# A mathematical model of Eternal Cosmology

***Cosmologies*: 1. observational, 2. astrophysical, 3. mathematical**

At present *Big Bang* is well established. But there is no "standard" physical picture for the origin of the universe, and no initial conditions for the next scenario , *inflationary universe,* that is derived by *trial and error approach* to get a reasonable exit to the final *Big Bang.* With this aim, a special scalar field was proposed, the "potential" for which is not known**.** Nevertheless, the physical model of inflation scenarios is observationally successful.

Thus, inflation looks pretty attractive but it is not fully defined ***mathematical*** model.
***H**ere I will discuss a well defined **model** but **not** its relations to "reality".*

Imagine an *Eternal Universe* consisting of ***3*** *basic components:* ***Gravity**, **Dark Matter,*** *and* ***G-DM - convertor*** that imitate ***DE*** *(to be explained later)***.** *The **3** characteristic "Eternal Constants" of this* **Dark World** are **G, c,** *and **cosmological** constant* $\Lambda$

The last one is not well defined experimentally but, in our model, it has ***no alternative.*** With these three, one can define the cosmological system of units of ***L,T, M.*** The prime Universe is pure geometrical and uses only ***Length*** and ***Time. Matter*** requires ***Mass,*** i.e. **G.**

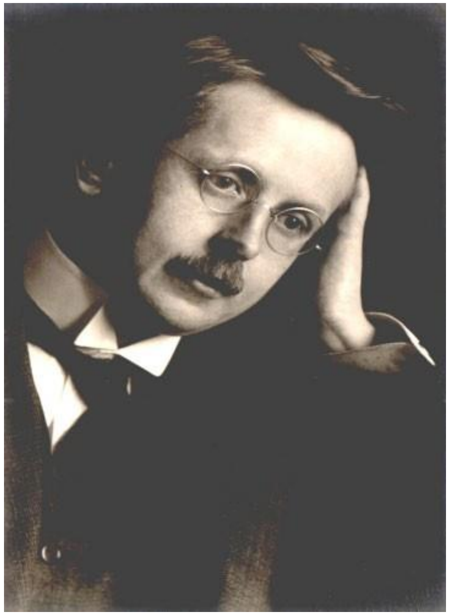
Presumably, in this world must exist gravity-matter waves, black holes and, most important, it *possibly* may give birth to new worlds, like ours... But it's long way to check these dreams.

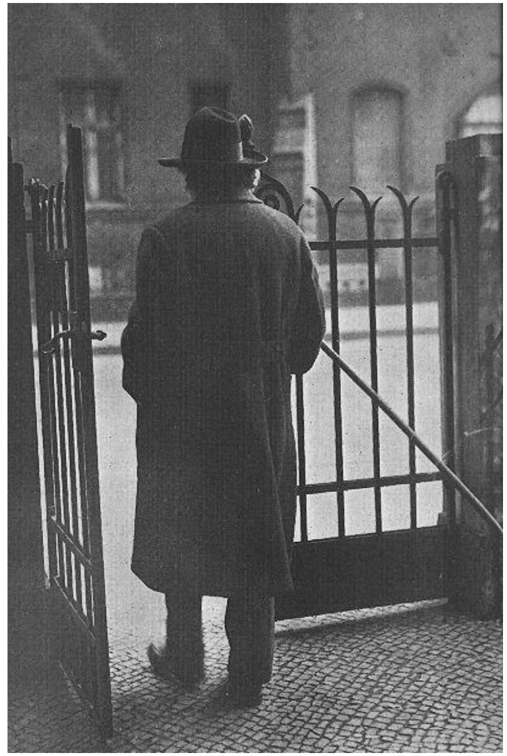
$$\gamma^i_{jk} \overset{\text{def}}{=} \Gamma^i_{jk}(g_{mn}) + \alpha\,(\delta^t_j\, a_k + \delta^i_k\, a_j)$$

$$\hat{\mathcal{L}} \equiv \sqrt{-\det(r_{kl})}$$

$$\sqrt{-g}\,\{R - 2\Lambda\,[\,\det(\delta^i_j + \lambda f^i_j)\,]^{\frac{1}{2}} - m^2\, a_i\, a^i\}$$

$$f_{01} = \dot{a}_1 - a'_0$$





1888 -1955

$$R - 3\,\alpha^2 a_i\, a^i$$

1879 -1955

$$-2\Lambda + \kappa\, e^{-2\alpha}\left[e^{-2\gamma}\dot{A}^2 - m^2 A^2\right]$$

