

# Level densities of heavy and superheavy nuclei

Azam Rahmatinejad

Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, Russia

December, 2020



# Outline

## 1 Introduction

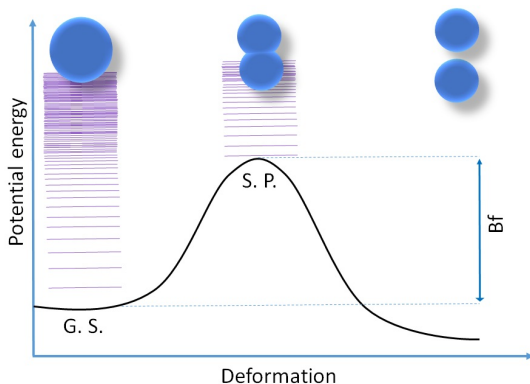
## 2 Studies

- Superfluid formalism
- Systematics
- Survival probabilities
- Spin and parity distributions



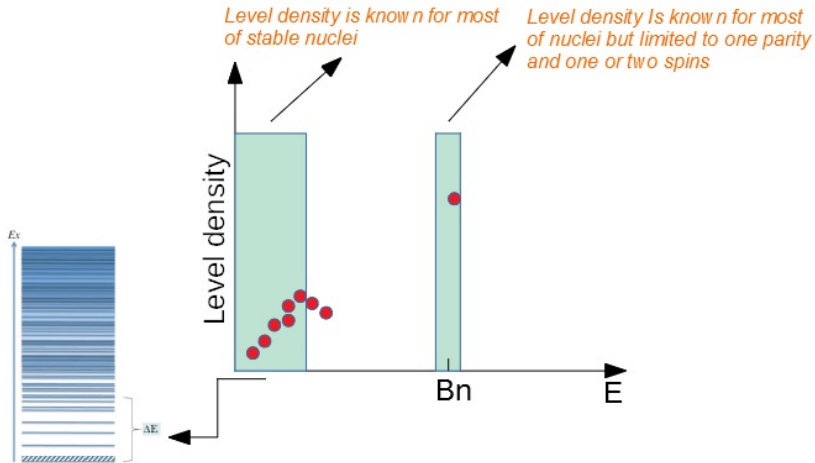
# Why level density is important in superheavy region?!

Level density is the number of levels per energy unit (MeV).



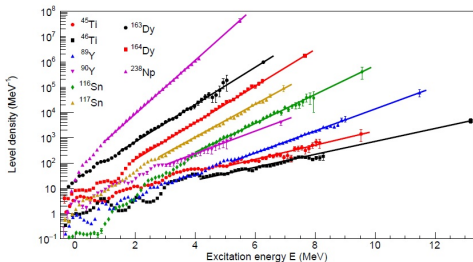
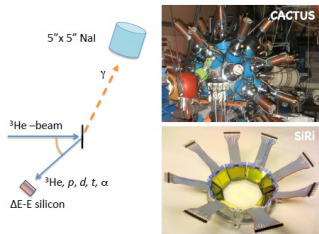
$$\frac{\Gamma_f}{\Gamma_n} \propto \frac{\rho_f}{\rho_n}$$





<https://www.nndc.bnl.gov/ensdf/>.





T. Renstrøm *et al.*, Phys. Rev. C **98**, 054310 (2018).

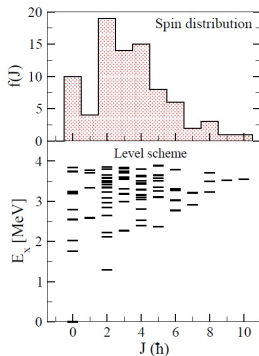
R. Chankova *et al.*, Phys. Rev. C **73**, 034311 (2006).

