Introduction to the Standard Model

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MAX-PLANCK-GESELLSCHAFT

MAX-PLANCK-INSTITUT
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HEIDELBERG

Selection of recommended books

- Peskin & Schröder – Quantum Field Theory
- Srednicki – Quantum Field Theory
- Sheng & Li – Gauge theory of elementary particle physics
- Schwartz – Quantum Field Theory and the Standard Model
- Weinberg – The Quantum Theory of Fields
- Griffith – Introduction to Elementary Particles
- Halzen & Martin – Quarks and Leptons

Feel free to ask me for other resources if you are in need.

Outline

I. Basic introduction and concepts
   - Symmetries
   - Quantum Electrodynamics (QED)
   - Spontaneous symmetry breaking

II. A model of leptons
   - Non-Abelian gauge theories
   - Electro-weak unification

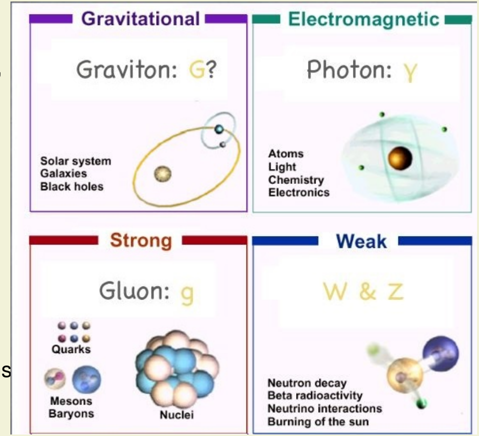
III. The three generation Standard Model
   - Flavor structure and CP violation
   - Puzzles and problems of the Standard Model

Introduction

Four fundamental forces

Gravitational
apples,
stellar systems,
galaxies,
the Universe,
black holes

Strong
nuclei,
baryons&mesons
α-decays,
confinement

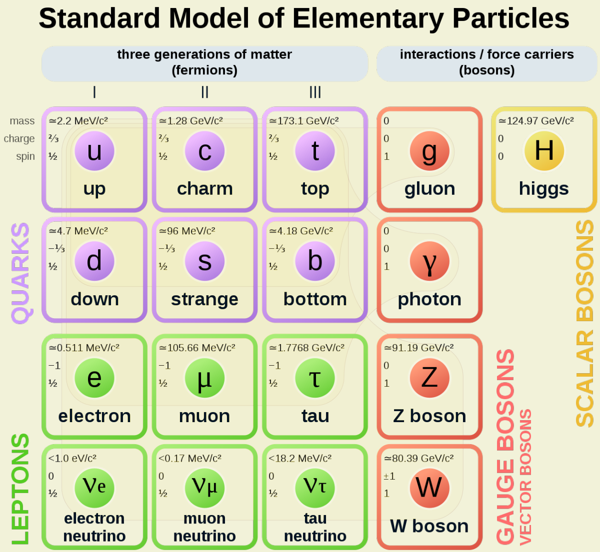


Electromagnetic
atoms, crystals,
tables, walls,
radio, x-ray,
sound, γ-decays

Weak
β-decays,
parity violation,
CP violation,
neutrino
interactions,
solar energy

Major achievement of the Standard Model (SM):
Electro-weak unification

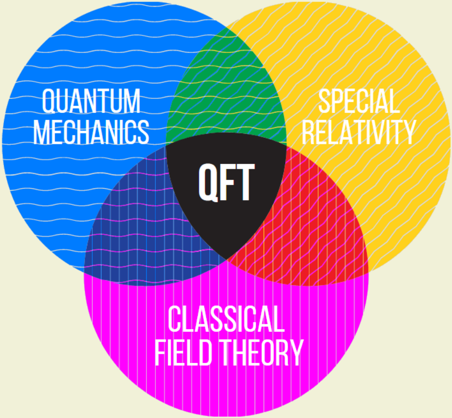
Fundamental matter



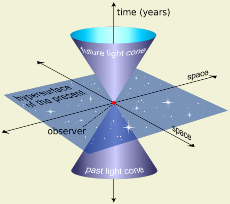
Fundamental concepts

The SM is a relativistic quantum field theory (QFT).

$|\psi\rangle$, $\mathcal{H}$
unitarity



$x^\mu = (t, \vec{x})$,
$\mathrm{SO}(1,3)$
causality



$\vec{E}$, $\vec{B}$, $\mathcal{L}$
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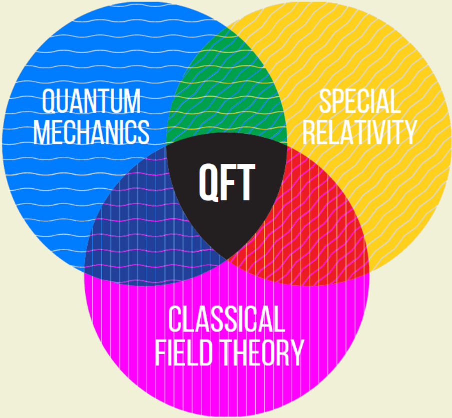

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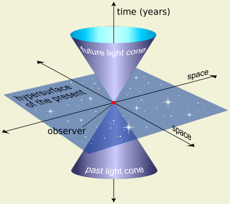

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Absolute key in the formulation: **Symmetries!**

Noether (1918):

Continuous symmetries $\Leftrightarrow$ conservation laws.

Symmetries and conservation laws

Symmetry / invariance		conservation law
time translation invariance	$\leftrightarrow$	conservation of energy
spatial translations	$\leftrightarrow$	conservation of momentum
rotational invariance	$\leftrightarrow$	angular momentum conservation
gauge invariance	$\leftrightarrow$	conservation of "charge"

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“Objects” are *representations* of symmetries.

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spin-1/2 "spinor" is $\mathbf{2}$-dim. representation of $\mathrm{SU}(2)$.

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SM gauge symmetry is $\mathrm{SU}(3)_{\mathrm{color}} \otimes \mathrm{SU}(2)_{\mathrm{left}} \otimes \mathrm{U}(1)_{\mathrm{hypercharge}}$

Objects: elementary quantum fields (particles); charged under these symmetries.

Poincaré / Lorentz group representations

Invariance under full Poincaré group of $3+1$-dim. space-time

- translations (space and time)
- rotations (space)
- boosts

$\left.\begin{array}{l}\text{rotations (space)}\\ \text{boosts}\end{array}\right\}$ Lorentz group $\mathrm{SO}(1,3)$
$\eta^{\mu\nu} = \pm\,\mathrm{diag}\,(+,-,-,-)$

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Skipping (a lot of interesting) technicalities, the representations of the Lorentz group can be classified as representations of

$$\mathfrak{sl}(1,3) \cong \mathfrak{sl}(2,\mathbb{C}) \cong \mathfrak{su}(2) \oplus \mathfrak{su}(2)$$

(this makes life very easy, because this behaves spimply like the well known "spin"$\times$"spin")

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Label of representation	common name	typical symbol
$(0,0)$	scalar" (invariant)	ϕ
$(1/2,0)$	"left-handed Weyl spinor"	ψ_L
$(0,1/2)$	"right-handed Weyl spinor"	ψ_R
$(1/2,1/2)$	"vector"	A_μ
$(1/2,0)\oplus(0,1/2)$	"Dirac spinor" (composite!)	Ψ

Important things to note

We work in natural units, i.e. set

$$\hbar = c = 1 \; .$$

Thus, dimensions work out as

$$[E] = [p] = [m] = \frac{1}{[L]} = \frac{1}{[t]} = \mathrm{GeV} \; .$$

To restore units use dimensional analysis and

$$\boxed{\hbar\, c \approx 200\,\mathrm{MeV}\,\mathrm{fm} \; .}$$

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Disclaimer for the whole lecture

Given the scope of this lecture we will to a large extent skip the subjects of:

- gauge-fixing / ghosts ,
- renormalization ,
- quantum (loop) corrections , *. . . and many more details. . .*

So be aware that our discussion will be largely **superficial**, noting that an accurate formulation of quantum field theory, renormalization, the treatment of spontaneously broken gauge symmetries, *etc.* requires much more care.