1882 - 1944

$$\sqrt{1 - \lambda^2\, \dot{a}^2}$$

Clever is the God but evil is He  not

# Ideas *of* Einstein, Weyl, Eddington

## *on* **generalizing** *the* **geometry** *of* **General Relativity** *to* **incorporate photon**

[ forgotten discovery ]

**1919--1923**

***Almost* Riemannian connection(!)**, the geodesics of which coincide with those of Riemannian, *up to parameterizations*

$$\gamma^i_{jk} = \Gamma^i_{jk}(g_{mn}) + \alpha\,(\delta^t_j\, a_k + \delta^i_k\, a_j)\,; \qquad r \equiv g^{ik}\, r_{ik} = R - 3\,\alpha^2 a_i\, a^i$$

$$2\kappa\,\mathcal{L}^{(4)}_{\mathrm{geo}} = \sqrt{-g}\,\{R - 3\,\alpha^2 a_i a^i - 2\Lambda\,[\det(\delta^i_j + \lambda f^i_j)\,]^{\frac{1}{2}}\,\}$$

$$f_{ij} \equiv \partial_i\, a_j - \partial_j\, a_i\,, \quad f_{01} = \dot{a}_1 - a'_0 \qquad R \equiv g^{ik}\, R_{ik}$$

Only **2 free parameters** remain for vecton, if $\Lambda$ is known.

$$2\kappa\,\mathcal{L}^{(4)}_{gAs} = \sqrt{-g}\left[R - 2\Lambda - \kappa\left(\frac{1}{2}F_{ij}F^{ij} + m^2 A_i\, A^i + \partial_i\psi\,\partial^i\psi + v(\psi)\right)\right]$$

This can be derived from the nonlinear Lagrangian for small $a_i$ proportional to $A_i$

The **most natural** base for units is:

$\Lambda \quad \kappa \stackrel{\mathrm{def}}{=} 8\pi G$ the speed of light *c*

(in our units it is 1).

ATF Arxiv: 0812.2616

ATF Arxiv: 1003.0782

ATF arXiv: 1905.09390

**`General spherical' metric:** $ds_4^2 = e^{2\alpha}dr^2 + e^{2\beta}d\Omega^2(\theta,\phi) - e^{2\gamma}dt^2 + 2e^{2\delta}drdt$

Reduction to **cosmological**, **static** (or **wave**) solutions has the **main constraint**

$$-\dot{\beta}' - \dot{\beta}\beta' + \dot{\alpha}\beta' + \dot{\beta}\gamma' = \frac{1}{2}[\dot{\psi}\psi' + A_0A_1] \propto T_{01}^{(m)} = \partial\mathcal{L}^{(m)}/\partial g^{01} \quad \text{for} \quad g^{01} \to 0$$

This is one of **Einstein's equations** corresponding to **delta-variations**

Separation of variables (dim. reduction to black holes, cosmologies, and waves)

$$\alpha = \alpha_0(t) + \alpha_1(r), \quad \beta = \beta_0(t) + \beta_1(r),$$

Altogether 7 types of solutions: 3 cosmologies + 3 `dual' static + 1 `self dual' wave

General anisotropic: $\beta' = \gamma' = 0$ FLRW cosmology: $\dot{\alpha} = \dot{\beta}\,, \quad \gamma' = 0$

We call the '*special' anisotropic* the cosmology with $\beta' = \gamma'$ and $\dot{\alpha} = 0$, which is dual to FRLW. The *flat isotropic* cosmology is obtained from the general anisotropic one if in addition $\bar{k} = k = 0$ and $\dot{\alpha} = \dot{\beta}$.

