(High-energy) Astrophysical Neutrinos

Lecture 3

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IFIC, CSIC-U. València

JINR-ISU Baikal Summer School 2021

Some recommended bibliography
(in chronological order)

T. K. Gaisser, *Cosmic rays and particle physics*, 1990, Cambridge University Press

V. S. Berezinskiĭ et al., *Astrophysics of cosmic rays*, 1990, Elsevier

M. Kachelrieß, *Lecture notes on high-energy cosmic rays*, arXiv:0801:4376 [astro-ph]

J. K. Becker, *High-energy neutrinos in the context of multimessenger astrophysics*, Phys. Rept. 458:173, 2008

M. S. Longair, *High energy astrophysics*, 2011, Cambridge University Press

L. A. Anchordoqui et al., *Cosmic neutrino Pevatrons: a brand new pathway to astronomy, astrophysics, and particle physics*, JHEAp 1:1, 2014

M. Ahlers and F. Halzen, *IceCube: neutrinos and multimessenger astronomy*, Prog. Theor. Exp. Phys. 12A105, 2017

M. Spurio, *Probes of multimessenger astrophysics*, 2018, Springer

Plan of lectures

- I Historical remarks and general comments from a multi-messenger perspective
- II Flavor and multi-messenger relations
- III Cosmic acceleration, energetics and sources

Bonus: new physics searches with HE astrophysical neutrinos

What if the background radiation at the source is very dense?

A photon background can be generated by synchrotron emission from accelerated electrons or it can be constituted by thermal photons

Sources can be opaque to high-energy gamma rays, which would produce $e^{\pm}$ pairs and start a cascade process

pair production

$$\gamma + \gamma_b \to e^+ + e^-$$

$$s = 4\,E_\gamma\,\varepsilon_\gamma \geq 4\,m_e^2 \longrightarrow E_\gamma \geq 260\ \text{GeV}\left(\frac{\text{eV}}{\varepsilon_\gamma}\right)$$

These $e^{\pm}$ pairs can upscatter background radiation via inverse Compton

inverse Compton

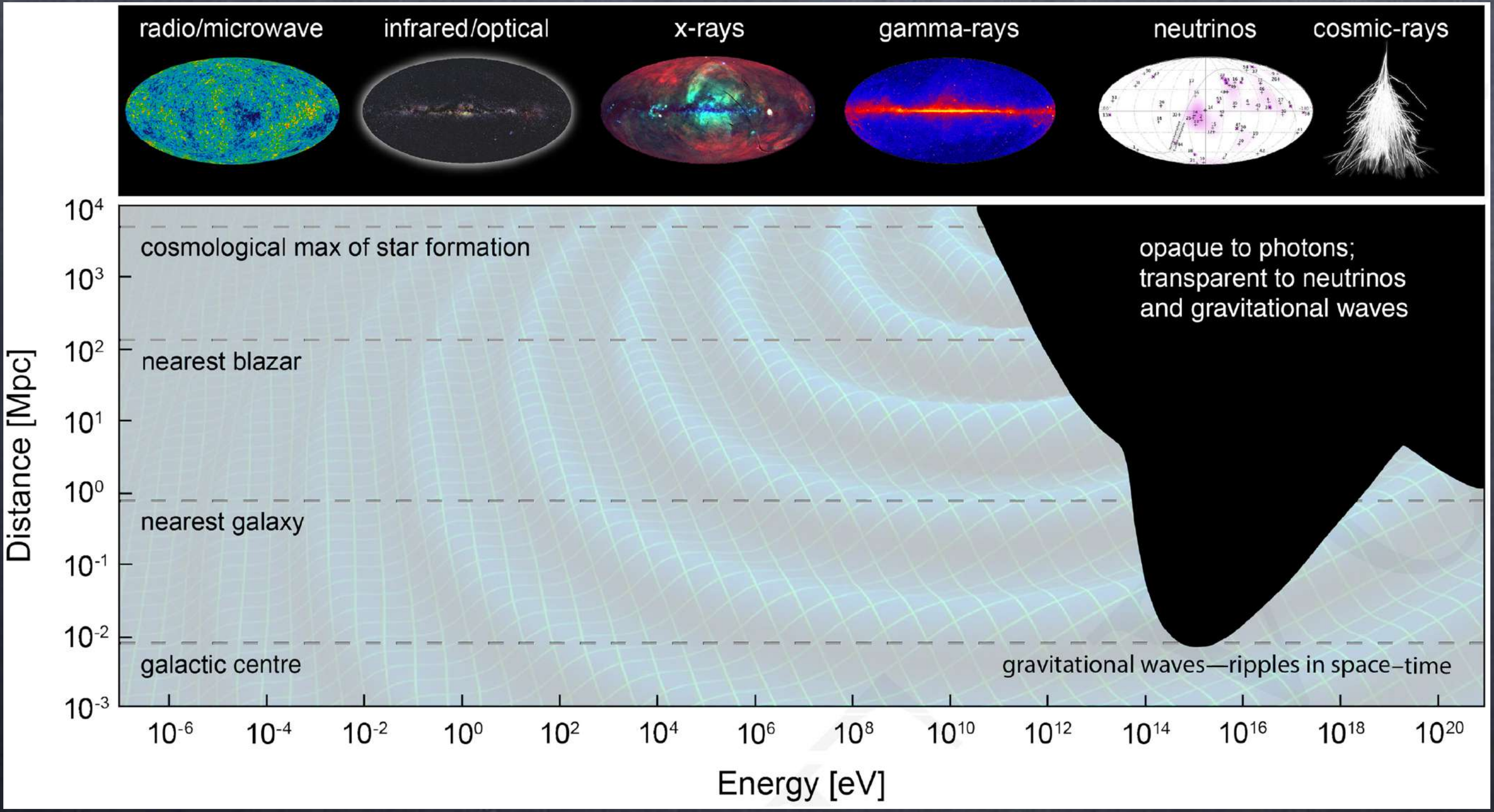
$$e^{\pm} + \gamma_b \to e^{\pm} + \gamma$$

Thompson regime

$$E_\gamma \simeq \begin{cases} \frac{4}{3}\gamma^2\,\varepsilon_\gamma \simeq 50\ \text{GeV}\left(\frac{\varepsilon_\gamma}{1\ \text{eV}}\right)\left(\frac{E_e}{100\ \text{GeV}}\right)^2 & ; \quad E_e < m_e^2/\varepsilon_\gamma \quad (E_\gamma < E_e) \\ \frac{E_e}{2} & ; \quad E_e > m_e^2/\varepsilon_\gamma \end{cases}$$

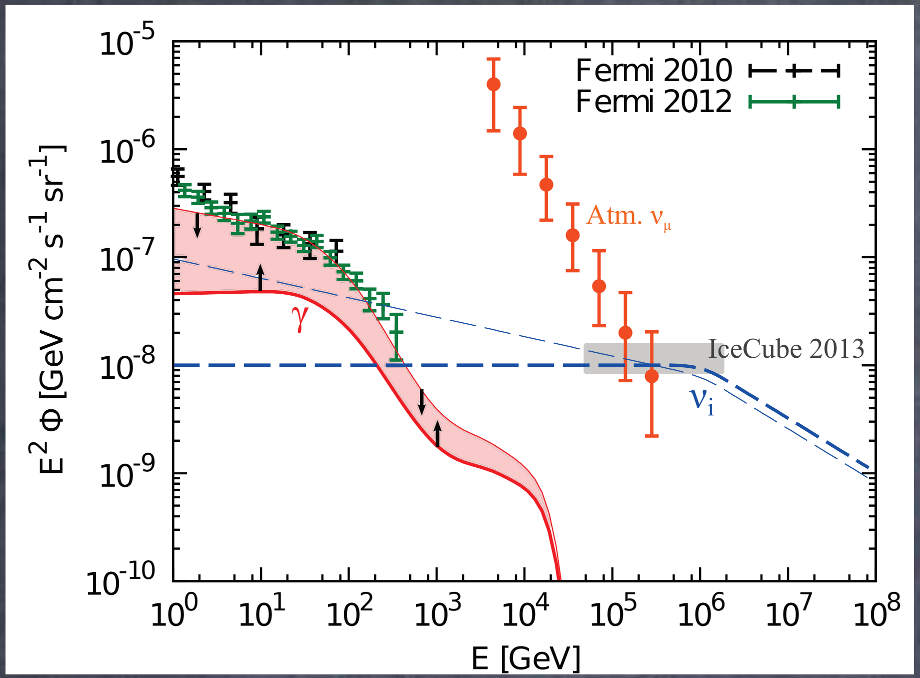
Klein-Nishina regime

When propagating on diffuse photon backgrounds (from CMB to IR), high-energy gamma rays would produce $e^{\pm}$ pairs, which would produce further gamma rays via inverse Compton onto background photons, until energies fall below ~100 GeV



I. Bartos and M. Kowalski, 2017

Constraints on the diffuse neutrino spectrum from gamma-ray cascades



K. Murase, M. Ahlers and B. C. Lacki, Phys. Rev. D88:121301, 2013

To avoid overshooting cascade limit

$$\phi_\nu \propto E_\nu^{-\beta} \quad ; \quad \beta < 2.2$$

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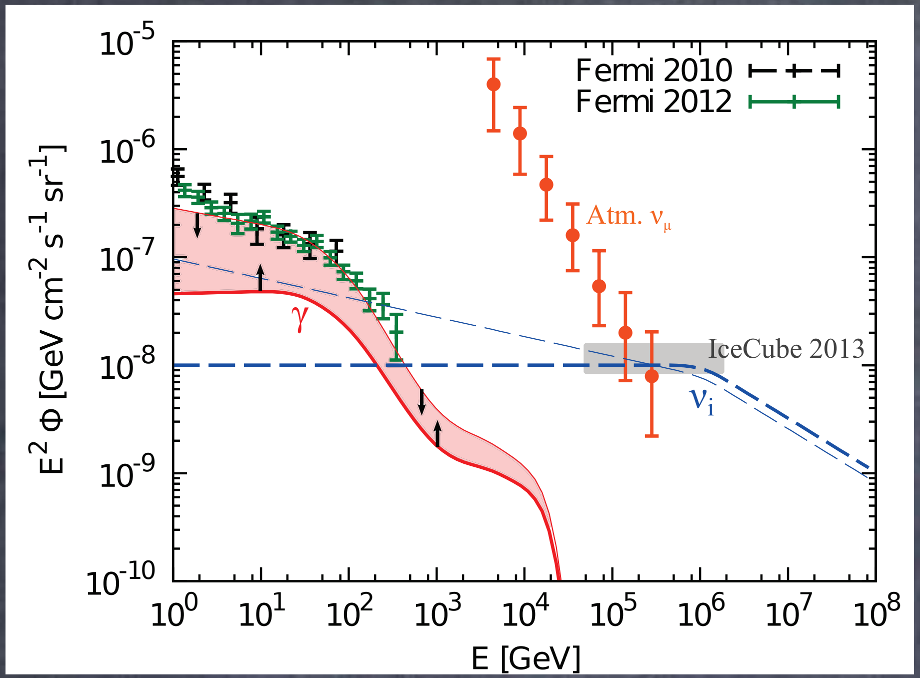

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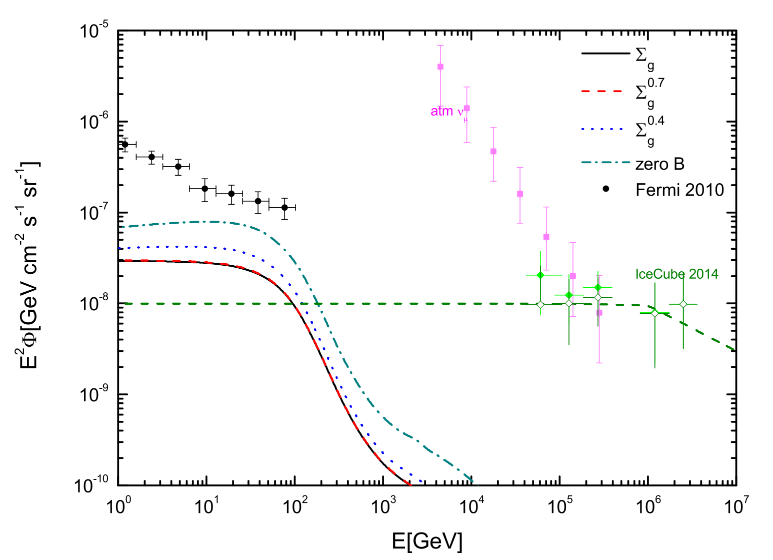

X.-C. Chang and X.-Y. Wang, Astrophys. J. 793:131, 2014

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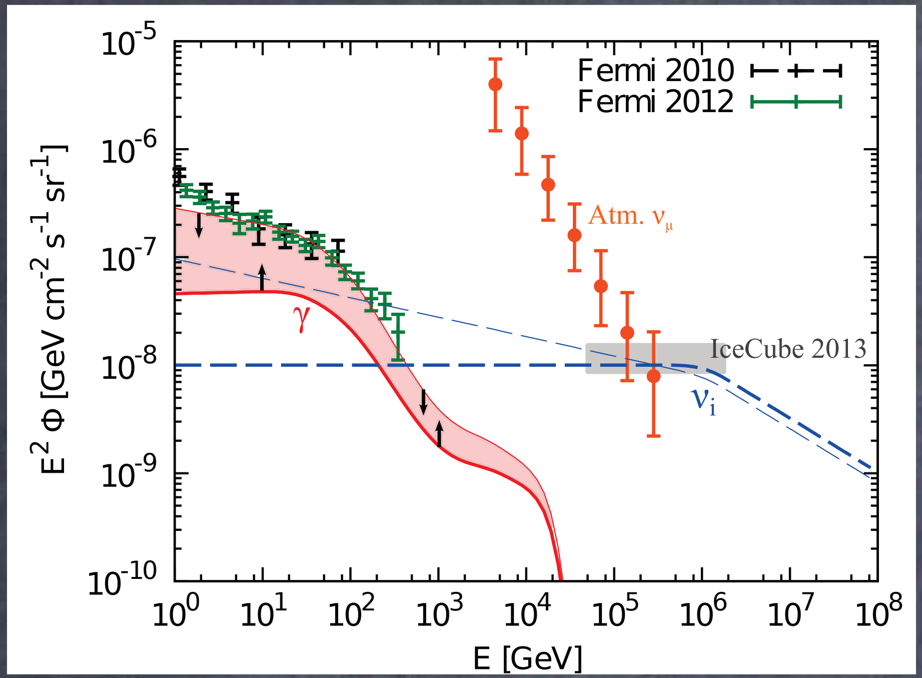