Dirac Theory / Quantum Electrodynamics (QED)

Quantum Electrodynamics (QED)

QED is the protoype of a quantum field theory (QFT).

For any (Q)FT (schematically):

Partition function $\mathcal{Z} = \int \mathcal{D}\phi \, \mathrm{e}^{\mathrm{i}S}$ with $S = \int \mathrm{d}^4x \, \mathcal{L}(\phi, \partial_\mu \phi)$.

Stationary action $S \Leftrightarrow$ equations of motion

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \phi\right)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \, .$$

(Euler-Lagrange equations)

Quantum Electrodynamics (QED)

Consider Dirac fermion $\Psi(x)$ charged under global symmetry.

Transformation under *global* $\mathrm{U}(1)$ ransformation:

$$\Psi(x) \mapsto \mathrm{e}^{\mathrm{i}q}\Psi(x)\ .$$

Symmetry and Lorentz invariant Lagrangian ($m \in \mathbb{R}$ w.l.o.g.):

$$\mathcal{L} = \mathrm{i}\overline{\Psi}\gamma^\mu\partial_\mu\Psi - m\,\overline{\Psi}\Psi\ .$$

Gamma matrices $\mathbb{C}^{4\times4}$, with $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$.
In "chiral" / "Weyl" basis:

$$\gamma^\mu := \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}\ ,$$

with $\sigma^\mu = (\mathbb{1}, \sigma^i),\ \overline{\sigma}^\mu = (\mathbb{1}, -\sigma^i)\ ,$
and Pauli matrices $\sigma^{i=1,2,3}$.

Definitions: $\overline{\Psi} := \Psi^\dagger\,\gamma^0, \quad \gamma^5 := \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$.

Conserved $\mathrm{U}(1) \Rightarrow$ conserved charge q, with conserved current

$$j^\mu(x) \ = \ \overline{\Psi}\gamma^\mu\Psi\ , \quad \text{with} \quad \partial_\mu j^\mu = 0\ .$$

Quantum Electrodynamics (QED)

QED is the theory of a Dirac fermion $\Psi(x)$ charged under *local* $\mathrm{U}(1)$.

Transformation under *local* $\mathrm{U}(1)$ gauge transformation:

$$\Psi(x) \mapsto \mathrm{e}^{\mathrm{i}\alpha(x)}\Psi(x) \qquad \text{and} \qquad A_\mu(x) \mapsto A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x)\ .$$

The corresponding gauge and Lorentz invariant Lagrangian:

$$\mathcal{L}_{\mathrm{QED}} = \mathrm{i}\overline{\Psi}\gamma^\mu D_\mu \Psi - m\,\overline{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\ .$$

Here:

$$D_\mu := \partial_\mu - \mathrm{i}eA_\mu(x) \qquad \text{and} \qquad F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu .$$

Quantum Electrodynamics (QED)

Euler-Lagrange equation of motion for A_μ

$$\partial_\mu F^{\mu\nu} = e j^\nu \; , \qquad \text{and} \qquad \partial_\mu \tilde{F}^{\mu\nu} \equiv \frac{1}{2}\partial_\mu \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = 0 \; .$$

(Maxwell's equations!)

Euler-Lagrange equation of motion for $\overline{\Psi}$:

$$\overline{\Psi}\left(\mathrm{i}\gamma^\mu \partial_\mu - m\right)\Psi = -e A_\mu \overline{\Psi}\gamma^\mu \Psi \; .$$

(Dirac equation)

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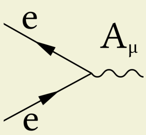

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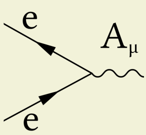

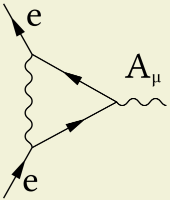

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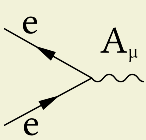

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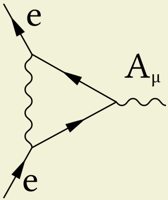

$$-\mathrm{i}e\overline{\Psi}\left(\gamma^\mu F_1(q^2) - \frac{[\gamma^\mu,\gamma^\nu]\,q_\nu F_2(q^2)}{4\,m}\right)\Psi$$

$$a_e = F_2(q^2\to 0) = \frac{\alpha}{2\pi} + \ldots$$

$$\vec{\omega}_s = g_e\frac{e}{2m}\vec{B} \equiv (a_e+1)\frac{e}{m}\vec{B}$$

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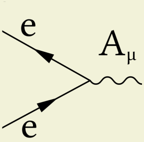

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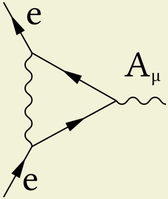

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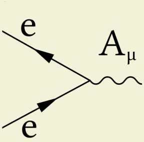

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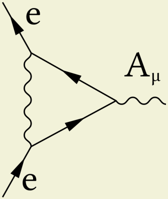

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$$a_e^{\mathrm{th}} = 1.159\,652\,182\,032(13)(12)(720) \times 10^{-3}$$
$$a_e^{\mathrm{exp}} = 1.159\,652\,180\,91(26) \times 10^{-3}$$

Quantum Electrodynamics (QED)

Recall: Dirac fermion is composite $(1/2,0)\oplus(0,1/2)$!

Let's expose this more clearly: $\Psi(x) := \begin{pmatrix}\psi_L(x)\\ \psi_R(x)\end{pmatrix}$.

$\curvearrowright$ can rewrite Dirac equation as two coupled equations:

$$\begin{aligned}\psi_R^\dagger\,\mathrm{i}\sigma^\mu\partial_\mu\,\psi_R &= m\,\psi_R^\dagger\psi_L\;,\\ \psi_L^\dagger\,\mathrm{i}\overline{\sigma}^\mu\partial_\mu\,\psi_L &= m\,\psi_L^\dagger\psi_R\;.\end{aligned}$$

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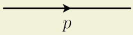
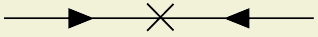
The independent equations for $\psi_{L,R}$ are coupled only by the *Dirac-mass* term



$$m\,\overline{\Psi}\Psi \;=\; m\,\psi_R^\dagger\psi_L + m\,\psi_L^\dagger\psi_R\;.$$



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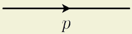

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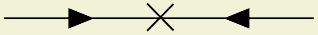
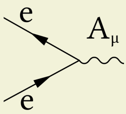


Including the *vector-like* electron-photon vertex



$$-eA_\mu\overline{\Psi}\gamma^\mu\Psi \;=\; -\,eA_\mu\left(\psi_R^\dagger\sigma^\mu\psi_R + \psi_L^\dagger\overline{\sigma}^\mu\psi_L\right)\ .$$

Note: $\mathrm{U}(1)$ acts *vector-like*($\equiv$same on left and right) on the fermions

$$\psi_{L,R}(x)\mapsto \mathrm{e}^{\mathrm{i}\alpha(x)}\psi_{L,R}(x)\ .$$

$\Rightarrow$ QED automatically conserves parity transformation "$\psi_L \leftrightarrow \psi_R$".