Mind that different gauge choices for $\gamma$ and $\alpha$

2-D gravity: $$2\kappa\,\mathcal{L}_g^{(2)} \equiv e^{2\beta+\alpha+\gamma}\,[\,e^{-2\alpha}(2\beta'^2 + 4\beta'\gamma') - e^{-2\gamma}\,(2\dot\beta^2 + 4\dot\beta\dot\alpha) + 2\bar{k}\,e^{-2\beta}\,]$$

$$2\kappa\,\mathcal{L}_a^{(2)} = -\,e^{2\beta+\alpha+\gamma}\{2\,\Lambda\,[1 - \lambda^2\,e^{-2(\alpha+\gamma)}\,(\dot a_1 - a_0')^2\,]^{\frac{1}{2}} + \kappa\,m^2(e^{-2\gamma}\,a_0^2 - e^{-2\alpha}\,a_1^2)\}$$

Linear appr.: $$2\kappa\,\mathcal{L}_A^{(2)} = -\,e^{2\beta+\alpha+\gamma}\{2\,\Lambda - \kappa\,[\,e^{-2(\alpha+\gamma)}F_{01}^2 + m^2\,(e^{-2\gamma}A_0^2 - e^{-2\alpha}A_1^2\,)]\}$$

1-D reduction: $$L_c = L_g + L_s + L_v\,, \quad \text{where} \quad L_g \equiv -e^{2\beta+\alpha-\gamma}(2\dot\beta^2 + 4\dot\beta\dot\alpha) - 6k\,e^{\alpha+\gamma}$$

$$L_s = \kappa\,e^{2\beta+\alpha+\gamma}\left[e^{-2\gamma}\dot\psi^2 - v(\psi)\right] \quad L_v = e^{2\beta+\alpha+\gamma}\{-2\Lambda + \kappa\,e^{-2\alpha}\left[e^{-2\gamma}\dot A^2 - m^2A^2\right]\}$$

Scalaron for comparison. $2\kappa\,\mathcal{L}_g^{(1)} \equiv L_g$, plus similarly defined $L_v\,,\ L_s$ $\quad \lambda^{-1} \sim c\,\Lambda$

In 1-D model, vecton is effectively relativistic particle in the ***gauge*** $\alpha+\gamma = 0$ !

$$L_g + L_a \equiv L_g - e^{2\beta+\alpha+\gamma}\left[2\,\Lambda\sqrt{1 - \lambda^2\,\dot a^2\,e^{-2(\alpha+\gamma)}} + \kappa\,m^2a^2e^{-2\alpha}\right]$$

A scalar model for DME,
to compare with `inflaton'. $$L_{sa} \equiv -\,e^{2\beta+\alpha+\gamma}\left[2\,\Lambda\sqrt{1 - \lambda^2\,\dot\varphi^2\,e^{-2\gamma}} + \kappa\,m^2\varphi^2\right]$$

$\dot a\,/\Lambda c = \bar v/c$ NB: Vecton as relativistic particle

Rfs. On integrable 2D models of gravity with matter: ATF, VdA: arXiv: 0505060; 0612258; 0902.4445.

This *nonlinear vecton* model is most interesting for cosmology as describing minimal realistic models. In D =1+1 it can give a unified description of BH, as well as gravity–matter waves. Here we mainly present qualitative discussions of D=1+0 cosmological models and approach to exactly solving their linear approximation. **1-D theory is:**

$$L_a \equiv -2e^{2\beta}\left[e^{\alpha-\gamma}(\dot{\beta}^2 + 2\dot{\beta}\dot{\alpha}) + \Lambda\sqrt{e^{2(\alpha+\gamma)} - \lambda^2\dot{A}^2} + \frac{1}{2}\mu^2 A^2 e^{-\alpha+\gamma}\right]$$

In the gauge $\gamma = -\alpha$. and with notation $\alpha = \rho - 2\sigma \text{ and } \beta = \rho + \sigma$

$$\mathcal{L}_c = -2e^{2\beta}\left[3e^{2\alpha}(\dot{\rho}^2 - \dot{\sigma}^2) + \Lambda\sqrt{1 - \lambda^2\dot{A}^2} + \frac{1}{2}\mu^2 A^2 e^{-2\alpha}\right]$$

$$\mathcal{H} = \bar{c}\sqrt{p_A^2 + M_A^2\,\bar{c}^2} + \mu^2 A^2 e^{2(\beta-\alpha)} + \frac{1}{24}e^{2(\beta+\alpha)}\,(p_\sigma^2 - p_\rho^2)$$

**This is 0,** if there are no other fields.

$$M_A \equiv 2\lambda^2\Lambda e^{2\beta} \quad \lambda^{-1} \equiv \bar{c}$$

**Vecton** looks like a **massive particle in a gravitational field.** It can be regarded as a **relativistic particle if** we rewrite it like (v/c) squared, expressing $\lambda^2$ in terms of **cosmological constant.** (see the previous page)