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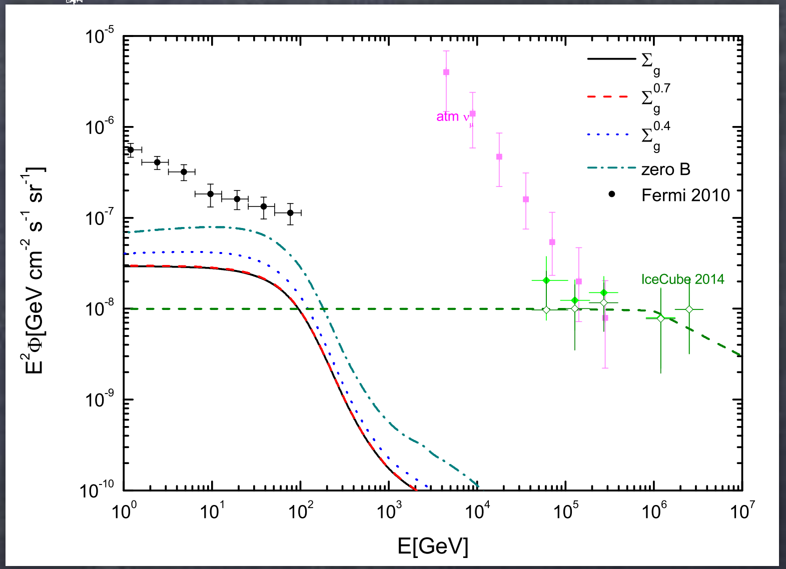
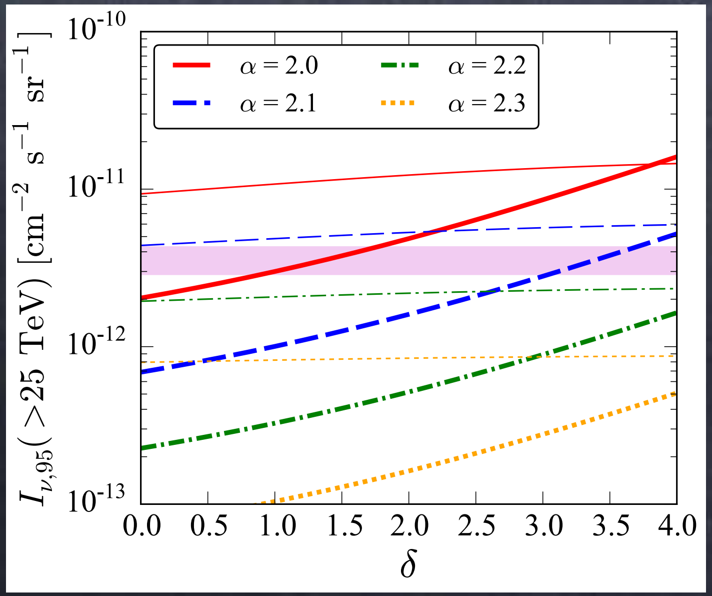


X.-C. Chang and X.-Y. Wang, Astrophys. J. 793:131, 2014

Cross-correlation of gamma-rays and galaxies: tighter limits



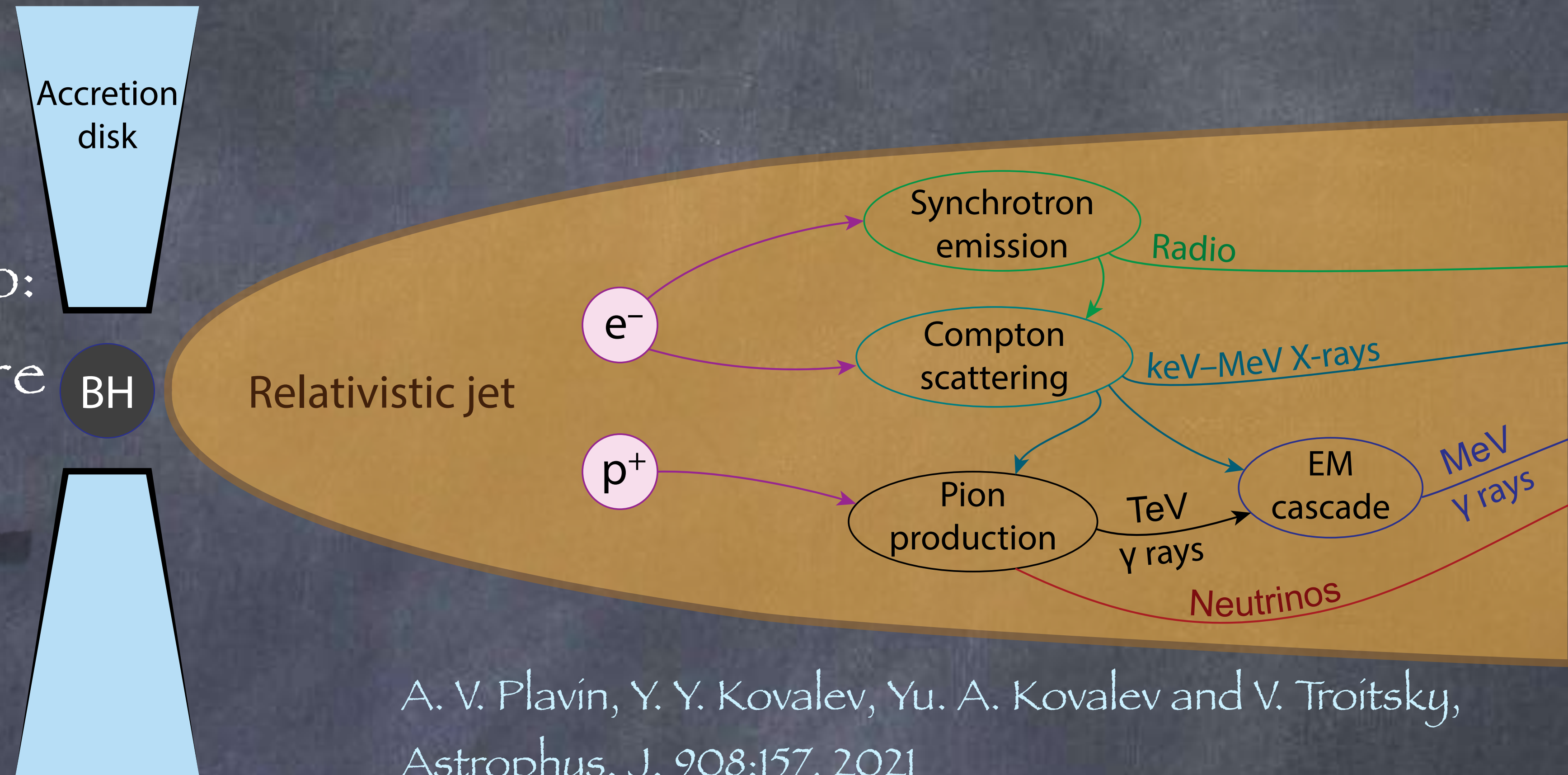
S. Ando, I. Tamborra and F. Zandanel, Phys. Rev. Lett. 115:221101, 2015

Exercise:

What does the fact that the transition from the Thompson to the Klein-Nishina regime in inverse Compton is at $E_e = m_e^2/\varepsilon_\gamma$ indicate?

Thus, correlations between neutrinos and photons of similar energies are not always obvious.

Consider the following scenario: electrons and protons are accelerated at the source.



A. V. Plavin, Y. Y. Kovalev, Yu. A. Kovalev and V. Troitsky, Astrophys. J. 908:157, 2021

Electrons moving in the magnetic field of the source (or near the source) emit (radio) synchrotron radiation, which in turn, is upscattered via ICS to keV-MeV energies by the same electrons. This X-ray radiation can act as the target for photo-production of neutrinos (and photons). High-energy photons, however, have a very high optical depth in the radiation field, so they cannot escape and hence, cannot be correlated with the neutrino flux of similar energy. The cascade process would produce a gamma-ray flux at lower energies.

CR/Gamma-ray/Neutrino Connection

Correlation between the spectrum of accelerated primaries and secondary radiation

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Correlation between the spectrum of accelerated primaries and secondary radiation

Assuming protons and electrons have the same spectrum, synchrotron radiation from relativistic electrons

$$P(\omega) \propto F\left(x = \frac{\omega}{\omega_c}\right) \quad ; \quad \omega_c = \frac{3\,e\,B}{2\,m}\,\gamma^2$$

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$$\frac{dN_p}{dE_p} \propto E_p^{-\alpha_p} \quad \rightarrow \quad P_{\rm tot}(\omega) \propto \int P(\omega)\, E_e^{-\alpha_p}\, dE_e \propto \omega^{-(\alpha_p-1)/2} \int F(x)\, x^{(\alpha_p-3)/2}\, dx$$

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Adding synchrotron losses

$$\frac{dN_\gamma}{dE_\gamma} \propto E_\gamma^{-(\alpha_p/2+1)}$$

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Adding synchrotron losses

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$$-\frac{dE}{dt} = P = \int \frac{dP(\omega)}{d\omega}\,d\omega = \frac{2}{3}\,\alpha\,E^2\,\frac{e^2\,B^2}{m^4}$$

$$\tau_{\rm synch} = \frac{E}{|\,dE/dt\,|} = \frac{3}{2\,\alpha\,E}\left(\frac{m^2}{e\,B}\right)^2$$

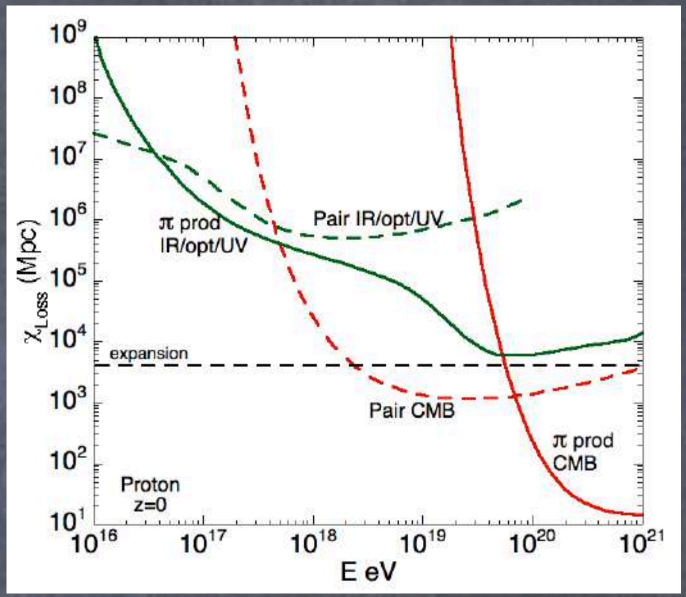
Exercise:

Can we observe PeV electrons from the center of the Milky Way?

Use energy losses via synchrotron radiation and consider the average galactic magnetic field.

Propagation of cosmic rays in the extragalactic medium

Energy losses of protons and nuclei



D. Allard, Astropart. Phys. 39:33, 2012

Adiabatic losses (redshift) $\frac{1}{E}\frac{dE}{dt} = -H(z) \rightarrow E = \frac{E_0}{1+z}$

Losses due to interactions with the background

$$\frac{1}{E}\frac{dE}{dt} = -\langle y\, n\, \sigma \rangle$$

y = fraction of energy lost per interaction

Electron positron pair production: $p + \gamma_b \rightarrow p + e^+ + e^-$

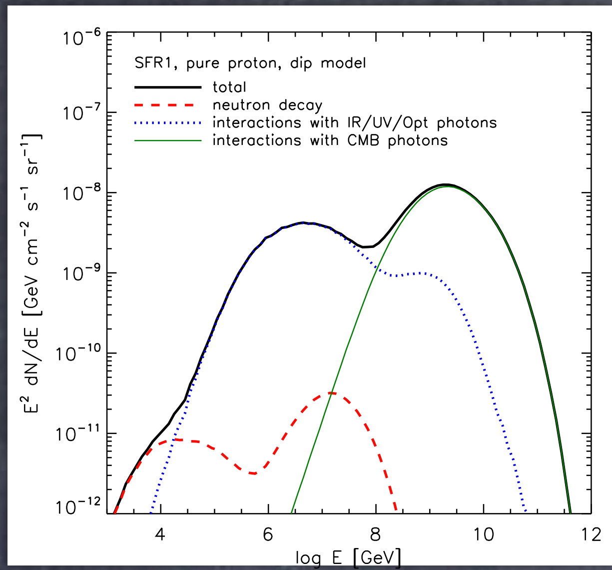
$$E_p \geq \frac{m_e\, m_p}{\varepsilon_\gamma} \simeq 2 \times 10^{18}\ \text{eV} \left(\frac{2.725\ \text{K}}{\varepsilon_\gamma}\right) \quad ; \quad y \simeq \frac{2\, m_e}{m_p} \simeq 10^{-3}$$

Pion production (GZK suppression) $p + \gamma_b \rightarrow \Delta \rightarrow \begin{cases} \pi^+ + n \\ \pi^0 + p \end{cases}$

$$E_p \geq \frac{m_\Delta^2 - m_p^2}{4\, \varepsilon_\gamma} \simeq 7 \times 10^{20}\ \text{eV} \left(\frac{2.725\ \text{K}}{\varepsilon_\gamma}\right) \quad ; \quad y \simeq \frac{m_\pi}{m_p} \sim 0.14$$

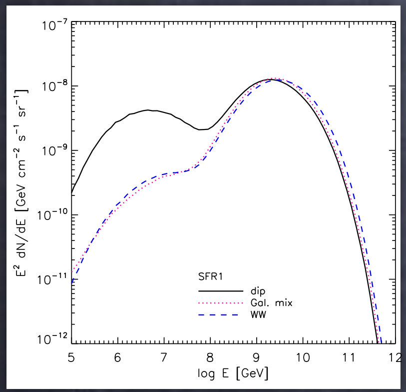
Neutrinos from CR interactions off the EBL and the CMB: cosmogenic neutrinos