Quantum Electrodynamics (QED)

Nobel prize Feynman, Schwinger, Tomonaga 1965

Important general lessons

- Conservation of the (electric) current tied to global symmetry.

Gauge principle:

- Gauge invariance dictates interactions and (partially) the degrees of freedom.
- Gauge invariance strictly prohibits mass for A_μ (photon).
- Observables are gauge invariant.
- Gauge "symmetry" should be viewed as a redundancy: States related by gauge transformations are physically indistinguishable.
- (not shown): Gauge invariance holds up in the quantum theory ("anomaly freedom").

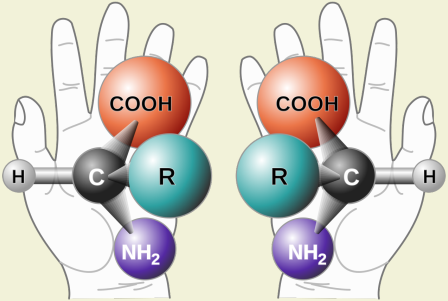
Spontaneous symmetry breaking

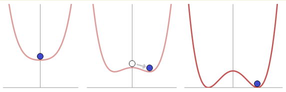
(in “scalar QED” or “Abelian Higgs model”)

Spontaneous symmetry breaking (SSB)

"SSB": System (Lagrangian) has a symmetry, but present state (typically ground state, or vacuum) is not symmetric.

A very common phenomenon in nature.





SSB for gauge theories is the worst name ever invented because symmetry is actually not broken but just *non-linearly* realized.

Example: Scalar QED

Consider complex scalar field charged under $\mathrm{U}(1)$ gauge

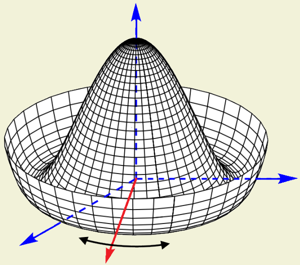
$$\phi(x) \mapsto \mathrm{e}^{\mathrm{i}\alpha(x)}\phi(x) \qquad \text{and} \qquad A_\mu(x) \mapsto A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x)\ .$$

Invariant Lagrangian density:

$$\mathcal{L}_{\mathrm{sQED}} = (D_\mu\phi)^*\,(D^\mu\phi) - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\ .$$

with potential $(\mu^2 > 0, \lambda > 0)$

$$V(\phi) = -\mu^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4\ .$$



Equations of motion have classical solution,

$$\langle 0|\phi(x)|0\rangle = const. =: v\ ,$$

"vacuum expectation value" (VEV)

that minimizes the potential energy.

Example: Scalar QED

Thus, it makes sense to expand theory around $\langle\phi(x)\rangle = v$:

$$\phi(x) \;=\; \mathrm{e}^{\mathrm{i}\,\xi(x)/v}\,(v+\sigma(x))\ .$$

Expanding the Lagrangian, there are terms

$$\mathcal{L}_{\mathrm{sQED}} \supset \dots + e^2\phi^*\phi A_\mu A^\mu + \mathrm{i}\,e\,\left[\phi^*\left(\partial_\mu\phi\right) - \phi\left(\partial_\mu\phi^*\right)\right]A^\mu + \dots$$
$$\rightarrow \quad \dots + e^2v^2A_\mu A^\mu + \dots$$

This is indicative of the fact that SSB generates a mass term for the gauge boson

$$m_A = \sqrt{2}e\,v\ .$$

We can pick a gauge to eliminate $\xi(x)$ from the theory. In this gauge (called "unitary gauge") the "wanna-be Goldstone" mode $\xi(x)$ becomes the *longitudinal* mode of the, then massive, gauge boson.

This is at the heart of the Brout-Englert-Higgs mechnanism.
You will show this in the ***exercises***.

Spontaneous symmetry breaking (SSB)

- For continuous global symmetries:

  Each spontaneously broken symmetry generator implies massless scalar degree of freedom (Nambu-Goldstone boson).

  Nobel prize Nambu 2008

- For gauge theories:

  SSB can happen but the Goldstone bosons are unphysical degrees of freedom.
  Via the so-called Brout-Englert-Higgs mechanism the "wanna-be" Goldstone bosons provide a consistent way to generate masses for gauge boson without spoiling gauge invariance.

  Nobel prize Englert and Higgs 2013

Non-Abelian gauge theories

Non-Abelian symmetry groups

$\mathrm{U}(1)$ is a group "generated" by charge q, $U \in \mathrm{U}(1)$ with $U = \mathrm{e}^{\mathrm{i}q\alpha}$.

Instead of exponentiating numbers, we can exponentiate matrices!

$$\text{generators} = \text{matrices} .$$

Example: $\mathrm{SU}(n)$. "Unitary": $U^\dagger U = \mathbb{1}$. "Special": $\det U = 1$.

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All matrices in $\mathrm{SU}(n)$ can be written as

$$U \;=\; e^{\mathrm{i}\mathrm{T}^a \alpha^a} \qquad (a = 1, \ldots, n^2 - 1) \;.$$

The fixed matrices $[\mathrm{T}^a]_{ij}$ are called *generators* and they *can* be chosen traceless and hermitean. (the dimensions of ij decide dimension of representation $\boldsymbol{r}$)

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For $n = 2$, well known $\mathrm{SU}(2)$ with $a = 1, 2, 3$ and the generators T^a being the Pauli matrices:

$$\mathrm{T}^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad \mathrm{T}^2 = \frac{1}{2}\begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} , \qquad \mathrm{T}^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

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In general, the generators do *not commute*≡*non-Abelian*:

$$\left[\mathrm{T}^a, \mathrm{T}^b\right] = \mathrm{i}\, f^{abc}\, \mathrm{T}^c\;.$$

f^{abc} are called the structure constants. For $\mathrm{SU}(2)$: $f^{abc} = \varepsilon^{abc}$.

Non-Abelian gauge theories – Yang-Mills theories

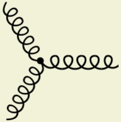
Exactly as in the case of $\mathrm{U}(1)$, we can also demand invariance under local $\mathrm{SU}(n)$:

$$\Psi(x) \mapsto U(x)\Psi(x) \equiv \mathrm{e}^{\mathrm{i}\mathrm{T}^a\alpha^a(x)}\Psi(x)\ .$$

The gauge covariant derivative then is given by

$$[D_\mu]_{ij} := \mathbb{1}_{ij}\partial_\mu - \mathrm{i}gA_\mu(x)^a\mathrm{T}^a_{ij}.$$

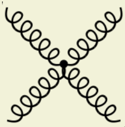
The field strength tensor then is given by



$$\mathrm{i}g\,F_{\mu\nu} := \mathrm{i}g\,F^a_{\mu\nu}\mathrm{T}^a := [D_\mu, D_\nu]\ ,$$

and spelled out

$$F^a_{\mu\nu} = \left[\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\,f^{\ a}_{bc}A^b_\mu A^c_\nu\right]\ .$$



The gauge-kinetic term is

$$\mathcal{L}_{\mathrm{YM}} \ = \ -\frac{1}{4}\,F^a_{\mu\nu}\,F^{\mu\nu,a}\ .$$

$\Rightarrow$ Non-Abelian gauge theories have self-interacting gauge bosons.