May this particle be a source of **“BIG BANG”**? **Possibly, YES!?**

**But this is not a simple problem!**

## Anisotropic variables, 1D Lagrangian and Hamiltonian

$$3\,\rho \equiv (\alpha + 2\beta)\,, \quad 3\,\sigma \equiv (\beta - \alpha)\,, \quad \alpha = \rho - 2\sigma$$

Vecton contribution $\quad 3\,A_{\pm} = e^{-2\rho+4\sigma}(\dot{A}^2 \pm m^2 e^{2\gamma} A^2)$

$$\mathcal{L}_c = e^{3\rho-\gamma}(-6\dot{\rho}^2 + 6\dot{\sigma}^2 + \dot{\psi}^2) - 6k\, e^{\rho-2\sigma+\gamma} -$$

Scalaron added for comparison $\quad - e^{3\rho+\gamma}\, v(\psi) + e^{3\rho-\gamma}\, 3A_{-}$

**Vecton plus scalaron Hamiltonian (constraint)**

$$\mathcal{H}_c \equiv -6\dot{\rho}^2 + 6\dot{\sigma}^2 + \dot{\psi}^2 + 6k\, e^{2\gamma-2(\rho+\sigma)} + e^{2\gamma}\, v(\psi) + 3\,A_{+} = 0$$

*Note all positive terms!*

The simplest models to solve: 1. remove scalaron and vecton terms; 2. take potential equal to cosmological constant. Then find all nontrivial solutions with anisotropy, curvature, and `dark energy'.

Elementary ***integrating*** and ***approximating***

some `*non-integrable'* models of very early Universes

with **vector** (scalar) **DM** is our program of **formal mathematical cosmology**).

1. The dynamics of spherical cosmology with `**scalaron**' interacting to gravity is given by **3** nonlinear second-order differential equations depending on `time', **2** metric functions and scalaron. These equations can be reduced to gauge invariant ones.

2. Replacing `time' by `metric' allows to **explicitly integrate general isotropic flat model** in **any gauge,** with **arbitrary potentials depending on metric.** **Anisotropic corrections** are **asymptotically small** in a rather general scalaron theory.

3. Restrictions on the potentials arise from our **positivity criterion** of the exact solutions, that are ***canonical momenta squared*** and on conditions controlling scenario (contracting, bouncing), which must be imposed on **characteristic** functions.

4. An inverse problem – finding, with a given scenario, proper expressions for $V(\alpha)$ that, in fact, proved to be **approximately** (**only!**) constant in the inflationary domain.

5. Presently, I am working on applying a similar approach to *anisotropic models* with a `**vecton**' in frame of a more complex perturbation theory. It look promising.

# E.O.M. for the anisotropic scalaron plus linear vecton

$$\ddot{\rho} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\rho} - e^{2\gamma}\,v(\psi)/2 =$$

Note gauge $3\,\dot{\rho} - \dot{\gamma}$ = 0

Main equation for gravity

$$= 2k\,e^{2\gamma - 2(\rho+\sigma)} + (3A_+ - A_-)/4\,,$$

Anisotropy equation

$$\ddot{\sigma} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\sigma} = k\,e^{2\gamma - 2(\rho+\sigma)} + A_-\,,$$

Matter equations

$$\ddot{A} + (\dot{\rho} + 4\dot{\sigma} - \dot{\gamma})\dot{A} + e^{2\gamma}\,m^2 A = 0\,.$$

Note $(3\,\dot{\rho} - \dot{\gamma})$

gauge dependence

$$\ddot{\psi} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\psi} + e^{2\gamma}\,v'(\psi)/2 = 0\,,$$

Notice the simplicity of the scalaron equations, especially, in the dependence on the gauge parameter. So it is natural that *these equations can be **exactly solved** in the isotropic limit*, when the anisotropy and curvature term vanish. We show in a moment why and how both can be derived by a sort of **perturbation theory**. The vecton case requires a more ingenious construction, which we hope to present in a near future.

