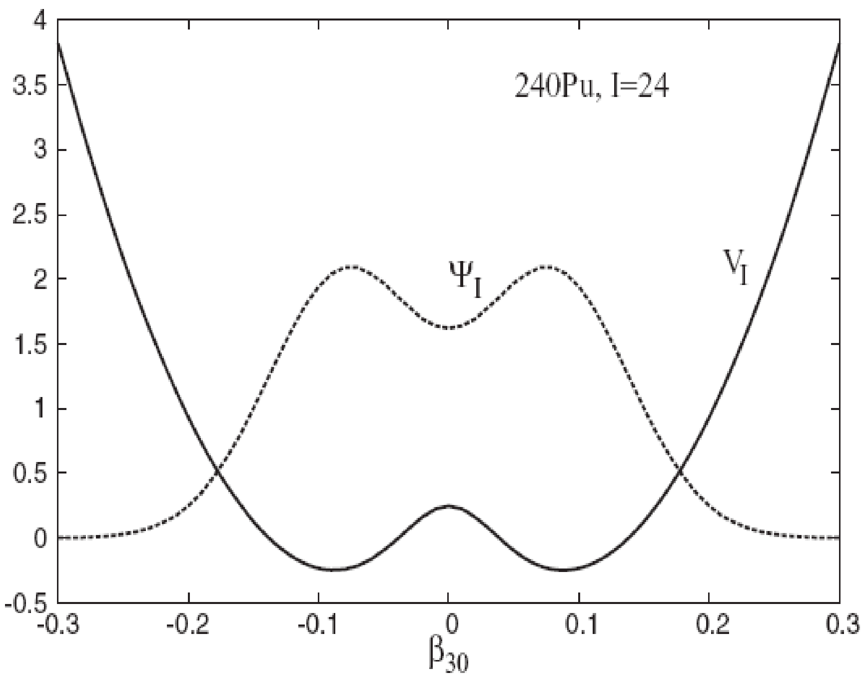
Having a parameterized wave function we can substitute it into the Schrodinger equation and obtain the potential:

$$V_I = \frac{\hbar^2}{2B}\frac{d^2\Psi_I}{d\beta_{30}^2}/\Psi_I + E_I^*,$$

which gives us

$$V_I(\beta_{30}) = \frac{\hbar\omega}{2}\left\{-1 + s_3^2(I)\left[1 + \beta_{30}^2/\beta_m^2(I)\right] - 2s_3^2(I)\frac{\beta_{30}}{\beta_m(I)}\frac{\exp\left[s_3^2(I)\beta_{30}/\beta_m(I)\right] - \exp\left[-s_3^2(I)\beta_{30}/\beta_m(I)\right]}{\exp\left[s_3^2(I)\beta_{30}/\beta_m(I)\right] + \exp\left[-s_3^2(I)\beta_{30}/\beta_m(I)\right]}\right\} + E_I^*,$$

