

Nuclear equation of state phase transitions, and their astrophysical implications

Evgeni Kolomeitsev

*BLTP JINR, Dubna
Matej Bel University, Slovakia*

...per collisionis ad astra ...

Why do we collide heavy nuclei?

QCD in action

Phase diagrams of nuclear matter

Giant nuclei in space – neutron stars

The Standard model. The QCD

1968: SLAC u up quark	1974: Brookhaven & SLAC c charm quark	1995: Fermilab t top quark	1979: DESY g gluon
1968: SLAC d down quark	1947: Manchester University s strange quark	1977: Fermilab b bottom quark	1923: Washington University* γ photon
1958: Savannah River Plant ν_e electron neutrino	1962: Brookhaven ν_μ muon neutrino	2000: Fermilab ν_τ tau neutrino	1983: CERN W W boson
1937: Cavendish Laboratory e	1937: Caltech and Harvard μ	1976: SLAC τ	1983: CERN Z

Degrees of freedom for strong interaction:

fermions: **quarks**

bosons: **gluons**

SU(3) gauge symmetry (color)

- QCD: symmetries

Gell-Mann, Ne'eman “Eightfold way”
flavour SU(3) and chiral symmetry
(accidental symmetries)

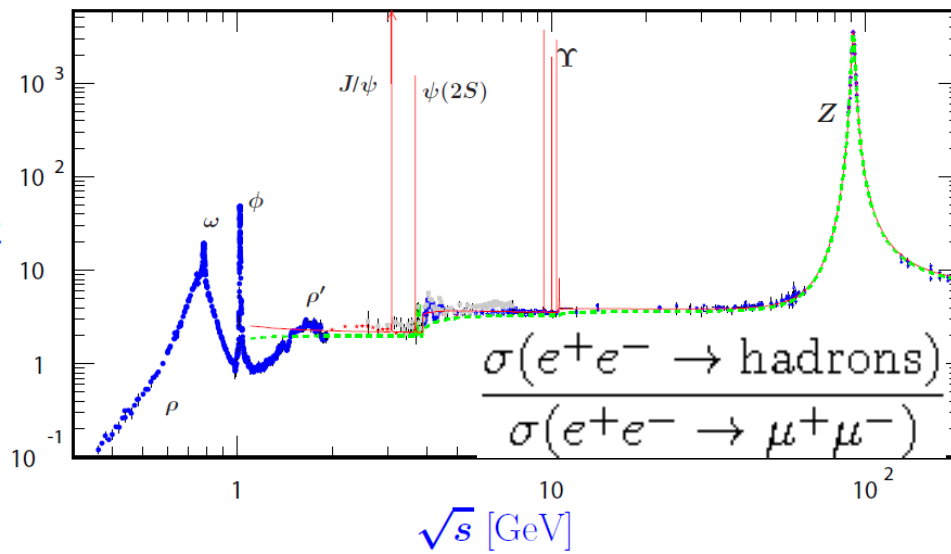
small masses of u,d,s quarks

Georgi, Manohar Large N_c counting ($3 \gg 1$)

- QCD: dynamics

Fritzsch, Gel-Mann, Leutwyler
hadron currents vs. quark currents

**Jets, DIS, pp, p-bar-p,
parton distribution functions**



QCD with light quarks

✓ **quarks** $q_L(x) = \frac{1+\gamma_5}{2} \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix}$ ✓ **gluons** $G^a(x) \leftarrow \text{SU}(N_c) \text{ gauge bosons}$
 $D_\mu(G) = \partial_\mu - i \frac{g}{2} G_\mu^a(x) \lambda_a$

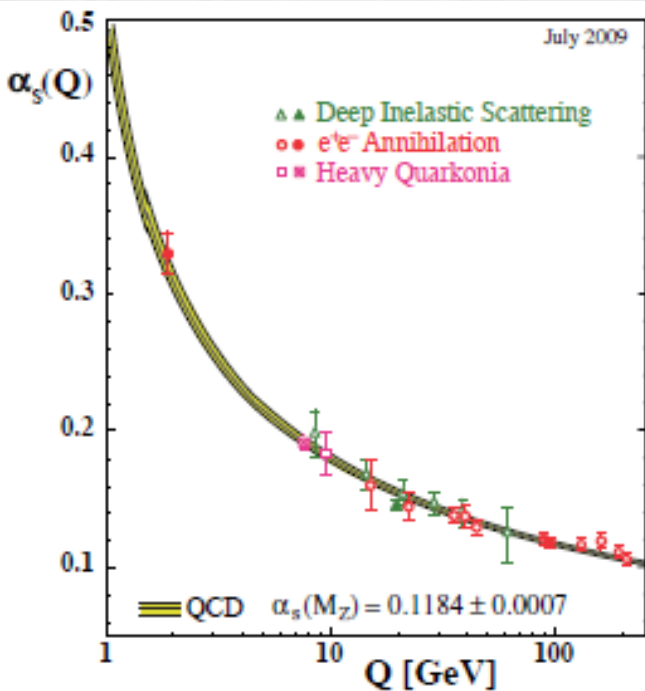
$$\mathcal{L}_{\text{QCD}}(x) = \bar{q}_L(x) i \gamma^\mu D_\mu(G) q_L(x) + \bar{q}_R(x) i \gamma^\mu D_\mu(G) q_R(x) - \frac{1}{4} \sum_{a=1}^{N_c^2-1} G_a^{\mu\nu}(x) G_{\mu\nu,a}(x)$$

$$- \bar{q}_L(x) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} q_R(x) - \bar{q}_R(x) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} q_L(x)$$

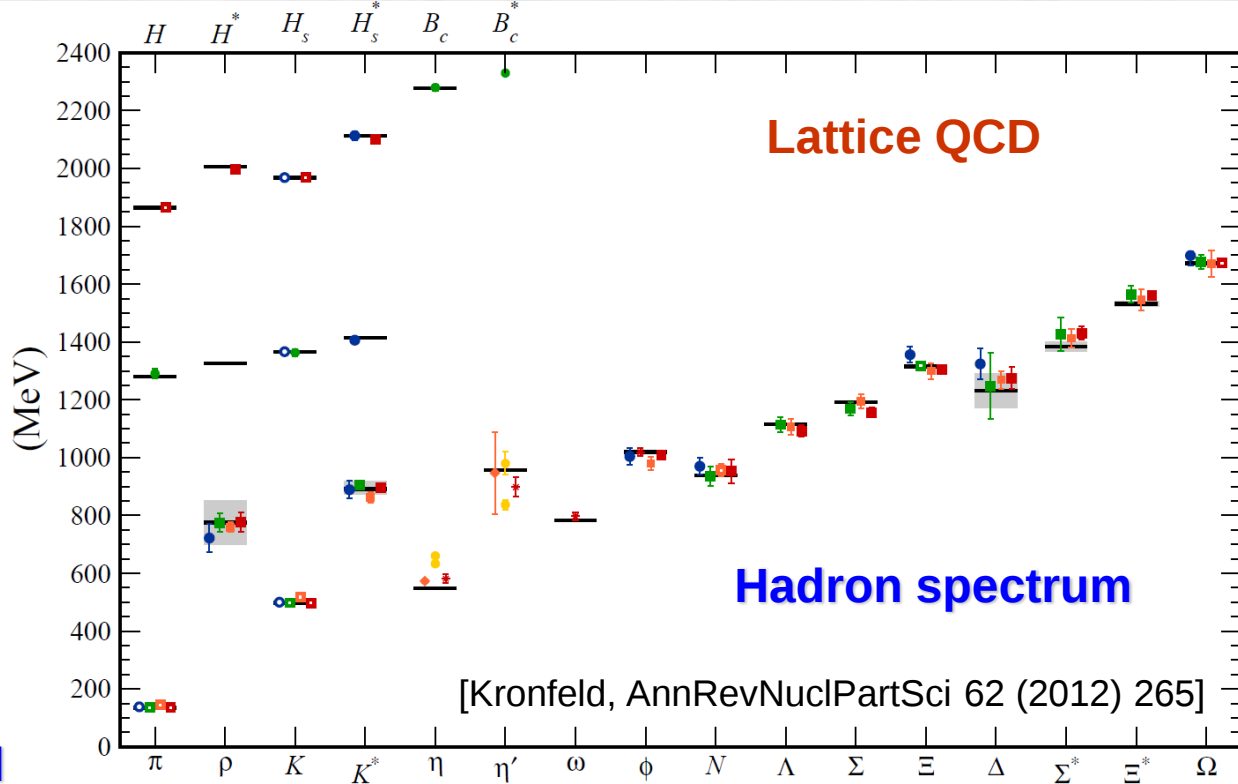
Accidental symmetries

- ✓ consider $m_{u,d,s}$ to be small
 - approximate $\text{SU}(3)_L \otimes \text{SU}(3)_R$ chiral symmetry
 - parity doublets in hadron spectrum (if not broken spontaneously !)
- ✓ consider number of colors $N_c = 3$ to be large: assume $g \sim N_c^{-1}$
 - contracted spin-flavor symmetry $\text{SU}(6)$

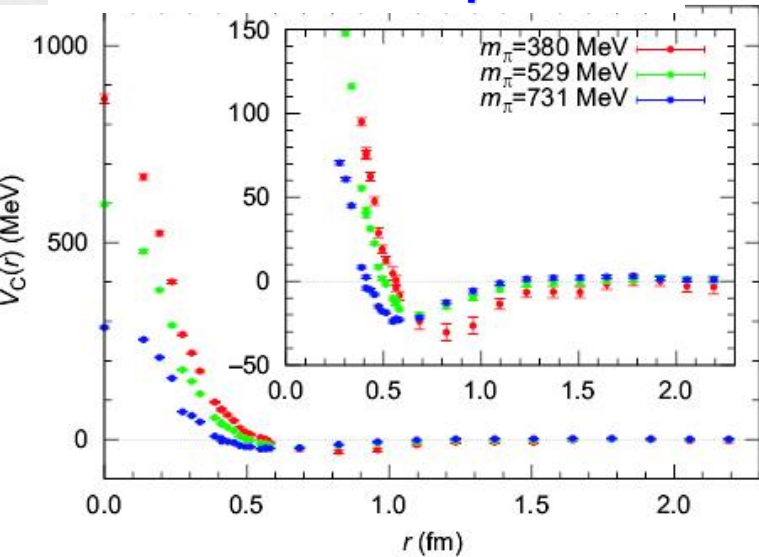
Running coupling constant



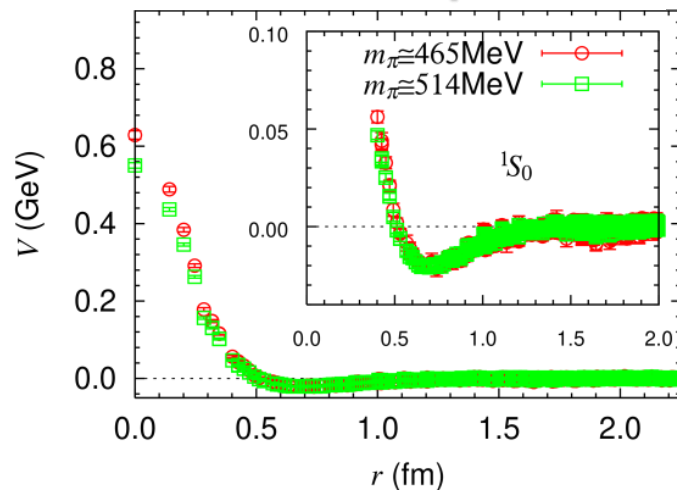
→ soft QCD is nonperturbative



nucleon-nucleon potential



Lambda-nucleon potential



Does the success of perturbative and nonperturbative QCD mean that we know everything about strong interaction?

2005

*FAIR-Discussion among H. Fritzsch, M. Gell-Mann (HEP community)
and hadron-physics community in Germany*

New facets of the QCD:

hadron spectroscopy

decay patterns

new 'exotic states';

Hadrogenesis – hadron molecules.

QCD phase diagrams

New phases (signals?)

Critical points (signals?)

equation of state

$T=0$ compact star physics constraints

$T \neq 0$ thermal excitation of relevant d.o.f

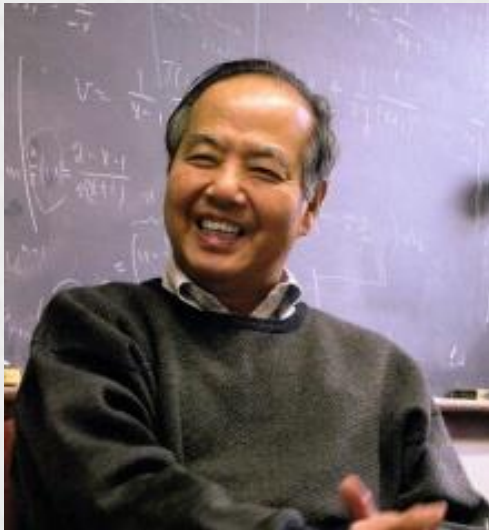


We have a beautiful piece of art and we are very curious how is it made?

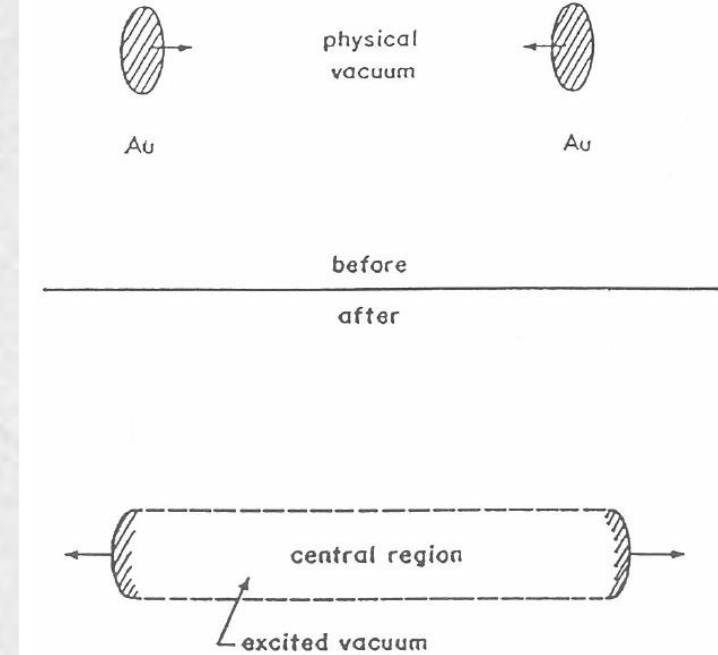


We suspect some structures and hidden elements under the covers?





Tsung-Dao Lee



The most crucial questions in an ultra-relativistic heavy ion collision are:

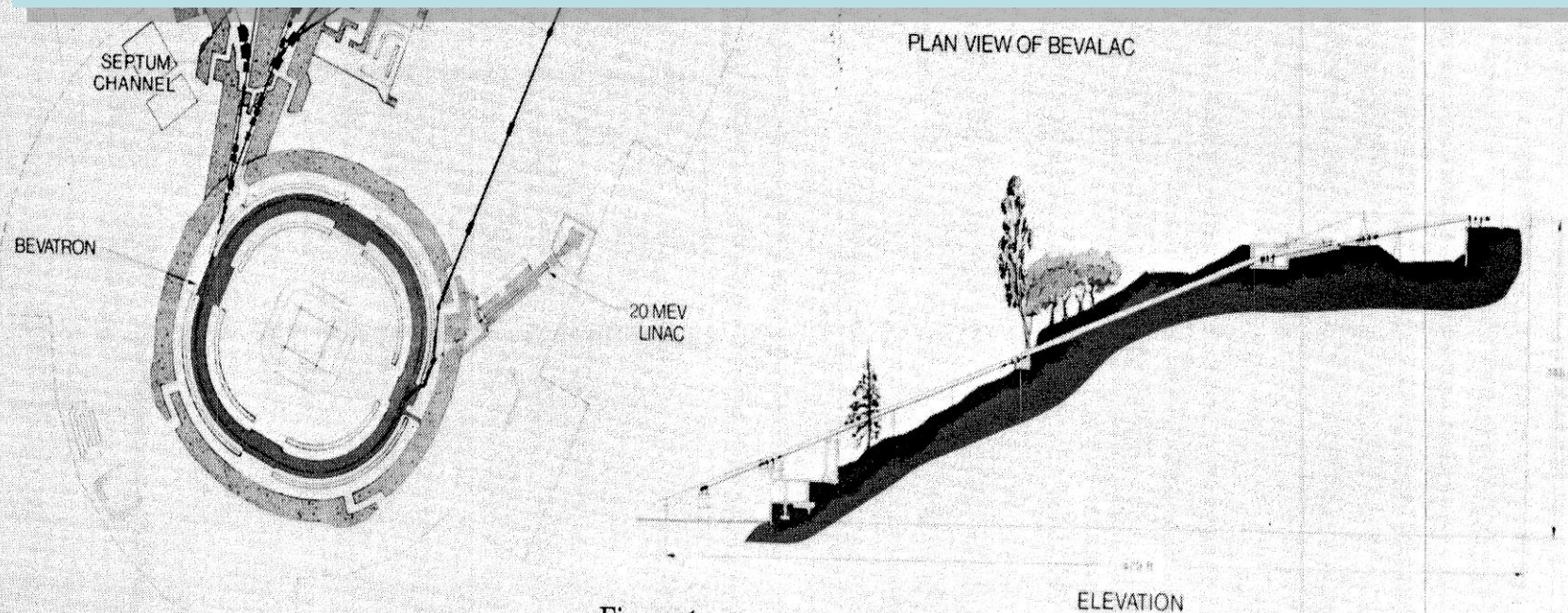
1. Can extended objects of very high energy densities ($\gg$ energy density inside a proton) be produced?
2. If produced, how can they be detected and how can we analyze their properties?

[in *Symmetry in Particle Physics*, 1984]

PH
EXF
ARIFUI
HE

S

The early 1974 Workshop at Bear Mountain, just north of New York City, on *BeV/nucleon collisions of heavy ions*, was a pivotal event in the conception of heavy ion physics, since after this workshop physicists began to take seriously the possibility of using heavy ion collisions as a tool to study the properties of matter under extreme conditions of high energy and baryon densities, to ask whether was there a “nuclear world quite different from the one we have learned to accept as familiar and stable?” [Gordon Baym, NPA956(2016) 1]



Shock waves and nuclear compression

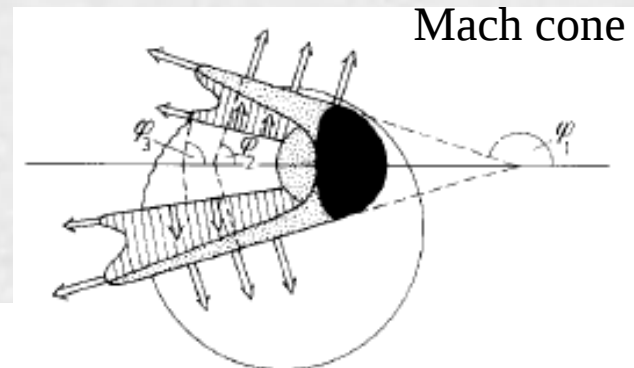
What happens when two nuclei collide? Interpenetration? Bounce?