***Important*** *equation of motion* **independent of scalar potential**

Vecton part is positive

$$\ddot\rho - \dot\rho\dot\gamma + 3\,\dot\sigma^2 + \dot\psi^2/2 =$$

$$= -k\,e^{2\gamma-2(\rho+\sigma)} - (3A_+ + A_-)/4$$

Gauge dependent **generalized** Hubble function[6] $H(t) \equiv \dot\rho$

Strong restrictions on the generalized Hubble function

$$\dot H(t) \equiv \ddot\rho(t) \geq 0, \quad \text{if} \quad \gamma'(\rho) \geq 3,\; v \geq 0,\; k \geq 0;$$

$$\dot H(t) \leq 0, \quad \text{if} \quad \gamma = 0,\, k \geq 0.$$

Canonical momenta for *graviton, scalaron and vecton*

$$(p_\rho,\, p_\psi,\, p_\sigma) = 2\,e^{3\rho-\gamma}(-6\dot\rho,\, \dot\psi,\, 6\,\dot\sigma)\,, \quad p_A = 2\,e^{\rho+4\sigma-\gamma}\,\dot A$$

*Define all momenta*

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1605.03948, 1506.01664

## *Exact solution of equations for the scalar inflaton in the isotropic limit: potentials in terms of arbitrary scenarios.*

*First introduce important* ***notation*** *and* ***dependent variables.***

$$(\dot\rho,\, \dot\psi,\, \dot\sigma) \equiv [\,\xi(\rho)\,,\eta(\rho)\,,\zeta(\rho)\,] = [\,\xi(\rho),\, \xi\,\psi'(\rho),\, \xi\,\sigma'(\rho)\,] \equiv \xi(\rho)[\,1,\, \chi(\rho),\, \omega(\rho)]$$

Characteristic functions of cosmology depending also on anisotropy and parameter *k*

$$\chi(\rho) \equiv \eta/\xi = \psi'(\rho) \text{ and } \omega(\rho) \equiv \zeta/\xi = \sigma'(\rho) \text{ are gauge invariant}$$

$$v(\psi) = \bar v[\rho(\psi)] \quad \text{for arbitrary } \bar v(\rho)$$

$$v'(\psi) = \frac{dv}{d\psi} = \frac{dv}{d\rho}\frac{d\rho}{d\psi} = \bar v'(\rho)\frac{\xi}{\eta} = \bar v'(\rho)/\chi(\rho)$$

**Gauge invariant Anzatz** *for* **solving all equations** *for* **vanishing anisotropy**

$$[\,x(\rho),\, y(\rho),\, z(\rho)\,] \equiv \exp\left(6\rho - 2\gamma\right)[\,\xi^2(\rho)\,,\eta^2(\rho),\, \zeta^2(\rho)\,]$$

Equations of motion for the **positive square-momentum type variables**

$$y'(\rho) + V'(\rho) - 6V(\rho) = 0\,, \qquad V \equiv e^{6\rho}\,\bar{v}(\rho)\,.$$ independent of k, $\sigma$

$$x'(\rho) - V(\rho) = 4k\,e^{4\rho-2\sigma}\,, \qquad z'(\rho) = 2k\,e^{4\rho-2\sigma}\,\sigma'(\rho)\,.$$

The constraint equation: $6\,x(\rho) = y(\rho) + V(\rho) + 6z(\rho) + 6k\,e^{4\rho-2\sigma}\,.$

$$y(\rho) = 6\left(C_y + \int V(\rho)\right) - V(\rho) = 6I(\rho) - V(\rho)$$ *defnition*

The solution of the $\psi$ equation

Solution of the *X* - eq.: $x(\rho) = \left(C_x + \int V(\rho)\right) + 4k\int e^{4\rho-2\sigma(\rho)}$

***Sigma*** equation $x(\rho)\,\sigma'^2(\rho) \equiv C_x - C_y + 2k\int \sigma'(\rho)\,e^{4\rho-2\sigma(\rho)}\,.$

The fundamental expressions for the solution with **vanishing anisotropy,**

and **exact relation** between the **fundamental cosmological functions**

$$\hat{r}(\rho) \equiv \dot{\psi}^2 e^{-2\gamma}/v(\psi) \;=\; \chi^2(1+6k\,e^{-2\rho}/\bar{v})\,[6(1-\omega^2)\,-\,\chi^2]^{-1}$$

