V.G.Kurakin, Lebedev Physical Institute, Moscow, Russia.

Beam Formation from Radionuclide Positron Source by the Charged Particles Beams Stochastic Optics Method

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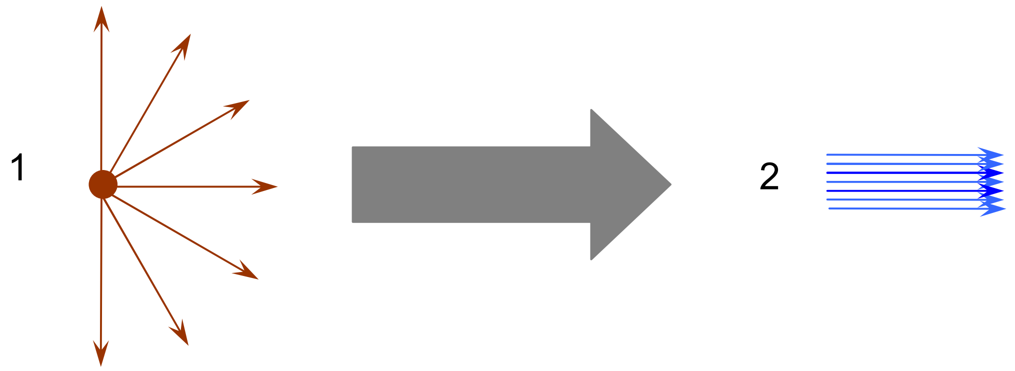
Nowadays positrons emitted by radionuclide positron sources are used for numerous purposes. The particles fly out within the whole 4pi solid angle that is not convenient for some applications and their further acceleration as well. The method of unidirectional beam formation from isotropic positron source is described and this one is based on the properties of charged particles beam partial reflection in the case of its incline incident on scattering medium.



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1. What is charged particles beams stochastic optics?

2. What happens when charged particles beam falls on incline plane border that separates vacuum and scattering medium?

3. The reflections lows in charged particles beams stochastic optics

4. Beam formation from isotropic charged particles source

When a beam of charged particles propagates in a medium, its transverse dimensions increase according to the three-second law due to multiple Coulomb scattering on the nuclei of the medium. Therefore, when the beam falls obliquely on a scattering medium with a flat boundary, the peripheral particles of the beam reach the interface as it moves and leave the medium. There is a partial reflection of the beam. In this talk, the regularities of such reflection are given followed by appropriate suggestion to use this phenomenon for beam formation from isotropic charged particles source.

Multiple Coulomb Scattering of charged particle in medium

$$P(x,y,\theta)dyd\theta = \frac{2\sqrt{3}}{\pi}\frac{1}{\Theta_s^2 x^2}\exp\left[-\frac{4}{\Theta_s^2 x}\left(\theta^2 - \frac{3y\theta}{x} + \frac{3y^2}{x^2}\right)\right]dyd\theta$$

Here $P(x,y,\theta)$ is the probability to find out the charge at depth x in intervals $(y,y+dy)$, $(\theta, \theta+d\theta)$ of transverse displacement and the angle between charge velocity and x axis. Quantity Θ_s may be expressed over the parameters that describe radiation processes in medium:

$$\Theta_s^2 = \left(\frac{E_s}{\beta cp}\right)^2 \frac{1}{X_0} \qquad E_s = \left(\frac{4\pi}{\alpha}\right)^{1/2} m_e c^2 = 21\ \text{MeV}$$