Nobel prize ’t Hooft and Veltman 1999
(for renormalization of spontaneously broken NA gauge theories)

A model of leptons: Electroweak unification

A model of Leptons

Consider now a theory of fermions, gauge bosons and scalars with gauge symmetry

$$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \qquad \text{“left”} \times \text{“hypercharge”}\,.$$

Fields:

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix},\ e_R,\ H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}\ .$$

	L	e_R	H
Lorentz	$(1/2,0)$	$(0,1/2)$	$(0,0)$
$\boldsymbol{r}$ $[\mathrm{SU}(2)_\mathrm{L}]$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$
q_Y $[\mathrm{U}(1)_\mathrm{Y}]$	$-1/2$	-1	$1/2$

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Note: All fermions we introduce are chiral Weyl spinors, but we use a trick to write them in terms of Dirac fermions

$$\Psi_L := P_L\Psi = \frac{1}{2}(\mathbb{1}-\gamma^5)\Psi = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} .$$

Crucial: *chiral* charge assignment for fermions

$\Leftrightarrow$ P violation in SM is “*explicit and maximal*”, (not spontaneous!)

Note: *chiral* charge assigments $\Rightarrow$ **no** possible fermion mass terms

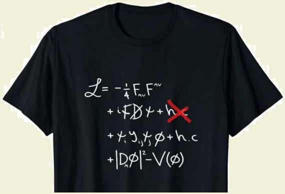
$$\psi_R^\dagger\psi_L\ , \quad \psi_L^\dagger\psi_R\ , \quad \psi_L\,\psi_L\ , \quad \psi_R\,\psi_R\ .$$

A model of Leptons

Electroweak Lagrangian:

$$\mathcal{L}_{\mathrm{EW}} = \mathcal{L}_{\mathrm{kinetic}} + \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{Yukawa}} \; .$$

A model of Leptons



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with

$$\begin{aligned}
\mathcal{L}_{\text{kinetic}} &= \mathrm{i}\,\overline{L}\gamma^\mu D_\mu L + \mathrm{i}\,\overline{e}_R\gamma^\mu D_\mu e_R + (D_\mu H)^\dagger (D^\mu H)\ , \\
\mathcal{L}_{\text{gauge}} &= -\frac{1}{4}\,B_{\mu\nu}\,B^{\mu\nu} - \frac{1}{4}\,W^a_{\mu\nu}\,W^{\mu\nu,a}\ , \\
\mathcal{L}_{\text{Higgs}} &= \mu^2\,H^\dagger H - \frac{\lambda}{2}\left(H^\dagger H\right)^2\ , \qquad (\mu^2 > 0, \lambda > 0) \\
\mathcal{L}_{\text{Yukawa}} &= -\,y_e\,\overline{L}\,H\,e_R + \text{h.c.}\ , \qquad (y_e \in \mathbb{R}, w.l.o.g.)
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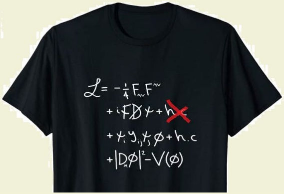

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\end{aligned}$$

Gauge covariant derivatives (depends on charges of resp. field)

$$D_\mu = \left(\partial_\mu - \mathrm{i}g' q_{\mathrm{Y}} B_\mu\right)\mathbb{1} - \mathrm{i}g\mathrm{T}^a W^a_\mu\;.$$

Potential is set up for spontaneous symmetry breaking.

A model of Leptons

$$H \;=\; \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_2+\mathrm{i}\,\varphi_3\\ \varphi_0+\mathrm{i}\,\varphi_1\end{pmatrix} \;=\; \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\,\mathrm{T}^a\,\xi^a(x)}\begin{pmatrix}0\\ h(x)\end{pmatrix}$$

$$V(H) \;=\; -\frac{\mu^2}{2}\left(\varphi_0^2+\varphi_1^2+\varphi_2^2+\varphi_3^2\right)+\frac{\lambda}{8}\left(\varphi_0^2+\varphi_1^2+\varphi_2^2+\varphi_3^2\right)^2 \;.$$

Note: Higgs potential is accidentially $\mathrm{SU}(2)\times\mathrm{SU}(2)\cong\mathrm{SO}(4)$ symmetric (custodial symmetry).

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Any VEV $\langle H\rangle$ with strength $|\langle H\rangle| = v/\sqrt{2} = \mu/\sqrt{\lambda} \neq 0$ breaks

$$\begin{array}{ccc}\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y} & \xrightarrow{\langle H\rangle} & \mathrm{U}(1)_\mathrm{em}\;,\\ \mathrm{T}^3 \;+\; Y & = & Q\;.\end{array}$$

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We can use $\mathrm{SU}(2)$ gauge rotation to absorb the Goldstone bosons $\xi^{a=1,2,3}(x)$("unitary gauge") to make H look like

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \ .$$

A model of Leptons

Taking into account the VEV, mass eigenstates arise as

$$W_\mu^\pm \;=\; \frac{1}{\sqrt{2}}\left(W_\mu^1 \pm \mathrm{i}\, W_\mu^2\right)\;, \qquad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}\;,$$

where

$$s_W \equiv \sin(\theta_W) = \frac{g'}{\sqrt{g^2 + g'^2}}\;, \quad c_W \equiv \cos(\theta_W) = \frac{g}{\sqrt{g^2 + g'^2}}\;.$$

The VEV induces physical gauge boson masses

$$m_{W^\pm} \;=\; \frac{g\,v}{2}\;, \quad m_Z = \frac{g\,v}{2\,c_W}\;, \quad m_A = 0\;.$$

Other often used relations

$$e = \frac{g\,g'}{\sqrt{g^2 + g'^2}}\;, \quad \frac{g^2}{8\,m_W^2} = \frac{G_\mathrm{F}}{\sqrt{2}}\;.$$

Nobel price Glashow, Salam, Weinberg 1979

Nobel price Rubbia and van der Meer 1984

A model of Leptons

After SSB the Lagrangian contains gauge boson couplings

$$A_\mu \times e\,Q\,\left(\overline{e}_L\gamma^\mu e_L + \overline{e}_R\gamma^\mu e_R\right) \qquad \equiv\; e\,A_\mu\,j^\mu_{\mathrm{em}}\;,$$

with $Q := \mathrm{ev}(\mathrm{T}^3) + Y\;.$

$$Z_\mu \times \frac{g}{c_W}\left(g_L\,\overline{e}_L\gamma^\mu e_L + g_R\,\overline{e}_R\gamma^\mu e_R\right) \qquad \equiv\; \frac{g}{c_W}\,Z_\mu\,j^\mu_{\mathrm{n.c.}}\;,$$

with $g_L := \mathrm{ev}(\mathrm{T}^3) - Q\,s_W^2\;, \qquad g_R := Q\,s_W^2\;,$

$$W^+_\mu \times \frac{g}{\sqrt{2}}\left(\overline{\nu}_L\gamma^\mu e_L\right) + \mathrm{h.c.} \qquad \equiv\; \frac{g}{\sqrt{2}}\,W^+_\mu\,j^\mu_{\mathrm{c.c.}} + \mathrm{h.c.}\;.$$

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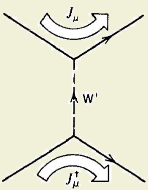
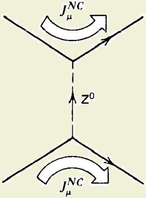
Historically important: for $q^2 \ll m^2_{W,Z}$





$$\rho := \frac{2\,\mathcal{M}_{\mathrm{c.c.}} \times j^\mu_{\mathrm{n.c.}} j_\mu^{\mathrm{n.c.}}}{\mathcal{M}_{\mathrm{n.c.}} \times j^\mu_{\mathrm{c.c.}} j_\mu^{\mathrm{c.c.}}}$$

$$= \frac{2\,\frac{g^2}{2}\,\frac{1}{m_W^2}}{\frac{g^2}{c_W^2}\,\frac{1}{m_Z^2}} = \frac{m_Z^2\,c_W^2}{m_W^2} = 1 \quad \mathbf{!}$$

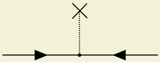
This is a consequence of custodial symmetry, i.e. the specific breaking of EWSB by $H = \mathbf{2}_{1/2}$.