$$\hat{r}(\rho) = \frac{6\,C_y}{V(\rho)} + \frac{6}{V}\int V(\rho) - 1 = \frac{6\,C_y}{V(\rho)} + \sum_1^\infty (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n\,\bar{v}(\rho)}$$

$$\chi^2 \equiv \frac{y}{x} \equiv \frac{6I_y(\rho) - V(\rho)}{I_x(\rho) + i_x(\rho)} \overset{\sigma\to 0}{=} 6 - \frac{V(\alpha) + 6ke^{4\alpha}}{I(\alpha) + ke^{4\alpha}} \overset{k\to 0}{=} 6 - \frac{V}{I}$$

$$I \equiv C_0 + \int V(\alpha)$$

$$\chi^2 = 6(1-\omega^2)\left[\sum_1^\infty (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n\,\bar{v}(\rho)} + \frac{6\,C_y}{V}\right] *$$

Perturbation expansion is

$$* \left[1 + \sum_1^\infty (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n\,\bar{v}(\rho)} + \frac{6}{V}\left(k\,e^{4\rho-2\sigma} + C_y\right)\right]^{-1}$$

All these formulas are **exact**, the second is **independent** on $\sigma$ and $k$

When $k = \sigma = 0$ , the exact equation for $\mathbf{r} = \hat{\chi}^2(1-\hat{\chi}^2)^{-1}$. $\hat{\chi}^2(\rho) \equiv \chi^2/6$.

$$\mathbf{r}'(\rho) + (6 + \bar{l}')\,\mathbf{r}(\rho) = -\bar{l}'(\rho)$$

Is equivalent to the equation for $\chi^2$ and we easily find **normalized potential**

$$\bar{v}(\alpha) = 6\,\xi^2 - \eta^2 = 6\,[1-\hat{\chi}^2(\alpha)]\,\exp\,[-6\int \hat{\chi}^2(\alpha)]$$

which is obviously **slowly varying in the inflationary domain**, i.e., for small $\hat{\chi}^2$

As in inflationary domain $\hat{\chi}^2(\alpha) \ll 1$ and $\mathbf{r}(\alpha) \ll 1$ but $[\chi^2(\alpha)]'$

and $\mathbf{r}'(\rho)$ must be positive, the potential cannot be really constant.

**Asymptotic hierarchy:** **flat isotropic, isotropic, anisotropic**

$$O(1) \subset O(e^{-2\alpha}) \subset O(e^{-4\alpha})$$

$\Theta(\alpha)$ **-- important function** for **constructing inflation** and **the exit** of **it .**

We define it as the left-hand of the equation for

$$[\chi^2(\alpha)]' = -(6-\chi^2)\,[\,\chi^2(\alpha) + \bar{l}'(\alpha)] \equiv (6-\chi^2)\,\Theta(\alpha)\,,$$

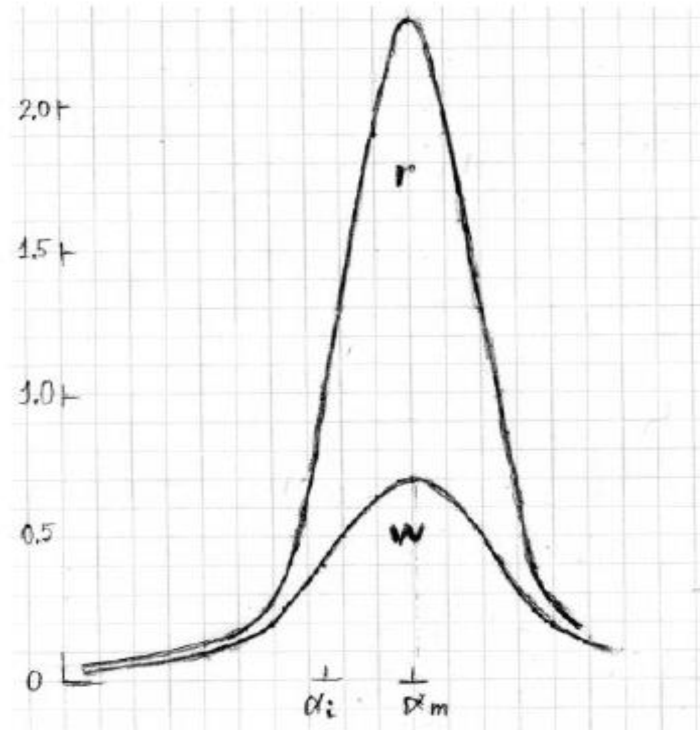
$$\Theta(\alpha) = -[\,\ln(6-\chi^2(\alpha)]' = [\,\ln(\mathbf{r}+1)]'$$