V. Berezinsky and G. Zatsepin, Yad. Fiz. 11:200, 1970

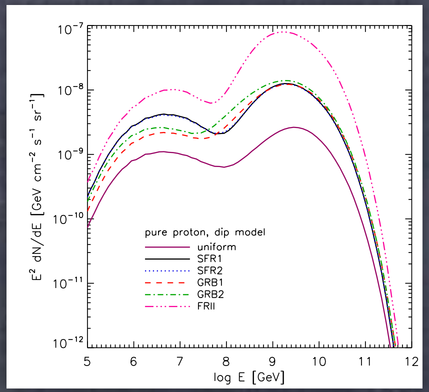


$$p + \gamma_{\mathrm{CMB/EBL}} \to \Delta \to \begin{cases} \pi^+ + n \\ \pi^0 + p \end{cases}$$

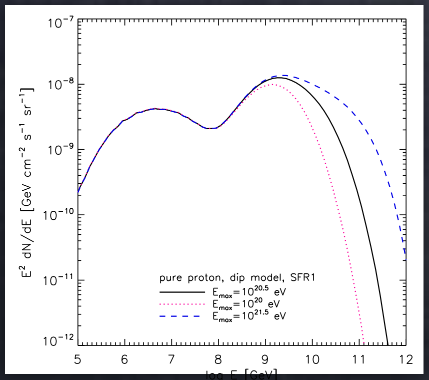
CR spectrum



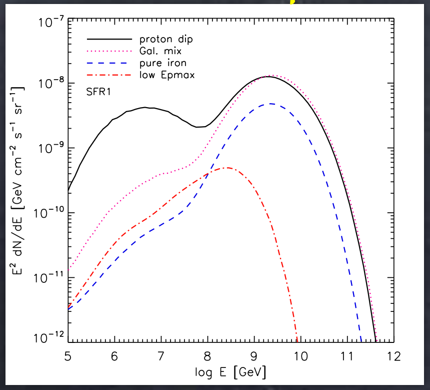
Redshift evolution



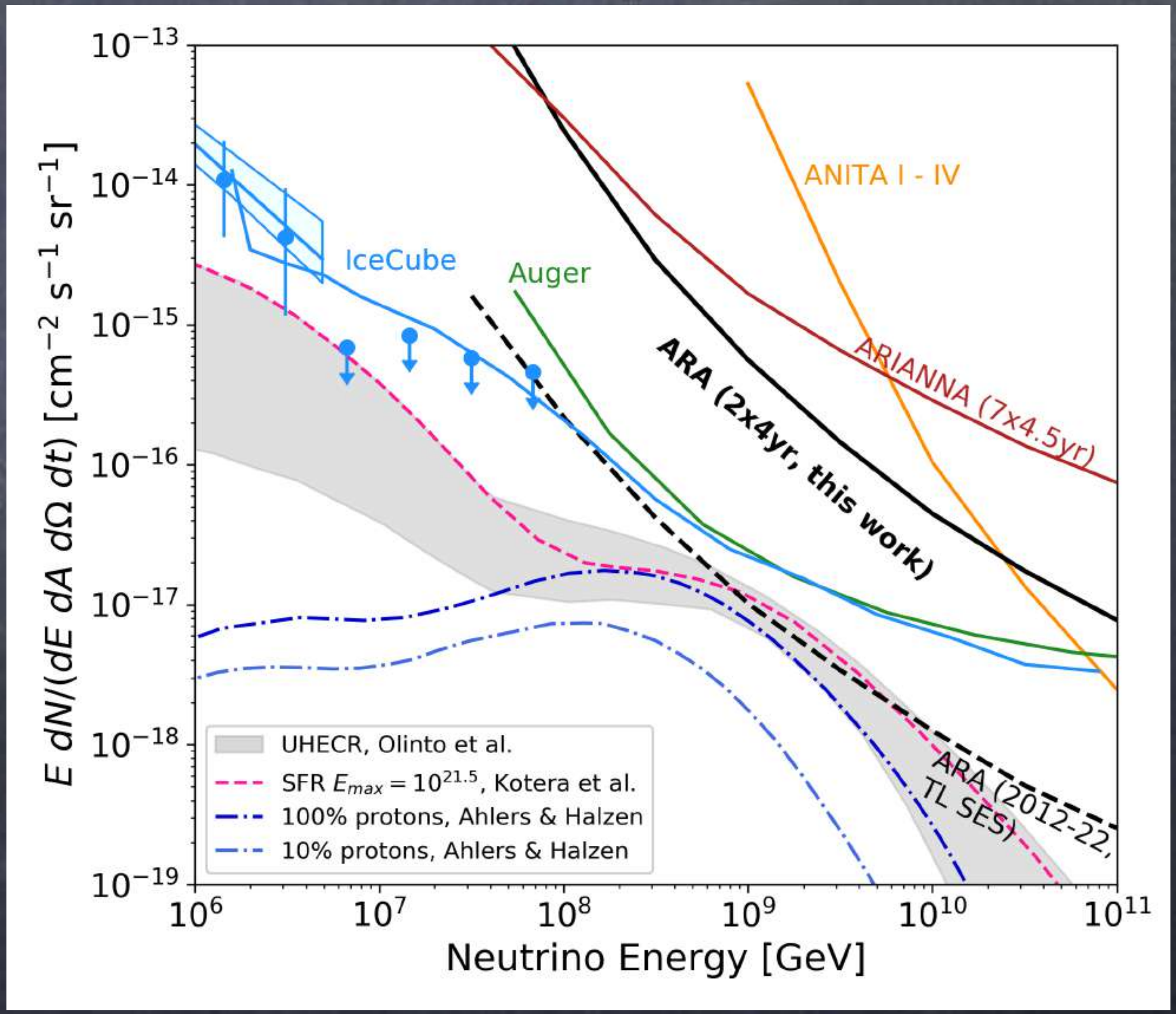
Maximum energy



Spectrum composition



K. Kotera, D. Allard and A. V. Olinto, JCAP 1010:013, 2010



P. Allison et al. [ARA Collaboration], Phys. Rev. D102:043021, 2020

Requirements for a cosmic accelerator

Geometry

accelerated particles should be maintained
within the object during the acceleration process

Larmor radius < size of the accelerator

$$\frac{dp}{dt} = Z e \left(\vec{v} \times \vec{B} \right) \quad \rightarrow \quad R_L = \frac{E}{Z e v B} \quad \rightarrow \quad E_{\rm max} \simeq 3 \times 10^{18} \text{ eV } (Z v) \left(\frac{B}{\mu \text{G}} \right) \left(\frac{L}{\text{kpc}} \right)$$

Power

sources must convert enough
energy into accelerated particles

Losses

the energy gained by a particle should be higher
than the energy lost by radiation or interaction

General energy arguments

Cosmic ray luminosity

$$\mathscr{L}_{\mathrm{CR}} = 4\,\pi\,\frac{\rho_{\mathrm{CR}}(E_{\mathrm{min}})\,V_{\mathrm{MWdisc}}}{\tau_{\mathrm{R}}}$$

$$\rho_{\mathrm{CR}}(E_{\mathrm{min}}) = \int_{E_{\mathrm{min}}} \frac{dn_{\mathrm{CR}}}{dE}\,E\,dE$$

$$\boxed{\mathscr{L}^{\mathrm{class}}_{\mathrm{tot}} \simeq \frac{dN_{\mathrm{class}}}{dt}\,\mathscr{L}_{\mathrm{class}}\,T_{\mathrm{class}}}$$

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From measurements of cosmic ray secondaries, we know the residence time of cosmic rays in the galaxy

$$\tau_{\rm R} \sim 10^7 \ {\rm yr}$$

$$\rho_{\rm CR}(E_{\rm min} \sim {\rm GeV}) \sim 1 \ {\rm eV/cm^3}$$

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$$\boxed{\mathscr{L}_{\rm CR} = 4\,\pi\,\frac{\rho_{\rm CR}(E_{\rm min})\,V_{\rm MWdisc}}{\tau_{\rm R}} \sim 3 \times 10^{40}\ {\rm erg/s}}$$

Let us consider the case of Supernova remnants

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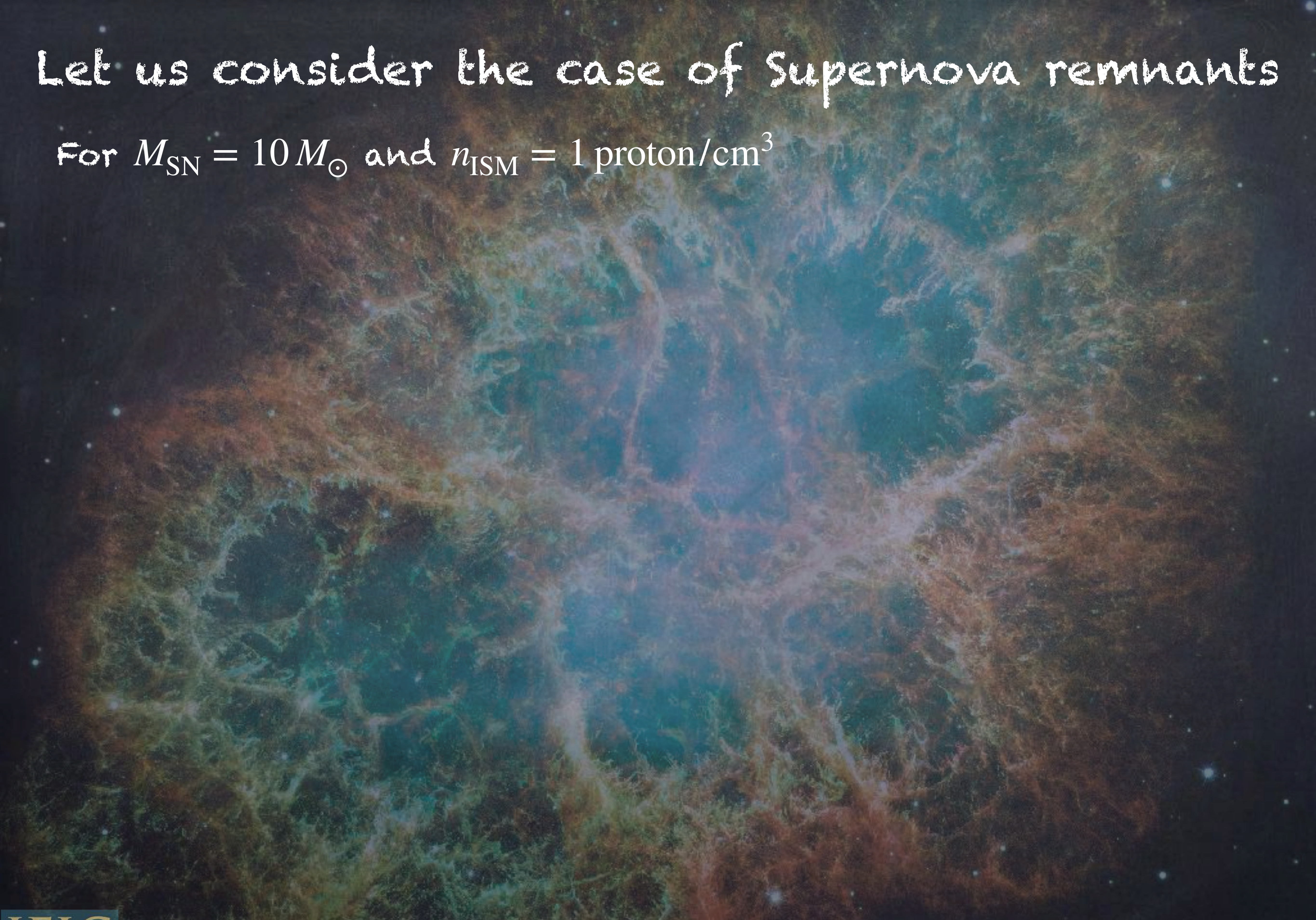
For $M_{\rm SN} = 10\, M_\odot$ and $n_{\rm ISM} = 1\,\text{proton/cm}^3$



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The characteristic time, $T_{\rm SNR}$, corresponds to the time it takes the expanding shell to sweep through the ISM such that $\rho_{\rm ISM} = \rho_{\rm SNR}$ (i.e., when it collects as much mass as that ejected)

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Taking 3 SN per century in the galaxy

$$\boxed{\mathscr{L}^{\rm SNR}_{\rm tot} \simeq \frac{dN_{\rm SNR}}{dt}\, \mathscr{L}_{\rm SNR}\, T_{\rm SNR} \sim 10^{42}\ \text{erg/s}}$$

Exercise:

What is (approximately) the current fraction of the volume of the disc of the Milky Way occupied by active SNRs?

Assume there are 3 SNs per century, a Milky Way disc with a radius of 15 kpc and a thickness of 300 pc, $M_{\rm SN} = 10\,M_{\odot}$, and $n_{\rm ISM} = 1\,\text{proton/cm}^3$.

Recall that only 1% of the gravitational energy is converted into kinetic energy.

But how are charged particles accelerated to very high energies at the sources?