Examples of the potentials



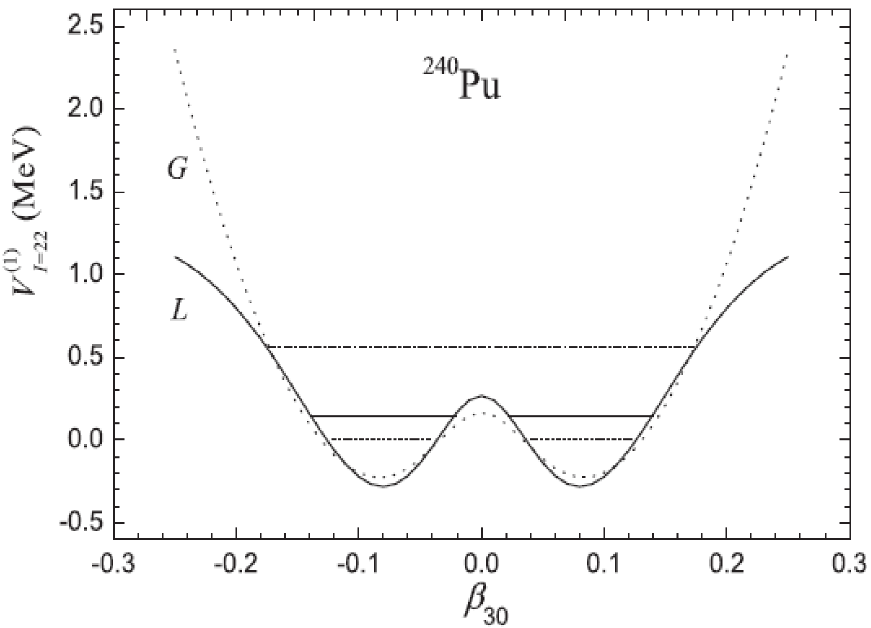


FIG. 3. The potentials $V_I^{(1)}$ as a function of β_{30} calculated with $I = 22$ using the Legendre ansatz (L) and the Gauss ansatz (G) for the wave functions of the ground state band. The straight solid line indicates the position of the first excited negative parity state. Dashed line indicates position of the lowest positive parity states. Dot-dashed line indicates position of the second excited positive parity states. The level energies are obtained using the Legendre ansatz.

The problem is to find the wave functions of the negative parity states of the lowest alternating parity band. In order to do this and to calculate the parity splitting, i.e. the shift of the energies of the negative parity states with respect to the positive parity states it is convenient to follow the prescription of the supersymmetric quantum mechanics and to introduce the supersymmetric partner potential:

$$V_I^{(2)}(\beta_{30}) = [W_I(\beta_{30})]^2 + \frac{\hbar}{\sqrt{2B}} \frac{dW_I(\beta_{30})}{d\beta_{30}} + E_I^*,$$

where

$$W_I(\beta_{30}) = -\frac{\hbar}{\sqrt{2B}} \frac{\left[\frac{d\Psi_I^{(1,+)}(\beta_{30})}{d\beta_{30}}\right]}{\Psi_I^{(1,+)}(\beta_{30})}$$