Z. Physik 226, 364—394 (1969)

Quasimolecular Nuclear Optical Potentials*

WERNER SCHEID and WALTER GREINER

Institut für Theoretische Physik der Universität Frankfurt (Main), Germany



Mach cone

PHYSICAL REVIEW D

VOLUME 8, NUMBER 12

Highly Excited Nuclear Matter*

Baumgardt et al 75

G. F. Chapline, M. H. Johnson, E. Teller, and M. S. Weiss

Lawrence Livermore Laboratory, University of California, Livermore, California 94550

Nuclear Physics A251 (1975) 502—529

SHOCK WAVES IN COLLIDING NUCLEI

MICHAEL I. SOBEL†

Nordita, Copenhagen

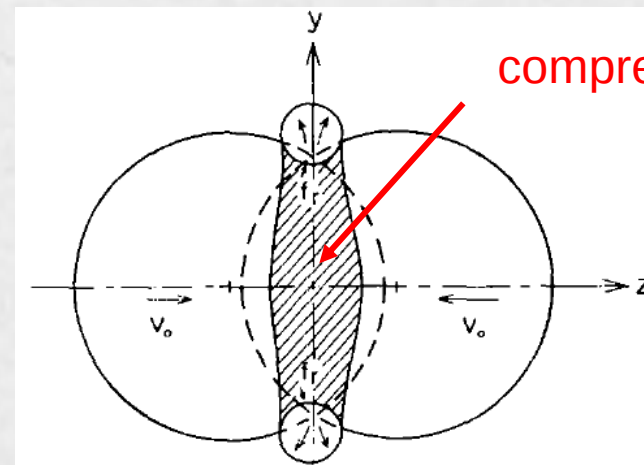
PHILIP J. SIEMENS and JAKOB P. BONDORF

The Niels Bohr Institute, Copenhagen

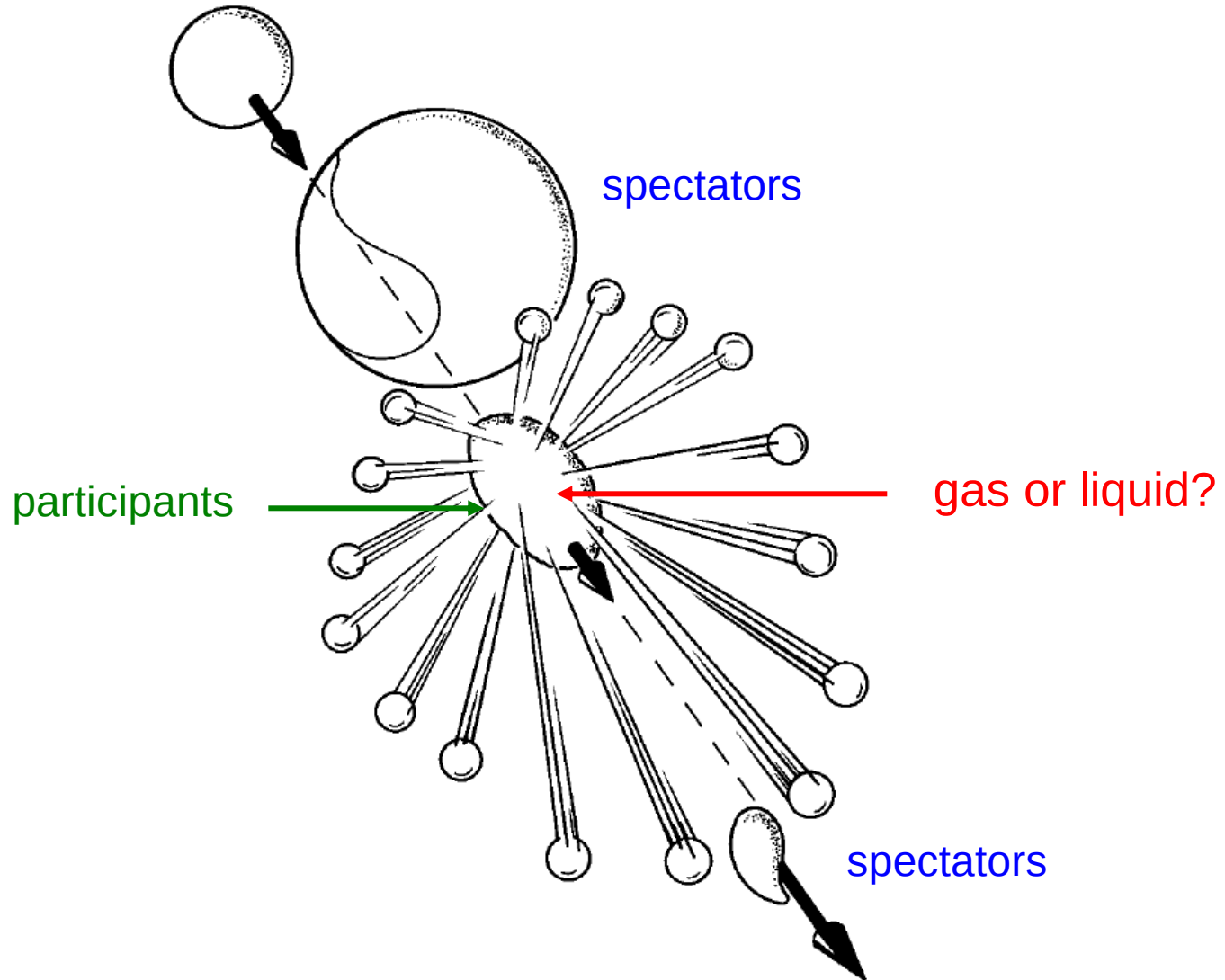
and

H. A. BETHE††

Nordita, Copenhagen



Collectivity



Sigma model

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix} \quad \text{[baryons: protons and neutrons]} \quad \frac{1}{2}^+$$

[mesons]

$$\begin{array}{ll} \pi = (\pi_1, \pi_2, \pi_3) & \text{pions} \quad 0^- \\ \sigma & \text{sigma-meson} \quad 0^+ \end{array}$$

$$\mathcal{L} = \bar{\psi} \gamma_\mu \partial^\mu \psi + \frac{1}{2} \partial_\mu \pi \partial^\mu \pi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \bar{\psi} [\gamma_5 (\tau \pi) + \sigma] \psi + U(\pi^2 + \sigma^2)$$

massless nucleons and mesons

Invariant under two transformations

vector transform.

$$\psi \rightarrow (1 - i \frac{\tau}{2} \Theta) \psi$$

$$\pi \rightarrow \pi + \Theta \times \pi$$

$$\sigma \rightarrow \sigma$$

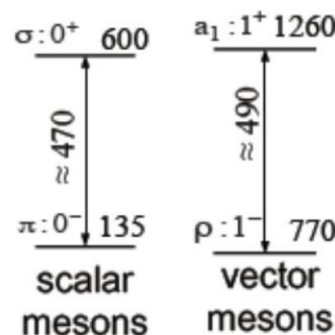
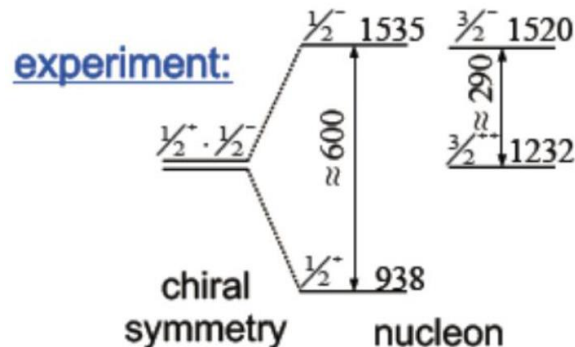
axial transform.

$$\psi \rightarrow (1 - i \gamma_5 \frac{\tau}{2} \Theta) \psi$$

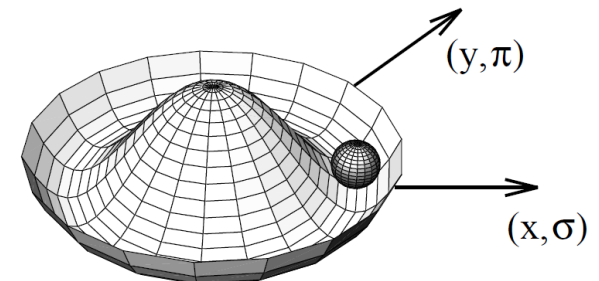
$$\pi \rightarrow \pi + \Theta \sigma$$

$$\sigma \rightarrow \sigma + \Theta \pi$$

mixes states with different parities



Symmetry should be spontaneously broken



massive nucleons, σ mesons, almost massless pions

Goldstone modes

+small explicit symmetry breaking

Medium = hot and/or dense hadron matter

$$U(\pi^2 + \sigma^2) \rightarrow U_{\text{eff}}(\boldsymbol{\pi}, \sigma; T, n_B) = U(\pi^2 + \sigma^2) + \text{Loops}(\text{medium})$$

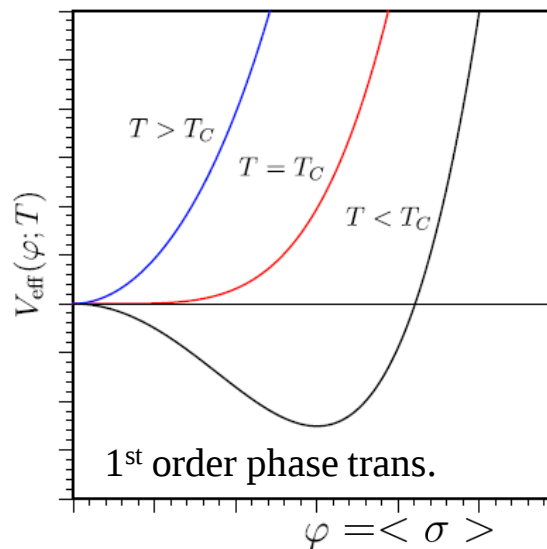
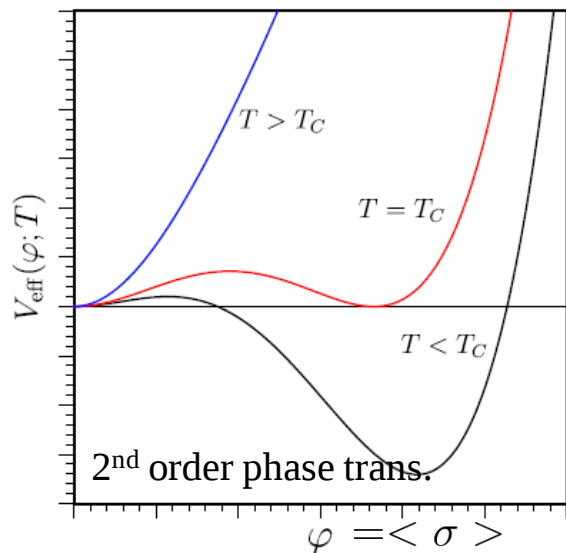
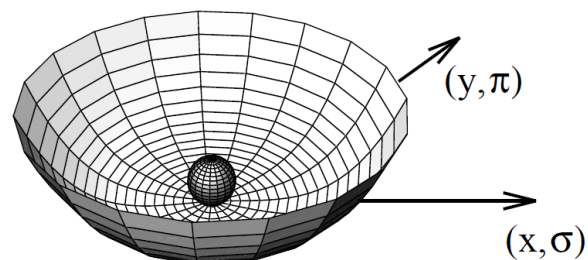
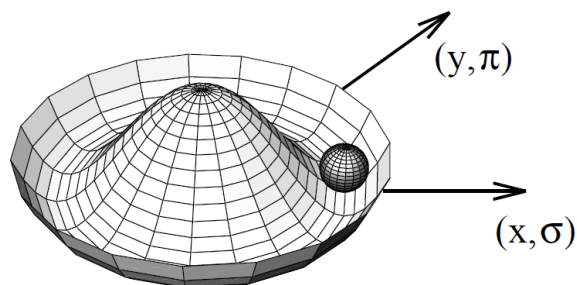
1971 Migdal

1972 Kirzhnits, A. Linde

1974 Weinberg, Dolan, Jackiw,

.....

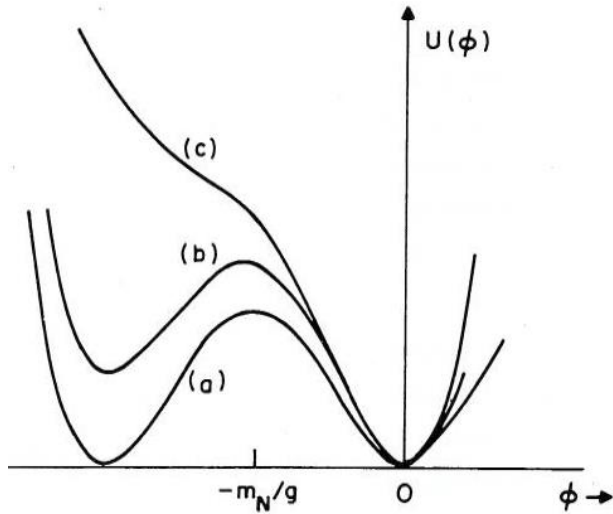
Chiral symmetry restoration



"Abnormal Nuclear States"

T.D. Lee, G.C Wick

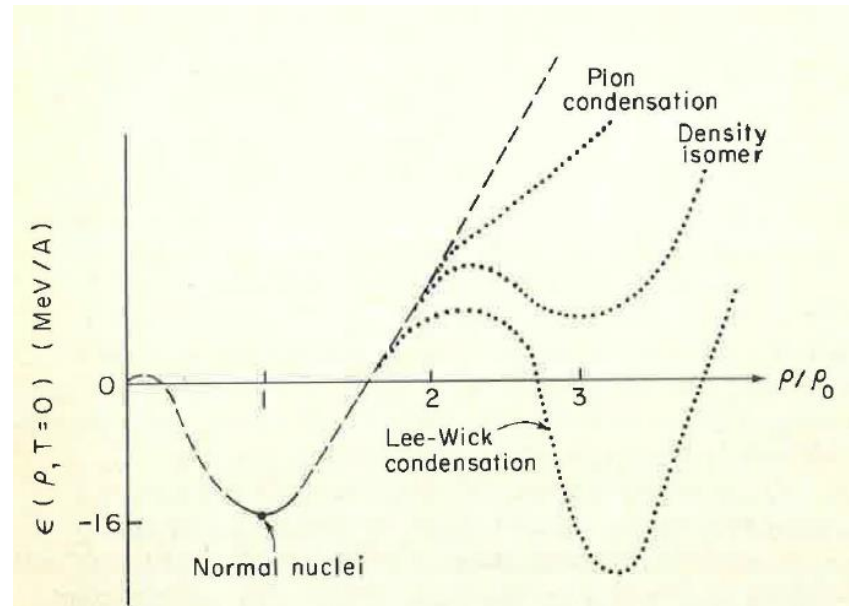
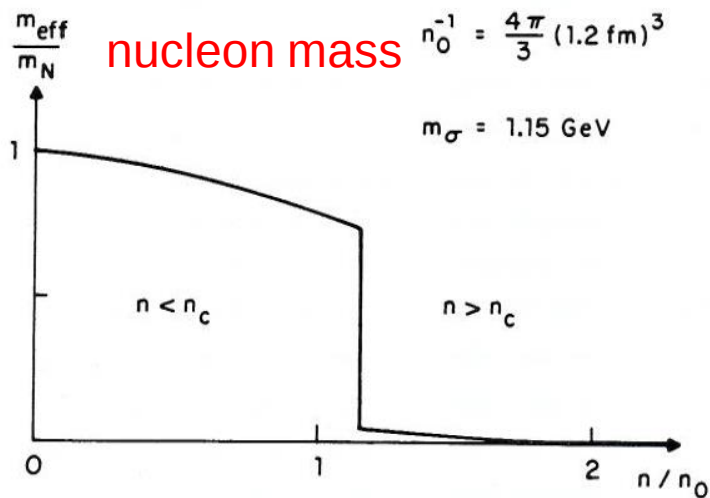
scalar field rearrangement



Tsung-Dao Lee



Gian Carlo Wick



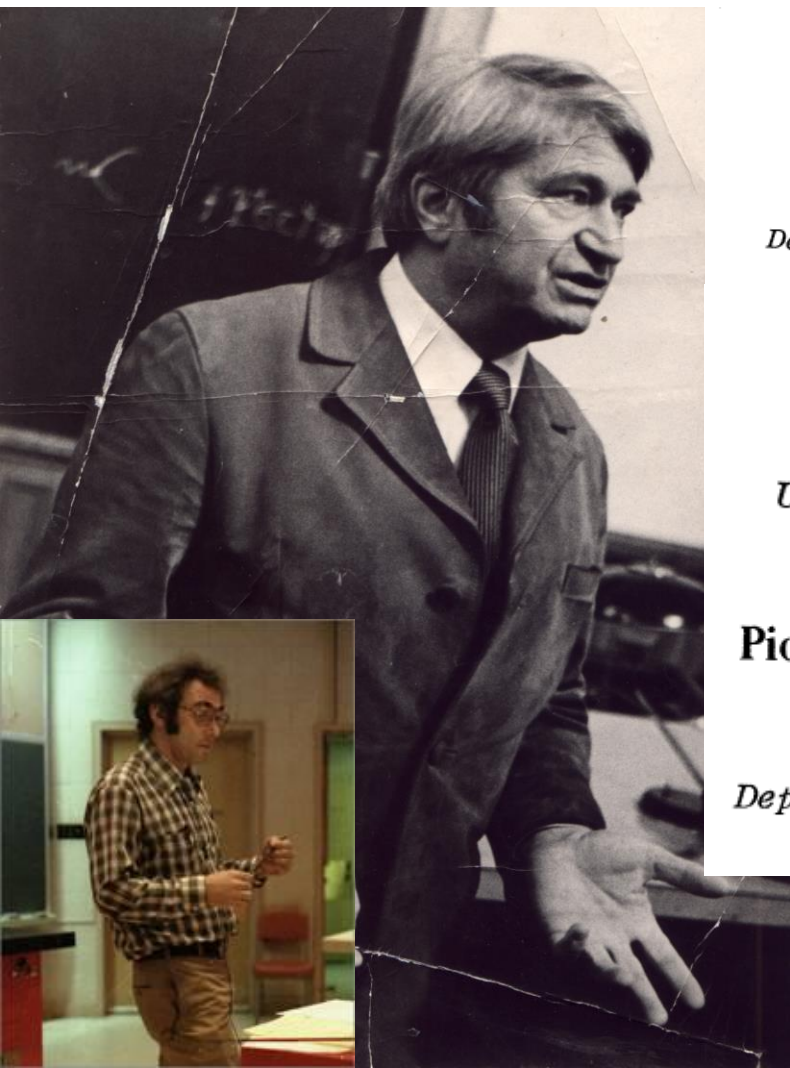
Stability of Vacuum and Limiting Fields

A. B. MIGDAL

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

Submitted June 21, 1971

Zh. Eksp. Teor. Fiz. **61**, 2209–2224 (December, 1972)



Condensed π^- Phase in Neutron-Star Matter*

R. F. Sawyer

Department of Physics, University of California, Santa Barbara, California 93106

(Received 29 March 1972)

π^- Condensate in Dense Nuclear Matter*

D. J. Scalapino

University of California, Santa Barbara, California 93106

(Received 17 April 1972)

Pion Condensation in Nuclear and Neutron Star Matter*

Gordon Baym

Department of Physics, University of Illinois, Urbana, Illinois 61801

(Received 13 April 1973)



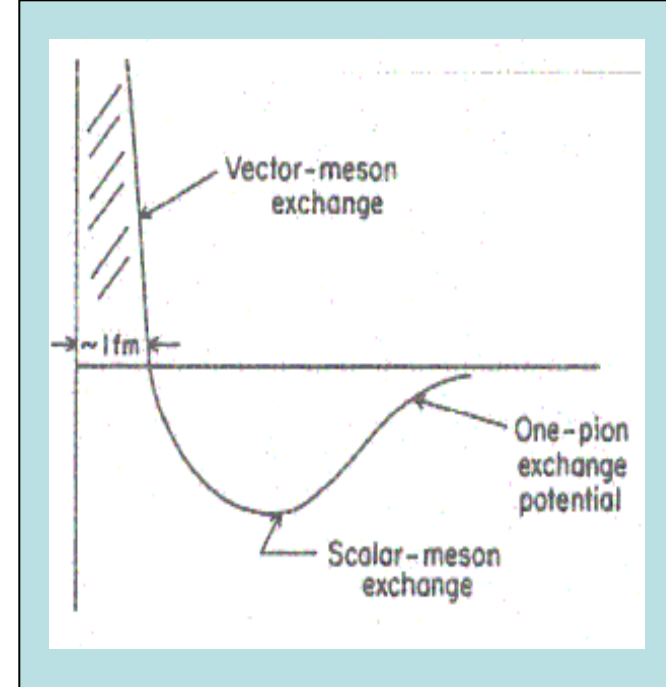
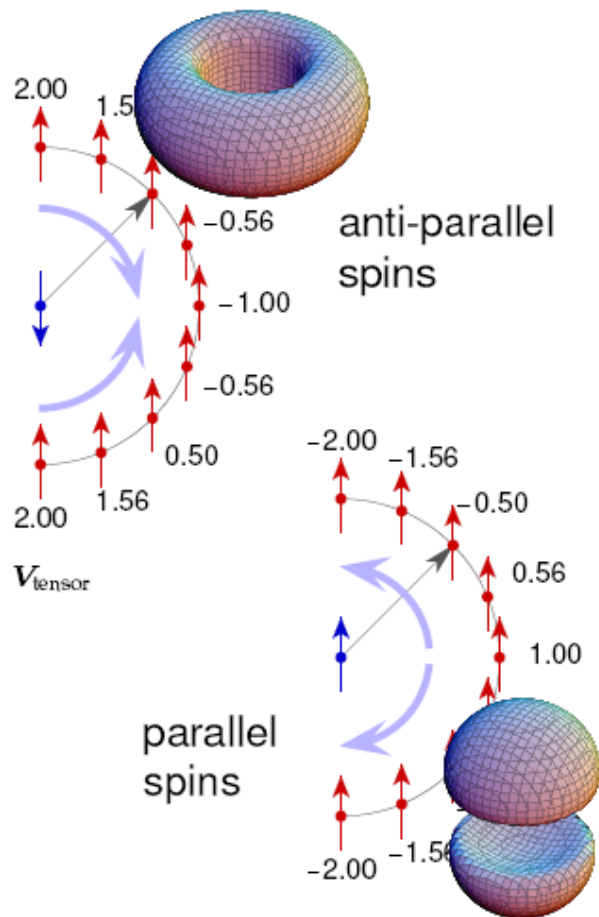
Baym