$$-\Theta'(\alpha) = \Theta^2 + (6+\bar{l}')\,\Theta + \bar{l}'' \equiv (\Theta-\Theta_+)(\Theta-\Theta_-)$$

$$(\chi^2)'' = (\Theta+6+\bar{l}')\,(\Theta' - \Theta^2) = -(\Theta+6+\bar{l}')\,[2\Theta^2 + (6+\bar{l}')\,\Theta + \bar{l}'']$$

Change sign of $(\chi^2)''$ is possible to gain only when $\bar{l}'' < 0.$

# A possible exit from inflation




$$\mathbf{w}(\alpha) \quad \text{and} \quad \mathbf{r}(\alpha) = \frac{\mathbf{w}(\alpha)}{1 - \mathbf{w}}$$

at the end of inflation

# Approximate solution of the anisotropy equation

## estimating $\sigma(\rho)$ in the weak anisotropy limit

***by asymptotically solving** the **Sigma*** equation

$$\sigma(\rho) = -3k \int_{\rho}^{\infty} \frac{e^{-2\rho}[\bar{v}(\rho)]^{-1}}{1 + \Sigma_1(\rho) + O(e^{-2\rho})} + O(e^{-4\rho})$$

$$\Sigma_1(\rho) = \sum_{1}^{\infty} (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n \, \bar{v}(\rho)} \,.$$

# Final remarks on the vecton theory

1. The structure of the linearized vecton theory is similar to scalaron case but anisotropy requires additional efforts.

2. With zero anisotropy, the equations for the linearized theory can be solved, otherwise they give a sort of useful `sum rules'. But, asymptotically small anisotropy approximation can probably be as effective as in the scalaron case.

3. The most difficult 'large vecton momentum' case for nonlinear vecton may also be treated asymptotically. It may be very interesting for transition from inflation to particle production processes, or else, for description of the bouncing phenomena.

3. The most interesting problem is to find solutions describing birth of a universe similar to our one and, finally, to derive black holes from dark matter as well as matter-- gravity waves in the eternal (mother?) universe.

Those who think of metaphysics as the most unconstrained are misinformed; compared with cosmology metaphysics is pedestrian and unimaginative.

S. Toulmin, British philosopher, 1922 -2009

# THE

# END

**THANK YOU FOR ATTENTION**

In general, our solutions may become negative even when the potential is positive

One has to require positivity at least in the classical domain \alfa > 0

Construction of **positive** general solutions

$$\bar{v}(\rho) > 0 \qquad y(\rho) \equiv \lambda \int_{\rho_0}^{\rho} e^{\lambda\rho}\, \bar{v}(\rho) \;-\; e^{\lambda\,\rho}\, \bar{v}(\rho) + e^{\lambda\,\rho_0} \bar{v}(\rho_0)$$

$$y(\rho) \;=\; - \int_{\rho_0}^{\rho} e^{\lambda\rho}\, \bar{v}'(\rho)\,, \quad y'(\rho) \;=\; -e^{\lambda\rho}\, \bar{v}'(\rho) > 0$$

$$\text{positive if } \bar{v}'\,(\rho) < 0$$

## First terms of the exact expression for the transition function

$$\chi^2 = (1-\omega^2)\left[\left(-\bar{l}' + \mathrm{o}(\bar{l}')\right) + 36\, C_y\, \frac{e^{-6\rho}}{\bar{v}(\rho)}\right] \ \mathrm{x}$$

$$\left[\left(1 - \frac{1}{6}\,\bar{l}' + \mathrm{o}(\bar{l}')\right) + 6\, \frac{e^{-2\rho}}{\bar{v}(\rho)}\left(k\, e^{-2\sigma} + C_y\, e^{-4\rho}\right)\right]^{-1}$$

$$\chi^2 = -\bar{l}'(\rho) + \mathrm{o}(\bar{l}') = -\chi\, v'(\psi)/v(\psi) + \ldots$$

$$\chi = -v'(\psi)/v(\psi) + \ldots \equiv -l'(\psi) + \mathrm{o}(l')$$

In fact we constructed a perturbative algorithm relating \rho and \psi pictures but it is rather complex. Nevertheless it is important as a matter of principle.