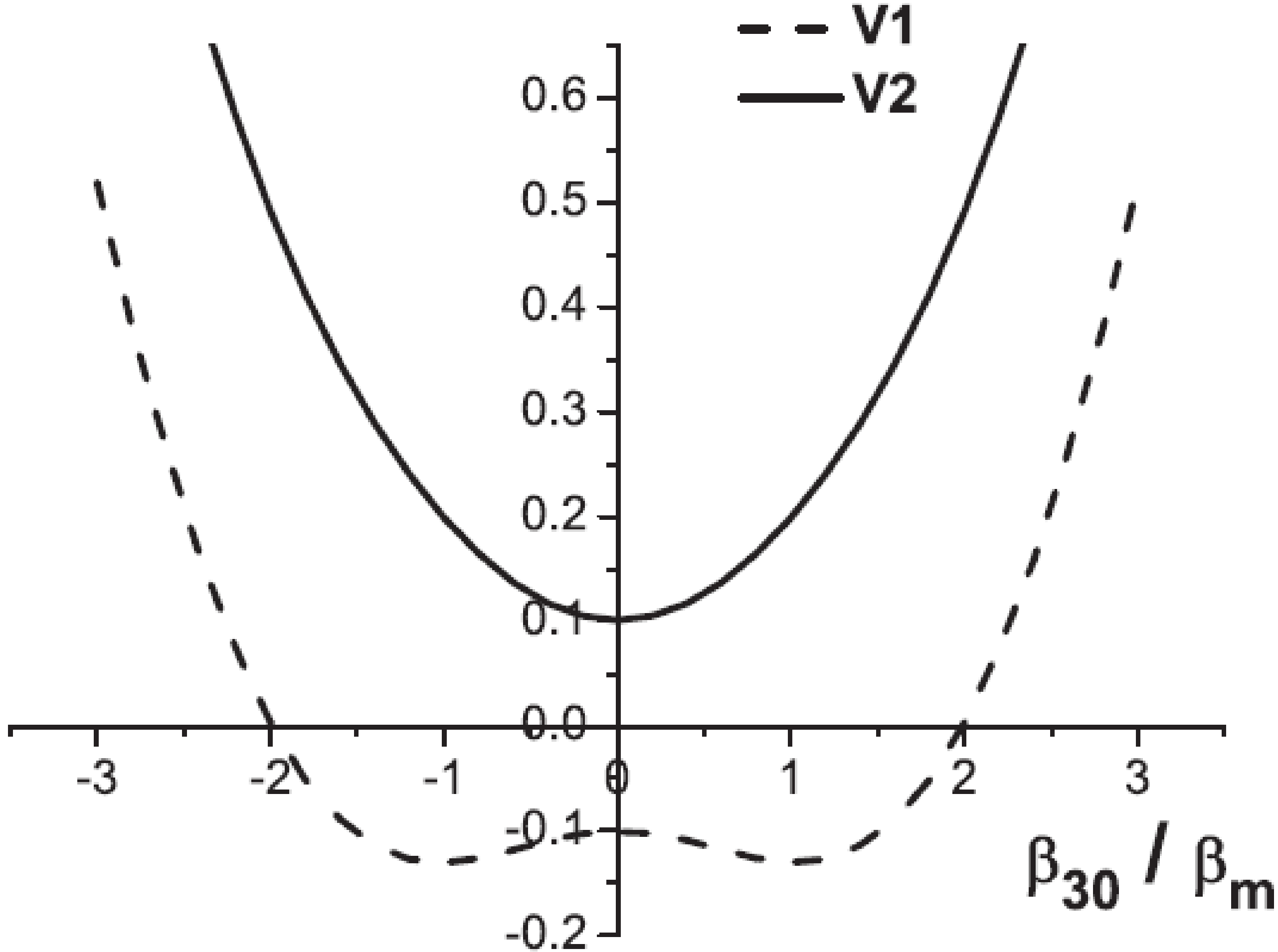


FIG. 1. The supersymmetric partner potentials $V_I^{(1)} \equiv V1$ and $V_I^{(2)} \equiv V2$ as functions of $\beta_{30}/\beta_m(I) \equiv \beta_{30}/\beta_m$ calculated for $I = 14$ with the parameters fixed for ^{240}Pu. The potential energy is counted from the excitation energy of the 14_1^+ state of ^{240}Pu.