Migdal

Scalapino

1974 Tbilisi

Tensor forces in NN interaction



$$V_{\text{tensor}} = V_t(r) \left(3 \frac{(\boldsymbol{\sigma}_1 \mathbf{r})(\boldsymbol{\sigma}_2 \mathbf{r})}{r^2} - \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2 \right)$$



Resummed, enhanced pion exchange

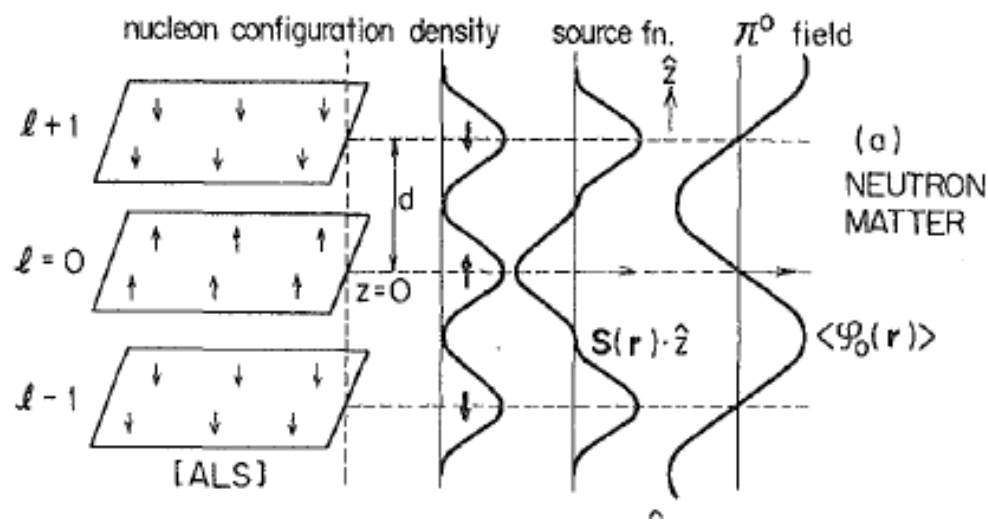
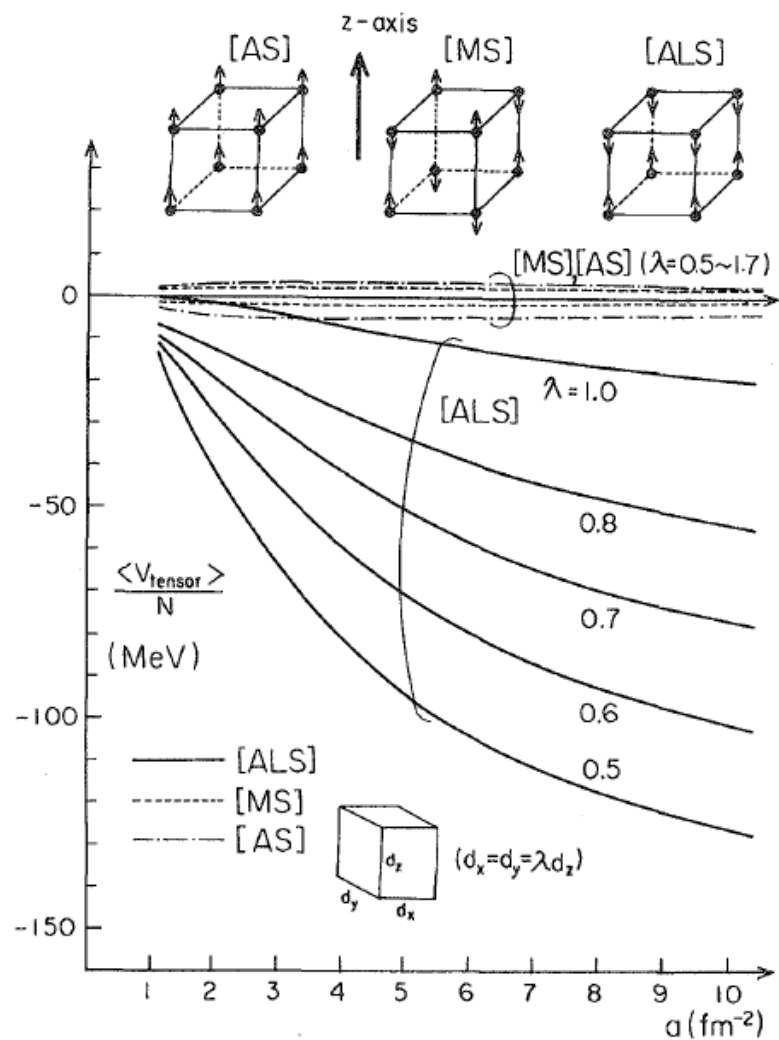
nucleon arrangement

Alternating-layer-spin configurations

Ryozo Tamagaki

Tatsuyuki Takatsuka

1972'-73'

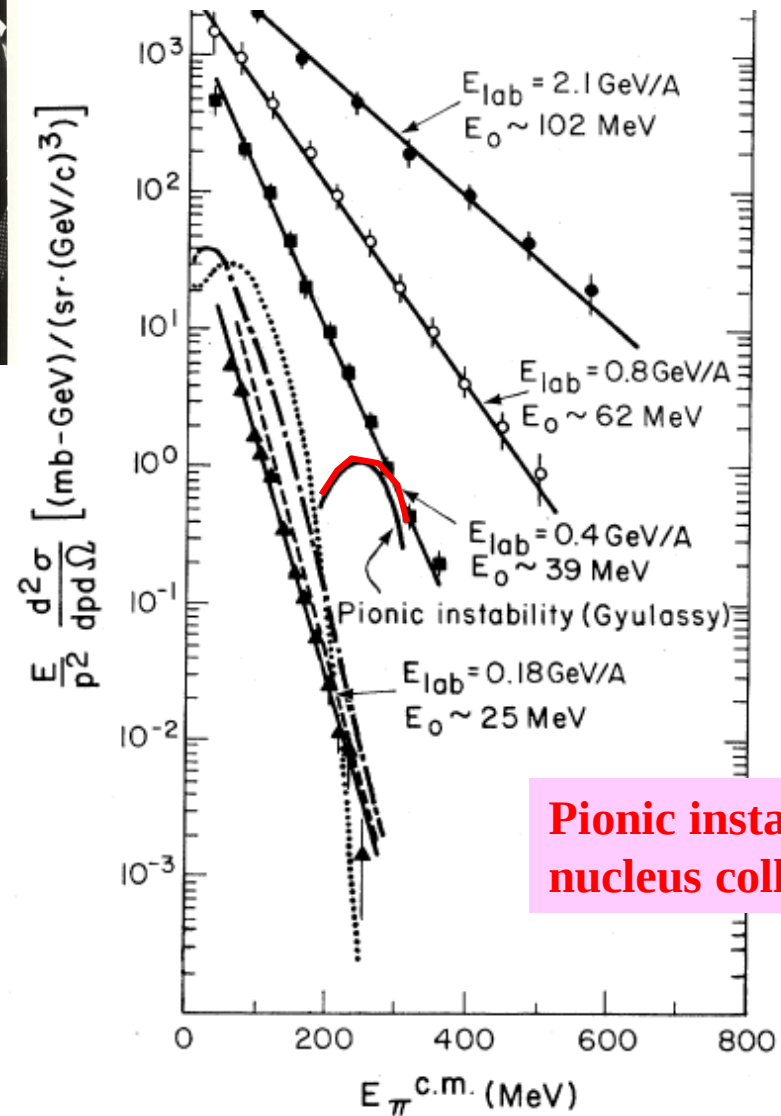


π^0 condensate

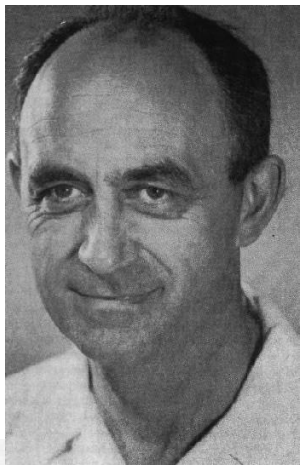


Pion condensation

A.B. Migdal, and D. Scalapino, R. Sawyer

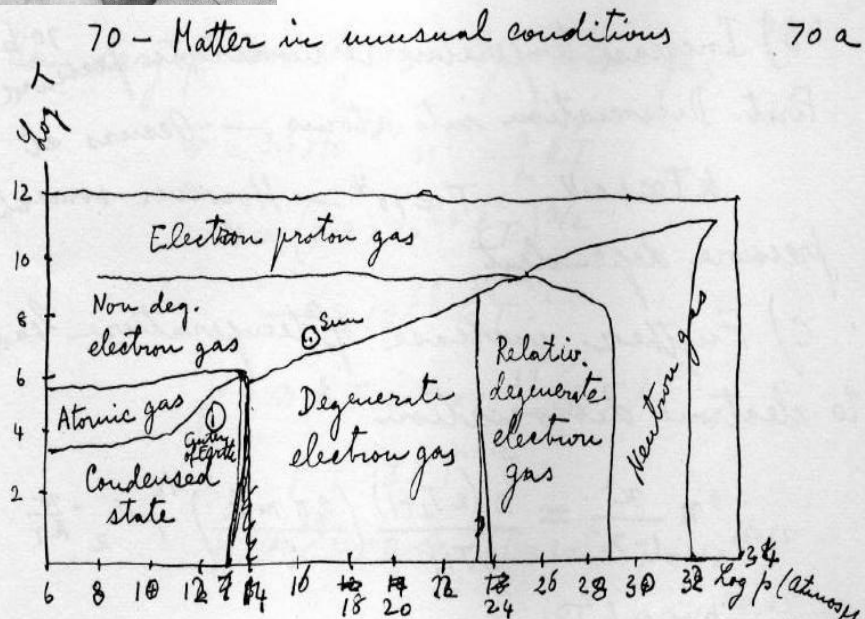


**Pionic instability in
nucleus collisions**



Phase diagrams

E.Fermi: Notes on
thermodynamics
and statistics, 1953



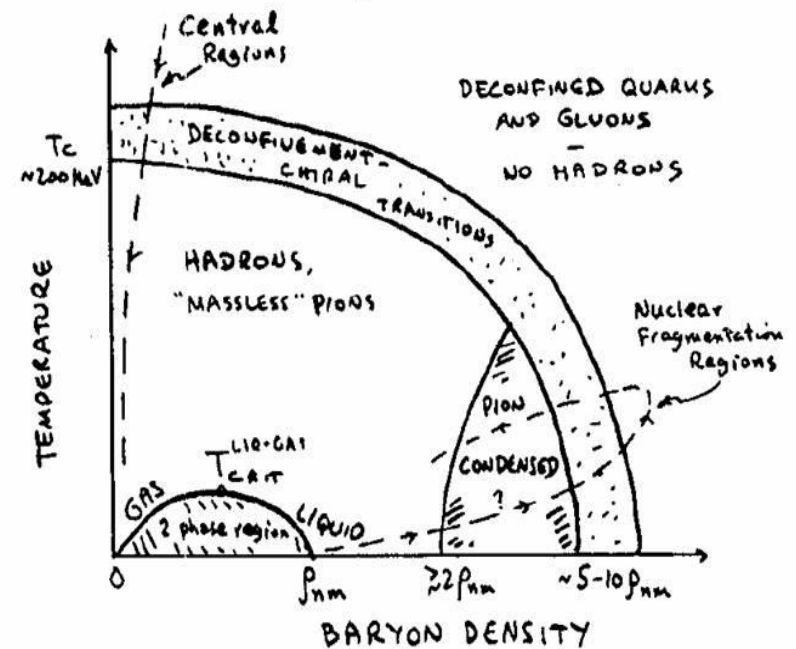
Start from ordinary condensed matter with
~~low~~ equation of state controlled by ordinary
chemical forces.

Condensed matter



G.Baym: Nucl.
Phys. A418
(1984) 433c

PHASE DIAGRAM OF NUCLEAR MATTER



Nuclear matter

Limiting hadronic temperature

PDG book → mass spectrum of hadronic states

$$\rho(m) \sim m^{-a} e^{+\frac{m}{T_H}} \quad a \simeq 3$$

Partition function of a hadron-resonance gas

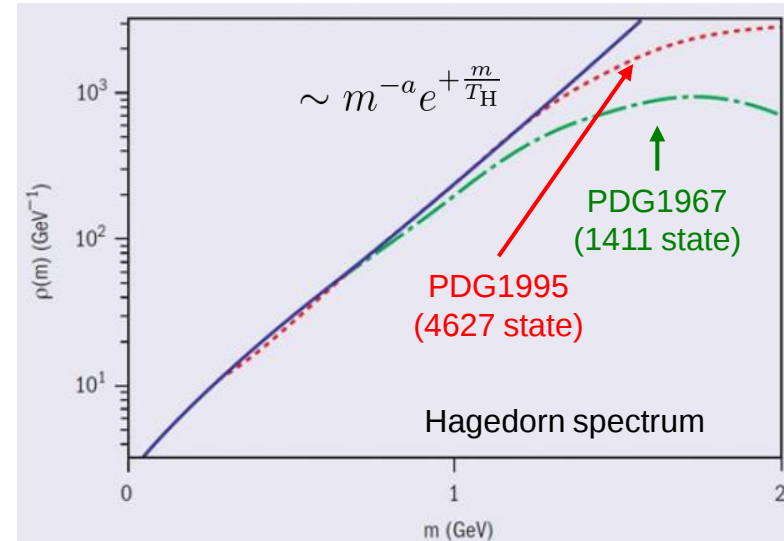
$$\log Z = \sum_i \frac{VTm_i^2}{2\pi^2} \rho(m_i) K_2(m_i/T) \simeq \frac{VT}{2\pi^2} \int dm m^2 \rho(m) K_2(m/T) \sim V \left[\frac{T}{2\pi} \right]^{3/2} \int \frac{dm}{m^{3/2}} e^{-\left(\frac{m}{T} - \frac{m}{T_H}\right)}$$

energy density $\epsilon(T) = -\frac{1}{V} \frac{\partial \log Z}{\partial (1/T)}$ diverges if $T \rightarrow T_H$ changes sign if $T > T_H$

↖ Hagedorn temperature

T_H is the highest possible temperature of matter:

it would require an infinite energy to reach it.

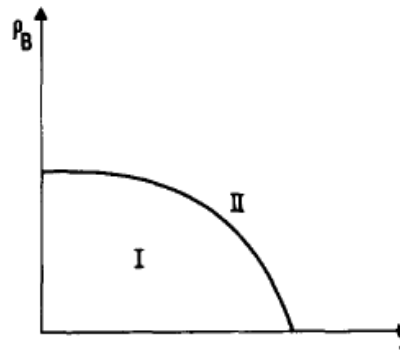


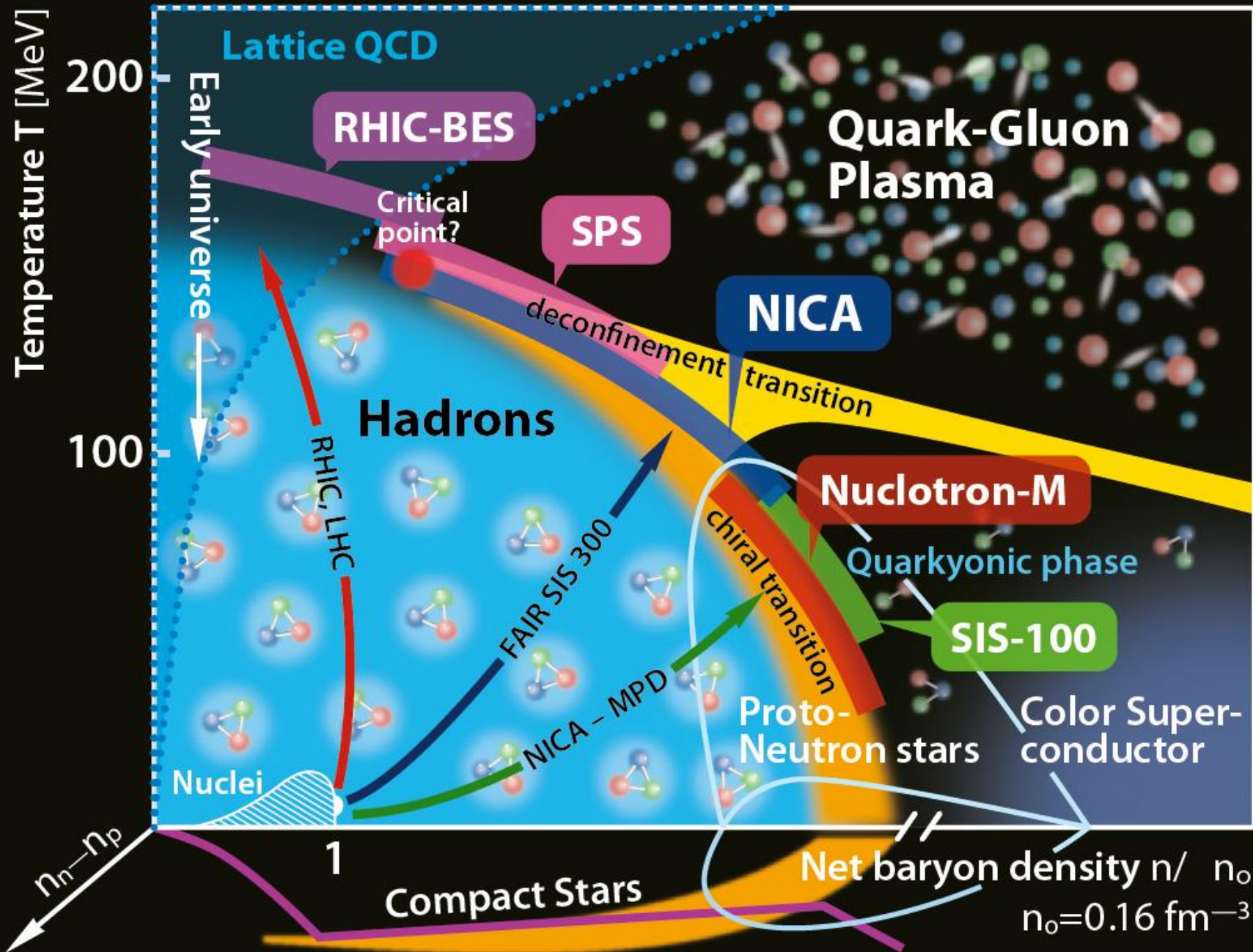
EXPONENTIAL HADRONIC SPECTRUM AND QUARK LIBERATION