One-shot acceleration

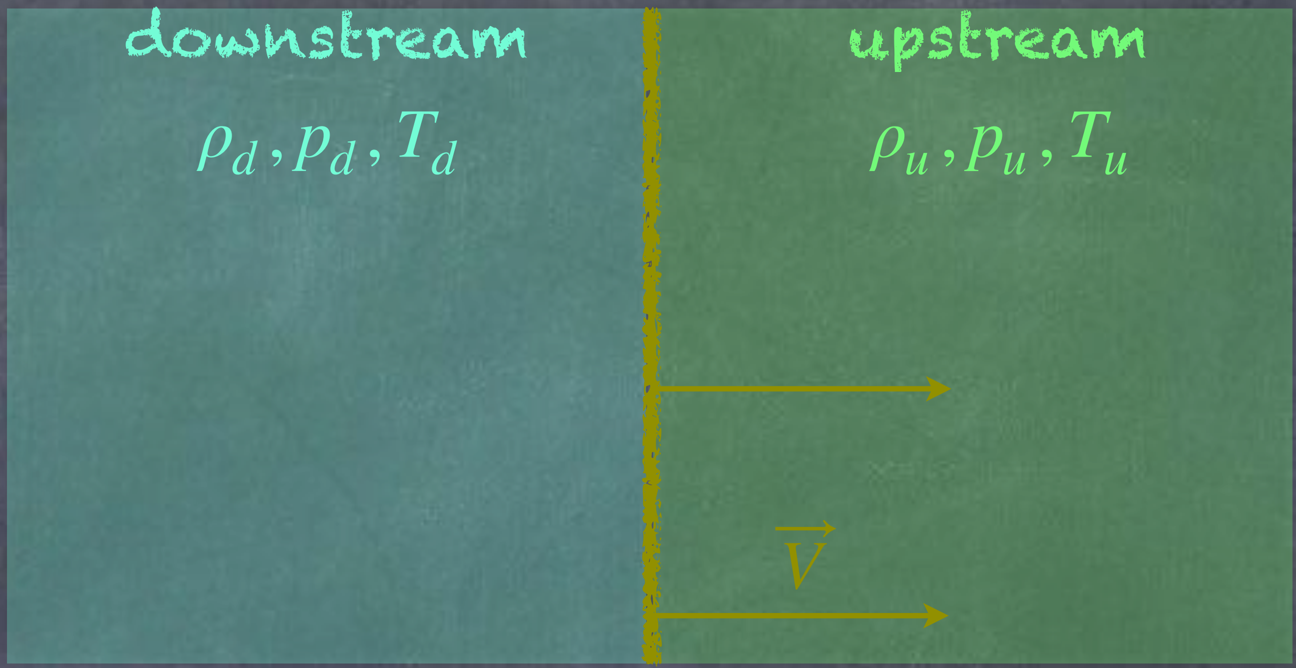
the particle escapes the accelerating region after the first iteration

Diffusive shock acceleration

repeated scattering of charged particles on magnetic irregularities back and forth across a shock front

Diffusive Shock Acceleration

Let us consider a strong shock wave propagating at a supersonic velocity V through stationary interstellar gas with density ρ_u , pressure p_u and temperature T_u (upstream). The density, pressure and temperature behind the shock (downstream) are ρ_d, p_d and T_d.



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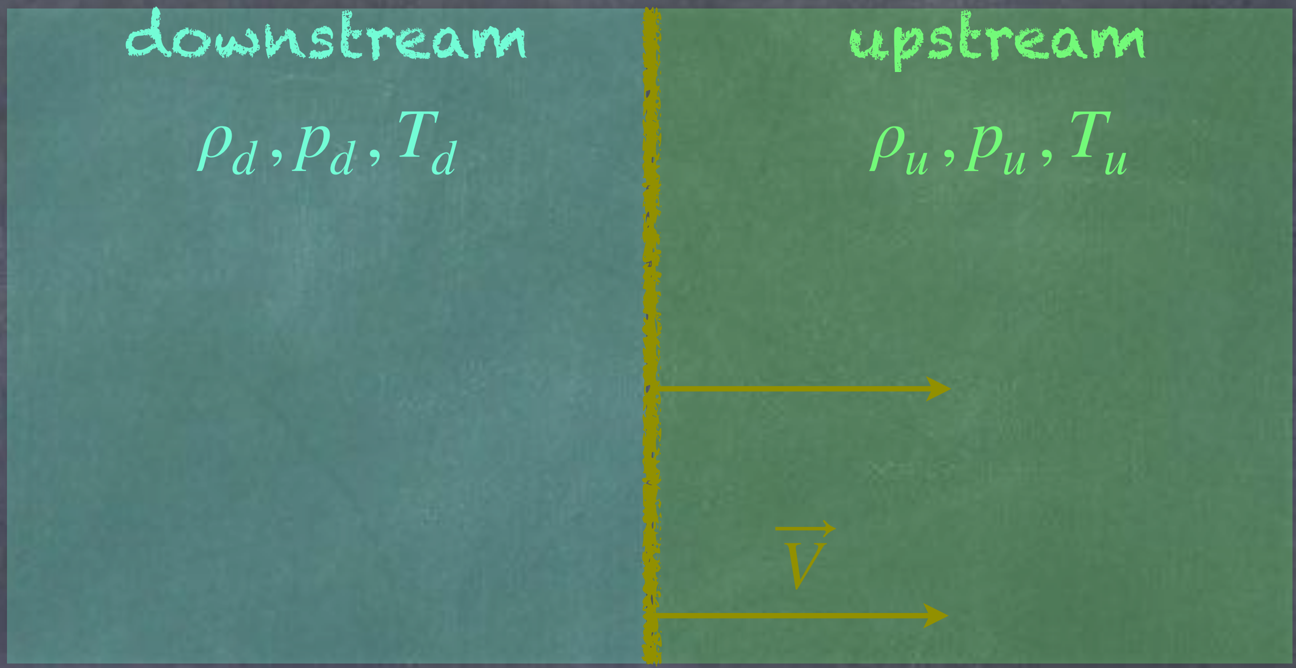

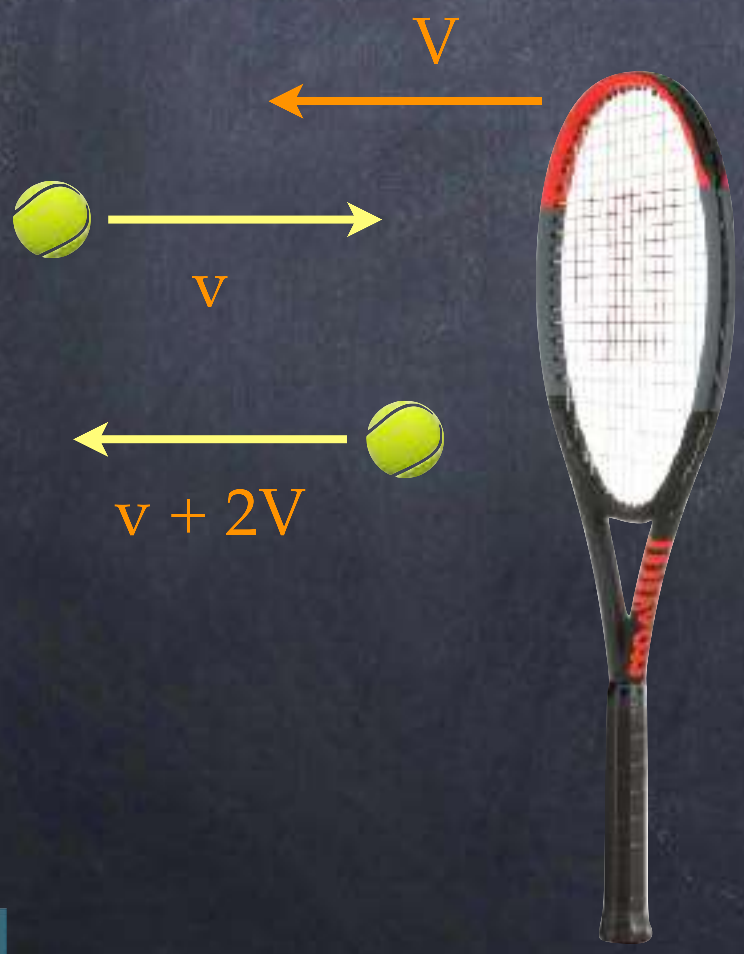

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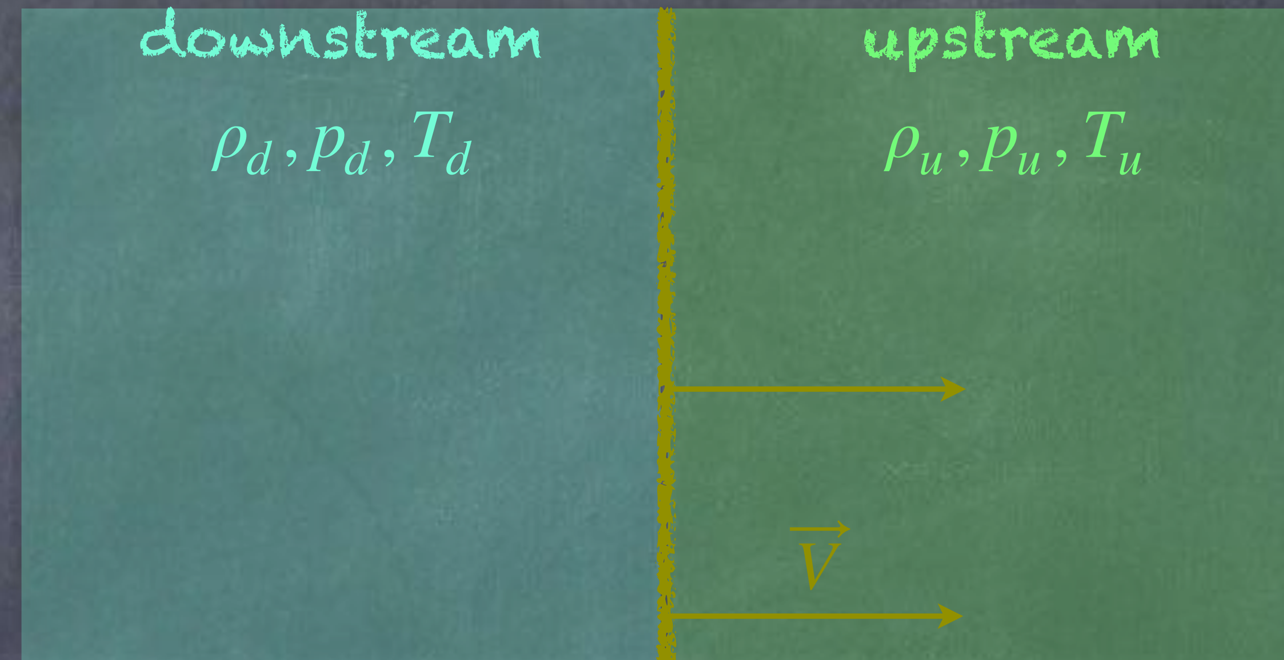
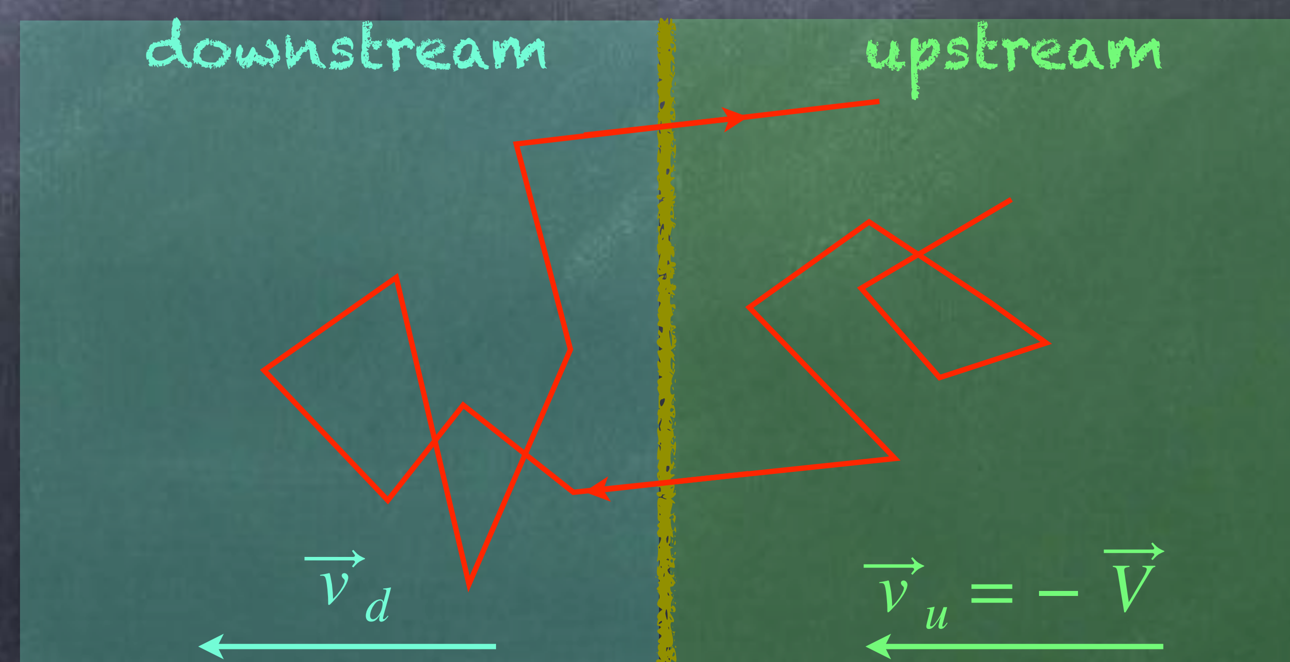


Rest frame of the shock front



Diffusive Shock Acceleration

Energy gain after a full cycle

$$E = E_0\,(1 + \xi)^k$$

Probability of remaining in the acceleration region

$$N = N_0\,(1 - P_{\rm esc})^k$$

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Number of particles accelerated to >E

$$N(> E) \propto \sum_{m=k}^{\infty} (1 - P_{\rm esc})^m = (1 - P_{\rm esc})^k \sum_{m=0}^{\infty} (1 - P_{\rm esc})^m = \frac{(1 - P_{\rm esc})^k}{P_{\rm esc}}$$

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$$N(>E) \propto \frac{1}{P_{\rm esc}} \left(1 - P_{\rm esc}\right)^{\frac{\ln(E/E_0)}{\ln(1+\xi)}}$$

$$\boxed{a^{\ln b} = b^{\ln a}}$$

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$$N(>E) \propto \frac{1}{P_{\rm esc}} \left(1 - P_{\rm esc}\right)^{\frac{\ln(E/E_0)}{\ln(1+\xi)}}$$

$$N(>E) \propto \left(\frac{E}{E_0}\right)^{-\beta} \qquad \beta \equiv \frac{P_{\rm esc}}{\xi}$$

$$\boxed{a^{\ln b} = b^{\ln a}}$$

Diffusive Shock Acceleration

Energy gain after a full cycle

$$E = E_0\,(1+\xi)^k$$

Probability of remaining in the acceleration region

$$N = N_0\,(1-P_{\rm esc})^k$$

Number of cycles needed to reach E

$$k = \frac{\ln\left(\frac{E}{E_0}\right)}{\ln\left(1+\xi\right)}$$

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(1st order) Diffusive Shock Acceleration

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$$\boxed{\xi = \gamma_L^2 \left(1 + \frac{2}{3}(v_u - v_d)\right)^2 - 1 \simeq \frac{4}{3}(v_u - v_d)}$$

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Probability of escape from the acceleration region

$$\boxed{P_{\rm esc} = \frac{n\, v_d}{n/4} = 4\, v_d}$$

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$$N(>E) \propto \left(\frac{E}{E_0}\right)^{-\beta} \qquad \frac{dN}{dE} \propto \left(\frac{E}{E_0}\right)^{-\beta-1}$$

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Euler equations

Mass conservation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\left(\rho \vec{v}\right) = 0$$

Momentum conservation:

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} + \frac{\vec{\nabla} p}{\rho} = \vec{f}$$

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$$\frac{\partial}{\partial x}\left(\rho v\right) = 0 \qquad \frac{\partial}{\partial x}\left(p + \rho v^2\right) = 0$$

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Jump conditions

$$\rho_u v_u = \rho_d v_d$$

$$p_u + \rho_u v_u^2 = p_d + \rho_d v_d^2$$

$$\rho_u v_u \left(e_u + \frac{v_u^2}{2} + \frac{p_u}{\rho_u}\right) = \rho_d v_d \left(e_d + \frac{v_d^2}{2} + \frac{p_d}{\rho_d}\right)$$

Using (for an ideal gas) $p \propto \rho^{\gamma}$ $c_s^2 = \frac{\gamma p}{\rho}$ $\mathscr{M} = \frac{v}{c_s}$ $e = \frac{1}{\gamma - 1}\frac{p}{\rho}$

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Leaky box model

the effect of diffusion

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Cosmic rays are confined within the disc of the galaxy and have certain (homogeneous) probability of escape

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In our galaxy $\tau_{\rm esc} \ll \lambda$, so if $\tau_{\rm esc} \propto E^{-\delta}$ $n \propto E^{-\beta-\delta}$

Exercise:

Show that for acceleration in magnetic gas clouds, the energy gain is second order in the relative speed, $V_{\rm cloud}$
(2nd order Fermi acceleration).