X_0 is radiation length

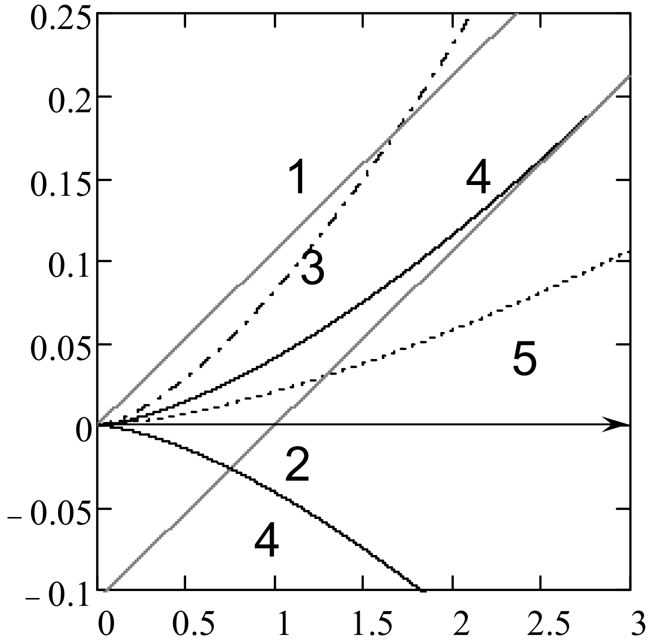
A probability to find out a beam scattered particle in transverse interval (0,y) is equal to

$$W(y) = \frac{1}{\sqrt{\pi}} \int_0^{\frac{\sqrt{3}y}{\Theta_s x^{3/2}}} \exp(-t^2)dt$$

This probability remains a constant along the line described by the expression

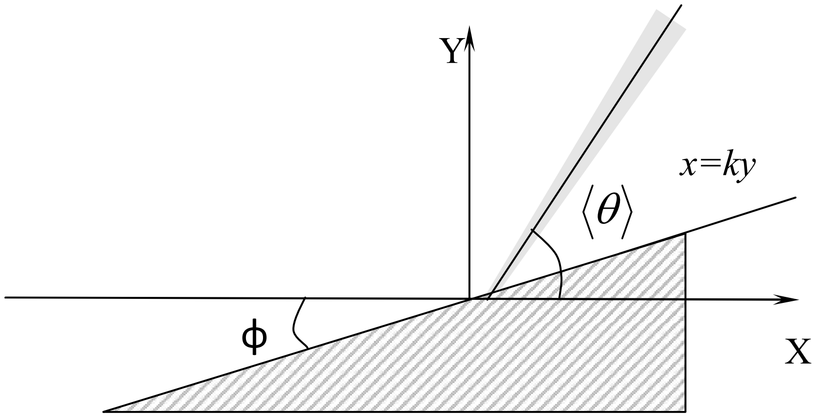
$$y = \frac{\kappa}{\sqrt{3}} \Theta_s x^{3/2}$$

This line is referred to as flow line



Flow lines in a scattering medium: (*1*) and (*2*) lines describe front and rear interfaces of the scattering medium $x = ky$ and $x = h + ky$, (*3*) flow line that corresponds to the total reflection, (*4*) flow line that is tangent to the interface (a symmetric line corresponding to negative transverse displacements is also shown), and (*5*) flow line that crosses the interface (ϴs2= 0.1 cm and h = 1 mm).

Incline incidence of charged particles beam on scattering medium



<θ> is the average scattering angle into vacuum.

Reflection angle

$$\chi = \langle\theta\rangle - \varphi = \frac{3}{2k} - \arctan\left(\frac{1}{k}\right) + \frac{\sqrt{3}}{\pi k}\int_0^\infty \exp\left[-\frac{4\xi^2 k^2}{3}\left(\arctan\frac{1}{k} - \frac{3}{2k}\right)^2\right]\frac{\exp(-\xi^2)}{\xi}\times$$

$$\frac{1-\exp\left[-\frac{4\pi^2 k^2 \xi^2}{3}\right]\exp\left[-\frac{4\pi k^2 \xi^2}{3}\left(\arctan\frac{1}{k} - \frac{3}{2k}\right)\right]}{erf\left[\frac{2k\xi}{\sqrt{3}}\left(\arctan\frac{1}{k} - \frac{3}{2k} + \pi\right)\right] + erf\left[-\frac{2k\xi}{\sqrt{3}}\left(\arctan\frac{1}{k} - \frac{3}{2k}\right)\right]}d\xi$$