N. CABIBBO
Istituto di Fisica, Università di Roma,
Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italy

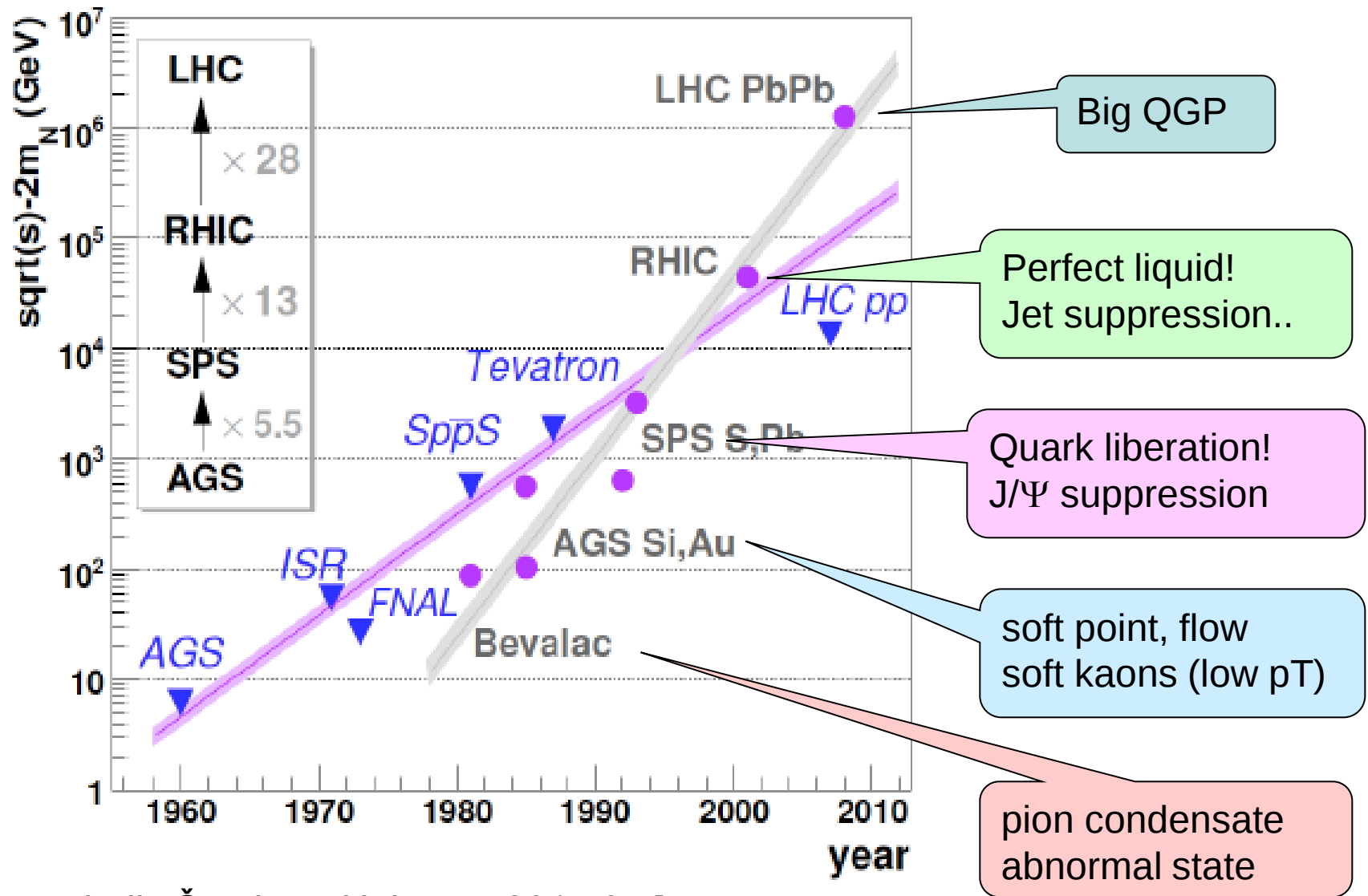
G. PARISI
Istituto Nazionale di Fisica Nucleare, Frascati, Italy

Received 9 June 1975





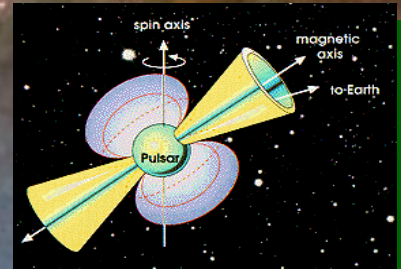
Citius, Altius, Fortius!



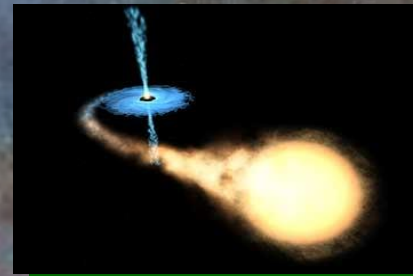
[Pasechnik, Šumbera Universe 2017,3,7]

Neutron Star Zoo

>2000 neutron stars in isolated rotation-powered pulsars
~ 30 millisecond pulsars



>100 neutron stars in accretion-powered X-ray binaries
~ 50 x-ray pulsar
intense X-ray bursters (thermonuclear flashes)



short gamma-ray bursts
neutron star -- neutron star,
neutron star -- black-hole mergers



soft gamma-ray repeaters – magnetars
(super-strong magnetic fields)



Measuring pulsar mass

Pulsar mass can be measured only in binary systems

$$\frac{(M_1 \sin i)^3}{(M_1 + M_2)^2} = \left(\frac{2\pi}{P_b}\right)^2 \frac{(a_1 \sin i)^3}{G}$$



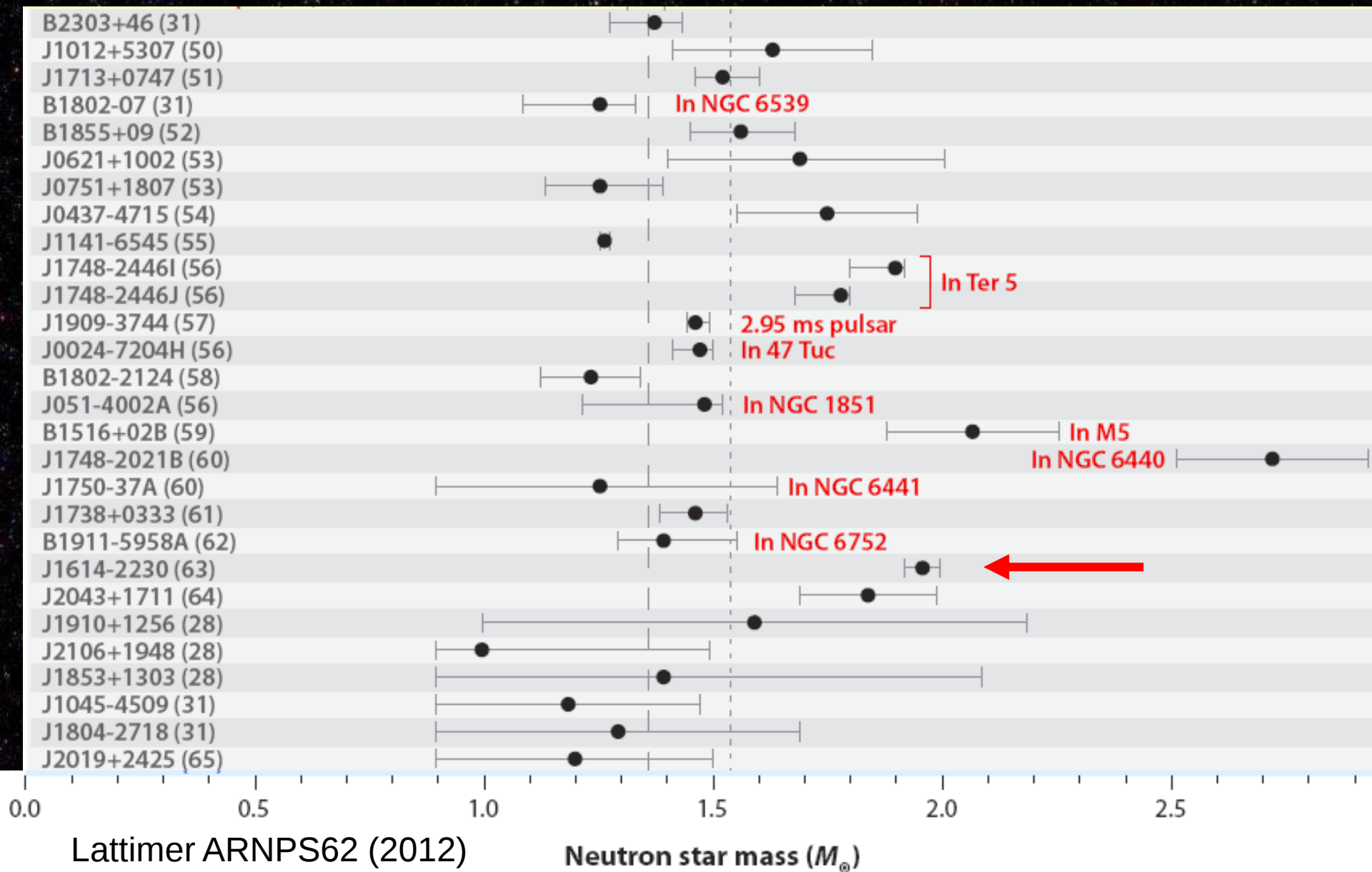
Newton gravity $\longrightarrow$ 5 Keplerian orbital parameters:
orbital period, semi-major axis length, excentricity, ...

Do not determine individual masses of stars and the orbital inclination.

Einstein gravity $\longrightarrow$ 5 potentially measurable post-Keplerian parameters:
orbit precession, Shapiro delay, gravitational redshift,

Measurement of any 2 post-Keplerian parameters allows to determine the mass of each star.

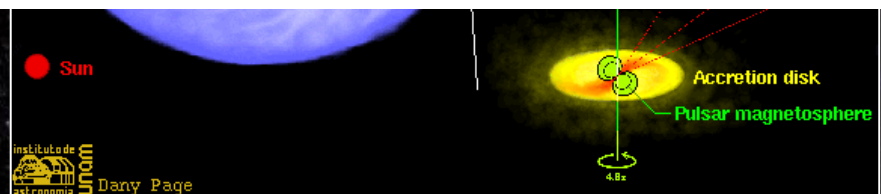
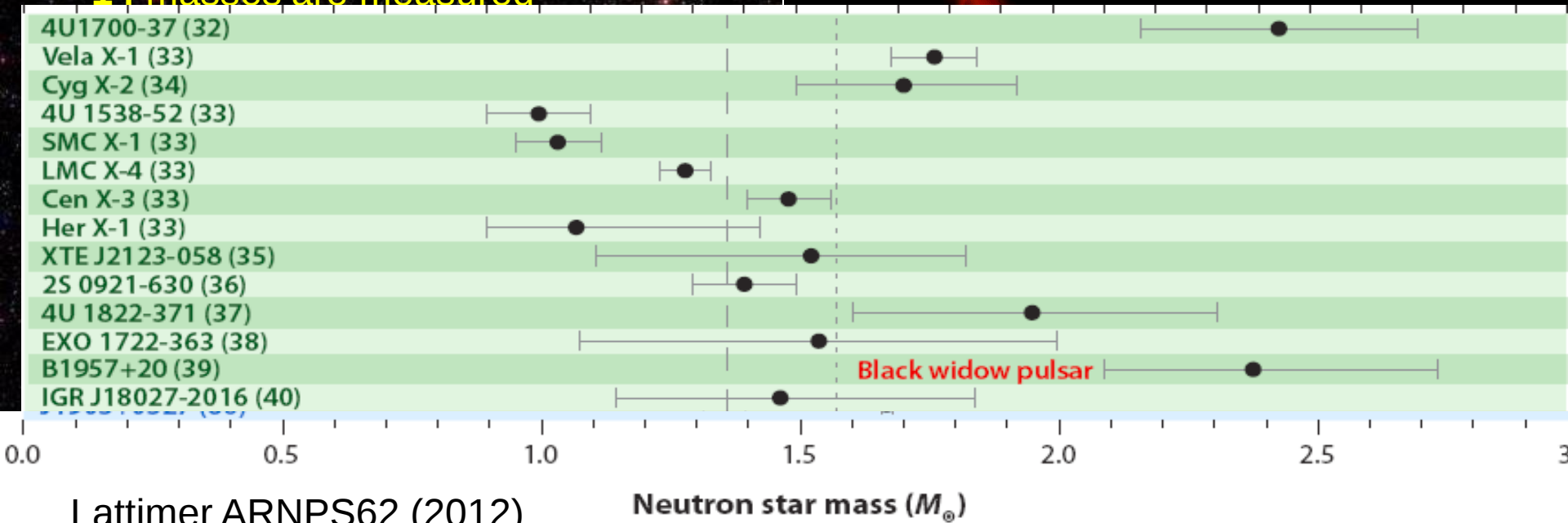
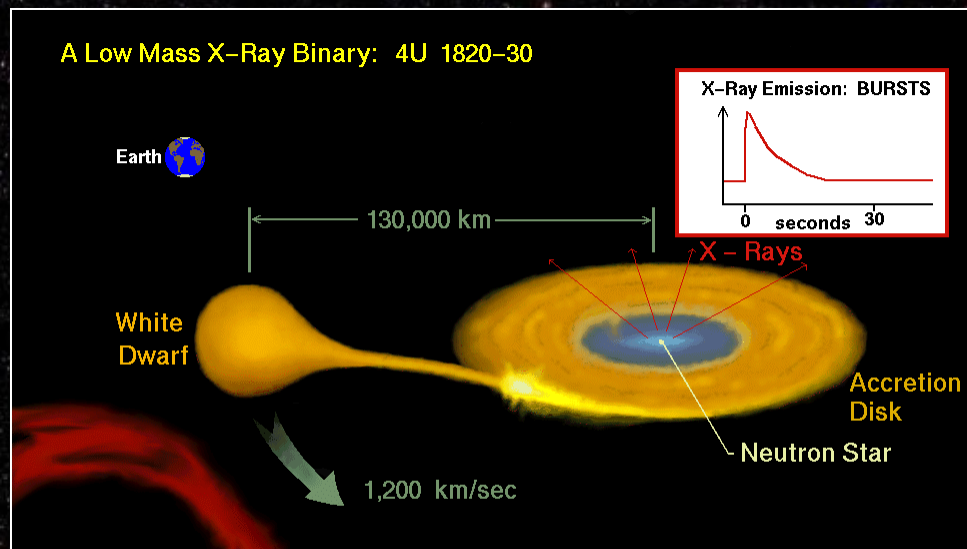
White dwarf -- neutron star binaries



Measuring pulsar mass

X-ray binaries

14 masses are measured



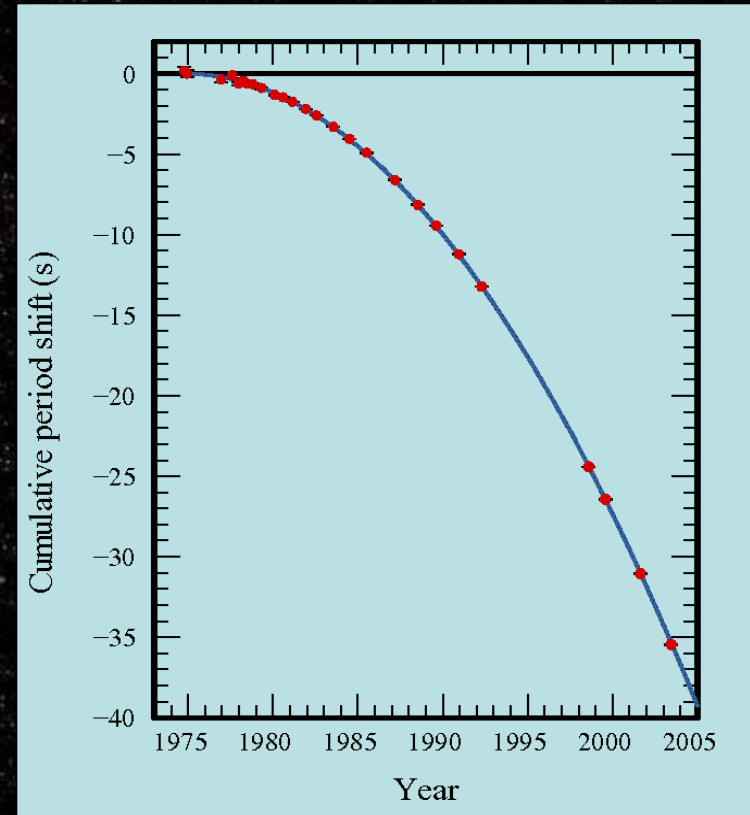
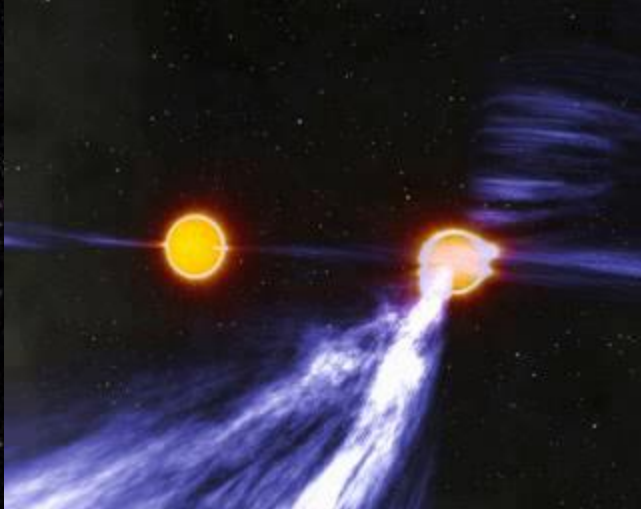
Measuring pulsar mass

Double neutron star binaries

1974 **PSR B1913+16** Hulse-Taylor pulsar

First precise test of Einstein gravitation theory

2003 **J0737-3039** first double pulsar



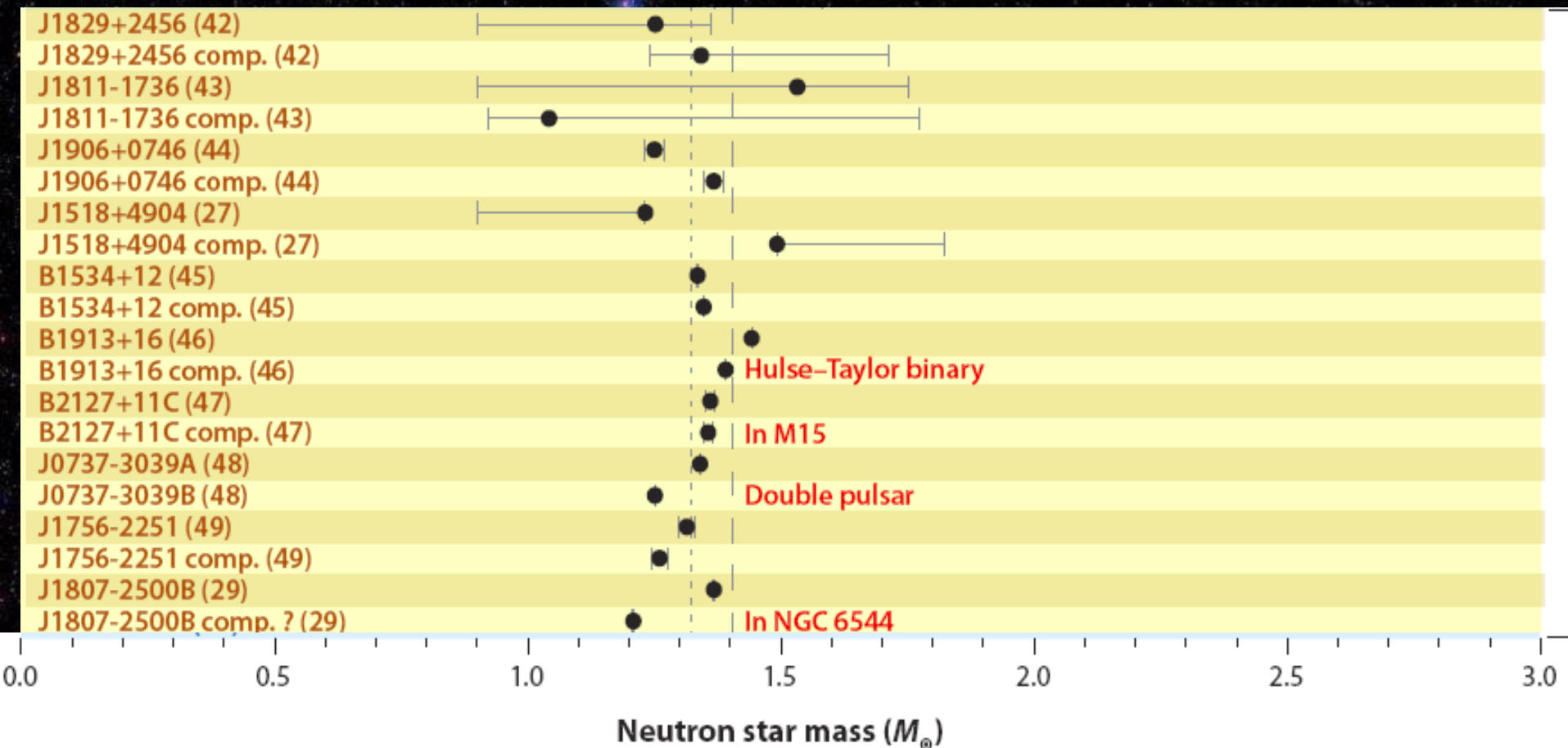
Pulsar A: $P^{(A)}=22.7$ ms, $M^{(A)}=1.338 M_{\text{sol}}$