H. Utsunomiya *et al.*, Phys. Rev. C **88**, 015805 (2013).

M. Guttormsen *et al.*, Phys. Rev. C **68**, 064306 (2003).

H.T. Nyhus *et al.*, Phys. Rev. C **85**, 014323 (2012).





$$\rho(U, J, \pi) = F_\pi(U) F_J(U, J) \rho(U)$$

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12a^{1/4}U^{5/4}} \exp(2\sqrt{aU}); U = aT^2$$

$$F_\pi = \frac{1}{2}$$

$$F_\pi(U) = \frac{1}{2} \tanh\left(\alpha \frac{U}{2}\right)$$

$$F_J(U, J) = e^{-\frac{J^2}{2\sigma^2}} - e^{-\frac{(J+1)^2}{2\sigma^2}} \approx \frac{2J+1}{2\sigma^2} e^{-\frac{J(J+\frac{1}{2})}{2\sigma^2}}$$

$$\rho_{def}(U, J, \pi) = \rho_{spher}(U, J, \pi) K_{coll}; K_{coll} \propto \sigma_\perp^2$$

S. I. Al-Quraishi, S. M. Grimes, T. N. Massey, and D. A. Resler, *Phys. Rev. C* **67**, 015803 (2003).

S.K. Singhal, H.M. Agrawal, *Nucl. Phys. A* **853**, 26 (2011).

T. V. Egidy, and D. Bucurescu, *Phys. Rev. C* **80**, 054310 (2009).



# Aims

- To obtain survival probabilities for superheavy nuclei.
- To examine calculation formalism for lighter mass region where experimental data on nuclear level density is available.
- To study spin and parity distributions and collective effects.



Nucleus is considered as a system of independent quasiparticles.

The thermal equilibrium is assumed between neutron and proton subsystems.

$$\Omega = -\beta \sum_{\tau=p,n} \sum_k (\varepsilon_{\tau k} - \lambda_{\tau} - E_{\tau k}) + 2 \sum_k \log[1 + \exp(-\beta E_{\tau k})] - \beta \frac{\Delta_{\tau}^2}{G_{\tau}}$$

The BCS equations, which determine the temperature dependence of  $\Delta_{\tau}$  and  $\lambda_{\tau}$ , are derived from  $\Omega$ .

$$N_{\tau} = \sum_k \left( 1 - \frac{\varepsilon_{\tau k} - \lambda_{\tau}}{E_{\tau k}} \tanh \frac{\beta E_{\tau k}}{2} \right), \quad \frac{2}{G_{\tau}} = \sum_k \frac{\tanh(\beta E_{\tau k})/2}{E_{\tau k}}$$

$\lambda_{\tau}$ ,  $\Delta_{\tau}$ : chemical potential, pairing gap.

$E_{\tau k} = \sqrt{(\varepsilon_k - \lambda)^2 + \Delta^2}$ : quasiparticle energies.

$G_{\tau}$ : The constant of the pairing interaction.





## Data

Mass, shell correction and deformations of nuclei with  $Z = 112 - 120$  at the ground state and at the saddle point.

Ground state								
Z	N	A	Mass (MeV)	$E_{tot}$ (MeV)	ELD (MeV)	$\delta E_{sh}$ (MeV)	$\beta_2$	$\beta_3$
112	165	277	151.79	-4.99	3.20	-8.18	0.2076	0.0000
112	166	278	152.43	-5.38	2.26	-7.64	0.2040	0.0000
112	167	279	154.31	-4.67	3.66	-8.32	0.2025	0.0000
112	168	280	155.25	-5.00	2.60	-7.60	0.1935	0.0018
112	169	281	157.40	-4.25	4.28	-8.53	0.1966	0.0000
112	170	282	158.31	-4.86	0.86	-5.71	0.1453	0.0000
112	171	283	160.24	-4.56	1.67	-6.23	0.1308	0.0006
112	172	284	161.41	-5.12	0.85	-5.97	0.1308	0.0006