A model of Leptons

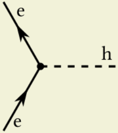
Fermion (Dirac) masses after SSB.

$$-\mathcal{L}_{\text{Yukawa}} \;=\; y_e\,\overline{L}\,H\,e_R + \text{h.c.} \;=$$

$$\xrightarrow{\text{unitary gauge}} \frac{y_e}{\sqrt{2}}\,\begin{pmatrix}\overline{\nu}_L & \overline{e}_L\end{pmatrix}\begin{pmatrix}0\\ v+h\end{pmatrix} e_R + \text{h.c.} =$$



$$\frac{y_e\,v}{\sqrt{2}}\,e_L^\dagger e_R + \frac{y_e}{\sqrt{2}}\,h\,e_L^\dagger e_R + \text{h.c.}$$



The VEV induces a Dirac mass for the charged lepton, and Higgs boson couplings proportional to mass/VEV

$$m_e \;=\; \frac{y_e\,v}{\sqrt{2}}\;, \qquad g_{he^+e^-} \;=\; \frac{y_e}{\sqrt{2}} \;=\; \frac{m_e}{v}\;.$$

Adding quarks

Quarks as chiral fermions, in fundamental representation $\mathbf{3}$ of $\mathrm{SU}(3)$, the gauge group of Quantum Chromodynamics (QCD).
Full gauge symmetry:

$$\mathrm{SU}(3)_{\mathrm{c(olor)}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}\ .$$

Quark fields:

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix},\ u_R,\ d_R\ .$$

	Q	u_R	d_R
Lorentz	$(1/2,0)$	$(0,1/2)$	$(0,1/2)$
$\boldsymbol{r}$ $[\mathrm{SU}(3)_{\mathrm{c}}]$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
$\boldsymbol{r}$ $[\mathrm{SU}(2)_{\mathrm{L}}]$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
q_{Y} $[\mathrm{U}(1)_{\mathrm{Y}}]$	$1/6$	$2/3$	$-1/3$

$$-\mathcal{L}_{\mathrm{Yuk.}} \;=\; y_u\, \overline{Q}_L^{\alpha}\, \tilde{H}\, u_R^{\alpha} + y_d\, \overline{Q}_L^{\alpha}\, H\, d_R^{\alpha} + \mathrm{h.c.}$$

(w.l.o.g. $y_u, y_d \in \mathbb{R}$)

$\tilde{H} := \mathrm{i}\sigma^2 H^*$ ($\tilde{H}$ transforms as $\mathbf{2}^* = \mathbf{2}$ of $\mathrm{SU}(2)_{\mathrm{L}}$ but has opposite hypercharge $-1/2$).

$\mathrm{SU}(3)$ is unbroken, but becomes strongly coupled at low energies to confine quarks into baryons and mesons $\rightarrow$ **QCD lecture**.

Nobel prize Gross, Politzer, Wilczek 2004
(for discovery of asymptotic freedom in QCD)

Parity violation in Nature

Weak interaction violates parity (by construction).

$$W_\mu^+ \times \frac{g}{\sqrt{2}} \left(\overline{\nu}_L P_L \gamma^\mu e_L\right) + \text{h.c.} \qquad P_L = \frac{1}{2}(\mathbb{1} - \gamma^5)$$

This describes experimental fact of "$(V - A)$" weak interactions.

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(\overline{u}\gamma^\mu(\mathbb{1} - \gamma^5)d\right) \times \left(\overline{\nu}\gamma_\mu(\mathbb{1} - \gamma^5)e\right)$$

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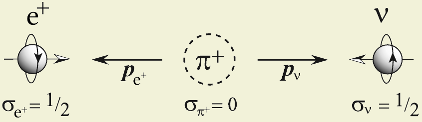
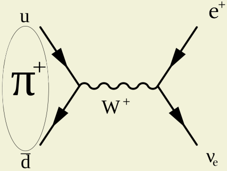
For example: 1) $\pi^+(= |u\overline{d}\rangle) \longrightarrow e^+ + \nu_e$





Parity violation in Nature

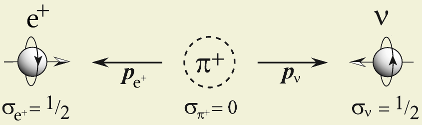
Weak interaction violates parity (by construction).

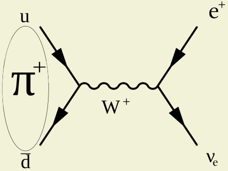
$$W_\mu^+ \times \frac{g}{\sqrt{2}} \left(\overline{\nu}_L P_L \gamma^\mu e_L\right) + \text{h.c.} \qquad P_L = \frac{1}{2}(\mathbb{1} - \gamma^5)$$

This describes experimental fact of "$(V - A)$" weak interactions.

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(\overline{u}\gamma^\mu(\mathbb{1} - \gamma^5)d\right) \times \left(\overline{\nu}\gamma_\mu(\mathbb{1} - \gamma^5)e\right)$$

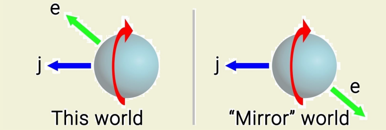
For example: 1) $\pi^+(= |u\overline{d}\rangle) \longrightarrow e^+ + \nu_e$





2) Famous '56
Wu experiment

$${}^{60}_{27}\text{Co} \rightarrow {}^{60}_{28}\text{Ni}^* + e^- + \overline{\nu}_e$$

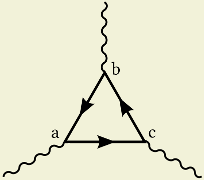


Nobel prize Lee and Yang 1957

Anomaly freedom of the SM

A symmetry of the classical action S is not necessarily a symmetry of the full quantum theory. Quantum corrections may not obey the symmetry.

$$\mathcal{Z} \xmapsto{\text{Sym.}} \int \mathcal{D}\Psi\mathcal{D}\overline{\Psi}\ \mathrm{e}^{\mathrm{i}\int\mathcal{A}}\ \mathrm{e}^{\mathrm{i}S[\Psi,\overline{\Psi}]}\ .$$



"Anomaly coefficient" $\mathcal{A}$

$$\mathcal{A} \propto \sum_{\text{chiral fermions}} \mathrm{tr}\left(\mathrm{T}^a\left\{\mathrm{T}^b,\mathrm{T}^c\right\}\right)\ .$$

- For gauge symmetries: Anomalies ***must*** cancel for consistency.
- For global symmetries: a very different issue. Anomalies are important for phenomenology, e.g. $\pi^0 \to \gamma\gamma$.
- Anomalies are always associated with chiral Fermions. Anomaly freedom automatic for symmetries that act vector-like (e.g. QED, QCD).
- SM: Anomalies cancel *within each generation*. **(exercises)**

$(\mathrm{SU}(3)\times\dots)$, $(\mathrm{SU}(2)\times\dots)$, $\mathrm{SU}(3)^3$, $\mathrm{SU}(2)^3$, $\mathrm{U}(1)\times\mathrm{SU}(2)^2$, $\mathrm{U}(1)\times\mathrm{SU}(3)^2$, $\mathrm{U}(1)^3$.