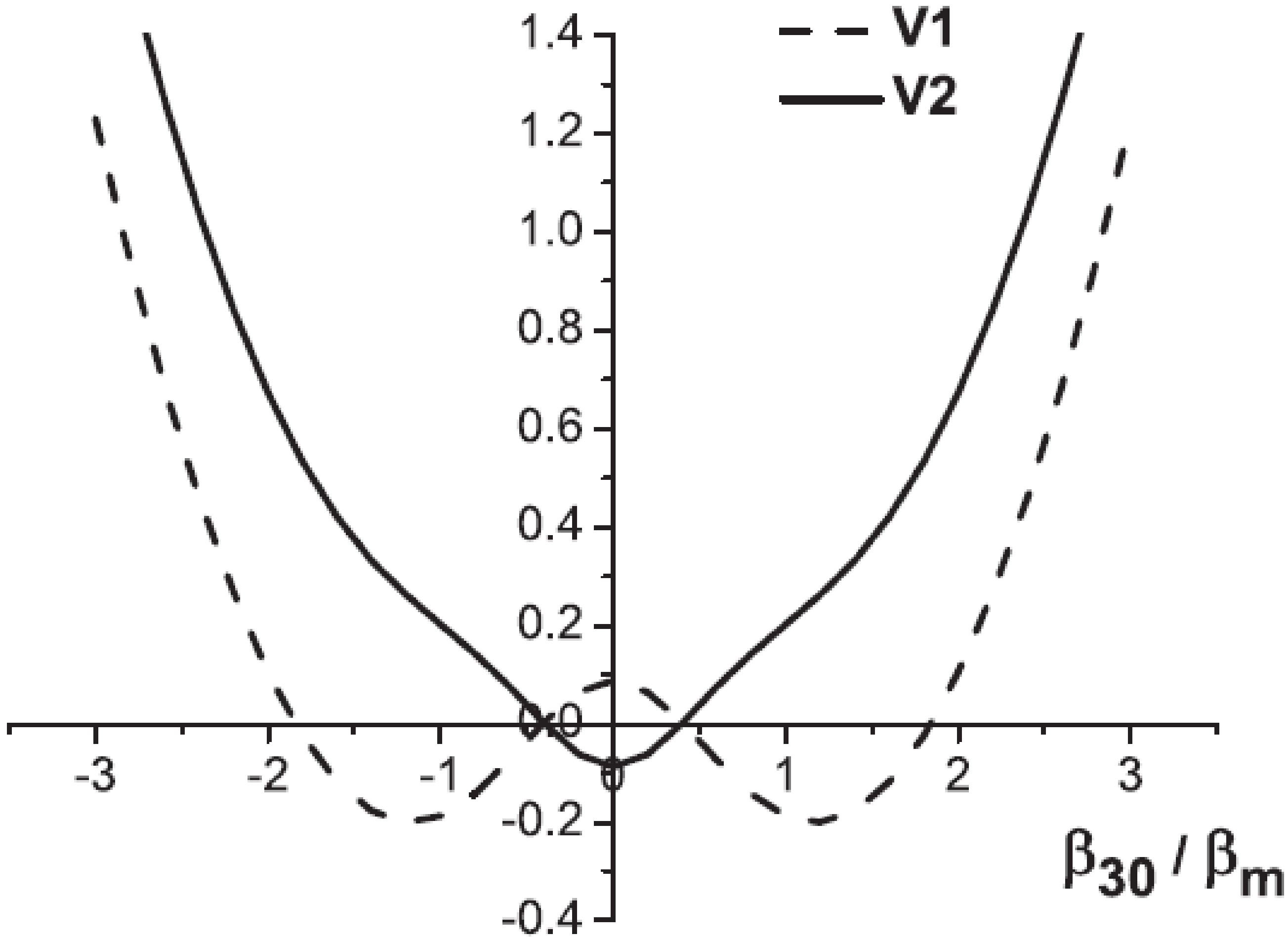


FIG. 2. The supersymmetric partner potentials $V_I^{(1)} \equiv V1$ and $V_I^{(2)} \equiv V2$ as functions of $\beta_{30}>/\beta_m(I) \equiv \beta_{30}/\beta_m$ calculated for $I = 20$ with the parameters fixed for ^{240}Pu. The potential energy is counted from the excitation energy of the 20_1^+ state of ^{240}Pu.

The mathematical technique of the supersymmetric quantum mechanics simplify calculations significantly.

$$H_I^{(2)} = -\frac{\hbar^2}{2B}\frac{d^2}{d\beta_{30}^2} + V_I^{(2)}(\beta_{30})$$

$$\Delta E(I) \approx \left\langle \Psi_{n=0,I}^{(2)} \middle| H_I^{(2)} \middle| \Psi_{n=0,I}^{(2)} \right\rangle.$$

$$\Psi_I^{(1,-)}(\beta_{30}) = \frac{1}{\sqrt{\Delta E(I)}} A_I^{+} \Psi_{n=0,I}^{(2)},$$

$$A_I^{+} = -\frac{\hbar}{\sqrt{2B}}\frac{d}{d\beta_{30}} + W_I(\beta_{30})$$

Interacting Boson Model

$$A_{2\mu}^{+} = \sum_{ss'} \Psi_{ss'} (\alpha_s^{+} \alpha_{s'}^{+})_{2\mu}$$

$$A_{2\mu} = \sum_{ss'} \Psi_{ss'} (\alpha_s \alpha_{s'})_{2\mu}$$

$$[A_{2\mu}, A_{2\mu'}^{+}]$$

$$[[A_{2\mu}, A_{2\mu'}^{+}], A_{2\mu''}^{+}] = \sum_{n} K_n \sum_{ss'} \Psi_{ss'}^{(n)} (\alpha_s^{+} \alpha_{s'}^{+})_{2\mu} \rightarrow K_1 A_{2\mu}^{+}$$

Boson representation of the SU(6) algebra

$$A_{2\mu}^{+} \to b_{2\mu}^{+}\sqrt{1-\frac{1}{N}\sum_{\nu} b_{2\nu}^{+} b_{2\nu}} \to b_{2\mu}^{+} S \frac{1}{\sqrt{N}}$$

$$A_{2\mu} \to \sqrt{1-\frac{1}{N}\sum_{\nu} b_{2\nu}^{+} b_{2\nu}}\; b_{2\mu} \to \frac{1}{\sqrt{N}} S^{+} b_{2\mu}$$

$$[A_{2\mu}, A_{2\mu'}^{+}] \sim b_{2\mu}^{+} b_{2\mu'}$$

Dyson boson representation

$$A_{2\mu}^{+} \rightarrow b_{2\mu}^{+}\left(1-\frac{1}{N}\sum_{\nu} b_{2\nu}^{+} b_{2\nu}\right)$$