## Data

Saddle point								
Z	N	A	Mass (MeV)	$E_{tot}$ (MeV)	ELD (MeV)	$\delta E_{sh}$ (MeV)	$\beta_2$	$\beta_3$
112	165	277	156.25	-0.53	2.13	-2.65	0.34	0.00
112	166	278	156.44	-1.37	1.70	-3.08	0.33	0.00
112	167	279	158.30	-0.68	2.16	-2.84	0.33	0.00
112	168	280	159.03	-1.22	0.86	-2.08	0.31	0.00
112	169	281	161.28	-0.38	2.33	-2.70	0.28	0.00
112	170	282	162.05	-1.12	1.51	-2.63	0.28	0.00
112	171	283	164.81	0.02	2.31	-2.29	0.28	0.00
112	172	284	165.76	-0.78	1.19	-1.97	0.28	0.00
112	173	285	168.73	0.33	2.91	-2.58	0.28	0.00
112	174	286	169.90	-0.47	1.45	-1.91	0.28	0.00



$$E_{\tau}(T) = \sum_k \varepsilon_{k,\tau} \left( 1 - \frac{\varepsilon_{k,\tau} - \lambda_{\tau}}{E_{k,\tau}} \tanh \frac{\beta E_{k,\tau}}{2} \right) - \frac{\Delta_{\tau}^2}{G_{\tau}},$$

$$U(T) = \sum_{\tau} E_{\tau}(T) - E_{\tau}(0).$$

$$S(T) = \sum_{\tau} \sum_k \left\{ \ln[1 + \exp(-\beta E_{k,\tau})] + \frac{\beta E_{k,\tau}}{1 + \exp(\beta E_{k,\tau})} \right\}.$$

$$\rho_i(U) = \frac{\exp(S)}{(2\pi)^{\frac{3}{2}} \sqrt{D}}$$

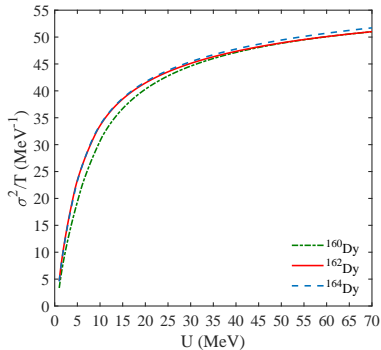


$$\rho(U) = \frac{\rho_i(U)}{\sqrt{2\pi\sigma^2}}$$

$$\sigma^2 = \frac{1}{2} \sum_{\tau=p,n} \sum_k m_{\tau k}^2 \cosh^{-2}(1/2\beta E_{\tau k}),$$

$m_{\tau k}$ : The single-particle spin projections.

$$\mathfrak{S} = \frac{\hbar^2 \sigma^2}{T}$$



Nucleus	$\mathfrak{S}_{r.b.}$ ( $\hbar^2/\text{MeV}$ )
<sup>160</sup> Dy	65.46
<sup>162</sup> Dy	66.83
<sup>164</sup> Dy	68.21

$$\mathfrak{S}_{r.b.} = 0.4MR^2$$



$$\rho_{\text{tot}}(U) = \sum_c \rho_i(U - U_c) \tau_c(U_c)$$

$$\tau_c(U_c) = 2I_c + 1$$

$$U_c = \hbar\omega_\beta(n_\beta + 1/2) + \hbar\omega_\gamma(2n_\gamma + |K|/2 + 1) + \frac{\hbar^2}{2\mathfrak{S}} [I_c(I_c + 1) - K^2]$$

$n_\beta, n_\gamma$ : the quantum numbers of harmonic oscillator energies.