Pulsar B: $P^{(B)}=2.77$ ms, $M^{(A)}=1.249 \pm 0.001 M_{\text{sol}}$

Orbiting period 2.5 hours

[Nature 426, 531 (2003), Science 303, 1153 (2004)]

Double neutron star binaries



Pulsar J1614-2230

$$M = (1.97 \pm 0.04) M_{\text{sol}}$$

Highest well-known mass of NS

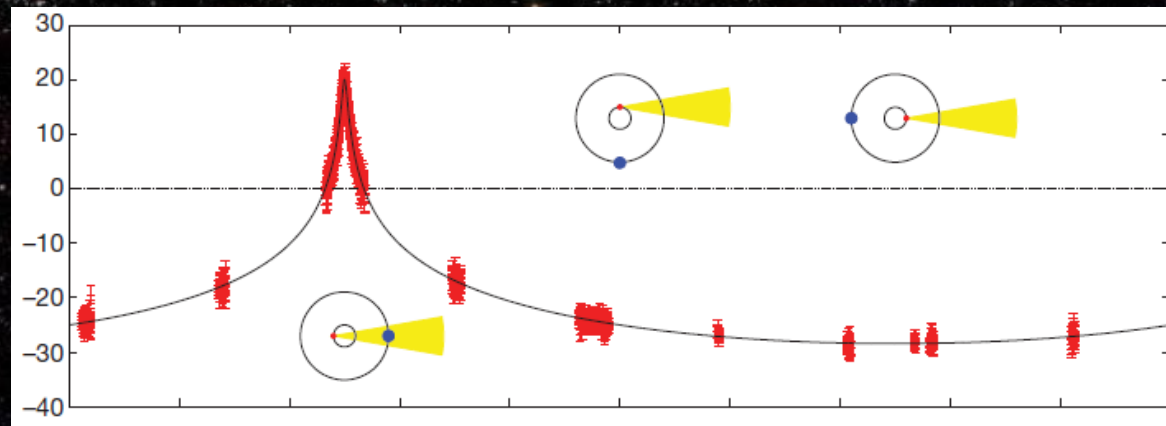
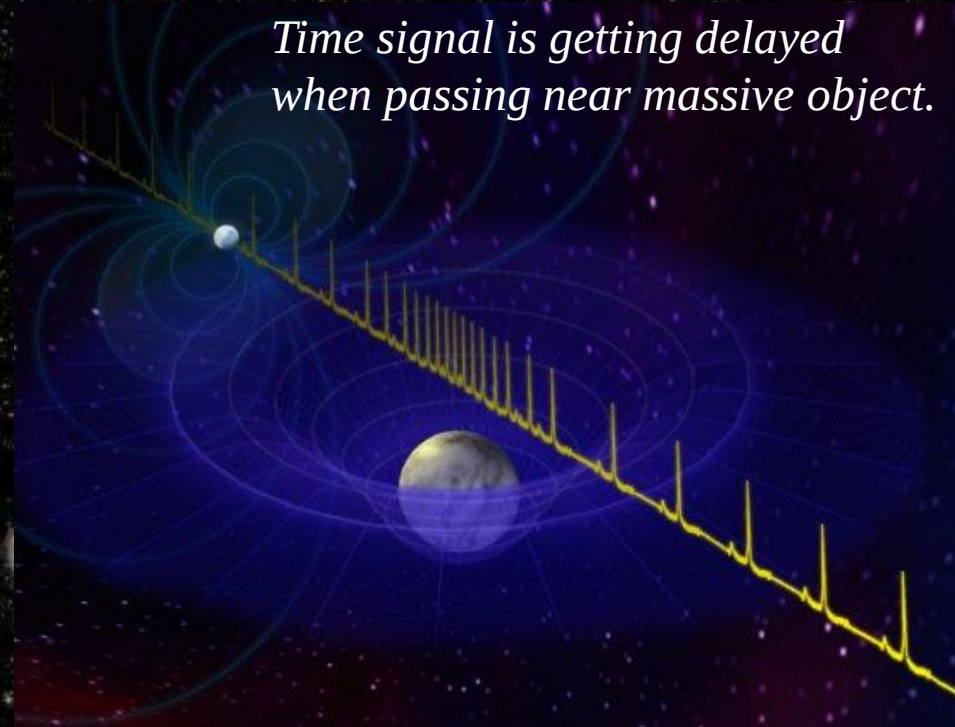
P. Demorest et al., Nature 467, 1081 (2010)

$$M = 2.14^{+0.20}_{-0.18} M_{\text{sol}} \quad (95\%)$$

H.T. Cromartie et al., Nat. Astr. (2019)

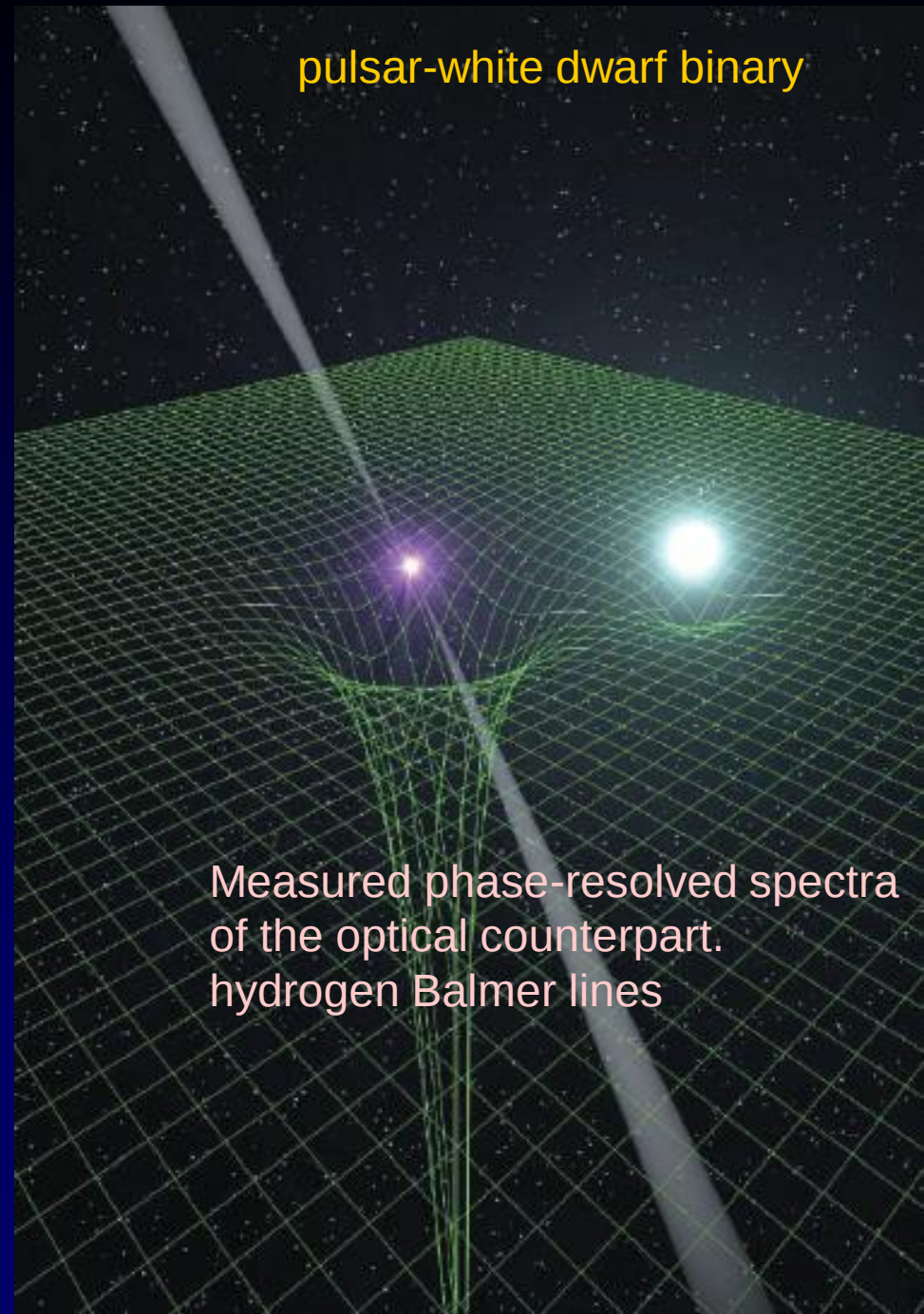
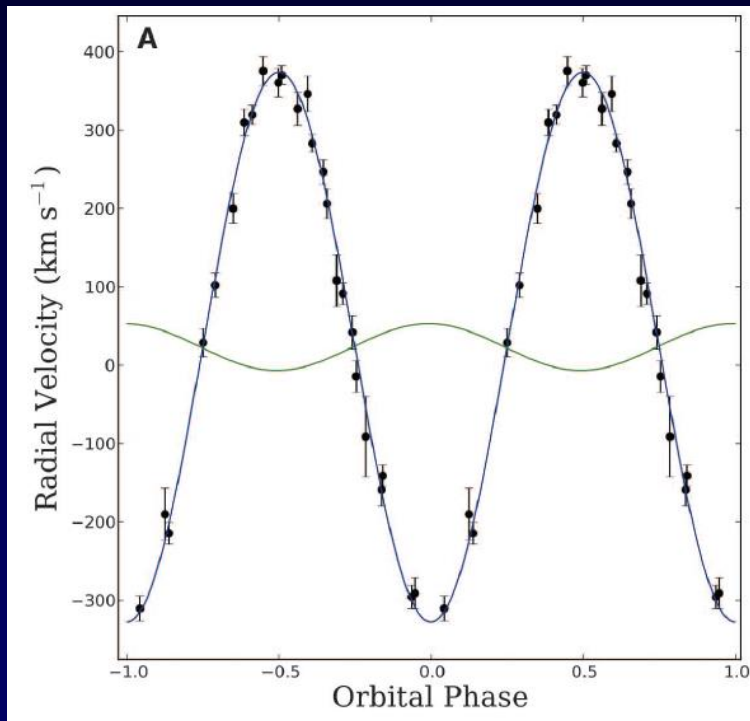
there are heavier, but far less precisely measured candidates)

Measured Shapiro delay with high precision

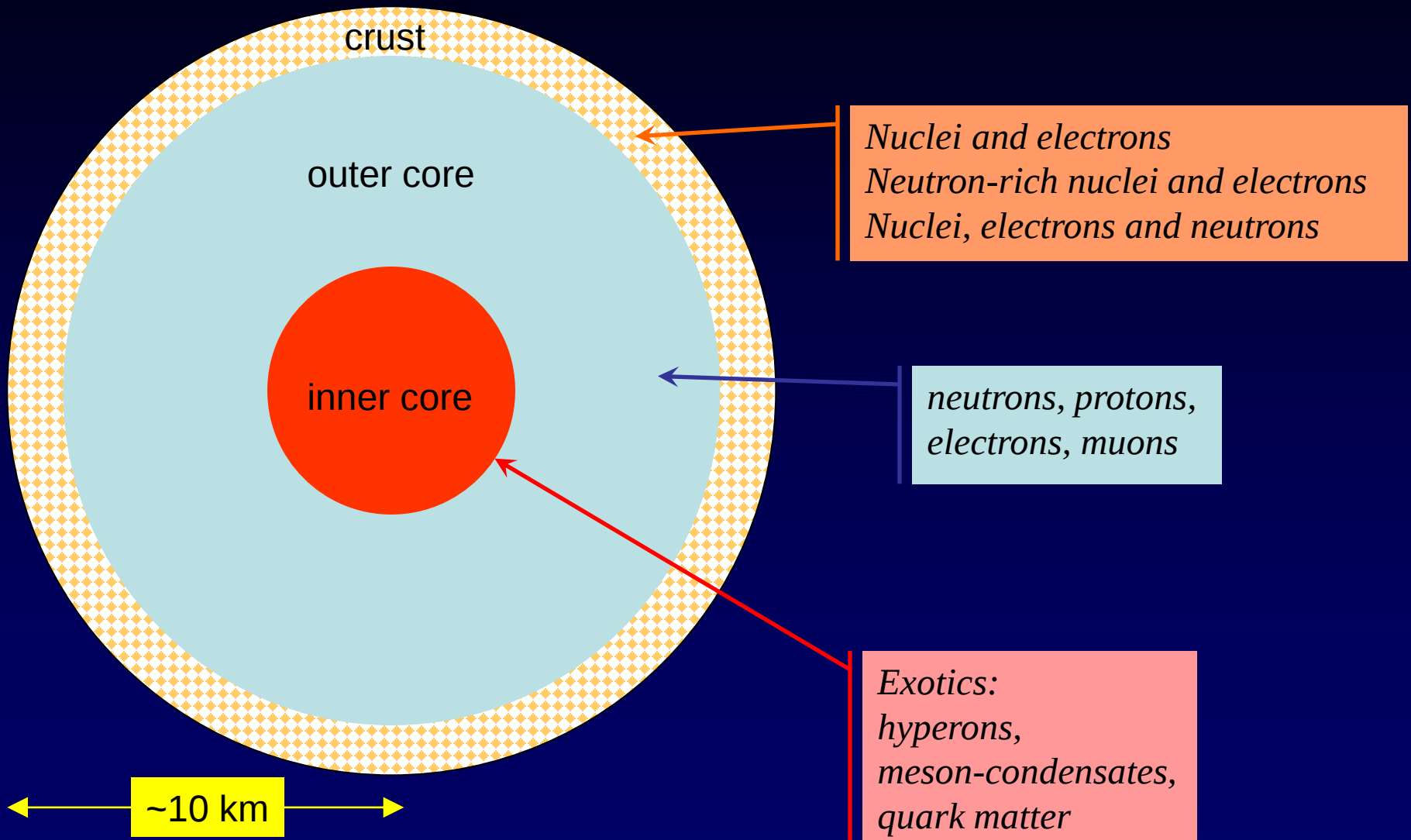


Pulsar J1614-2230

$$M = (2.01 \pm 0.04) M_{\text{sol}}$$



Cross section of a neutron star

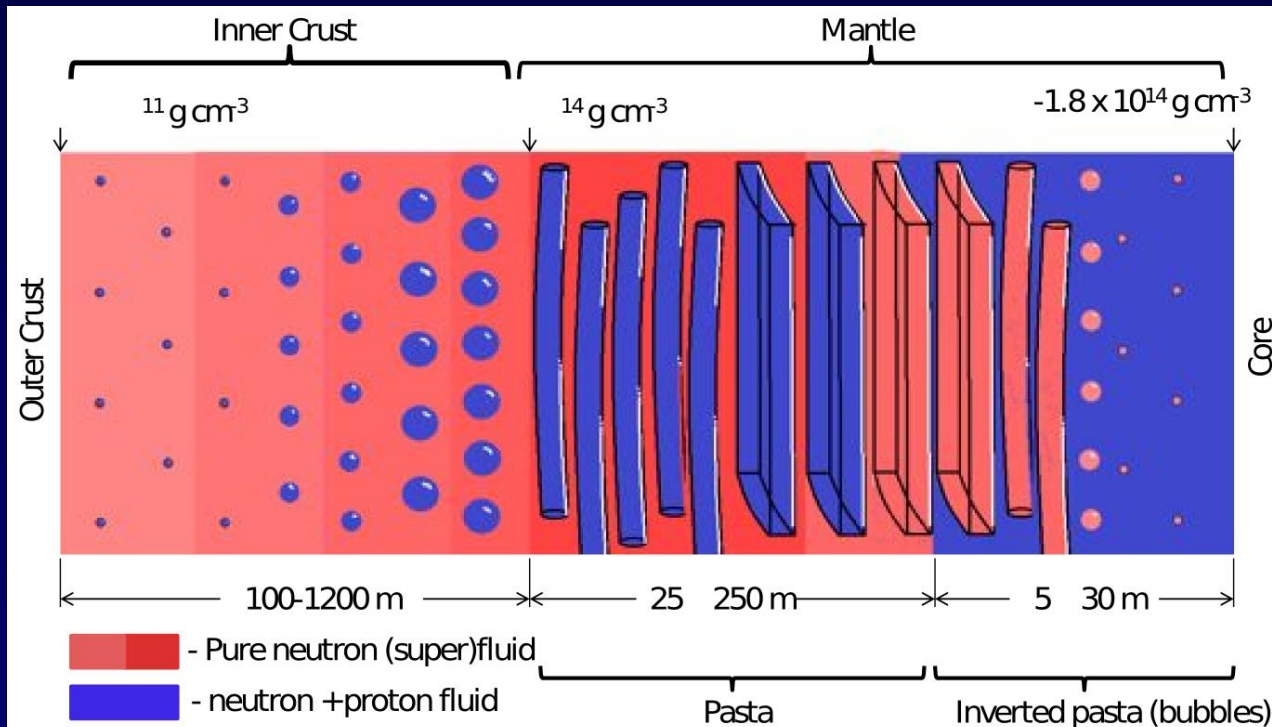
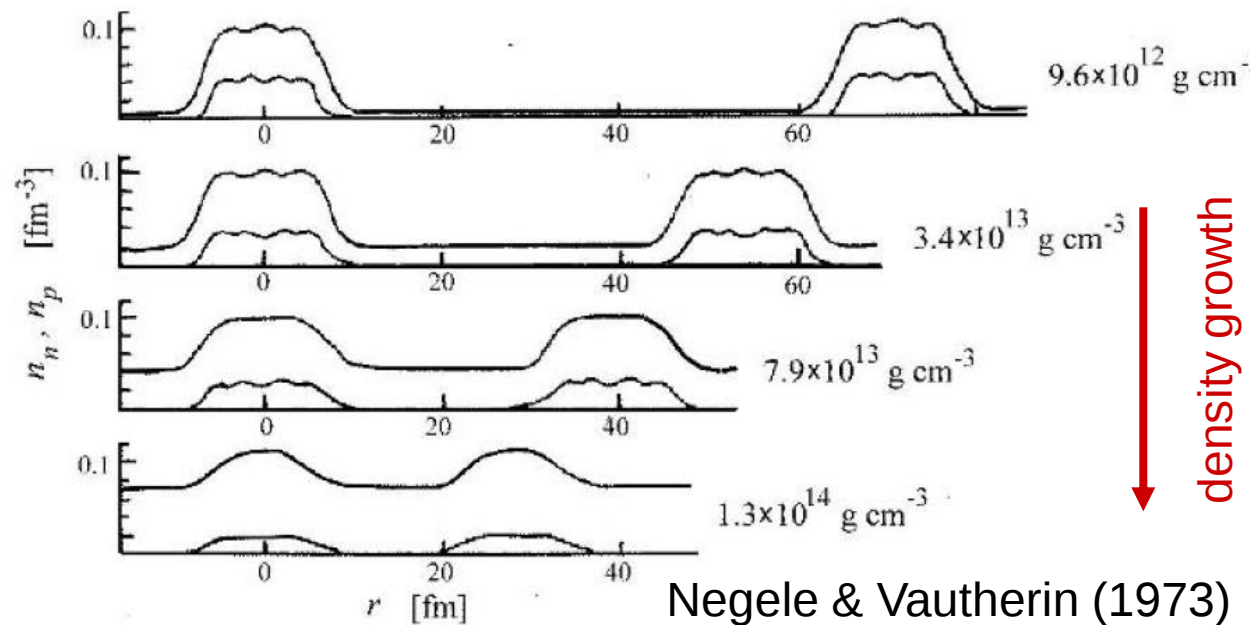