Note that the probability of collision with the cloud, $P(\cos\theta_{ud})$, is proportional to the relative velocity between the cloud and the particle (which travels at a speed close to the speed of light) and the particle moves out of the cloud randomly, $P(\cos\theta_{du}) = \text{constant}$.

In this case, in the rest frame of the particles bath, $v_d = V_{\rm cloud}$ and $v_u = 0$.

Diffusive Shock Acceleration maximum energy in SNRs

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Assume a density of particles released at position x_0 (downstream)

$$-v_d \frac{\partial n}{\partial x} = D_d \frac{\partial^2 n}{\partial x^2} + Q\,\delta(x - x_0)$$

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$$T_{\text{cycle}} = T_{\text{upstream}} + T_{\text{downstream}} = 4\left(\frac{D_u}{v_u} + \frac{D_d}{v_d}\right) \sim 20\,\frac{D}{v_u}$$

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Integrating the acceleration rate, dE/dt

$$\boxed{E_{\text{max}} = \frac{3}{20}\,v_u\,Z\,e\,B\,\left(v_u\,T_{\text{SNR}}\right)}$$

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Integrating the acceleration rate, dE/dt

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For $M_{\rm SN} = 10\,M_\odot$ and $n_{\rm ISM} = 1\,{\rm proton/cm^3}$, computing the time $T_{\rm SNR}$ the SNR remains active and using $B \sim 4\,\mu{\rm G}$ for the ISM magnetic field

$$E_{\rm max} \sim 40\,Z\ {\rm TeV}$$

Exercise:

Compare the maximum energy obtained for Bohm diffusion with the case of Kolmogorov diffusion,

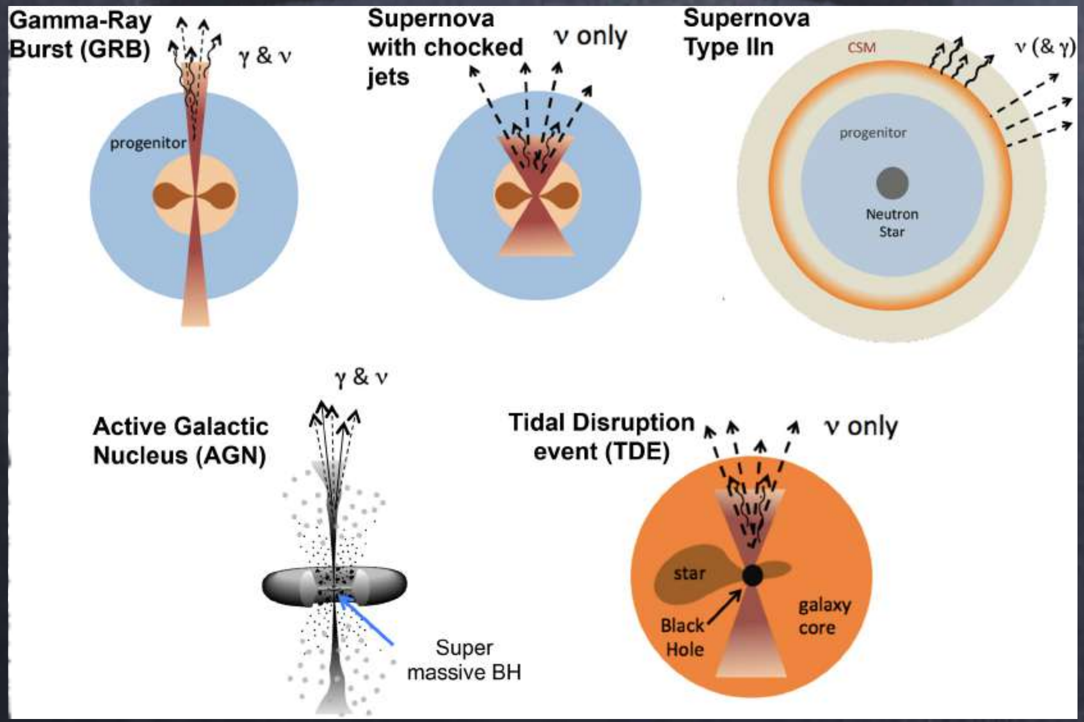
$$D_{\mathrm{Kolmogorov}} = D_{\mathrm{Bohm}} \left(\frac{1}{2\pi L E} \right)^{2/3}$$

Take L = 1 pc for the injection length for turbulence and typical values for SNRs in our galaxy.

Acceleration to E > 100 TeV

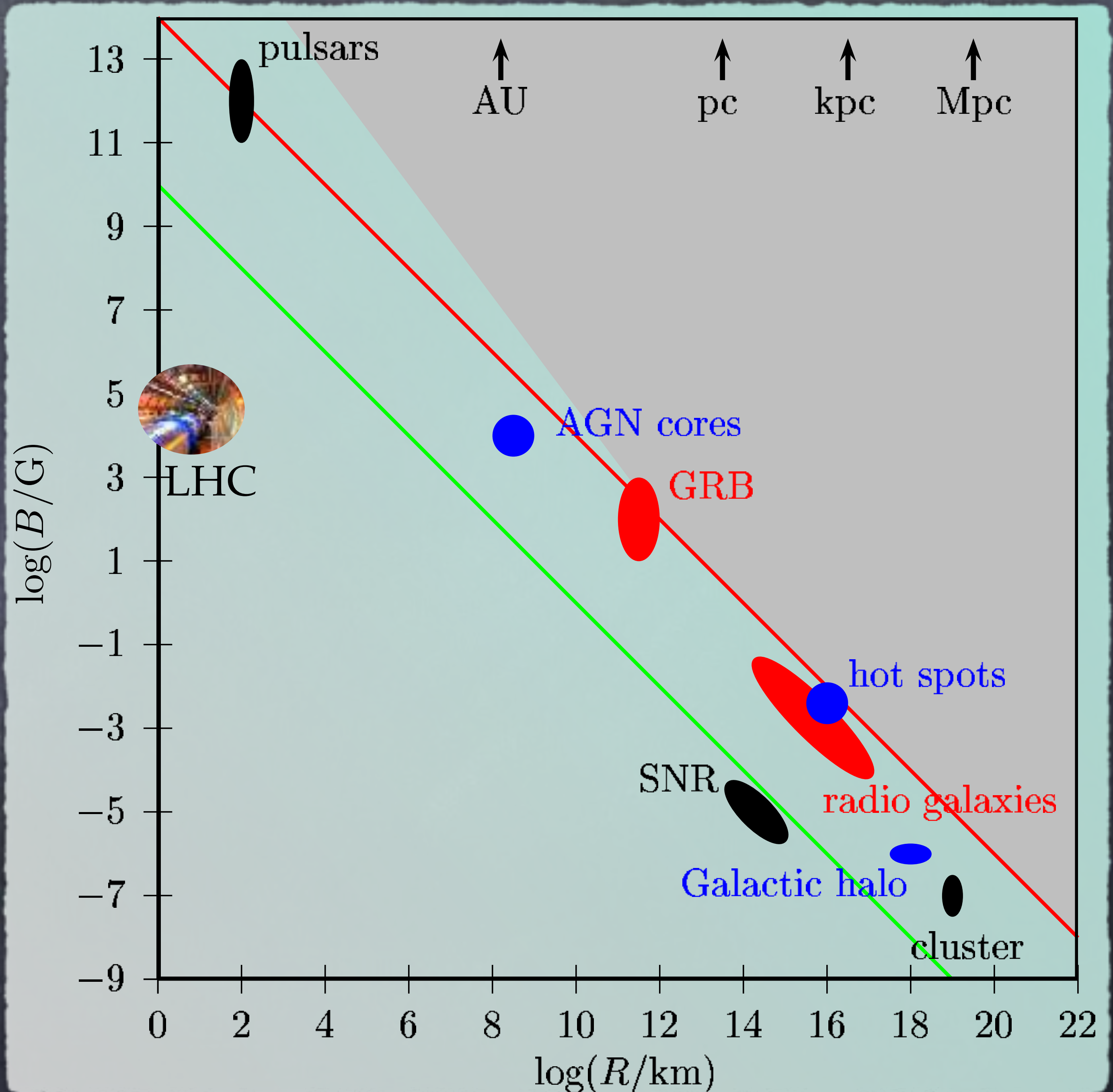
$$E_{\max} \simeq 3 \times 10^{18}\ \mathrm{eV}\ (Z v) \left(\frac{B}{\mu\mathrm{G}}\right) \left(\frac{L}{\mathrm{kpc}}\right)$$

Supernova Remnants
Active Galactic Nuclei
Gamma Ray Bursts
StarBurst Galaxies
Tidal Disruption Events
Pulsars...



I. Bartos and M. Kowalski, 2017

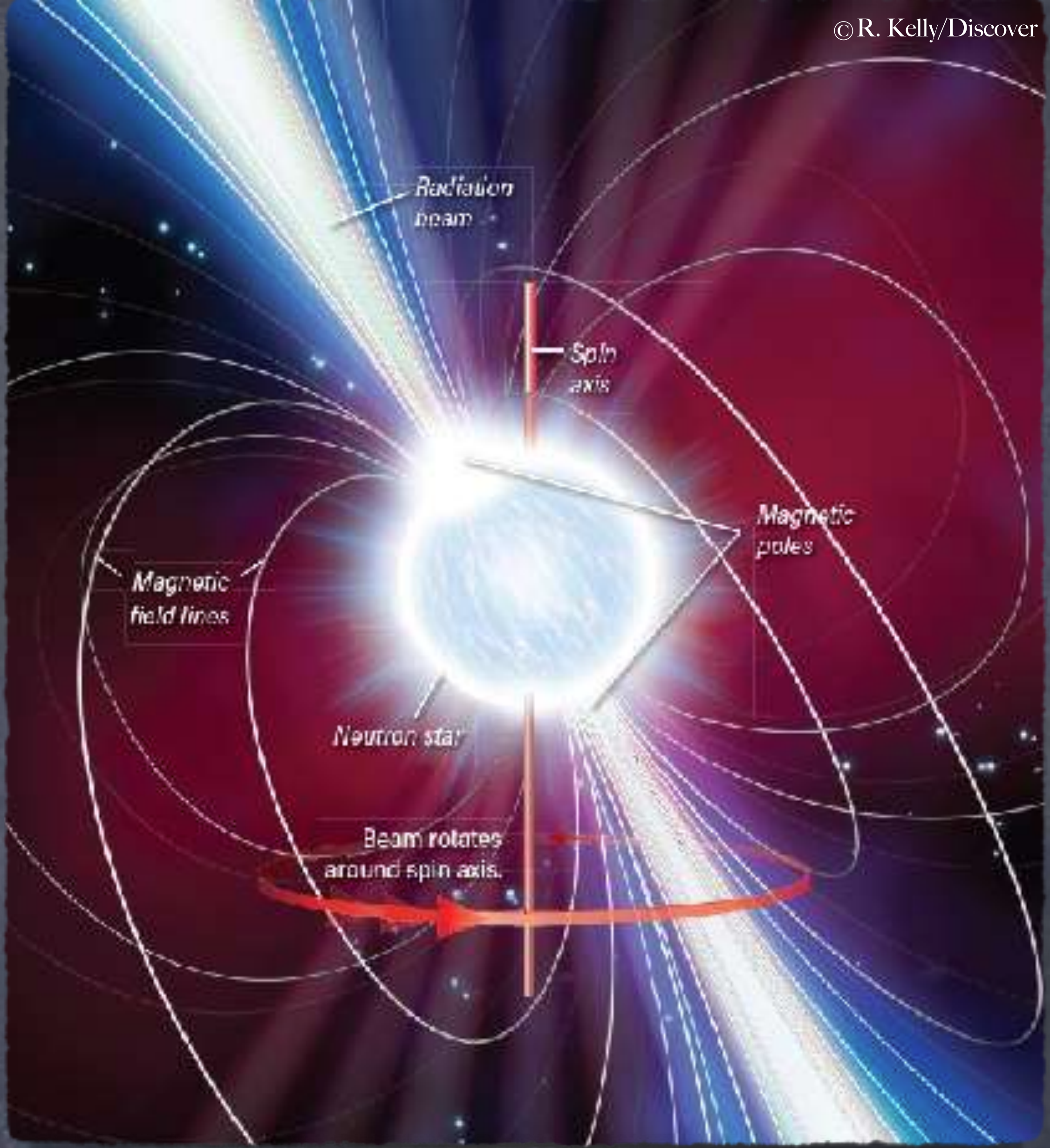
The Hillas plot



M. Kachelrieß and D. V. Semikoz, Prog. Part. Nucl. Phys. 109:103710, 2019

Pulsars

Fast rotating neutron stars that emit a beam of electromagnetic radiation, typically along its magnetic axis. They have very stable and short periods, very high magnetic fields and emit in radio. The radiation can only be observed when the axis is pointing towards the Earth.