$K$ : the projection of  $I_c$  on the symmetry axis.

$$\rho_{\text{tot}}(U) = \rho(U) K_{\text{coll}}$$

$$K_{\text{coll}} = K_{\text{rot}} K_{\text{vib}}$$

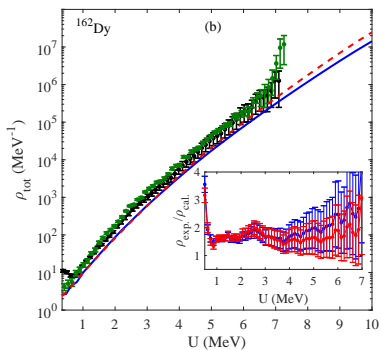
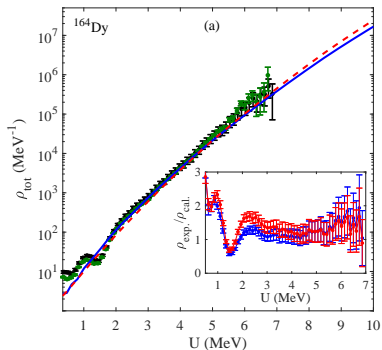
$$K_{\text{vib}} = \exp\left(0.0555A^{2/3} T^{4/3}\right)$$

$$K_{\text{rot}} = \begin{cases} 1, & \text{for spherical nuclei} \\ \mathfrak{S}_\perp T, & \text{for deformed nuclei,} \end{cases}$$

$$\mathfrak{S}_\perp = \mathfrak{S}_{r.b} f(\beta_2, \beta_4)$$

$$f(\beta_2, \beta_4) = 1 + \sqrt{5/16\pi} \beta_2 + (45/28\pi) \beta_2^2 + (15/7\pi \sqrt{5} \beta_2 \beta_4)$$





A. Rahmatinejad, T. M. Shneidman, N. V. Antonenko, A. N. Bezbakh, G. G. Adamian, and L. A. Malov, *Phys. Rev. C* **101**, 054315 (2020).

Green symbols: M. Guttormsen *et al.*, *Phys. Rev. C* **68**, 064306 (2003), and

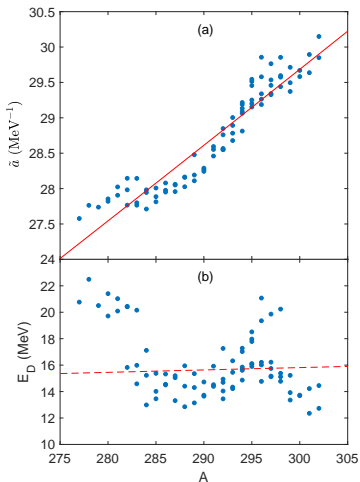
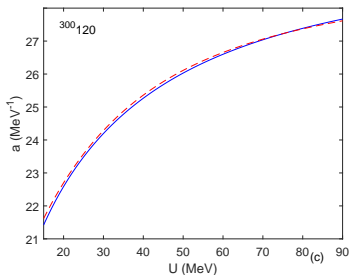
H.T. Nyhus *et al.*, *Phys. Rev. C* **85**, 014323 (2012).

Black symbols: T. Renstrøm *et al.*, *Phys. Rev. C* **98**, 054310 (2018).



$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12a[U]^{\frac{1}{4}}U^{\frac{5}{4}}} \exp(2\sqrt{a[U]U})$$

$$a(A, U) = \tilde{a}(A) \left[ 1 + \frac{1 - \exp(-\frac{U}{E_D})}{U} \delta E_{sh} \right]$$