Crust

Nucleus melting

Pasta structure

interplay of Coulomb energy and surface tension



saturation density

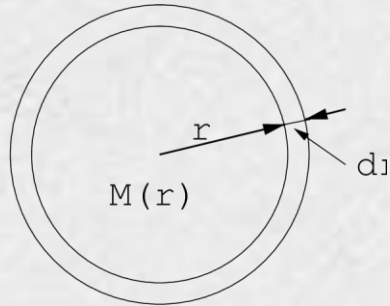
$$\rho_0 = 2.8 \times 10^{14} \frac{\text{g}}{\text{cm}^3}$$

$$M_{\text{crust}} \sim 0.1 M_{\text{sol}}$$

$$R_{\text{crust}} \sim 10^2 - 10^3 \text{ m}$$

Tolman-Oppenheimer-Volkov equation

Equilibrium condition for a shell in a non-rotating neutron star



$$S_{\Omega}(r) dp = dF_G \quad \text{Newton's Law}$$

$$4\pi r^2 dp = G \frac{M(r) dM}{r^2} \quad dM = 4\pi r^2 \varepsilon(p) dr$$

INPUT: equation of state (EoS)

$$\varepsilon = \varepsilon(p) \quad \text{or} \quad \begin{cases} p = p(n) \\ \varepsilon = \varepsilon(n) \end{cases}$$

boundary conditions: $\varepsilon(r=0) = \varepsilon_c, \quad M(r=0) = 0, \quad P(r=R) = 0$

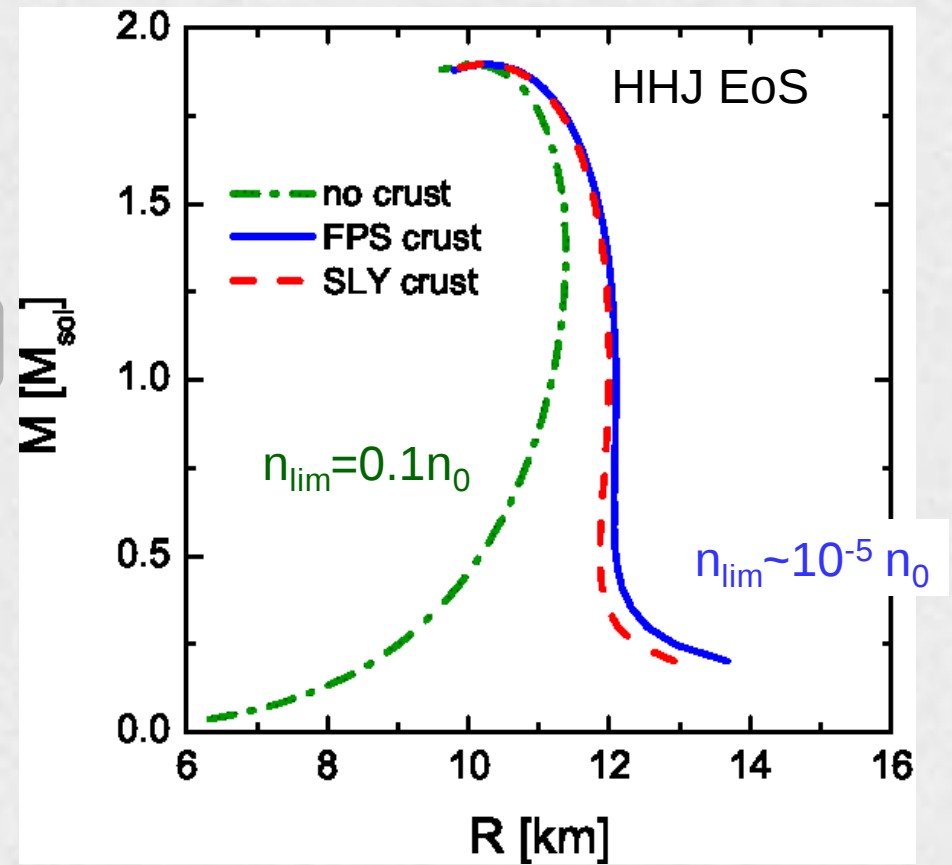
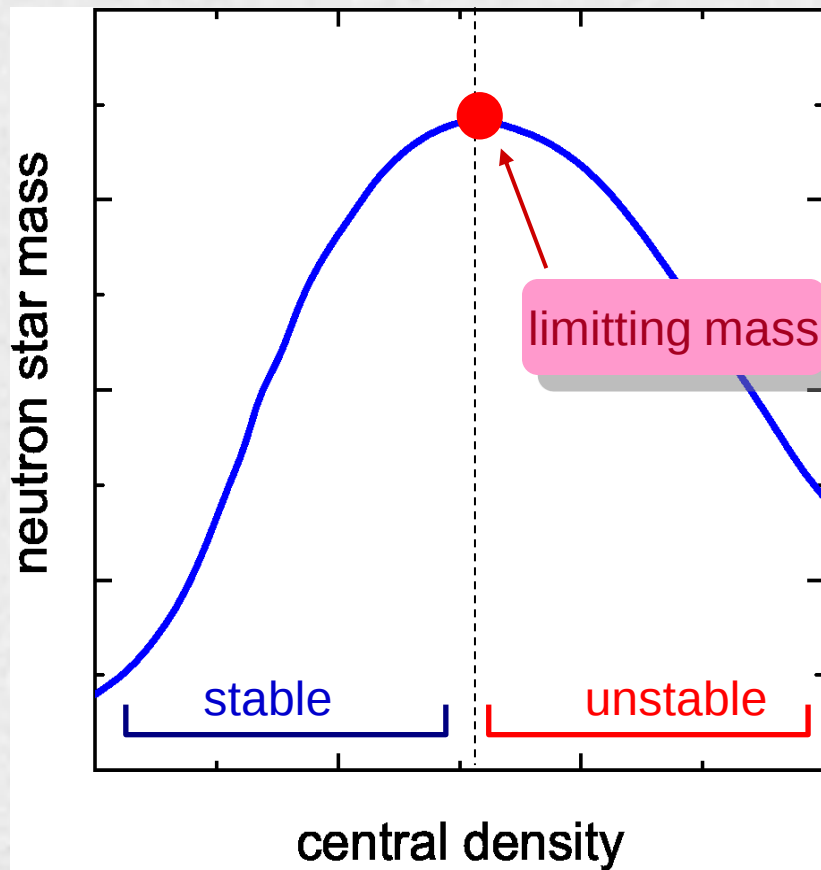
OUTPUT:

neutron star density profile, radius R and mass M

relativistic corrections

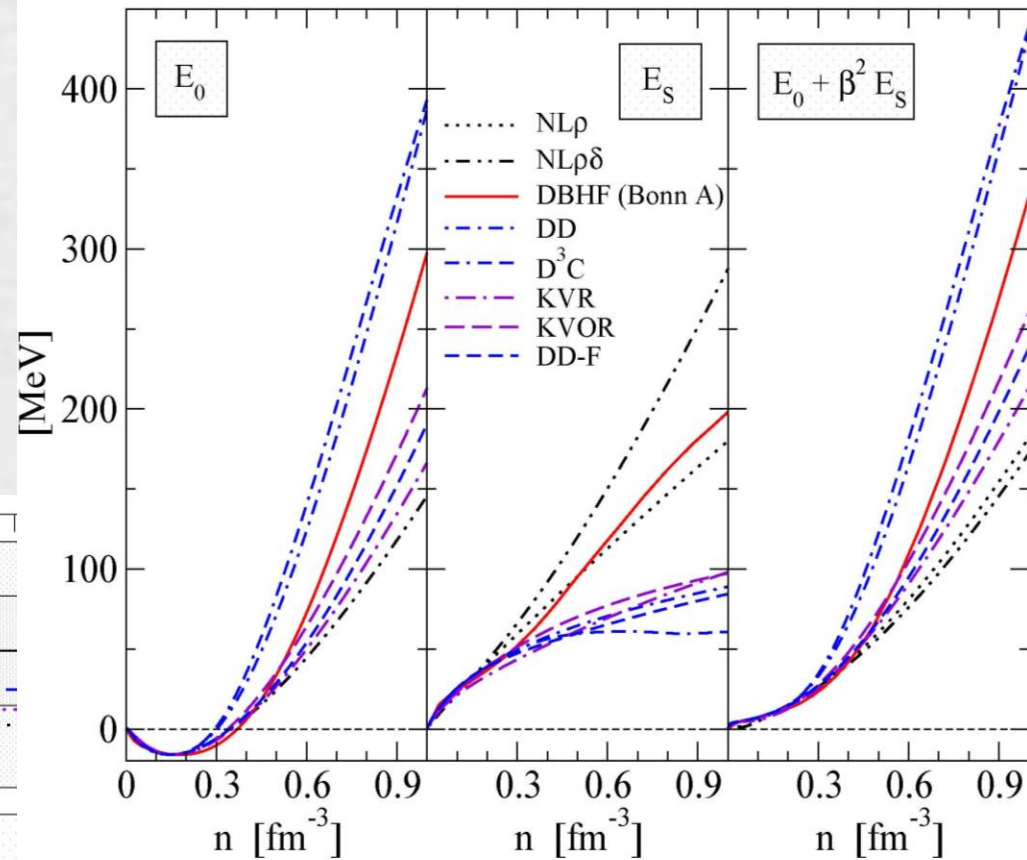
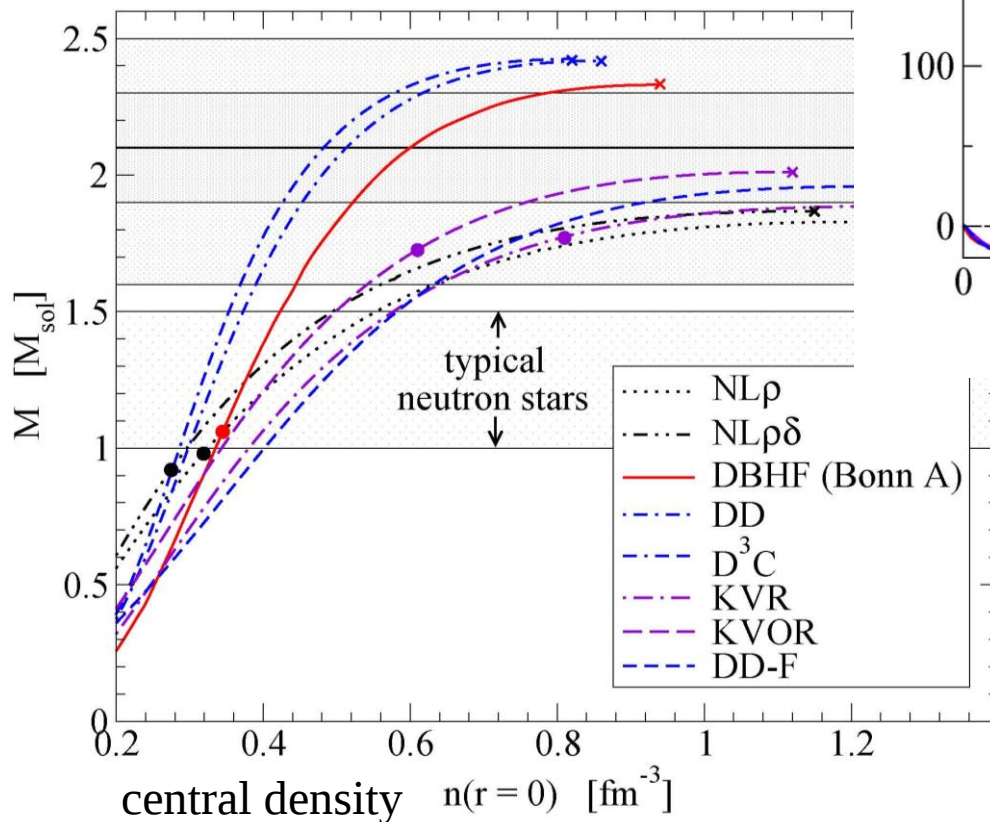
$$\frac{dp}{dr} = -\frac{G \rho M}{r^2} \left(1 + \frac{p}{\rho c^2}\right) \left(1 + \frac{4\pi P r^3}{M c^2}\right) \left(1 - \frac{2GM}{c^2 r}\right)^{-1}$$

Neutron star configuration



uncertainty in $R \sim 10^3$ m

Selection of Nuclear Equations of State



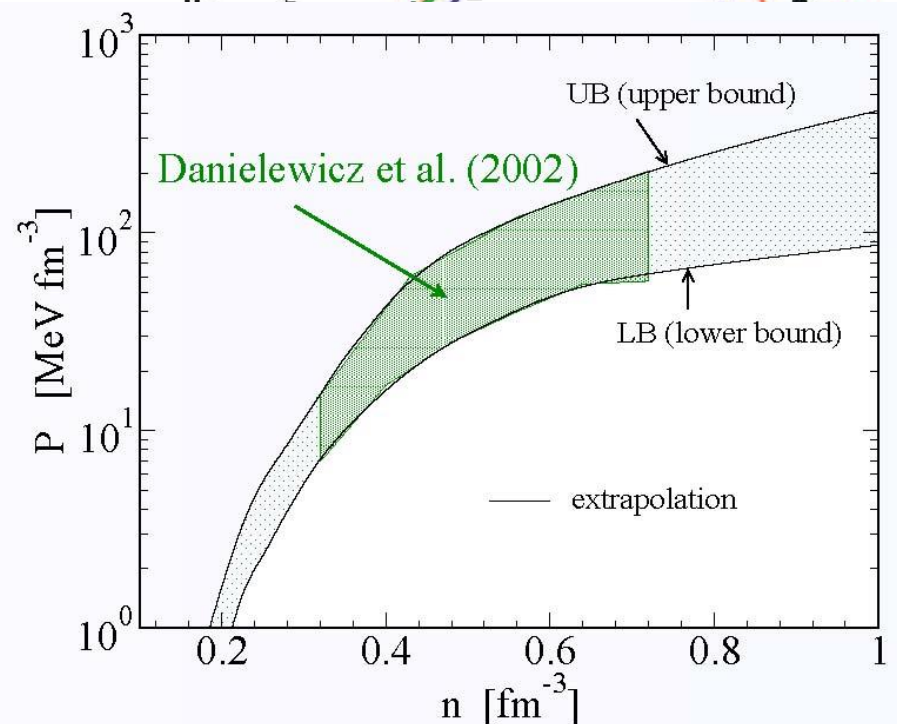
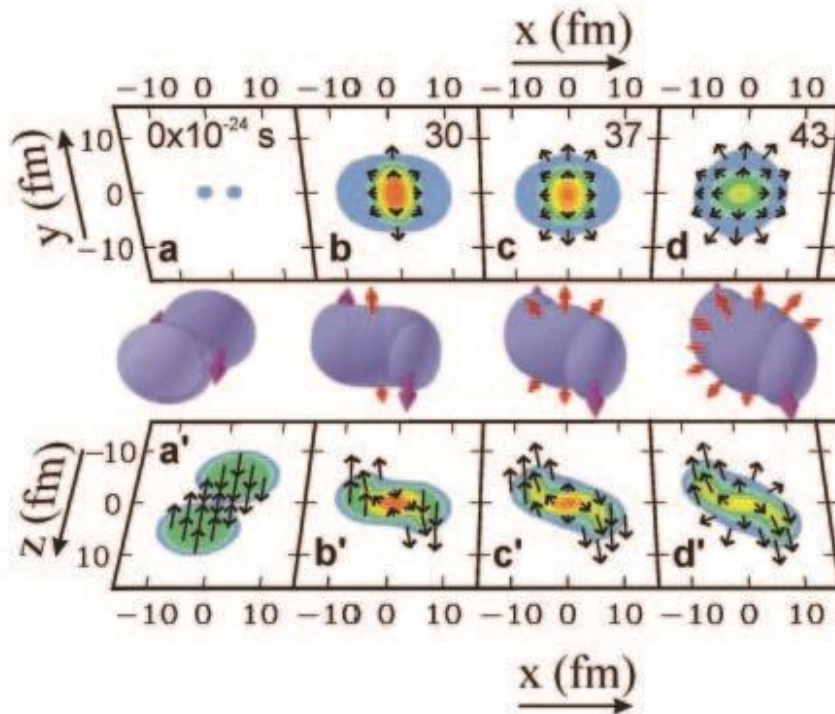
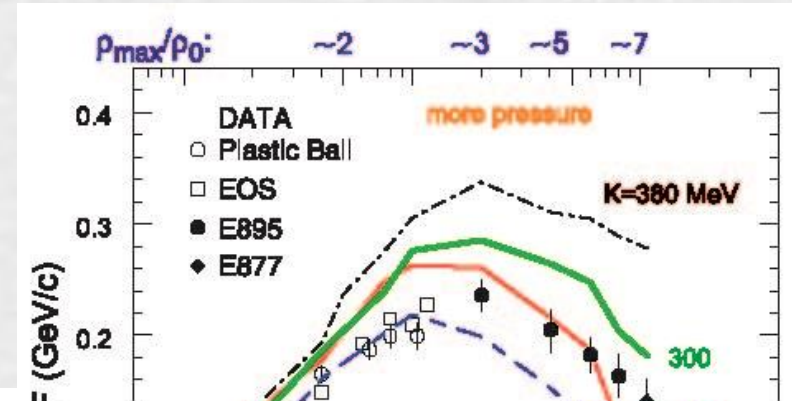
Nucleon part of EoS should be sufficiently stiff

Constraints from heavy-ion collisions

- Boltzmann kinetic equation
- Mean-field potential

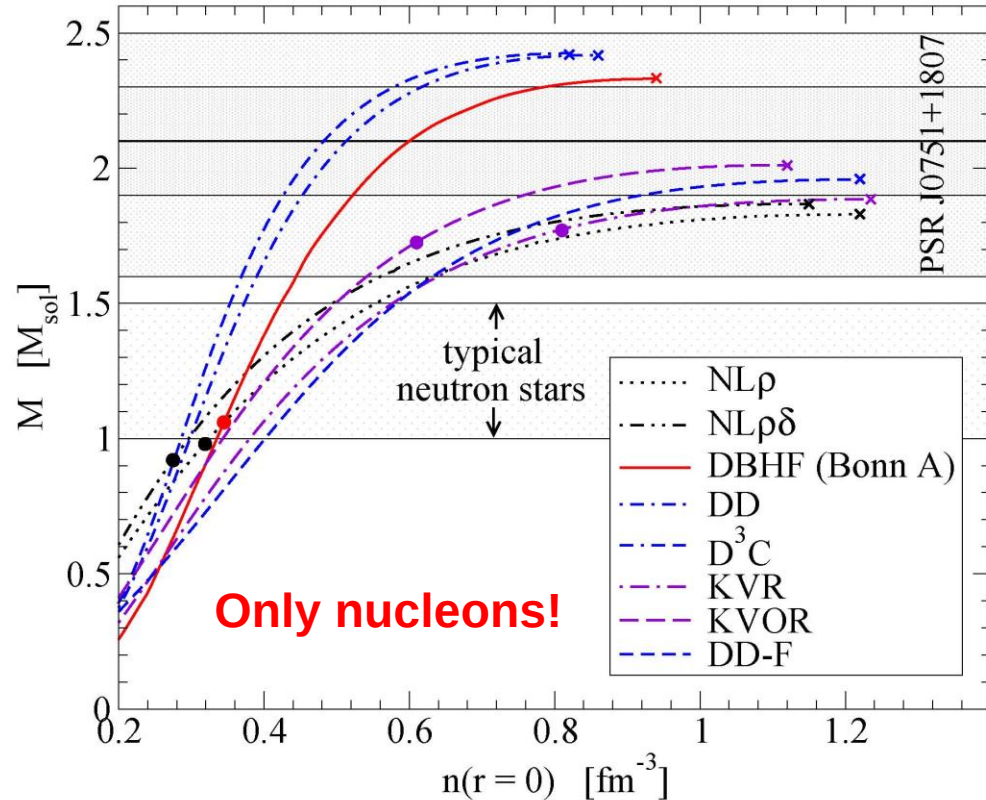
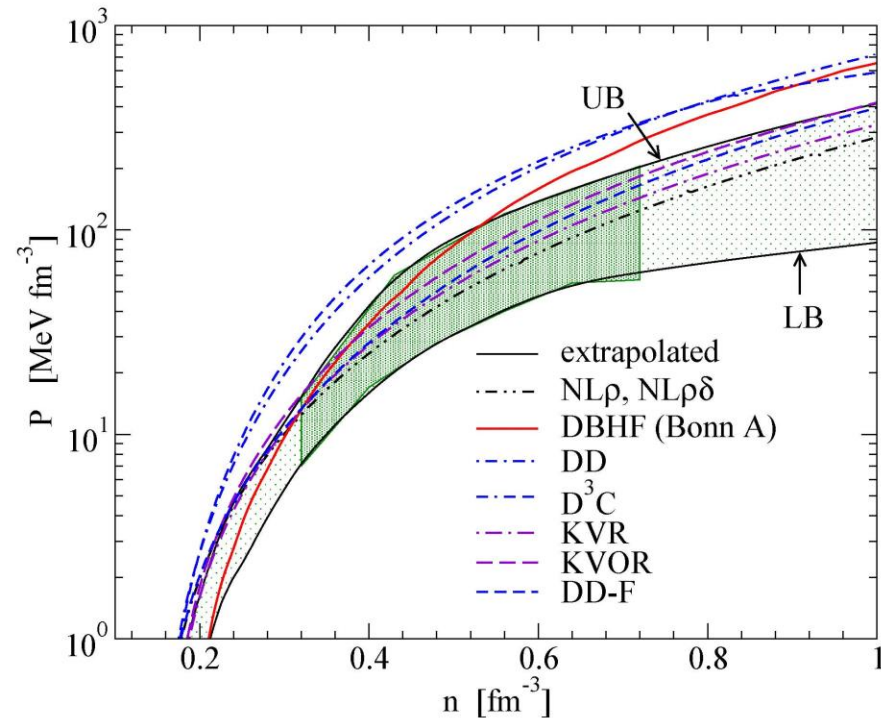
$$U = (a\rho + b\rho^v)/[1+(0.4\rho/\rho_0)^{v-1}] + \delta U_p$$

fitted to directed & elliptic flow



✓ constraints from heavy-ion collisions

✓ maximum mass constraints



Puzzle

Nucleon part of EoS should be sufficiently stiff.