This provides the relation to approximate standard formulas**.**

The small **anisotropic corrections** are derived in **asymptotic domain** of **large** ***\rho***

Here, we consider special anisotropic cosmology with $\dot{\alpha} = \beta' = \gamma' = 0$:

$$\mathcal{L}_c = e^{2\beta}\left[e^{-\gamma}(\dot{a}^2 - 2\dot{\beta}^2) - e^{\gamma}(2\Lambda + m^2a^2)\right] - 6ke^{\gamma}\,. \tag{76}$$

Denoting $\xi \equiv \dot{\beta}$, $b \equiv \dot{a}$, $v_0 \equiv 2\Lambda$, $v(a) \equiv v_0 + m^2a^2$, we find the constraint

$$e^{2\beta}\left[e^{-\gamma}(b^2 - 2\xi^2) + e^{\gamma}v(a)\right] + 6ke^{\gamma} = 0\,, \tag{77}$$

which is quadratic in the momenta: $(p_a,\, p_\beta) \equiv 2e^{2\beta-\gamma}(b\,, -2\xi)$. Equations of motion are

$$2\dot{b} + 2(2\xi - \dot{\gamma})b + e^{2\gamma}v'(a) = 0\,, \tag{78}$$

$$2\dot{\xi} + 4\xi^2 - 2\xi\dot{\gamma} - 2[v(a) + 3ke^{-2\beta}]e^{2\gamma} = 0\,. \tag{79}$$

Defining the new dynamical variables by $y(\beta) \equiv b^2\, e^{4\beta-2\gamma}$, $x(\beta) \equiv \xi^2 e^{4\beta-2\gamma}$ we find the gauge invariant equations and constraint similar to Eqs.(37-39) but there is no $\sigma$-equation:

$$x' - 2e^{4\beta}\,\bar{v}(\beta) + 6ke^{2\beta} = 0\,, \quad (a); \qquad y' + \bar{v}'(\beta)\,e^{4\beta} = 0\,, \quad (b)\,. \tag{83}$$

$$2x(\beta) = y(\beta) + V(\beta) + 6\,k\,e^{2\beta}\,, \quad \text{where} \qquad V(\beta) \equiv e^{4\beta}\,\bar{v}(\beta)\,. \tag{84}$$

Their exact solution is also similar (40) with $\sigma \equiv 0$

$$2x = 4I(\beta) + 6ke^{2\beta}, \quad y = 4I(\beta) - V(\beta)\,; \qquad I(\beta) \equiv \int V(\beta) + C \tag{85}$$

anisotropic model. It is sufficient to define our main characteristic of cosmology

$$\tilde{\chi}^2(\beta) \equiv y(\beta)/2x(\beta) = 1 - V(\beta)/4I(\beta)\,, \quad \text{for} \quad k = 0\,, \tag{86}$$

which satisfy equation similar to equation (70) for $\hat{\chi}^2(\alpha) \equiv \chi^2/6$:

$$(\tilde{\chi}^2)' = -(1 - \tilde{\chi}^2)\,(4\tilde{\chi}^2 + \bar{l}') \equiv (1 - \tilde{\chi}^2)\,\tilde{\Theta}(\beta)\,, \qquad \tilde{\chi}^2 < 1\,. \tag{87}$$

# Constructing and solving cosmologies of early universes with dark energy and dark matter

Alexandre T.Filippov, JINR.RU

**New interpretation of Einstein's 3 papers of 1923:** arXiv: **0812.2616**

[formulation of a **new unified theory** of **Dark Energy** and **vecton Dark Matter] .**

Also: **1003.0782, 1008.2333, 1011.2445, (vecton in cosmology as a relativistic particle);**

**112.3023, 1302.6372** , ( **general formulation in any dim**., **applied to cosmologies** );

**1403.6815, 1506.01664, 1605.03948**, **1905.0330,** ( **general and special solutions** ).

Older: **0612.258,** **0801.1312**, **0811.4501,** **0902.4445** **( 3 with Vittorio de Alfaro )**

in these publ. we proposed and studied **2-D Liouville - Toda models** describing

***static, cosmological*** and **wave solutions** of ***dilaton gravity coupled to matter*.**