$$\tilde{a} = a_1 A + a_2 A^2$$

$$a_1 = 0.09 \text{ MeV}^{-1},$$

$$a_2 = 2.89 \times 10^{-4} \text{ MeV}^{-1},$$

$$E_D \approx 15 \text{ MeV}.$$



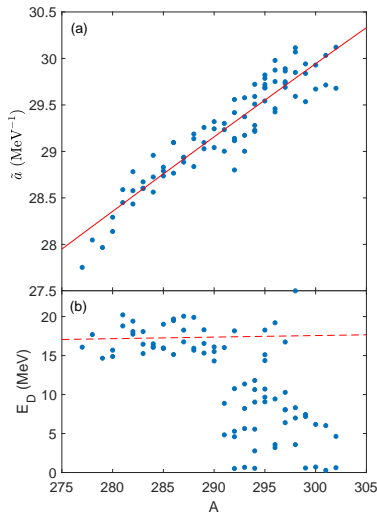
# Saddle point

$$|\delta E_{sh}| \leq 1.7 \text{ MeV}$$

$$\delta E_{sh} \rightarrow (\delta E_{sh} - \Delta)$$

$$\tilde{a} = a_1 A + a_2 A^2$$

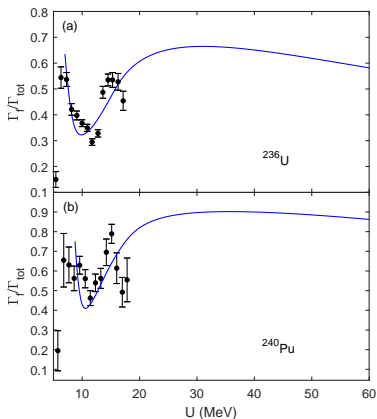
$$\begin{aligned} a_1 &= 0.1217 \text{ MeV}^{-1}, \\ a_2 &= -7.3 \times 10^{-5} \text{ MeV}^{-1}, \\ E_D &\approx 17 \text{ MeV}. \end{aligned}$$





$$\frac{\Gamma_n}{\Gamma_f} = \frac{gA^{2/3} \int_0^{U-B_n} \varepsilon \rho_{GS}(U - B_n - \varepsilon) d\varepsilon}{K_0 \int_0^{U-B_f} \rho_{SP}(U - B_f - \varepsilon) d\varepsilon}$$

$$\frac{\Gamma_f}{\Gamma_{tot}} = \frac{1}{1 + \Gamma_n/\Gamma_f}$$

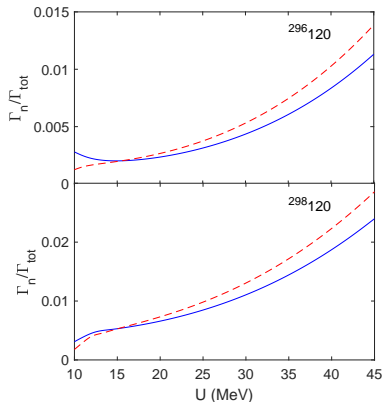
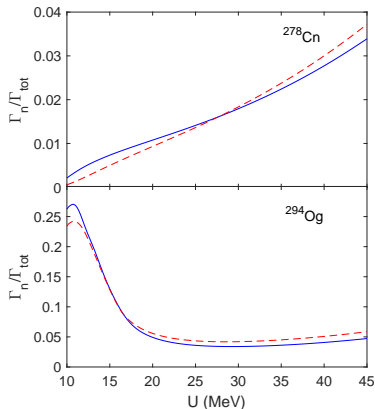


A. Rahmatinejad, A. N. Bezbakh, T. M. Shneidman, G. Adamian, N. V. Antonenko, P. Jachimowicz, M. Kowal, arXiv:2005.08685 [nucl-th].

E. Cheifetz, H. C. Britt, and J. B. Wilhelmy, Phys. Rev. C **24**, 519 (1981).



$$\frac{\Gamma_n}{\Gamma_f} = \frac{4A^{2/3}a_f(U - B_n - \Delta_n)}{K_0a_n[2a_f^{1/2}(U - B_f - \Delta_f)^{1/2} - 1]} \times \exp\left[2a_n^{1/2}(U - B_n - \Delta_n)^{1/2} - 2a_f^{1/2}(U - B_f - \Delta_f)^{1/2}\right]$$