Pressure in isospin symmetric matter should be no to high. (particle flow in HIC)

New degrees of freedom

Puzzles

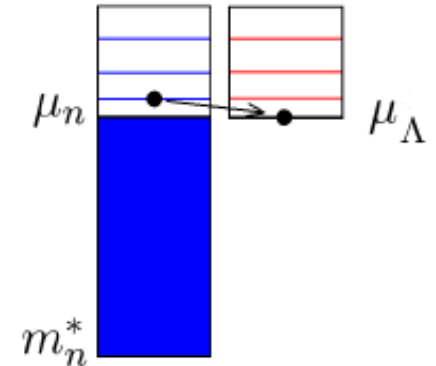
strangeness

Hyperon puzzle. H-N interaction is attractive in nuclei.

Within RMF framework hyperons appear in neutron stars and reduce $M_{\max} < 2M_{\text{sol}}$

Delta puzzle. New fermions appear, maximum NS mass drops further [Drago et al.]

Light di-baryons, light nucleon resonances?



Mesonic condensates

$$e^- \longrightarrow \pi_c^- + \nu_e$$

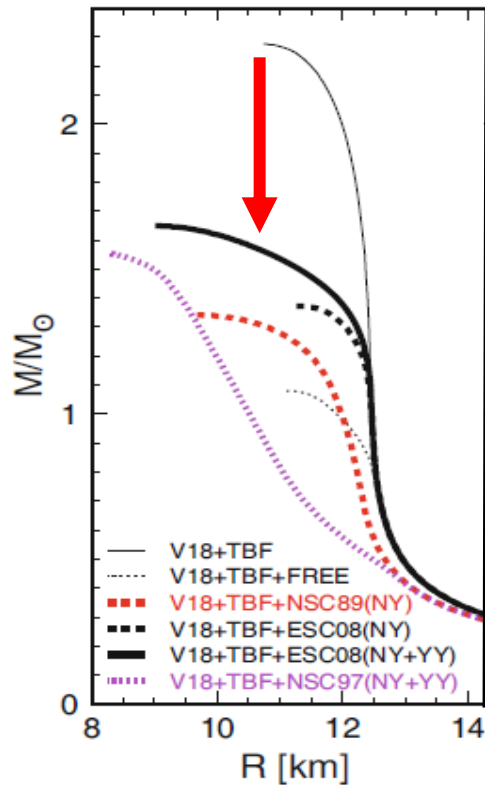
$$e^- \longrightarrow K_c^- + \nu_e$$

weak reactions start if

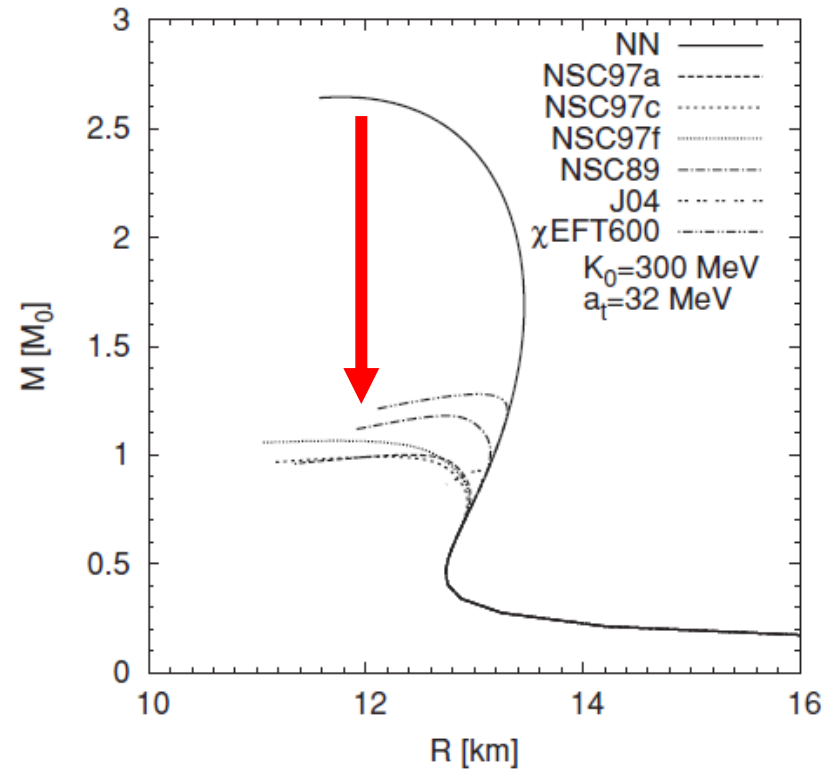
$$\mu_e \geq \min_p \omega_{\pi, K}(p)$$

“Hyperon puzzle”

If we allow for a population of new Fermi seas (hyperon, Δ baryons, ...) EoS will be softer and the NS will be smaller



[Rijken, Schulze, EPJA52 (2016) 21]



[Dapo, Schaefer, Wambach PRC 81 (2010) 035803]

Simple solutions: -- **make nuclear EoS as stiff as possible** [flow constraint]
 -- **suppress hyperon population (increase repulsion/reduce attraction)**

against phenomenology of YN, NN, YY interaction in vacuum
 +hypernuclear physics

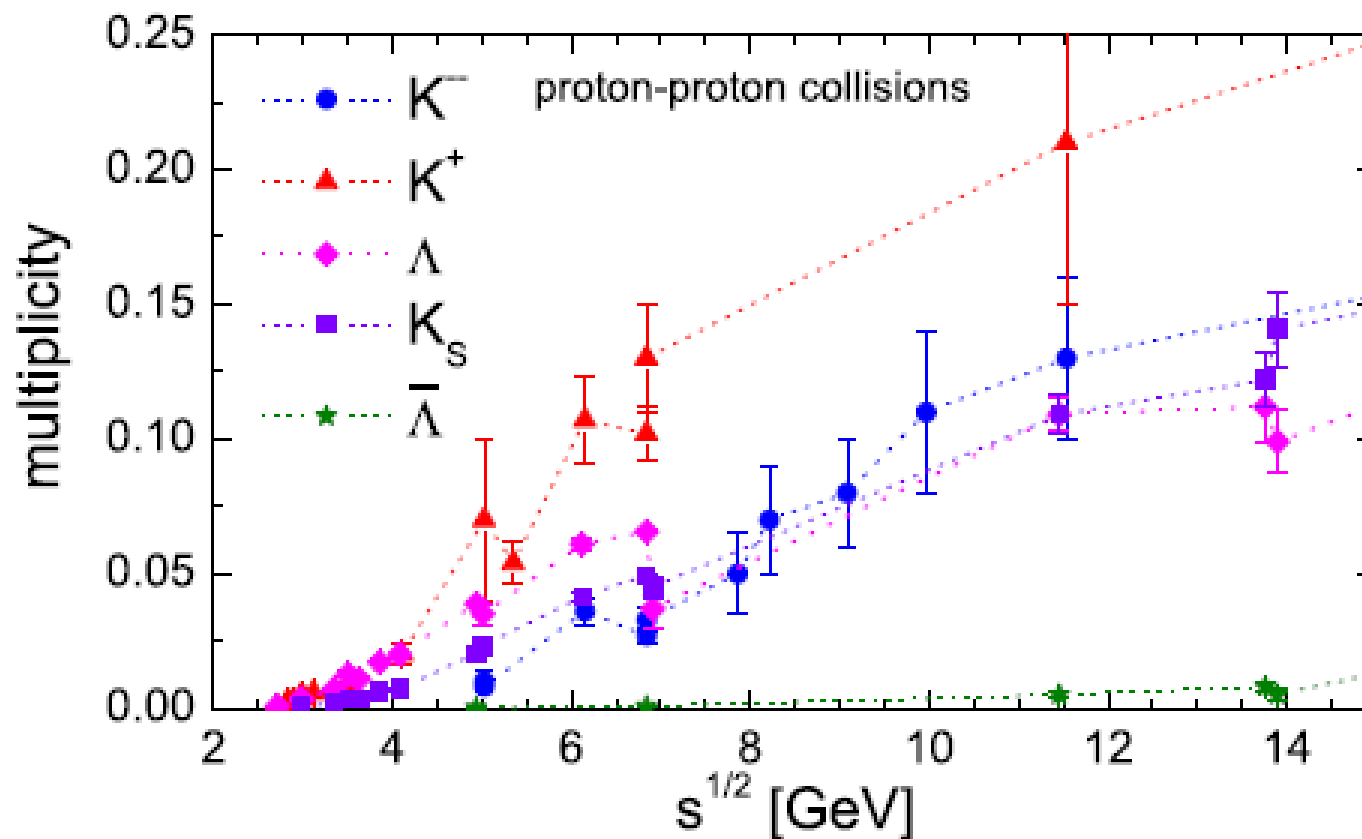
Strangeness is HIC interesting because

- ✓ It is a **tag** on a hadron, saying that it was not in colliding nuclei but is **produced** in the course of collision.
- ✓ **Strange quarks** like **baryons**: $K, \Lambda, \Sigma, \Xi, \Omega, \dots$, **anti-strange quarks** like **mesons** $K, \dots$
 - strangeness/anti-strangeness **separation** in baryon-rich matter
- ✓ Strangeness is **conserved** in strong interaction
 - Strangeness production **threshold is high**,
 - sensitive to possible **in-medium effect**.
QGP signal? (Rafelski-Mueller conjecture)

Strangeness is HIC difficult because

- ✓ Strangeness **production cross sections** poorly known
(new data from HADES on pp, COSY on pn, ANKA)
- ✓ Limited exp. information about **elementary reactions** among strange particles
- ✓ **Strong couplings** among various strange species. Complicated dynamics

Input. Elementary processes



Quality of data at high energies $> 4\text{GeV}$!

Change of the production mechanism at 4-5 GeV

Strangeness production rate at AGS-SPS

- **strangeness production** K^+ and K^0 evolution calculated from

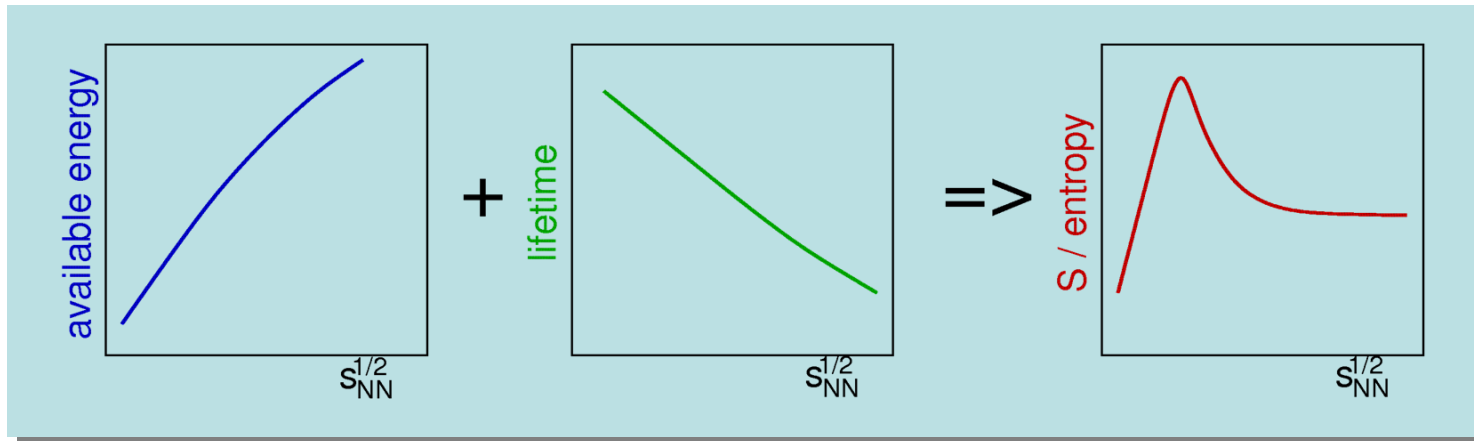
$$\frac{d\rho_K}{d\tau} = \rho_K \left(-\frac{1}{V} \frac{dV}{d\tau} \right) + \mathcal{R}^+ - \mathcal{R}^-$$

expansion rate annihilation rate production rate

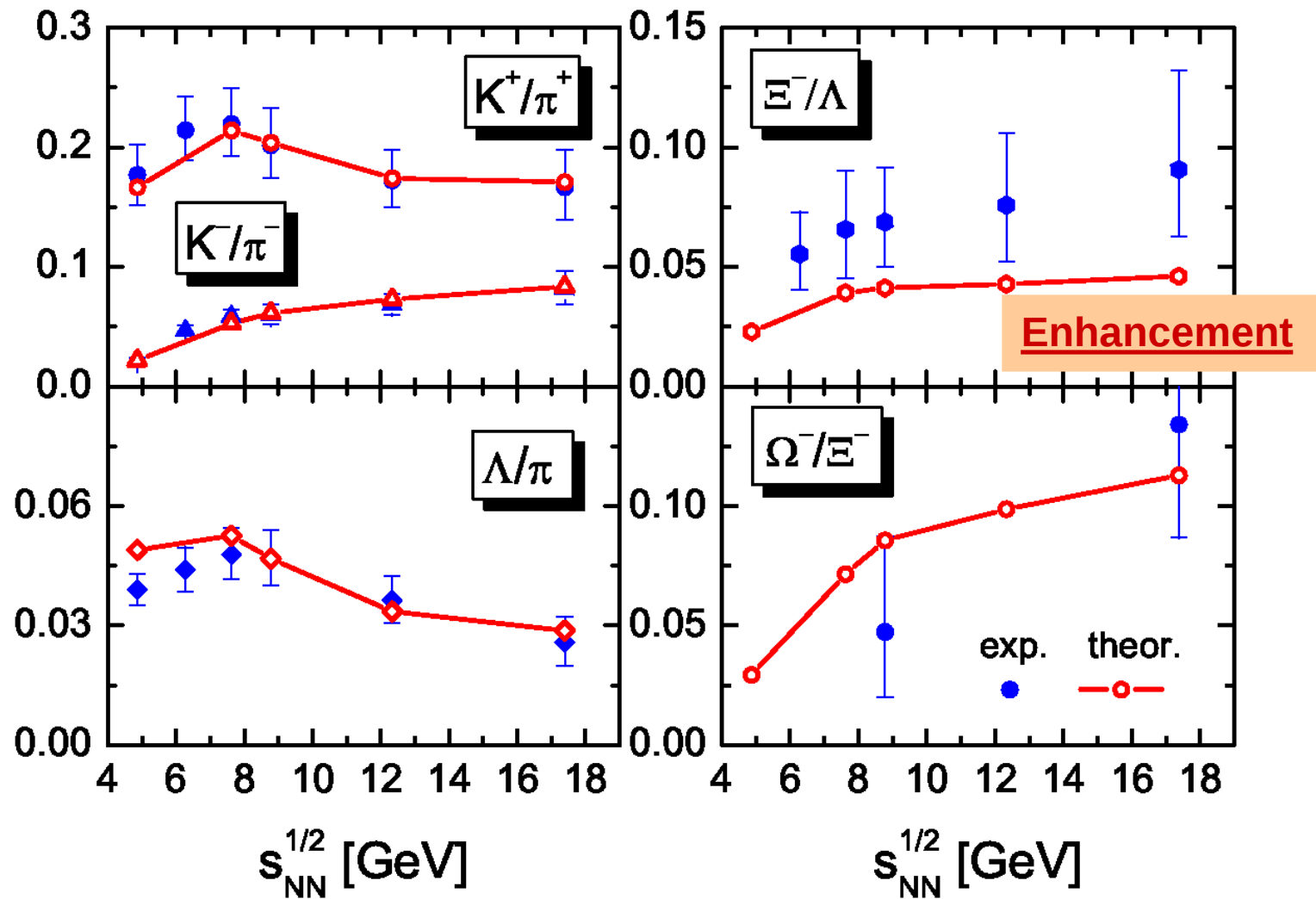
calculate from known cross-sections and evolved densities

Initial strangeness: number of K^+ from pp collisions final state: chem. freeze-out parameters

- **fireball expansion time vs. energy** parameterized the space-time evolution
a data/experience (HBT, spectra) driven ansatz for expansion



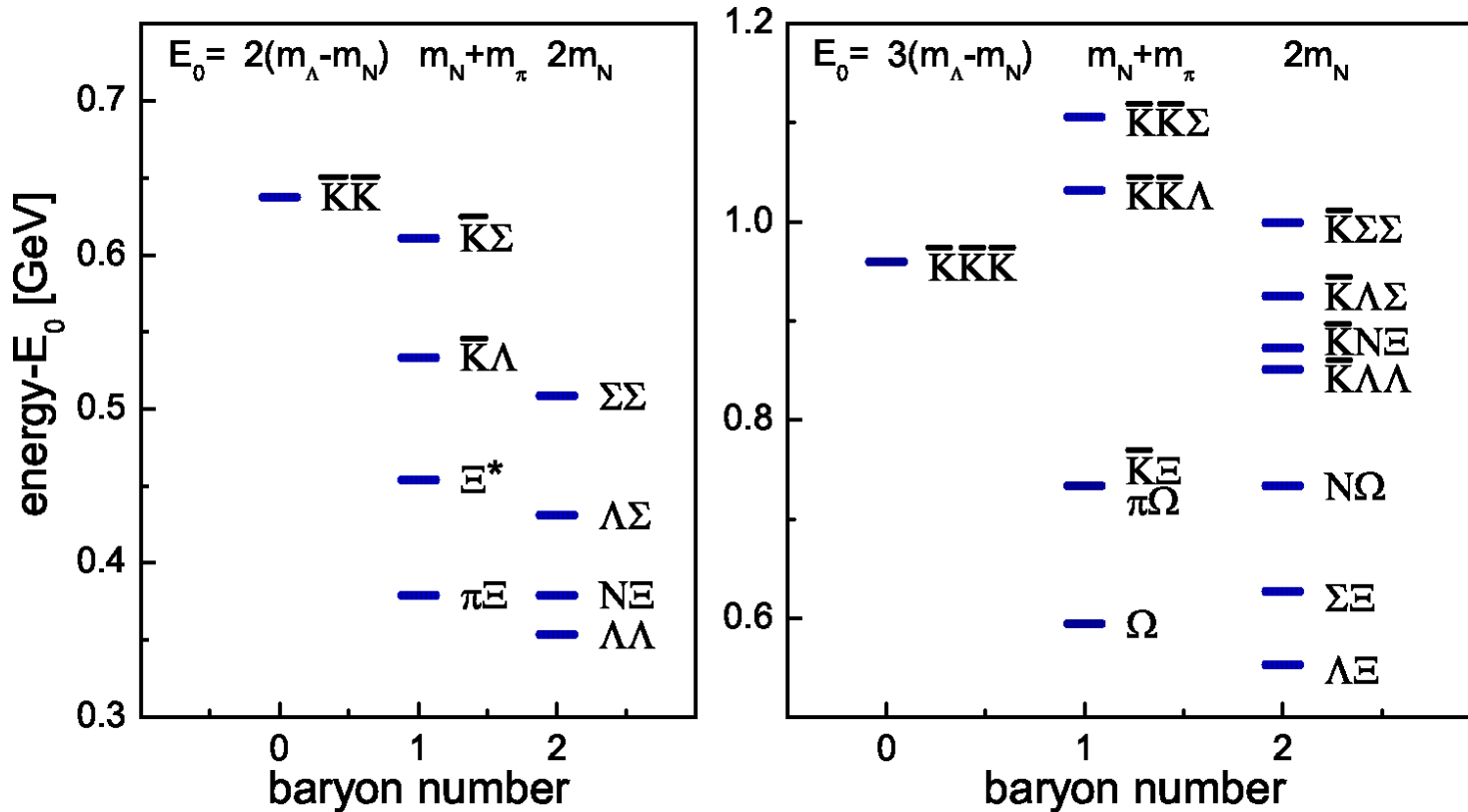
● results



The time needed for a strangeness production is about **15-20 fm/c**.
 In hydro the typical expansion time is **<10 fm/c!**.