This unveils a very delicate balance of charges in the SM whose origin we do not understand.

The three generation Standard Model

Three generations

Empirically it is a fact that matter fermions come in three copies which are identical representations under all symmetries.

"Who ordered that?" (I.I. Rabi 1936)

Modelling: simply $(Q_L, u_R, d_R, L, e_L)_{\boldsymbol{i}}$ with $i = 1, 2, 3$.

Three "generations" or "families" of different "flavors".

$$-\mathscr{L}_{\text{Yuk.}} = \overline{Q}^i \widetilde{H}\, y_u^{ij}\, u_{\text{R}}^j + \overline{Q}^i H\, y_d^{ij}\, d_{\text{R}}^j + \overline{L}^i H\, y_e^{ij}\, e_{\text{R}}^j + \text{h.c.}\,,$$

$$y_u,\; y_d,\; y_e \;\in\; \mathbb{C}^{3\times 3}\,.$$

Many new parameters and possibility of physical complex couplings!

Naming scheme of the mass eigenstates:

Quarks			Leptons		
u(p)	c(harm)	t(op)	$\boldsymbol{\nu_1}$	$\boldsymbol{\nu_2}$	$\boldsymbol{\nu_3}$
d(own)	s(trange)	b(ottom)	$\boldsymbol{e}$	$\boldsymbol{\mu}$	$\boldsymbol{\tau}$

Three generations

Use “bi-unitary diagonalization” (singular value decomposition) for general matrices:

$$y_f \;=\; V_L^f\,\lambda_f\,V_R^{f\dagger}\;, \quad \text{where} \quad \lambda_f \;=\; \mathrm{diag}(\lambda_{f,i},\dots) \in \mathbb{R}\;.$$

This allows us to diagonalize the mass (and Higgs coupling) terms by a basis change of the fermion fields in flavor space.

$$\begin{aligned} u'_L &= V_L^{u\dagger}\,u_L\,, \qquad & d'_L &= V_L^{d\dagger}\,d_L\,, \qquad & L' &= V_L^{e\dagger}\,L\,,\\ u'_R &= V_R^{u\dagger}\,u_R\,, \qquad & d'_R &= V_R^{d\dagger}\,d_R\,, \qquad & e'_R &= V_R^{e\dagger}\,e_\mathrm{R}\,. \end{aligned}$$

This diagonalizes the mass and Higgs-coupling terms, but:

$$\frac{g}{\sqrt{2}}W_\mu^+\left(\overline{u}_L\,\gamma^\mu\,d_L\right)+\mathrm{h.c.} \;=\; \frac{g}{\sqrt{2}}W_\mu^+\left(\overline{u}'_L\left[V_L^{u\dagger}V_L^d\right]\gamma^\mu\;d'_L\right)+\mathrm{h.c.}\,.$$

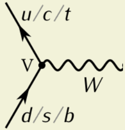
Note: The Higgs and Z couplings are flavor diagonal.

$\Rightarrow$ There are no “Flavor changing neutral currents” (FCNC’s) in the Standard Model (at tree level!).

Three generations

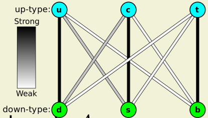
The ***flavor changing*** transitions between different generations of left-chiral fermions at the W-vertex

$$\frac{g}{\sqrt{2}} W_\mu^+ \left(\overline{u}_L \left[V_L^{u\dagger} V_L^d \right] \gamma^\mu \ d_L \right) + \text{h.c.}\, .$$



We define the unitary Cabbibo-Kobayashi-Maskawa matrix

$$V_{\text{CKM}} \ := \ V_L^{u\dagger} V_L^d \ .$$



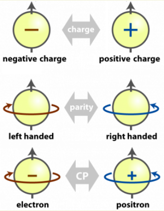
V_{CKM} can be parametrized by 4 parameters (**exercises**).
"Standard": 3 angles$+1$ *complex* phase; Wolfenstein: 4 $\mathcal{O}(1)$ parameters $\lambda,\ A,\ \rho,\ \eta$.

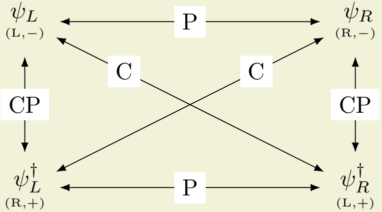
$$V_{\text{CKM}} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{td} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - \mathrm{i}\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-\mathrm{i}\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Having *physical* complex phases requires *at least* three generations and implies the explicit violation of CP (matter anti-matter asymmetry).


Nobel prize Kobayashi and Maskawa 2008

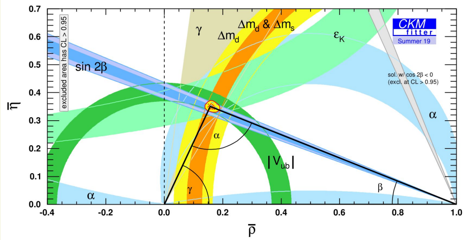

CP Violation in the SM





Experimentally, CP violation is well established in K, B and D mesons.

Nobel prize Cronin and Fitch 1980



For example:

$$\mathrm{BR}(B^+ \to D^0 K^+) \neq$$
$$\mathrm{BR}(B^- \to \overline{D}^0 K^-)\ .$$

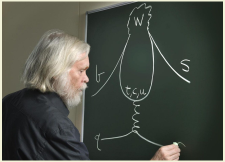
... *and many more*

Quark flavor physics is precision science now!

Ongoing: Search for CP violation in lepton sector.

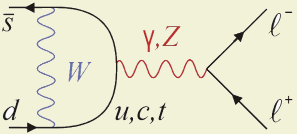
FCNC's in the SM

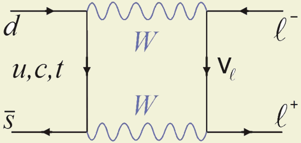
In the SM there are no Flavor Changing Neutral Currents (FCNC's) at tree level.



However, via loops neutral flavor change $d \to d - type$ or $u \to u - type$ can happen.

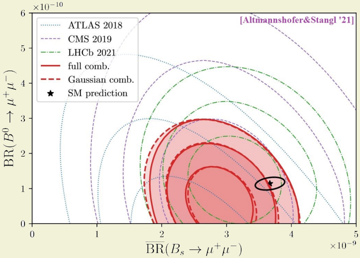
For example: $K_{\text{long}} = |d\bar{s}\rangle \to \ell^+\ell^-$ (and many others).





Hence, such processes are naturally very suppressed and therefore offer crucial tests of the SM.

Most recently:



John Ellis picture credit Claudia Marcelloni, CERN

CP Violation in the strong interactions?

$\mathrm{SU}(3)_c$ gauge interactions allow for the presence of a so-called θ-term

$$\mathscr{L}_\theta \;=\; \theta\,\frac{g^2}{32\pi^2}\,G^a_{\mu\nu}\,\widetilde{G}^{\mu\nu,a}\;.$$

This term is P and T odd, hence, also CP violating.
The corresponding CP-odd basis invariant is

$$\overline{\theta} \;:=\; \theta \;+\; \arg\det y_u\,y_d\;.$$

Thus, next to the Jarlskog invariant as source of CP violation from weak interactions (CKM), also strong interactions can violate CP. Unlike CP violation from weak interactions, which practically *always* comes with flavor violation, this type of CP violation has nothing to do with flavor changes.

Strong CP violation induces an electric dipole moment of the neutron

$$d_n \approx \left(1.5\times 10^{-16}\,e\cdot\mathrm{cm}\right)\overline{\theta}\;.$$

The non-observation of d_n implies $\overline{\theta} \lesssim 10^{-10}$.