$$A_{2\mu} \rightarrow b_{2\mu}$$

$$\sqrt{1-\frac{1}{N}\sum_{\nu} b_{2\nu}^{+} b}$$

$$\theta\left(1-\frac{1}{N}\sum\nolimits_{\nu} b_{2\nu}^{+} b_{2\nu}\right)$$

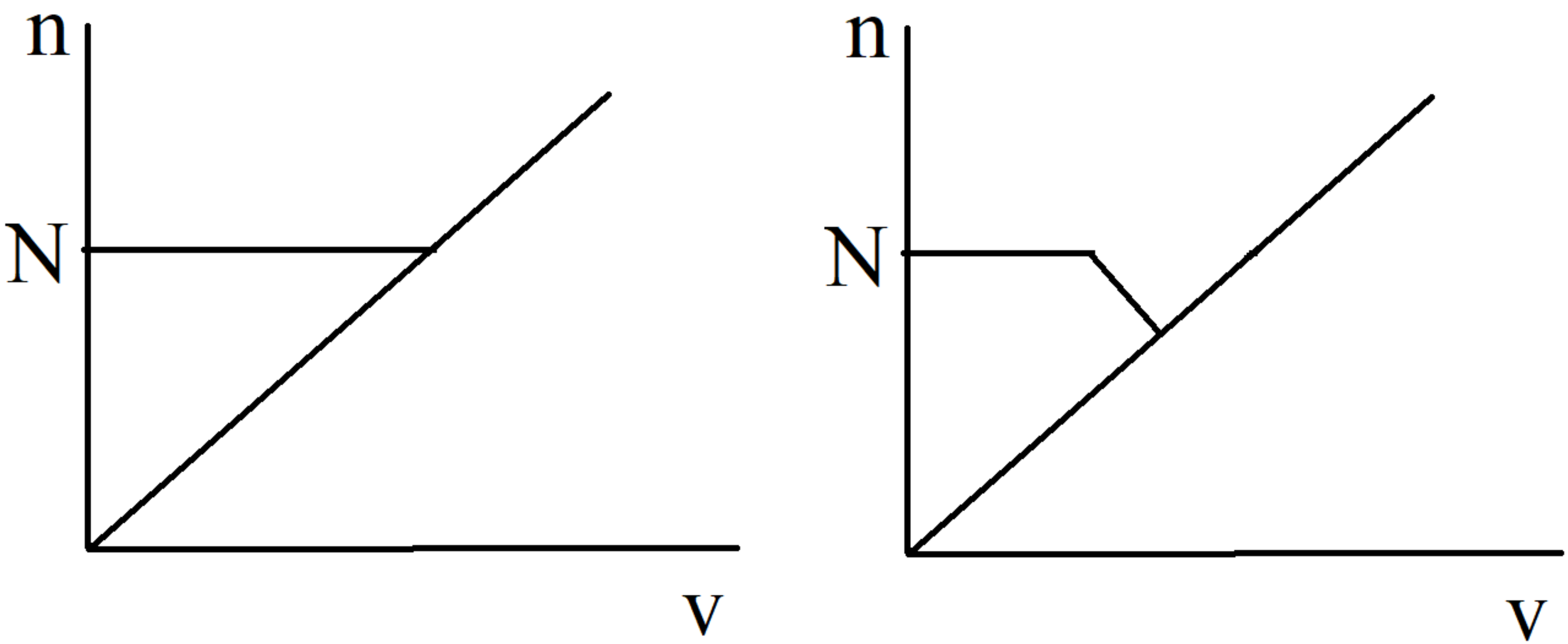
$$\left(1+e^{\frac{\sum_{\nu} b_{2\nu}^{+} b_{2\nu}-N}{a}}\right)^{-1}$$

$$A^+_{2\mu} \to \Phi^{-1} b^+_{2\mu} \Phi$$

$$A_{2\mu} \to b_{2\mu}$$

$$\Phi(N,v)/\Phi(N+1,v+1) = \left(1 - \frac{N+v}{2K}\right)\left(1 - \frac{N}{N_{max}}\right)$$

$$\Phi(N,v)/\Phi(N+1,v-1) = \left(1 - \frac{N-v-3}{2K}\right)\left(1 - \frac{N}{N_{max}}\right)$$