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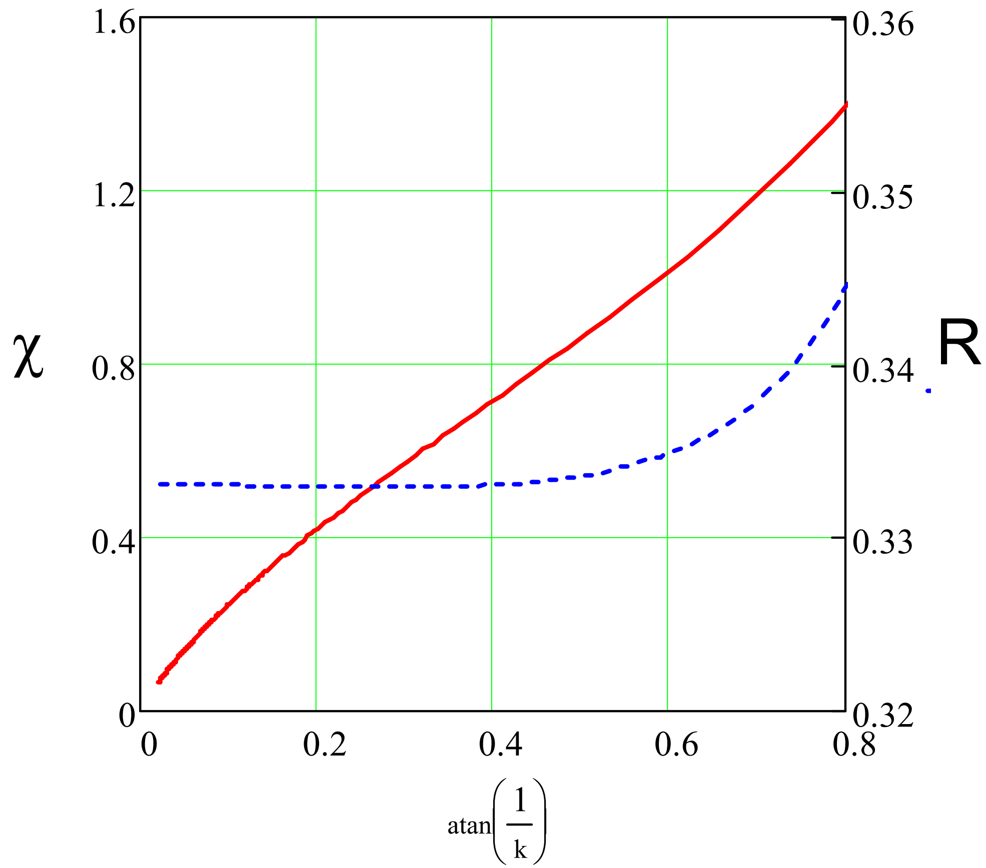

Reflection beam angle and beam reflection coefficient (doted line) versus incident beam angle. These are independent on scattering medium material and beam parameters

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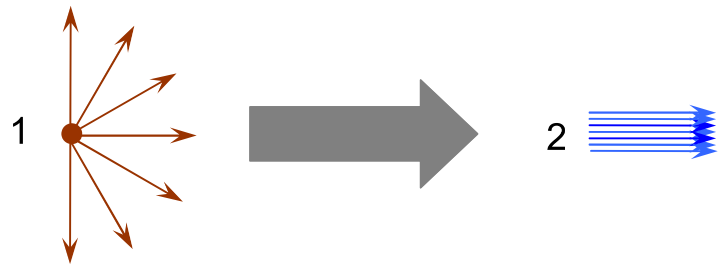
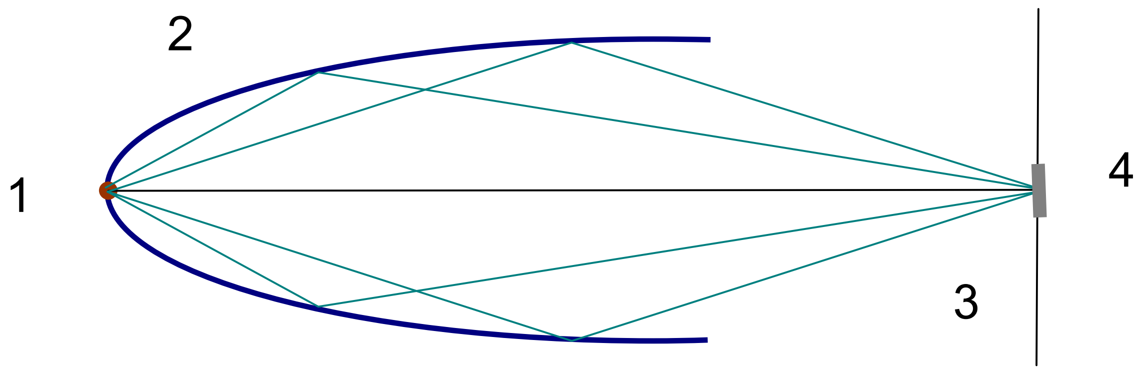