$\pi^g$ : The same parity as the whole nucleus at the ground state

$\pi^s$ : The parity opposite to the ground state

$$P(n) = \frac{f^n}{n!} e^{-f}$$

$$f = \sum_{k \in \pi_s} \frac{1}{1 + \exp(\beta E_k)}$$

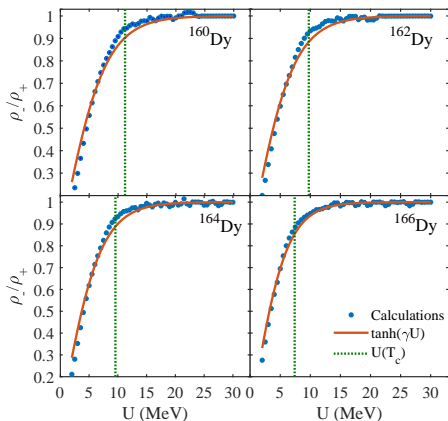
$$\frac{\rho^s}{\rho^g} = \frac{\beta^s}{\beta^g} \frac{Z^s}{Z^g} \sqrt{\frac{C^g}{C^s}} e^{(\beta^s - \beta^g) U}$$

D. Mocolj *et al.*, *Phys. Rev. C* **75**, 045805 (2007).



$$\frac{\rho_-}{\rho_+} = \tanh \gamma U$$

Nucleus	$\gamma$ (MeV <sup>-1</sup> )
<sup>160</sup> Dy	0.133
<sup>162</sup> Dy	0.143
<sup>164</sup> Dy	0.147
<sup>166</sup> Dy	0.172



S. I. Al-Quraishi, S. M. Grimes, T. N. Massey, and D. A. Resler, *Phys. Rev. C* **67**, 015803 (2003).

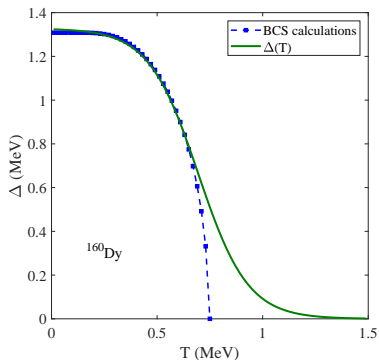
R. Razavi, O. Nouri, A. Rahmatinejad, S. Mohammadi, *Mod. Phys. Lett. A*, 2050315-1 (2020).



Thank you for your attention!



$$\Delta = \frac{\Delta_0}{1 + \exp\left(\frac{T - T_m}{f_m}\right)}$$



$$P(n) = \frac{f^n}{n!} e^{-f}$$

$$P^+ = \sum_n^{\text{even}} \frac{f^n}{n!} e^{-f} = e^{-f} \cosh f$$

$$P^- = \sum_n^{\text{odd}} \frac{f^n}{n!} e^{-f} = e^{-f} \sinh f$$

$$\frac{P^-}{P^+} = \tanh f$$

$$\frac{P^s}{P^g} = \tanh f', \quad f' = f_p + f_n$$

$$P^{s(g)} = \frac{Z^{s(g)}}{Z}$$

$$Z^s = \frac{Z}{1 + \frac{1}{\tanh f'}}$$

$$Z^g = \frac{Z}{1 + \tanh f'}$$



$$E_{\tau}(T) = \sum_k \varepsilon_{k,\tau} \left( 1 - \frac{\varepsilon_{k,\tau} - \lambda_{\tau}}{E_{k,\tau}} \tanh \frac{\beta E_{k,\tau}}{2} \right) - \frac{\Delta_{\tau}^2}{G_{\tau}},$$

$$U(T) = \sum_{\tau} E_{\tau}(T) - E_{\tau}(0).$$

$$S(T) = \sum_{\tau} \sum_k \{ \ln[1 + \exp(-\beta E_{k,\tau})] + \frac{\beta E_{k,\tau}}{1 + \exp(\beta E_{k,\tau})} \}.$$

$$C_{\tau}(T) = \frac{1}{2} \sum_k \cosh^{-2} \left( \frac{\beta E_{k,\tau}}{2} \right) \left[ \beta^2 E_{k,\tau}^2 - \beta \Delta_{\tau} \frac{d\Delta_{\tau}}{dT} \right].$$