n
N
v
n
N
v

Dynamical symmetries

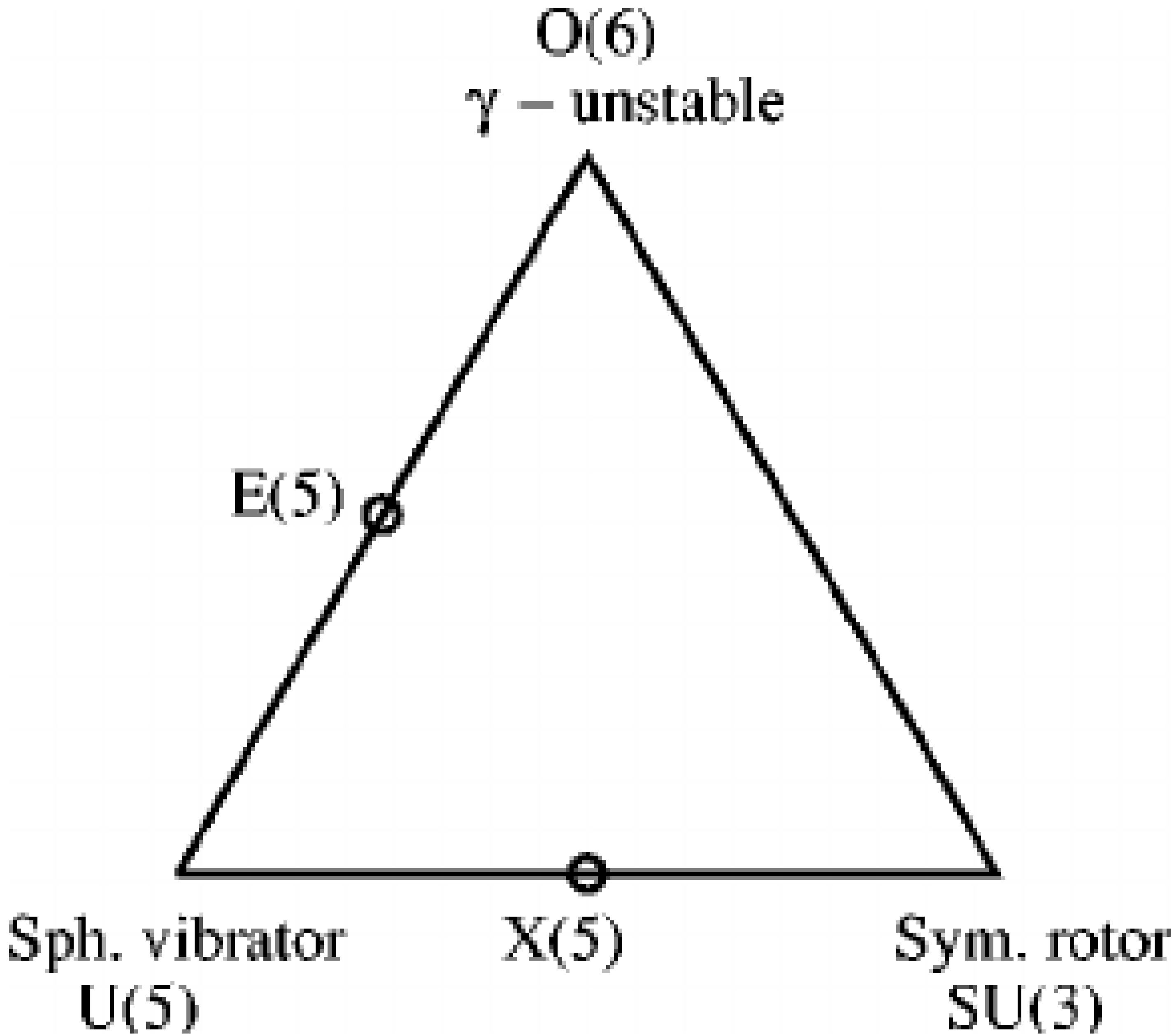
$$
\begin{array}{ccl}
 & \nearrow & U(5) \supset O(5) \supset O(3) \supset O(2) \\
U(6) & \rightarrow & O(6) \supset O(5) \supset O(3) \supset O(2) \\
 & \searrow & SU(3) \supset O(3) \supset O(2)
\end{array}
$$

Collective Hamiltonian

$$H = \varepsilon \sum_{\mu} d_{\mu}^{+} d_{\mu} + \kappa_1 \left(\sum_{\mu} d_{\mu}^{+} (-1)^{\mu} d_{-\mu}^{+} ss + h.c. \right)$$

$$\kappa_2 \left(\sum_{\mu} (d^{+} d^{+})_{2\mu} d_{\mu} s + h.c. \right)$$

$$+ \frac{1}{2} \sum_{L=0,2,4} c_L \sum_{\mu} (d^{+} d^{+})_{2\mu} (dd)_{2\mu}.$$


O(6)
γ – unstable
E(5)
Sph. vibrator
U(5)
X(5)
Sym. rotor
SU(3)

Conclusion.

- Pseudospin symmetry.

  This symmetry is supported by the experimental data and justified theoretically.

- The mathematical technique of the Supersymmetric Quantum Mechanics simplify significantly a solution of the nuclear structure problems with double minimum potentials.

- The Interacting Boson Model which is based on the SU(6) group is introduced to describe collective nuclear properties.