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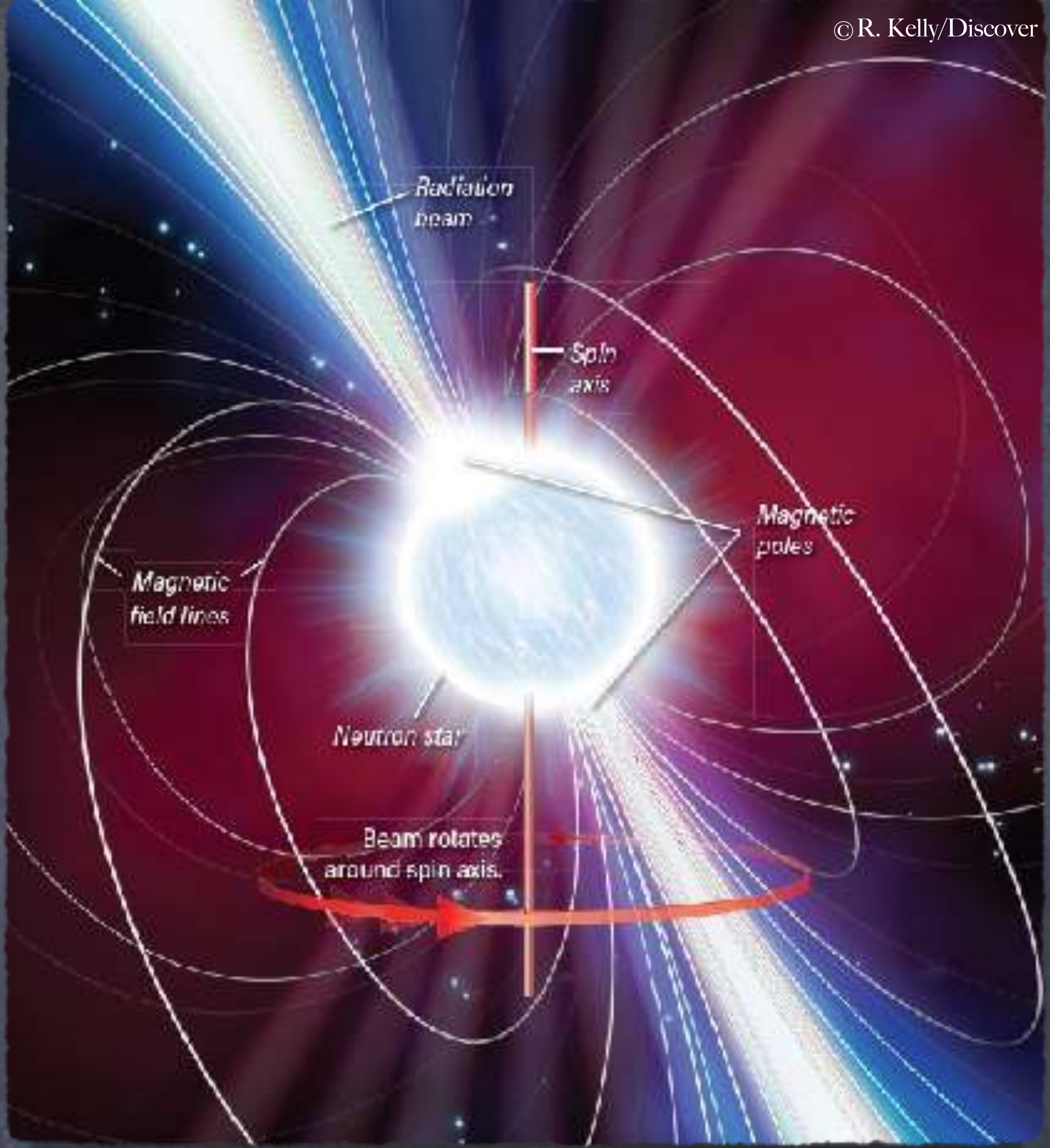

Estimating the mili-second period by angular momentum conservation

$$M R^2 \omega = M R_{\rm NS}^2 \omega_{\rm NS}$$

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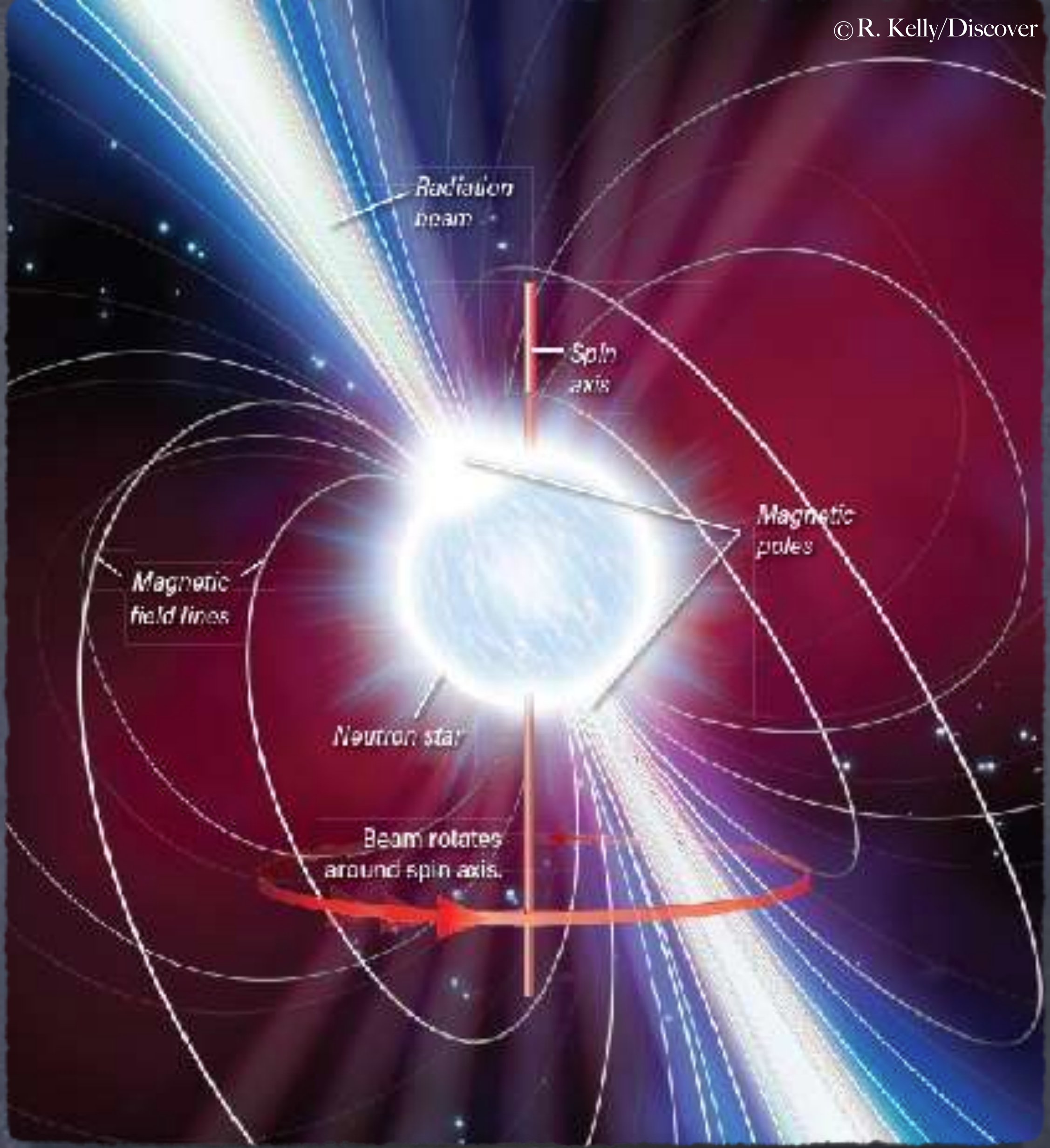

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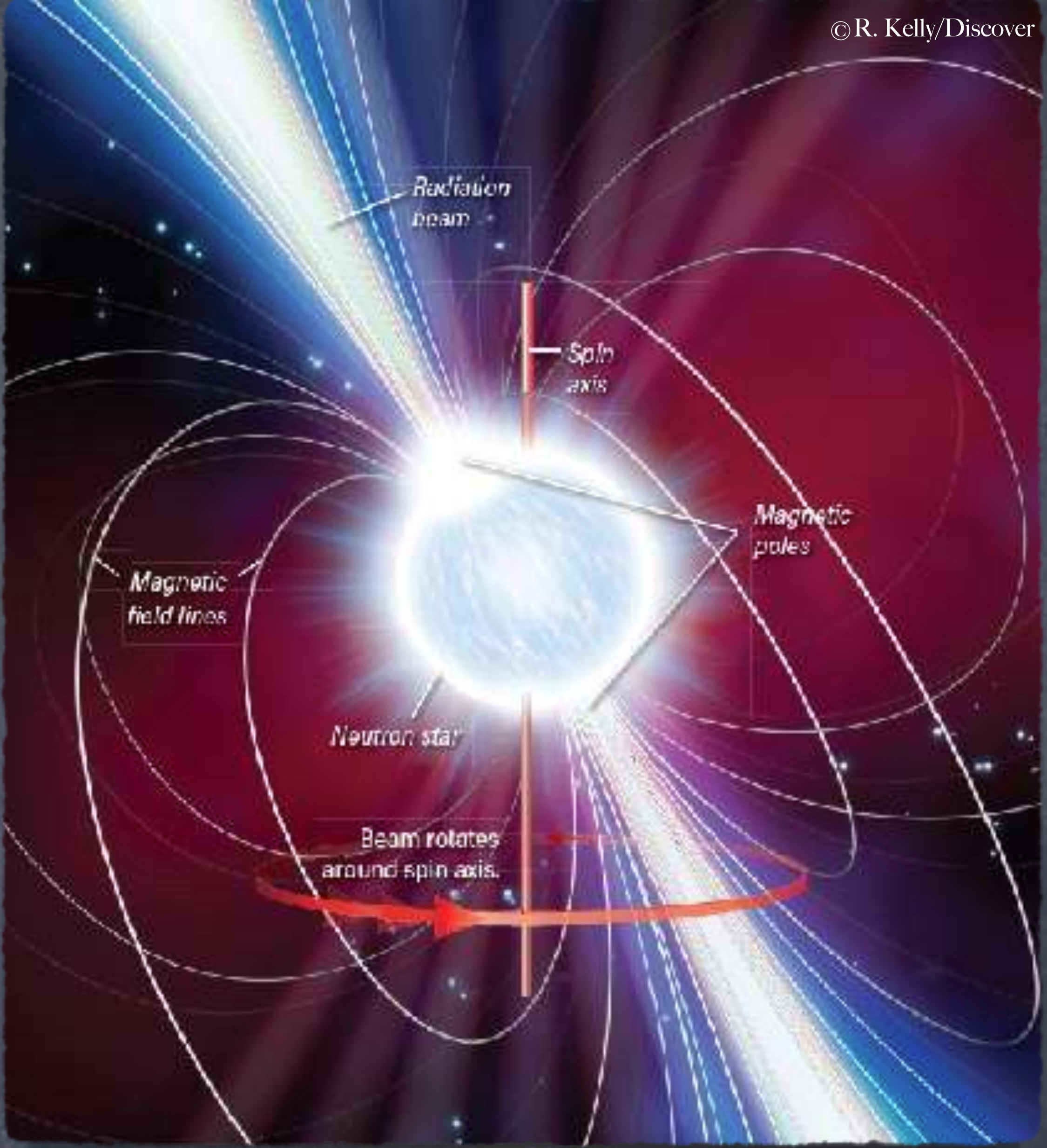

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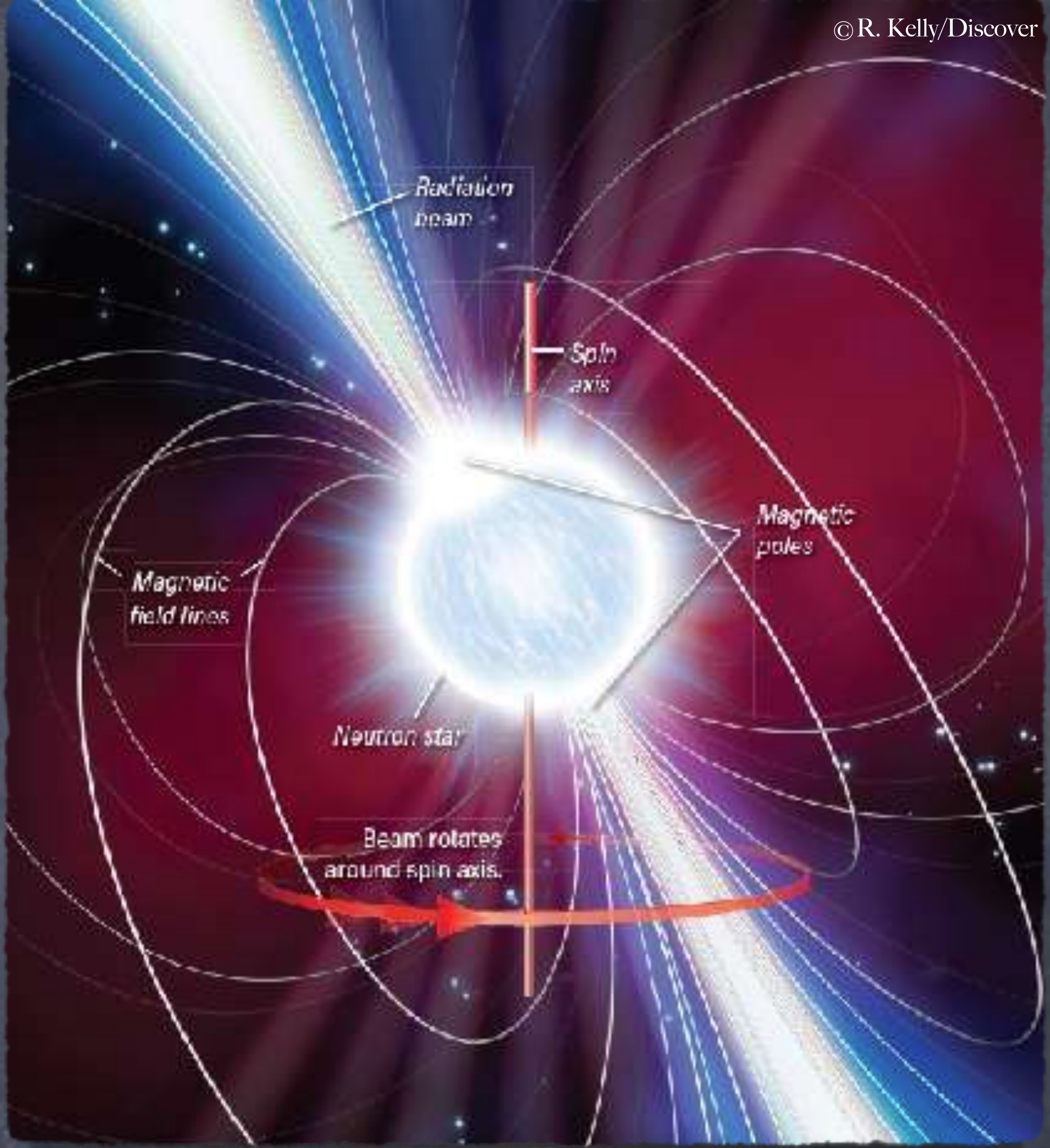