$$U^s(\mathcal{g}) = - \frac{\partial \ln Z^s(\mathcal{g})}{\partial \beta}$$

$$U^{\mathcal{g}} = U + \left[ (1 - \tanh(f)) \frac{\partial f}{\partial \beta} \right]$$

$$U^s = U + \left[ (1 - \coth(f)) \frac{\partial f}{\partial \beta} \right]$$

$$S^s(\mathcal{g}) = \beta U^s(\mathcal{g}) + \ln Z^s(\mathcal{g})$$

$$S^{\mathcal{g}} = \beta U^{\mathcal{g}} + S - \log[(1 + \tanh(f))]$$

$$S^s = \beta U^s + S - \log[(1 + \coth(f))]$$

$$C^s(\mathcal{g}) = -\beta^2 \frac{\partial^2 \ln Z^s(\mathcal{g})}{\partial \beta^2}$$

$$C^{\mathcal{g}} = C - \beta^2 (1 - \tanh(f)) \left[ (1 + \tanh(f)) \left( \frac{\partial f}{\partial \beta} \right)^2 - \left( \frac{\partial^2 f}{\partial \beta^2} \right) \right]$$

$$C^s = C - \beta^2 (1 - \coth(f)) \left[ (1 + \coth(f)) \left( \frac{\partial f}{\partial \beta} \right)^2 - \left( \frac{\partial^2 f}{\partial \beta^2} \right) \right]$$



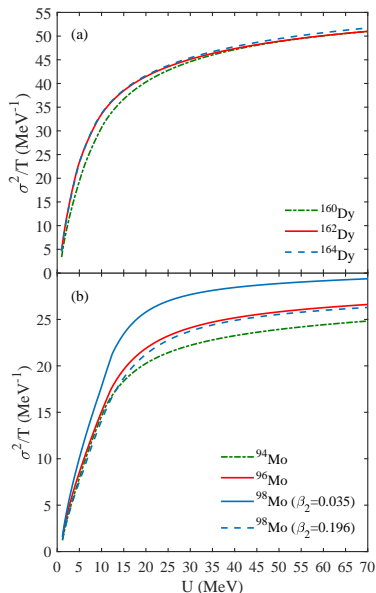


The spin cut-off factor is related to the effective moment of inertia

$$\mathfrak{S} = \frac{\hbar^2 \sigma^2}{T}$$

Nucleus	$\mathfrak{S}_{r.b.}$ ( $\hbar^2/\text{MeV}$ )	$\mathfrak{S}_{exp}$ ( $\hbar^2/\text{MeV}$ )
$^{160}\text{Dy}$	65.46	34.56
$^{162}\text{Dy}$	66.83	37.19
$^{164}\text{Dy}$	68.21	40.87
$^{94}\text{Mo}$	26.98	3.44
$^{96}\text{Mo}$	27.94	3.85
$^{98}\text{Mo}$	28.92	4.08

$$\mathfrak{S}_{r.b.} = 0.4MR^2, \quad \mathfrak{S}_{exp} = 3/E_{2+}.$$



Assumption of a decoupling between intrinsic and collective degrees of freedom

$$U = U_i + U_c$$

$$\rho_{\text{tot}}(U) = \int \rho_i(U_i) \rho_{\text{coll}}(U - U_i) dU_i$$

$$\rho_{\text{coll}}(U - U_i) = \sum_c \delta(U - U_i - U_c) \tau_c(U_c).$$

$$\tau_c(U_c) = 2I_c + 1$$

$$\rho_{\text{tot}}(U) = \sum_c \rho_i(U - U_c) \tau_c(U_c)$$



$$\rho_{\text{tot}}(U) \simeq \sum_c \left[ \rho_i(U) - U_c \frac{\partial \rho_i(U)}{\partial U} \right] \tau_c(U_c)$$

$$= \sum_c \left[ \rho_i(U) - \frac{U_c}{T} \rho_i(U) \right] \tau_c(U_c).$$

$$\rho_{\text{tot}}(U) \simeq \rho_i(U) \sum_c \exp\left(-\frac{U_c}{T}\right) \tau_c(U_c)$$

$$K_{\text{coll}} = \sum_c \exp\left(-\frac{U_c}{T}\right) \tau_c(U_c)$$

$$U_c = \hbar\omega_\beta(n_\beta + 1/2) + \hbar\omega_\gamma(2n_\gamma + |K|/2 + 1) + \frac{\hbar^2}{2\mathcal{I}_c} [I_c(I_c + 1) - K^2]$$

$n_\beta, n_\gamma$ : the quantum numbers of harmonic oscillator energies.