We don't understand why $\overline{\theta}$ should be so small and this is called the strong CP problem.

Total parameter count of the SM

"Classical" SM without neutrinos: 19

- Gauge sector: g, g', g_3 ,
- Higgs sector: v, λ ,
- Flavor sector:
  - Masses: y_u, y_c, y_t; y_d, y_s, y_b; y_e, y_μ, y_τ,
  - Mixings: θ^q_{12}, θ^q_{13}, θ^q_{23}, (in Wolfenstein parametrization λ, A, ρ)
  - Phases: δ_{CKM}, (in Wolfenstein parametrization η)
  - Exotic: $\overline{\theta}_{\mathrm{QCD}}$.

Including neutrinos with Dirac (Majorana) mass terms: $+7(9)$

- Neutrino flavor sector:
  - Masses: m_1, m_2, m_3;
  - Mixings: θ^ℓ_{12}, θ^ℓ_{13}, θ^ℓ_{23},
  - Phases: δ_{PMNS}, (+ϕ_1,ϕ_2 for Majorana neutrinos)

Global (accidental?) symmetries of the SM

There are also global symmetries in the SM that (unlike gauge 'symmetries') relate physically distinguishable states.

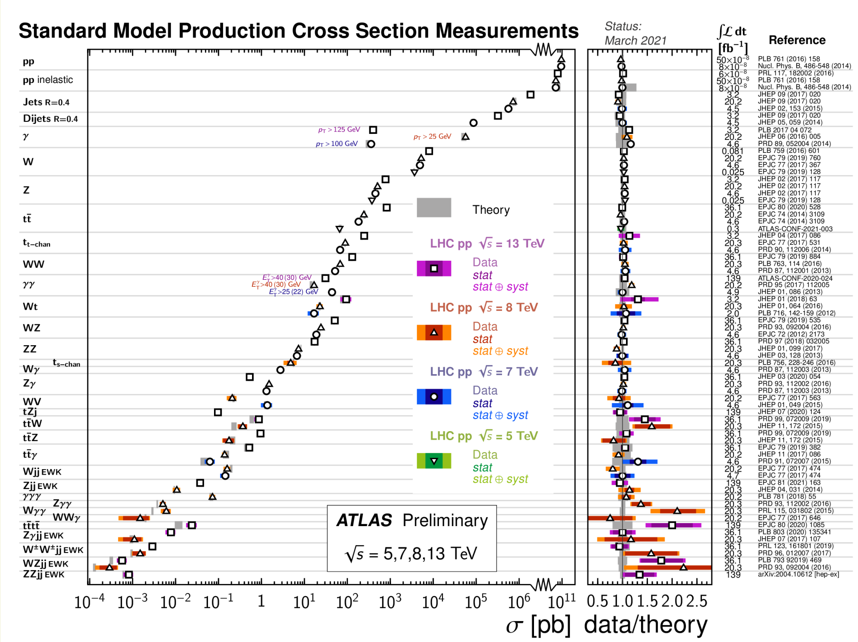
These global symmetries are "accidential". Either because they arise in the slipstream of gauge symmetries, or because parameters are "such and such".

- Custodial symmetry: $\mathrm{SO}(4) \xrightarrow{\langle H \rangle} \mathrm{SO}(3) \Rightarrow \rho = 1$ (at tree level). Broken by g'.
- Nuclear isospin $\mathrm{SU}(2)$: $n \leftrightarrow p$, because $m_u \approx m_d$.
- Chiral symmetry of QCD $\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}$ (explicitly(2x) and spontaneously broken) $\rightarrow$ QCD lecture.

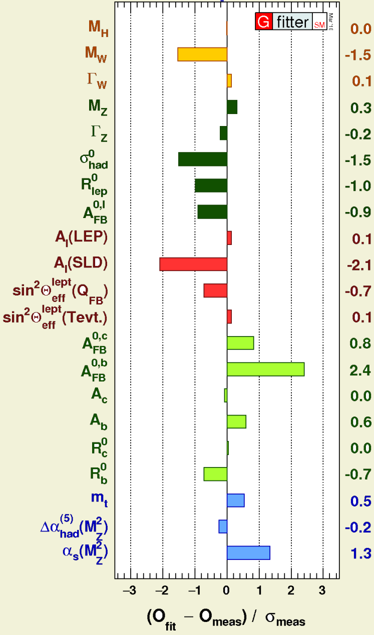
Nobel prize Gell-Mann 1969

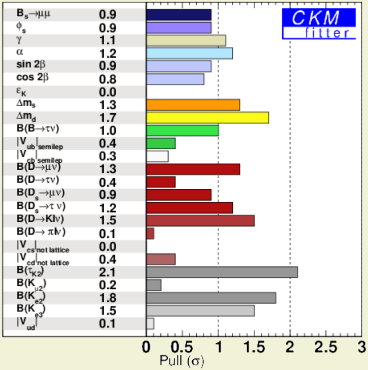
- Lepton family numbers $\mathrm{U}(1)_e$, $\mathrm{U}(1)_\mu$, $\mathrm{U}(1)_\tau$. Broken by neutrino masses $m_\nu \Rightarrow$ CLF (e.g.$\mu \rightarrow e\gamma$) heavily suppressed.
- Baryon(B)- and Lepton(L)-number symmetries $\mathrm{U}(1)_\mathrm{B}$&$\mathrm{U}(1)_\mathrm{L}$. Broken by chiral anomaly with $\mathrm{SU}(2)_\mathrm{L}$. $\mathrm{U}(1)_{\mathrm{B}-\mathrm{L}}$ is conserved in presence of 3 right-handed neutrinos.

The experimental success of the SM



The experimental success of the SM





SM is highly predictive and *shockingly* successfull.

Really, no deviations?

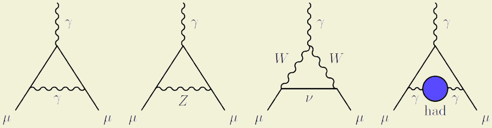
There will *always* be statistical fluctuations→deviations. . .

. . . significance is important!

For example: Long-standing $(3-4)\sigma$ deviation in muon anomalous magnetic moment $a_\mu \equiv \frac{1}{2}(g_\mu - 2)$.

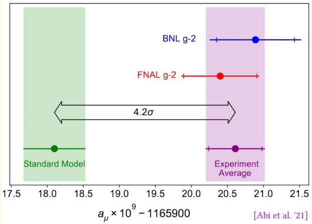
$$\frac{\delta a_\ell}{a_\ell} \sim \frac{m_\ell^2}{M_{\rm NP}^2}$$

This is an *indirect* test of *the whole* SM (and NP).



Discrepancy between SM theory prediction and measurement:

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 251(59) \times 10^{-11}\,.$$



Other increasinly significant deviations: Tests of “Lepton Flavor Universality” (LFU) (at LHCb/Belle(II)/Babar) $B \to K\mu\overline{\mu}/B \to Ke\overline{e}$, $B \to D\tau\overline{\nu}/B \to D\ell\overline{\nu}$, . . .