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$$\int \vec{B} \cdot d\vec{A} = \int \vec{B}_{NS} \cdot d\vec{A}_{NS} \qquad B_{NS} = 10^{12}\ \mathrm{G} \left(\frac{B}{10^3\ \mathrm{G}}\right) \left(\frac{R}{R_\odot}\right)^2 \left(\frac{10\ \mathrm{km}}{R_{NS}}\right)^2$$

Example of one-shot Acceleration

Let us consider a particle that moves in a non-uniform and static magnetic field. Because of the motion of the particle, it feels a time-varying magnetic field, which induces an electric field (Faraday's law)

$$\overrightarrow{\nabla} \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t}$$

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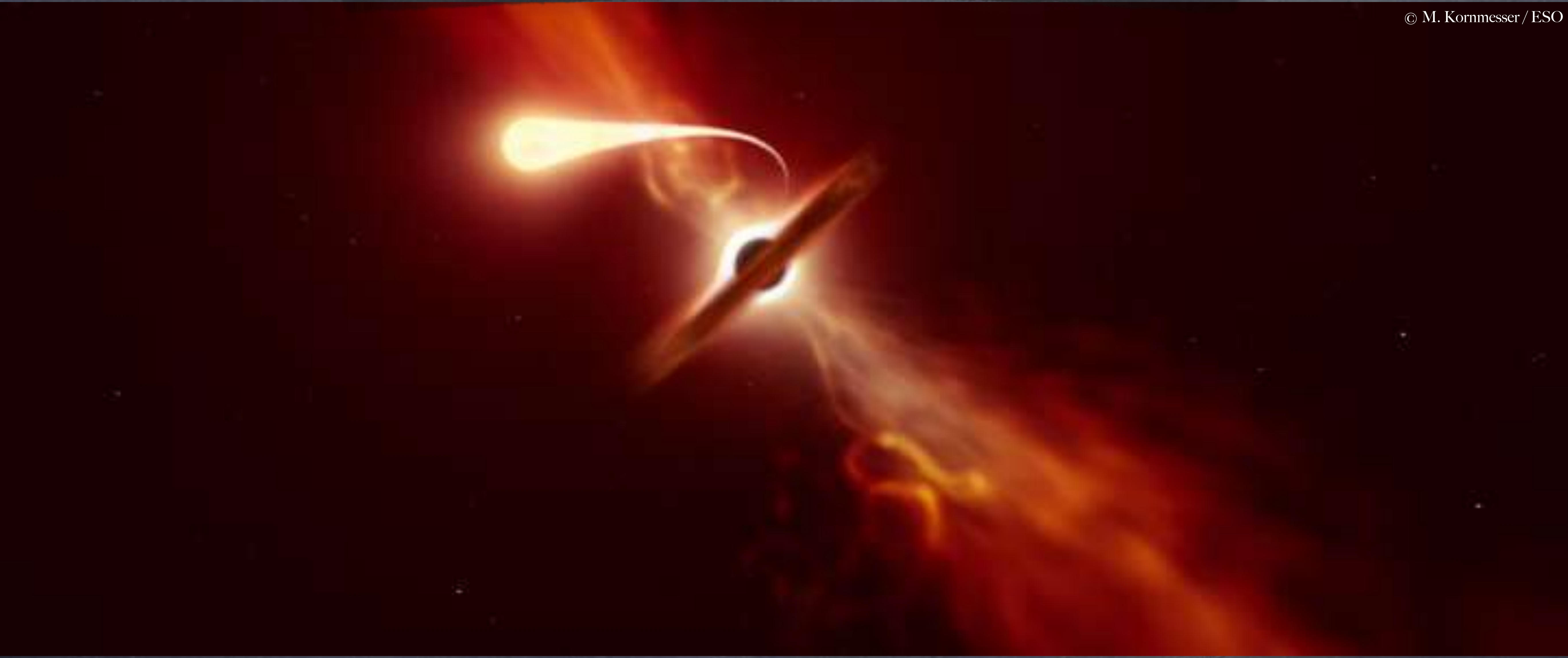
For the Crab pulsar:

$$P \sim 0.03\,\mathrm{s}\,, \quad R \sim 10\,\mathrm{km}\,, \quad B \sim 10^{11}\,\mathrm{G}$$

$$E_{\rm max} \sim Z \times 10^{18}\,\mathrm{eV}$$

Tidal disruption events: X-ray binaries



Neutron star or black hole with a companion star.
Once the companion exceeds the Roche volume of the binary system, an accretion disc forms around the compact object.

Gamma Ray Bursts

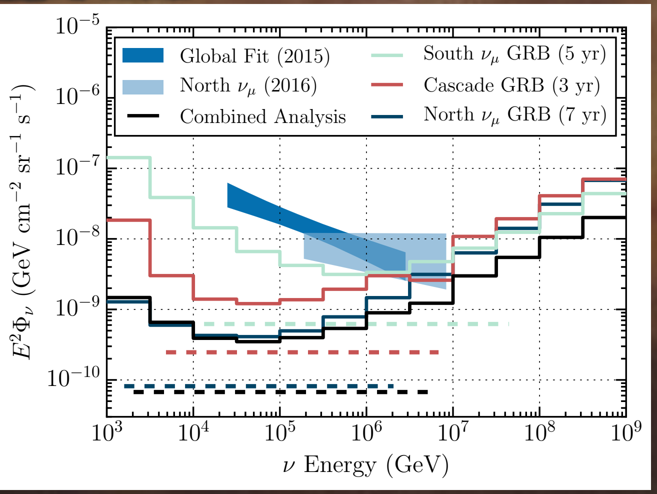
E. Waxman and J. N. Bahcall, Phys. Rev. Lett. 78:2292, 1997

The most luminous sources in the Universe: typical pγ accelerators

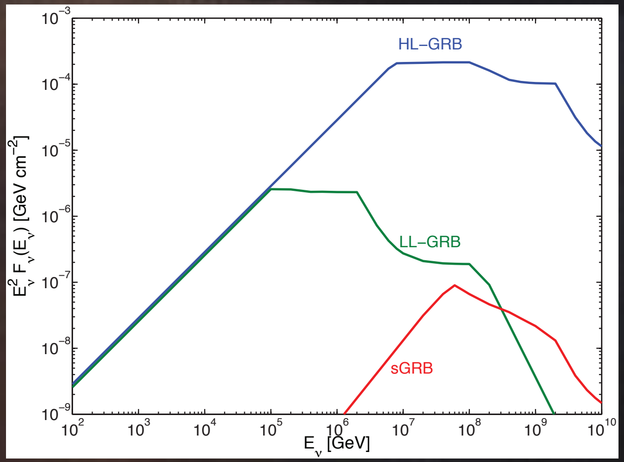
The photon spectrum (target for neutrino production) is constituted by synchrotron radiation of electrons in the internal shock fronts of the jet, with a break at ~250 keV, produced by cooling of electrons (or even by inverse Compton)

Assuming that the proton spectrum follows the electron spectrum of the source, the neutrino spectrum has two breaks:
at ~0.1-1 PeV (due to the break in the photon spectrum) and
at ~10-100 PeV (due to pion synchrotron losses)

No correlations found
Small fraction in IceCube
from High Luminosity GRBs



M. G. Aartsen et al. [IceCube Coll.], Astrophys. J. 843:112, 2017



I. Tamborra and S. Ando, JCAP 1509:036, 2015

Active Galact Nuclei

Powered by accretion around SMBHs in the center of galaxies, AGNs are less luminous than GRBs, but radiate for longer. Synchrotron radiation by accelerated electrons produces a IR/X-rays peak, and ICS of synchrotron, the MeV-GeV peak.

Smaller Lorentz factor for the jets and much lower minimum energy of the background radiation (UV): higher neutrino energies

F. W. Stecker et al., Phys. Rev. Lett. 66:2697, 1991

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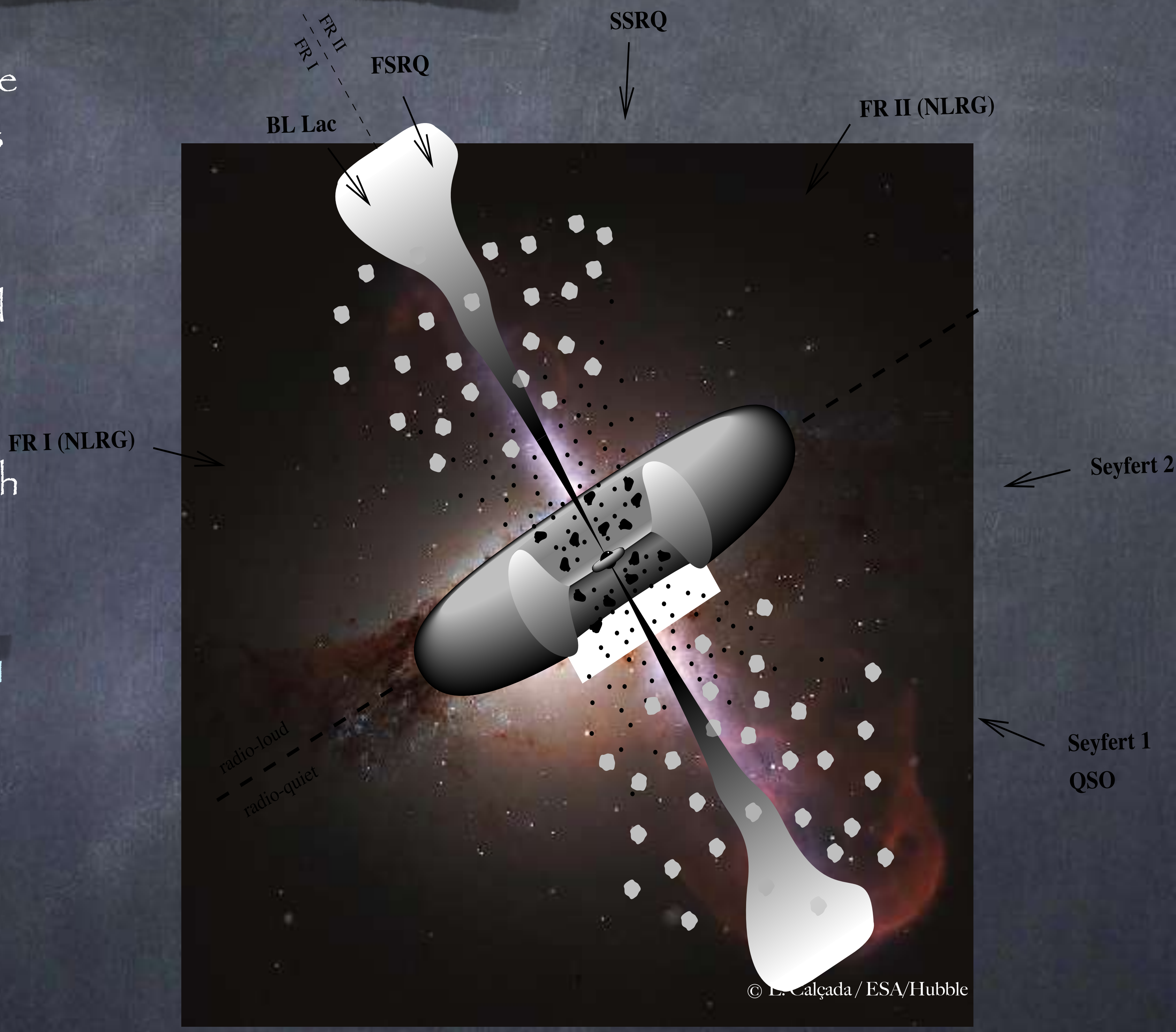