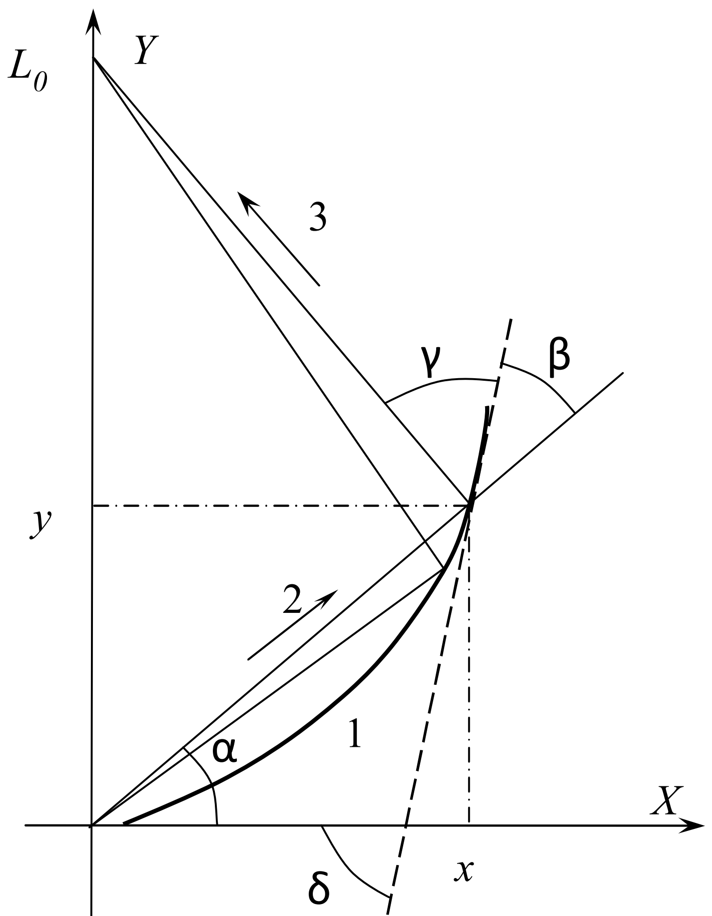
Beam formation from isotropic charged particles source

1 – isotropic source, 2 - beam

The schema suggested reminds a car headlight or searchlight, a point like positron source being located at its flat bottom center. The remaining part of this reflector is the element of surface of revolution with the axis that is perpendicular to the flat bottom and starts from point particles source.