Ongoing tests of Higgs properties

Recall: Higgs couplings to fermions

$$g_{hff} = \frac{m_f}{v} \propto m_f \ .$$

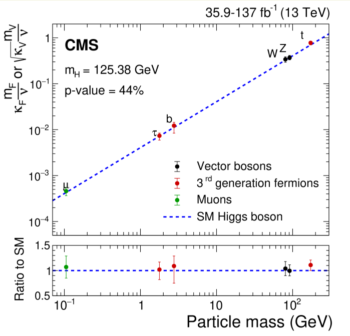
Higgs self-interactions, induced by SSB

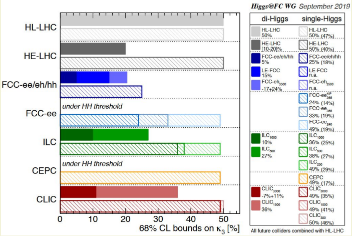
$$V(h) \ = \ \frac{m_h^2}{2}h^2 + \frac{\lambda}{2}vh^3 + \frac{\lambda}{8}h^4 + const.$$

$$v = \frac{\mu}{\lambda} \ , \quad m_h = 2\lambda v^2 \ .$$

The cubic and quartic self-couplings of the Higgs boson are predicted in the SM.

$$\kappa_3 := \lambda_{3h}/\lambda_{3h}^{\rm SM}$$





RG Evolution to high scales

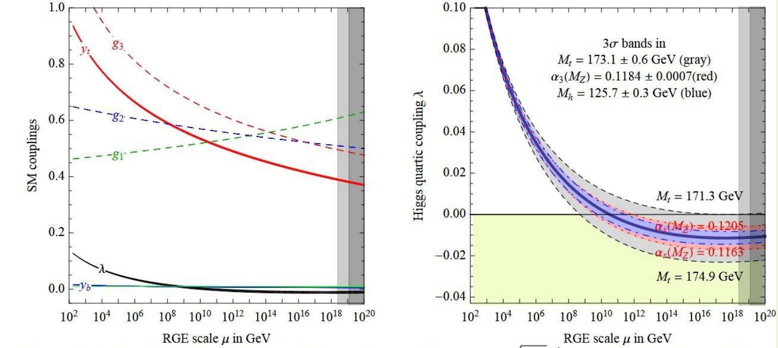
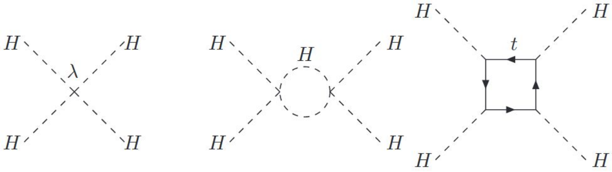
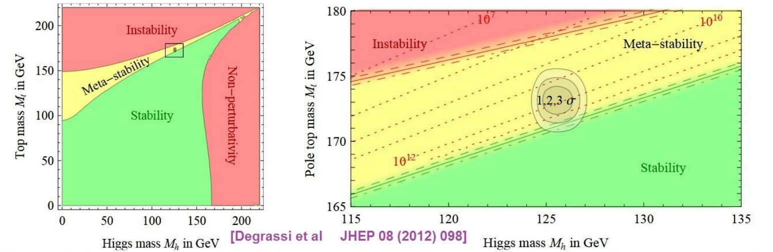


Figure 1: **Left**: *SM RG evolution of the gauge couplings* $g_1 = \sqrt{5/3}g'$, $g_2 = g$, $g_3 = g_s$, *of the top and bottom Yukawa couplings* (y_t, y_b), *and of the Higgs quartic coupling* λ. *All couplings are defined in the* $\overline{\text{MS}}$ *scheme. The thickness indicates the* $\pm 1\sigma$ *uncertainty.* **Right**: *RG evolution of* λ *varying* M_t, M_h *and* α_s *by* $\pm 3\sigma$.

[Degrassi et al JHEP 08 (2012) 098]



(Near?) Criticality of the SM



[Degrassi et al JHEP 08 (2012) 098]

Figure 5: *Regions of absolute stability, meta-stability and instability of the SM vacuum in the M_t–M_h plane (upper left) and in the λ–y_t plane, in terms of parameter renormalized at the Planck scale (upper right).* **Bottom***: Zoom in the region of the preferred experimental range of M_h and M_t (the gray areas denote the allowed region at 1, 2, and 3σ). The three boundary lines correspond to $\alpha_s(M_Z) = 0.1184 \pm 0.0007$, and the grading of the colors indicates the size of the theoretical error. The dotted contour-lines show the instability scale Λ in GeV assuming $\alpha_s(M_Z) = 0.1184$.*

Puzzles and problems of the Standard Model

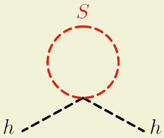
- Neutrino masses. $m_\nu = 0$ in the SM. $\Rightarrow$ SM ***must*** be extended!

  > Nobel prize Kajita and McDonald 2015
  > ("for discovery of neutrino oscillations, which shows that neutrinos have mass")

  Empirically $\sum m_\nu \lesssim 0.1\,\mathrm{eV}$. Easily remedied, e.g. Dirac/Majorana mass for ν:

  $$\mathcal{L}_{m_\nu} = y_\nu\, \overline{L}\, \widetilde{H}\, \nu_R\ , \quad \text{or} \quad \mathcal{L}_{m_\nu} = \frac{1}{\Lambda}\,(LH)^{\mathrm{T}}\,(LH)\ .$$

  Exact mechanism for m_ν still unclear. $\longrightarrow$ see **neutrino lectures**
- Strong CP problem ("Why no CP violation in strong interactions?").
- Flavor puzzle ("Why hierachical masses and mixings? Why three generations?").
- Charge quantization ("Why is Hydrogen neutral?").
- Vacuum stability? $\mathrm{U}(1)$ Landau pole at high scale?
- Electroweak hierarchy problem ("why $m_h \ll M_{\mathrm{Pl}}$").
- Baryon asymmetry of the Universe?
- What is Dark Matter?
- Computation of vacuum energies? (what is Dark Energy?)
- Unification with gravity?



Beyond the Standard Model

- Grand Unified Theories (GUTS)

$$\mathrm{SO}(10)\ /\ \mathrm{SU}(5)\ \supset\ \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\ .$$

$$\mathbf{16}=\mathbf{10}\oplus\overline{\mathbf{5}}\oplus\mathbf{1}=(\mathbf{3},\mathbf{2})_{1/6}\oplus(\overline{\mathbf{3}},\mathbf{1})_{-2/3}\oplus(\mathbf{1},\mathbf{1})_{1}\oplus(\overline{\mathbf{3}},\mathbf{1})_{1/3}\oplus(\mathbf{1},\mathbf{2})_{-1/2}\oplus(\mathbf{1},\mathbf{1})_{0}$$

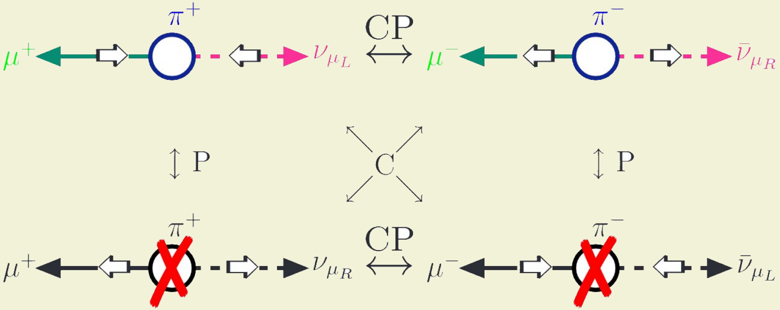
  - Would explain charge quantization and unification of gauge couplings!
  - Predicts proton decay (not observed. . .).
  - Unification of Higgs representations? 'Doublet-triplet splitting'.

- Supersymmetry $\quad\text{Bosons}\leftrightarrow\text{Fermions}$.

- . . . .

Thank You!

Backup slides



[Image credit: Fred Jegerlehner]

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