Multi-strangeness enhancement

Mass spectrum of strange hadronic states with different baryonic numbers

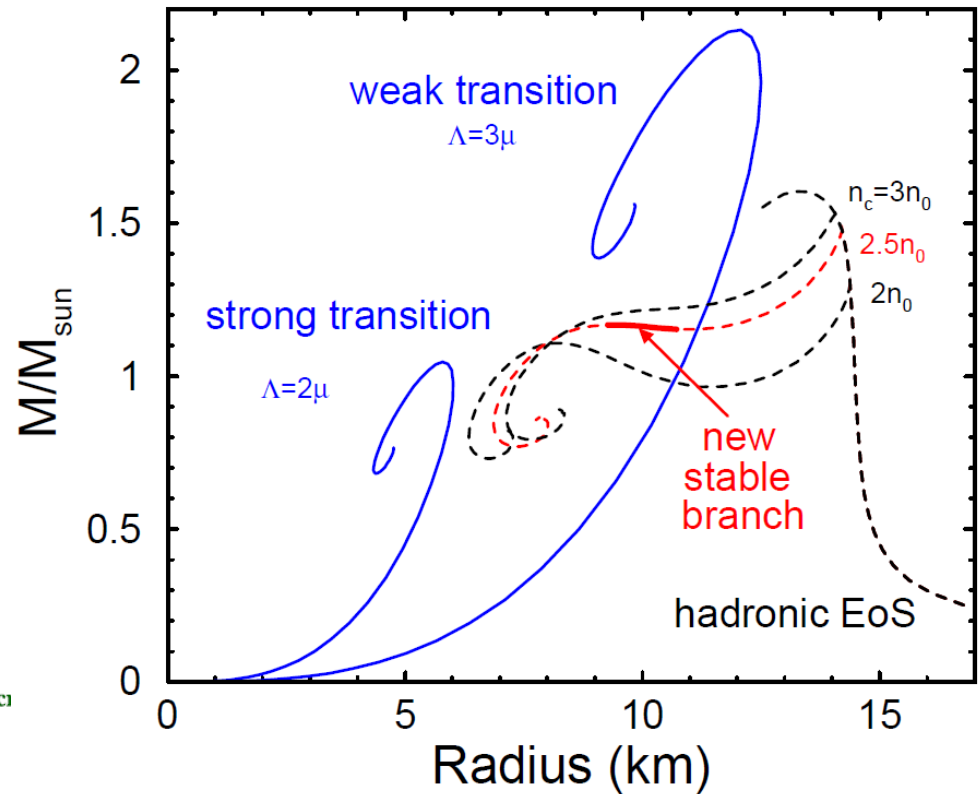


Once several s quarks appear in the hadronic system it is energetically favorable to “store” them together in a multistrange state Ξ or Ω . In both single and double baryon channels the states with the Ξ and Ω baryon are at the bottom of the spectrum. This means that Ξ s and Ω s would play a role of a strangeness reservoir, being filled with the decrease of temperature.

Quark Stars

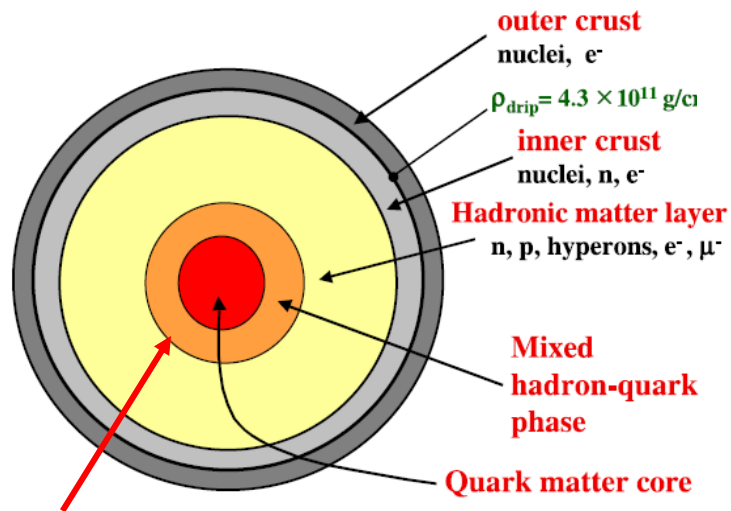
Perturbative QCD EoS

[Fraga, Pisarski, Schaffner-Bielich, NPA 702,217]



Λ renormalization subtraction point

no constraints from Lattice QCD!



Interface?

[Bombaci 0809.4228]

Propagator of transverse gluons

medium effects

$$D(\omega, k; T) = \frac{1}{\omega^2 - k^2 - \Pi(\omega, k; T)} \approx \frac{1}{\omega^2 - k^2 - m_g^2(T)}$$

HTL perturbative expression

$$m_g^2(T) = \frac{N_c}{6} g_{\text{eff}}^2(T) T^2$$

$$P = \frac{d}{6\pi^2} \int \frac{dk k^4}{\omega_g^*(k)} f_g^*(k) - B(m_g(T))$$

$$\varepsilon = \frac{d}{6\pi^2} \int dk k^2 \omega_g^*(k) f_g^*(k) + B(m_g(T))$$

$$\omega_g^*(k) = \sqrt{k^2 + m_g^2(T)}$$

$$f^*(k; T) = [\exp(\omega_g^*(k; T)/T) - 1]^{-1}$$

$$g_{\text{eff}}^2(T) = \frac{48\pi^2}{11 N_c \ln \left(\frac{T+T_s}{T_c/\lambda} \right)}$$

thermodynamical consistency

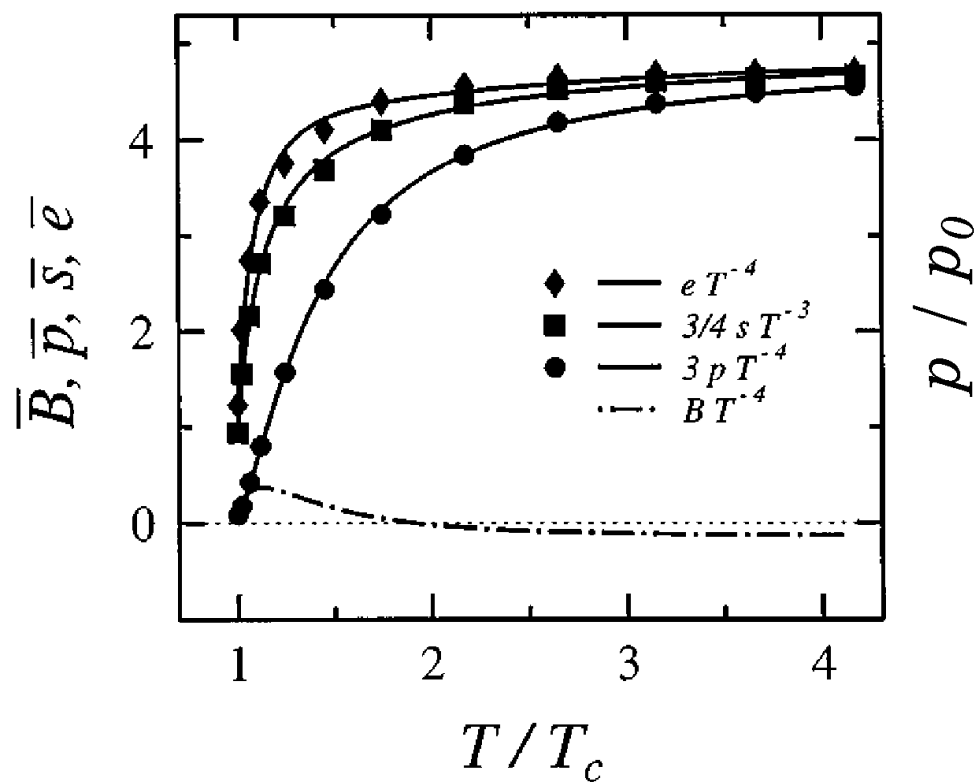
$$\frac{\delta P}{\delta m_g} = 0$$

$$B = B_0 - \frac{d}{2\pi^2} \int_{T_0}^T dT' m_g^*(T') \frac{dm_g^*}{dT'} \int \frac{dk k^2}{\omega_g^*(k)} f^*(k)$$

tune T_s, λ, B_0, d

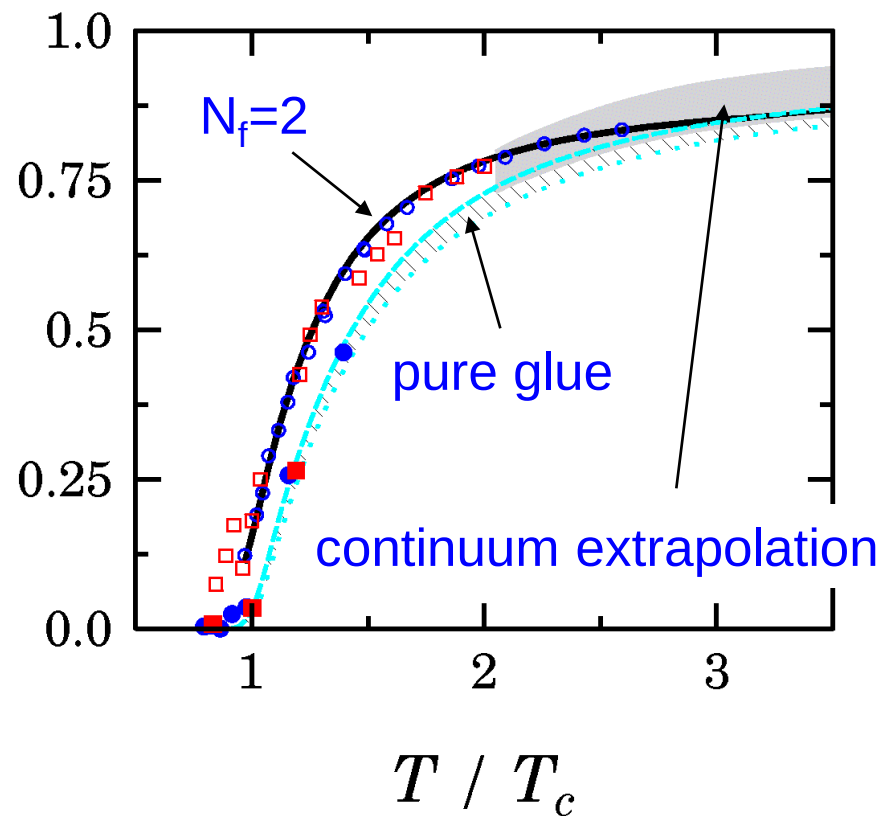
Peshier et al., PLB 337, 235 (1994)

gluons



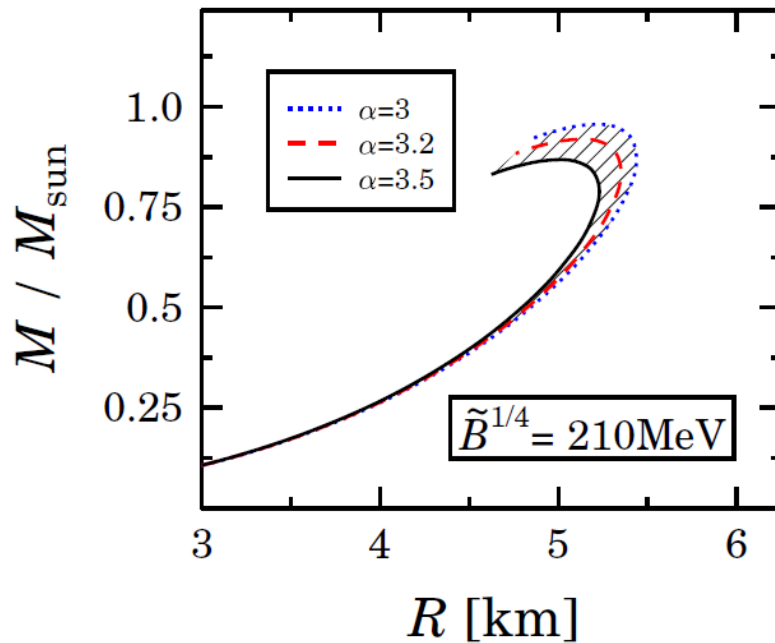
Peshier et al., PRD 66, 094003 (2002)

gluons + quarks



“rotation” to the μ axis

$$\frac{\partial s(T, \mu, m_i)}{\partial \mu} = \frac{\partial n(T, \mu, m_i)}{\partial T}$$



Quark stars are
 very small ($R < 6 \text{ km}$)
 and very light ($M < 1 \text{ Msol}$)

Matching between the realistic hadronic EoS and the quark EoS fitted to the
 lattice data is not always possible

[Ivanov et al. PRC72 (2005) 025804]

No hybrid stars are possible

Conclusion

Kaleidoscope of different facets of QCP which can be addressed in HIC and in space

On each facet, even if we do not use words “quarks” and “gluons” we deal with strong interaction, with QCD in dynamics.

NSs and HICs are the only sources of the information about properties of the strongly interacting matter under extreme condition.

They provide test for our theories and models in dynamical systems.

Problems in discussion:

Relativistic equation of state for high densities?

Inclusion of new particles (hyperons, Deltas)?

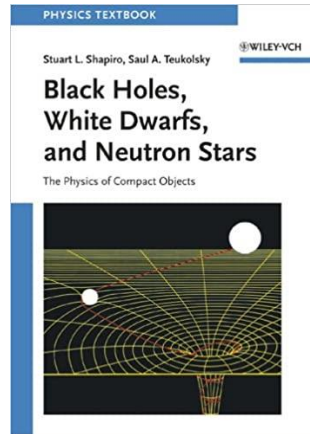
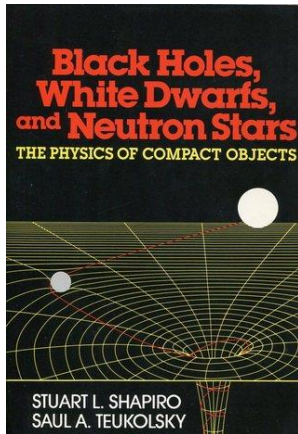
Hadrons in medium?

Quark-hadron phase transition?

We always have to expect unexpected!



Book of the Physics of Compact Stars



Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects

- Author(s): [Stuart L. Shapiro](#)
- [Saul A. Teukolsky](#)

First published: 27 July 1983

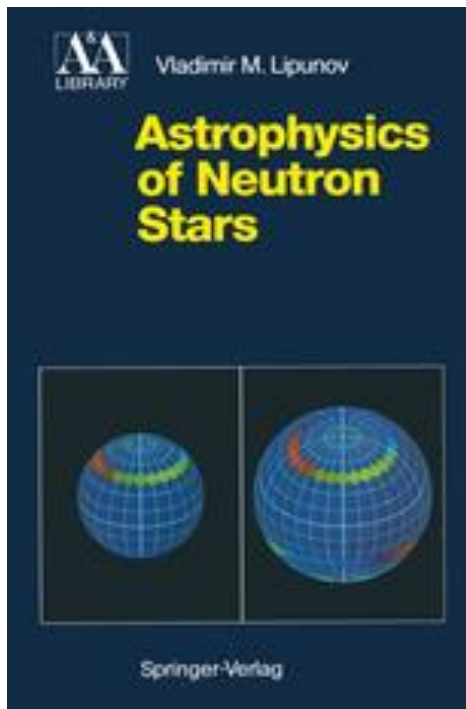
Print ISBN: 9780471873167 | Online ISBN: 9783527617661

| DOI: 10.1002/9783527617661

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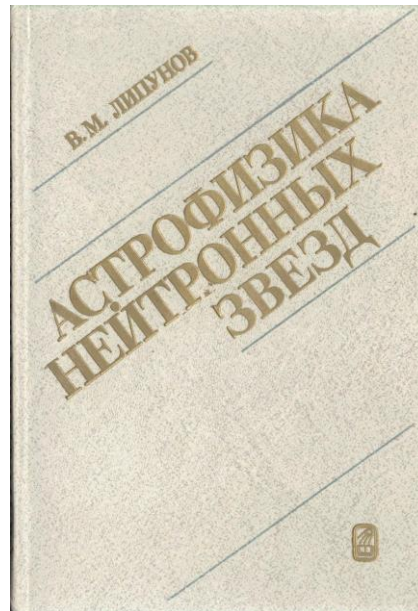
<http://alexandr4784.narod.ru/shcapt2.html>



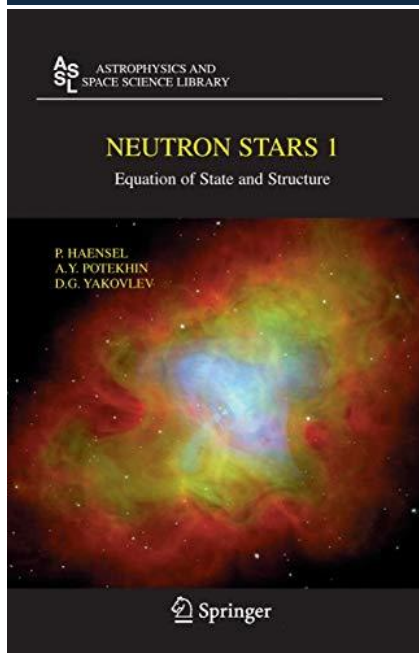
Astrophysics of Neutron Stars

Authors: **Lipunov**, Vladimir M.

<https://www.springer.com/gp/book/9783642763526>



<http://ind.pskgu.ru/ebooks/lipunov.html>

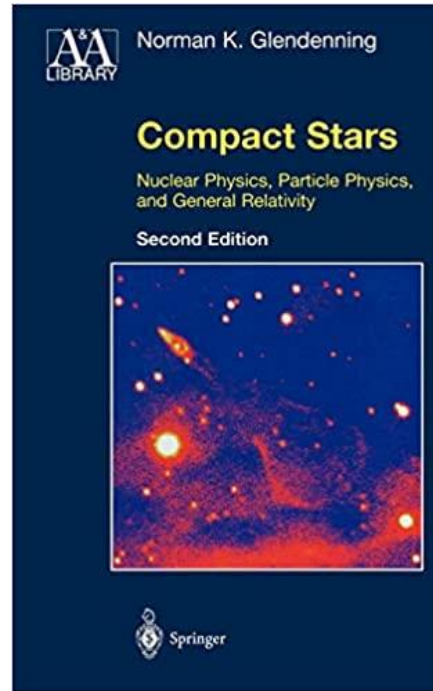
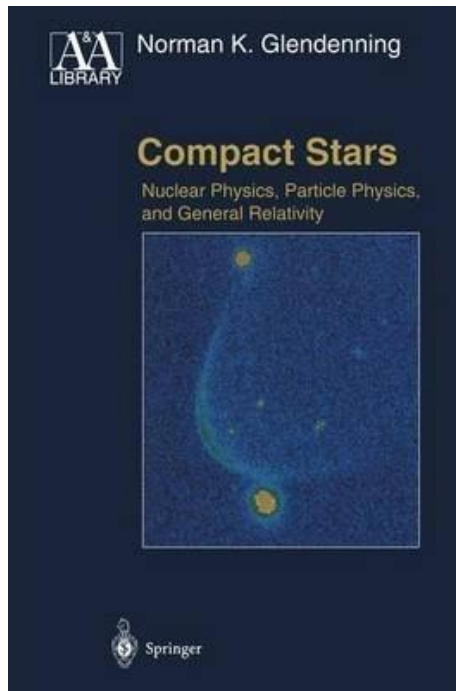


Neutron Stars 1

Equation of State and Structure

Authors: **Haensel**, P., **Potekhin**, A.Y., **Yakovlev**, D.G.

<https://www.springer.com/gp/book/9780387335438>



<https://www.amazon.com/Compact-Stars-Relativity-Astronomy-Astrophysics/dp/0387989773>