1- isotropic charged particles source, 2 – reflector, 3 – focal plane, 4- a sample



The illustration of beam formation from the point charged particle isotropic source with reflecting surface of rotational symmetry. 1 – reflecting surface, 2 – the particle flow from point isotropic source, 3 – the reflected particles flow.

$$\boxed{\arctan\left(\frac{dy}{dx}\right) + \chi\left(\arctan\left(\frac{dy}{dx}\right) - \arctan\left(\frac{y}{x}\right)\right) = \frac{\pi}{2} + \arctan\left(\frac{x}{L_0 - y}\right)}$$

Any solution of this equation defines the profile of the rotational surface reflecting the charged particles from point source into focus point with ordinate L_0. Taking into account that nonlinear dependence $\chi(\beta)$ might be approximated by linear dependence $\chi=\lambda\beta$, λ being equal to 1.5 at least in incident angles range (0.2, 0.6), the above equation can be transformed into

$$\frac{dy}{dx} = \tan\left(\frac{\lambda}{1+\lambda}\arctan(y/x) + \frac{\pi}{2(1+\lambda)} + \frac{1}{1+\lambda}\arctan(x/(L_0 - y))\right)$$

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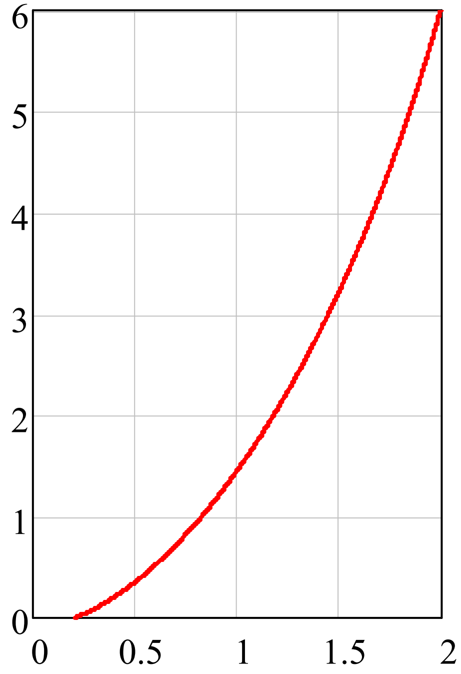

The shape of reflection surface that collects charged particles from the point source located at the point(0,0) into the focus point $L_0=20$ cm. The coordinate axis's are numbered in cm.

We define the collection efficiency as the ratio of the particle being reflected from reflection surface to all particles emitted to upper semi sphere in the Y-axis direction

$$\Delta = \int_{\vartheta,\varphi} nR(\vartheta)\sin\vartheta d\vartheta d\varphi \Big/ \int_0^{2\pi} d\varphi \int_0^{\pi/2} n\sin\vartheta d\vartheta$$

.

integration over the surface shown on previous slide gives

$$\Delta = 0.32$$

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Based on stochastic charged particles beams optics principles reflection lows are derived. One of them establishes the relation between the angles that form with a medium border the falling on medium and leaving it a charged particle beam. The second one is the formula for reflection coefficient. <u>Both reflection angle as well as reflection coefficient do not depend on beam energy and scattering medium material.</u> Based on these lows the scheme of beam formation from an isotropic source of charged particles is suggested. The scheme is based on the surface of rotational symmetry that collects the moving charged with velocities distribution in solid angle of 2π into a remote focus, the appropriate surface differential equation being derived and solved numerically.

Published papers on charged particles beams stochastic optics problems

Kurakin, V.G., Kurakin, P.V. Measuring the Energy of a Charged Particle Beam by Stochastic Electron Optics Methods. *Phys. Part. Nuclei Lett.* **17,** 578–580 (2020). https://doi.org/10.1134/S1547477120040287

Kurakin, V.G., Kurakin, P.V. Reflection and Refraction of a Charged Particle Beam in a Scattering Medium. *Tech. Phys.* **64,** 1749–1752 (2019). https://doi.org/10.1134/S1063784219120120

Kurakin, V.G., Kurakin, P.V. Phase Portrait of a Scattered Charged-Particle Bunch. *Tech. Phys.* **63,** 772–775 (2018). https://doi.org/10.1134/S1063784218050158

Kurakin, V.G., Kurakin, P.V. On the Theoretical Basics of Charged-Particle-Beam Stochastic Optics. *Phys. Part. Nuclei Lett.* **15,** 786–789 (2018). https://doi.org/10.1134/S1547477118070464

The end

Thank you!