$K$ : the projection of  $I_c$  on the symmetry axis.



# Yrast band

Quantum numbers:

$$K = 0, I = 0, 2, 4, \dots$$

$$n_{\beta} = n_{\gamma} = 0$$

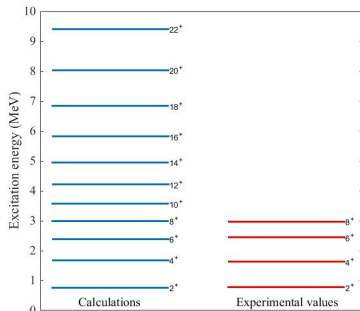
$$U_c = \frac{\hbar^2}{2\mathfrak{I}} [I(I+1)].$$

$$E(2^+) = \frac{2 \times 3}{2\mathfrak{I}}$$

$$\mathfrak{I}_{exp} = \frac{3}{E(2^+)}.$$

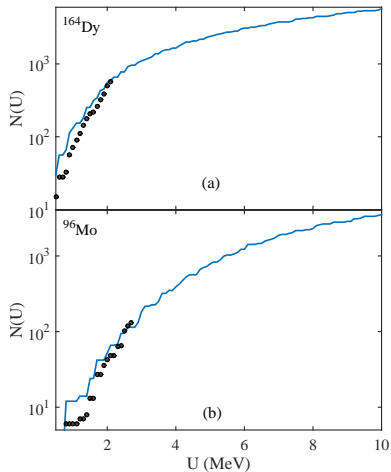
$$\mathfrak{I} = \mathfrak{I}_{r.b} (1 - a_1 e^{-a_2 I(I+1)})$$

$$a_1 = 0.89, a_2 = 0.006$$



D. Abriola, and A. A. Sonzogni, Nuclear Data Sheets **107**, 2423 (2006).

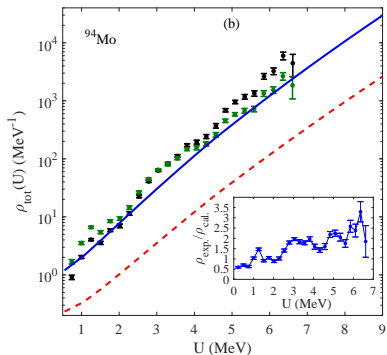
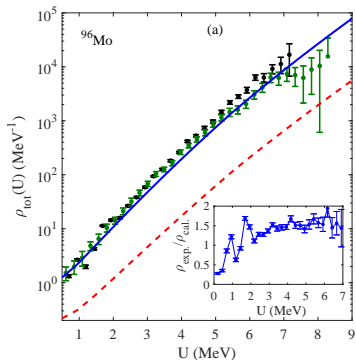




B. Singh and J. Chen, Nuclear Data Sheets **147**, 1 (2018).

D. Abriola, and A. A. Sonzogni Nuclear Data Sheets **109**, 2501 (2008).





Green symbols: R. Chankova *et al.*, Phys. Rev. C **73**, 034311 (2006).

Black symbols: H. Utsunomiya *et al.*, Phys. Rev. C **88**, 015805 (2013).



$$W(xn) = P(xn, E_{CN}) \prod_{i=1}^x G_{ni}(E_i)$$
$$G_{ni}(E_i) = \frac{\Gamma_n(E_i)}{\Gamma_n(E_i) + \Gamma_f(E_i)}$$