D. F. Torres and L. A. Anchordoqui, Rept. Prog. Phys. 67:1663, 2004

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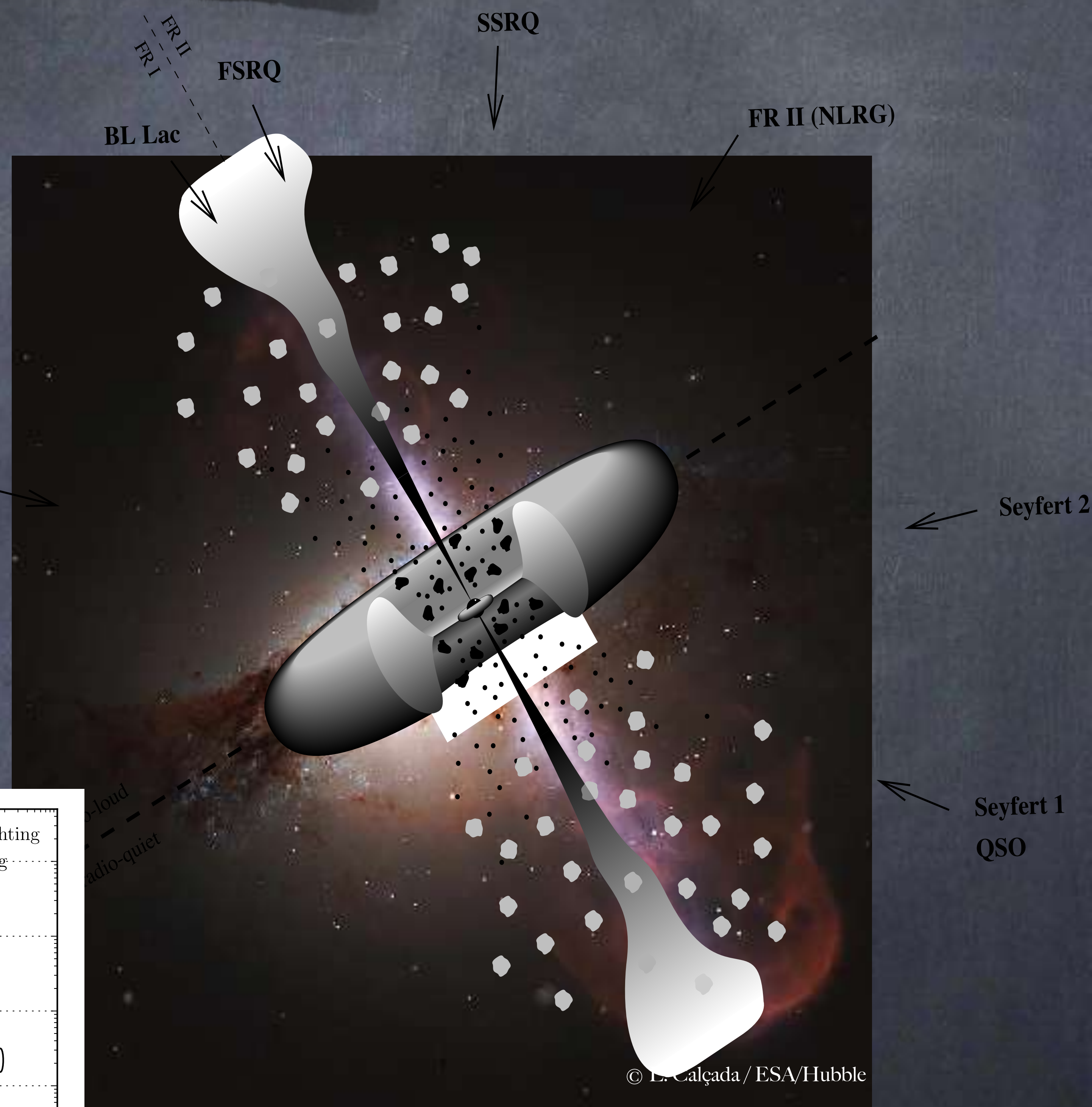
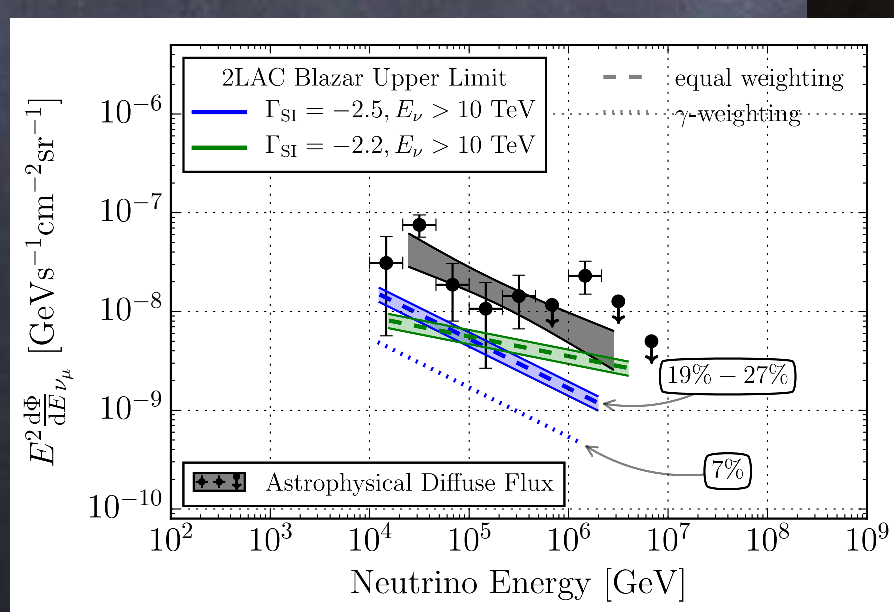


D. F. Torres and L. A. Anchordoqui, Rept. Prog. Phys. 67:1663, 2004

No correlations found
<27% contribution to the diffuse flux

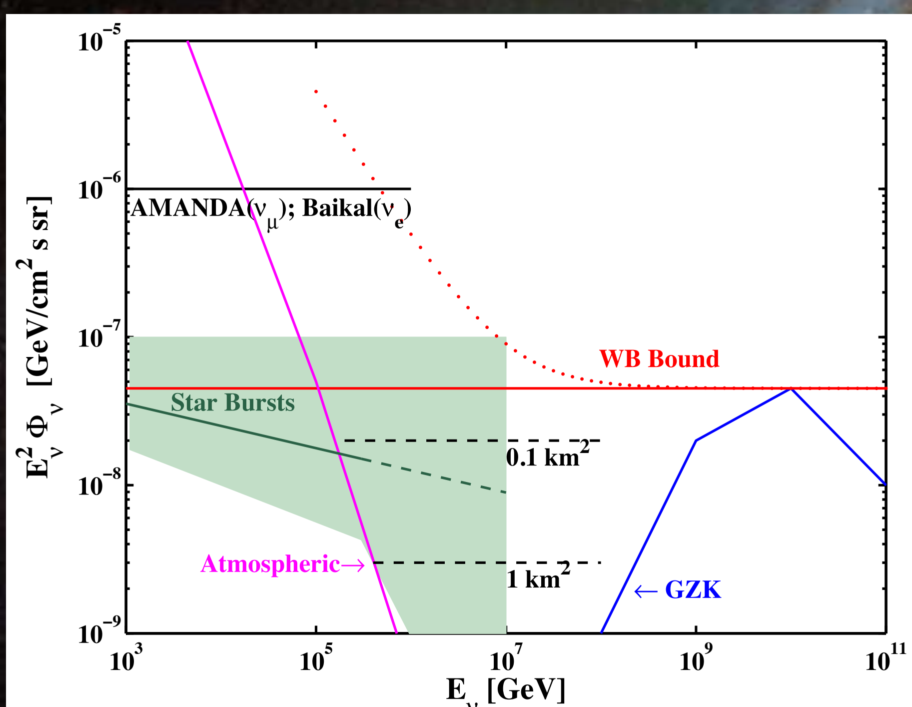


M. G. Aartsen et al. [IceCube Coll.], Astrophys. J. 835:45, 2017

StarBurst Galaxies

Galaxies with intense star formation rate and high gas density.
Neutrinos are produced by hadronic (pp) interactions of protons off ISM.

Located at large distances (> Mpc): only a dozen (< 20 Mpc) have been observed in gamma-rays, but the larger activity at high redshift makes them a good candidates for the diffuse neutrino flux



A. Loeb and E. Waxman, JCAP 0605:003, 2006

Normalization of the neutrino spectrum using synchrotron radiation

$$E_e^2 \frac{dN_e}{dE_e} \sim 2\,\nu_{\rm synch}\, L_{\nu,{\rm synch}} \qquad E_p \frac{d\mathscr{L}_p}{dE_p} = \frac{2}{3}\,\frac{1}{4}\, f\, E_e^2 \frac{dN_e}{dE_e}$$

$$E_\nu \frac{d\mathscr{L}_\nu}{dE_\nu} = \frac{2}{3}\,\frac{3}{4}\, f\, E_p \frac{d\mathscr{L}_p}{dE_p} = 6\,\nu_{\rm synch}\,\mathscr{L}_{\nu,{\rm synch}}$$

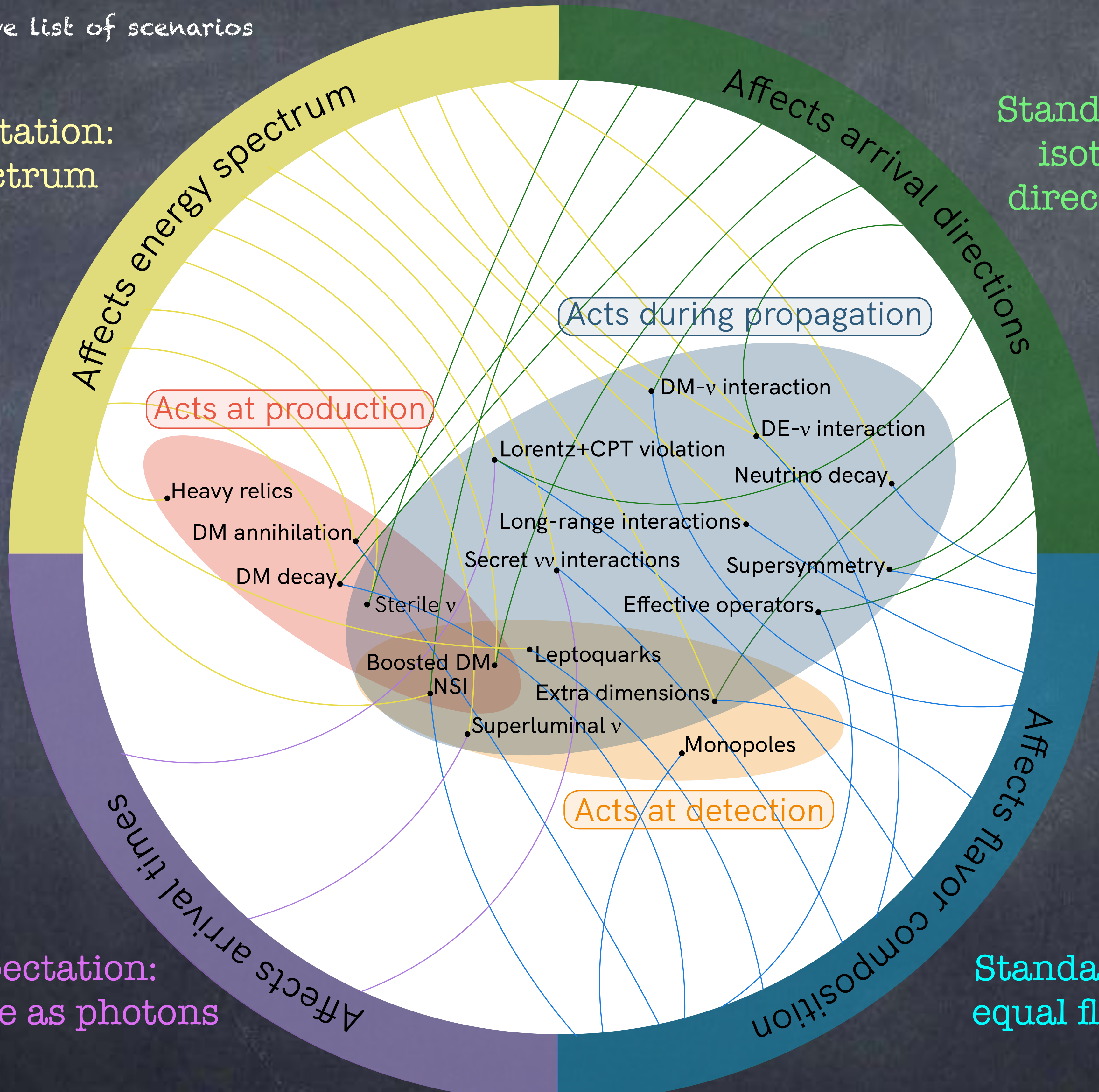
$$E_\nu^2 \frac{dN_\nu}{dE_\nu} = \frac{\zeta_z}{4\pi}\, t_{\rm H}\, 6\,\nu_{\rm synch}\, L_{\nu,{\rm synch}} \sim 10^{-7} \left(\frac{E_\nu}{\rm GeV}\right)^{2-\alpha_p} {\rm GeV\,cm^{-2}\,s^{-1}\,sr^{-1}}$$

Searching for new physics

Note: Not an exhaustive list of scenarios

Standard expectation:
power-law spectrum

Standard expectation:
isotropy (diffuse)
directional (sources)



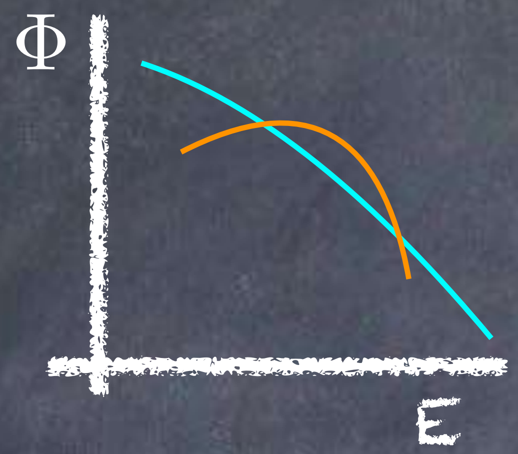
Standard expectation:
same arrival time as photons

Standard expectation:
equal flux of all flavors

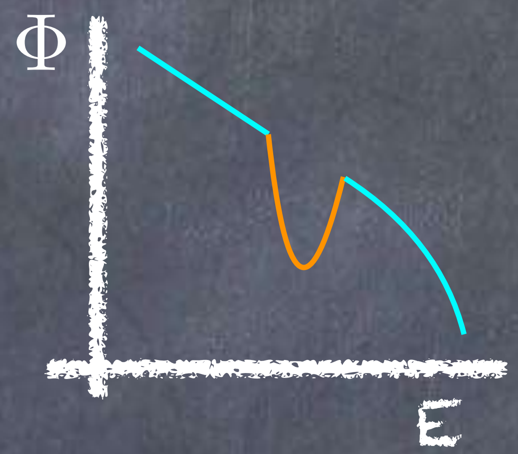
C. Argüelles, M. Bustamante, A. Kheirandish, SPR, J. Salvado and A. C. Vincent, PoS(ICRC2019)849, 2020

Neutrino-dark matter interactions

New signal



Features on spectrum



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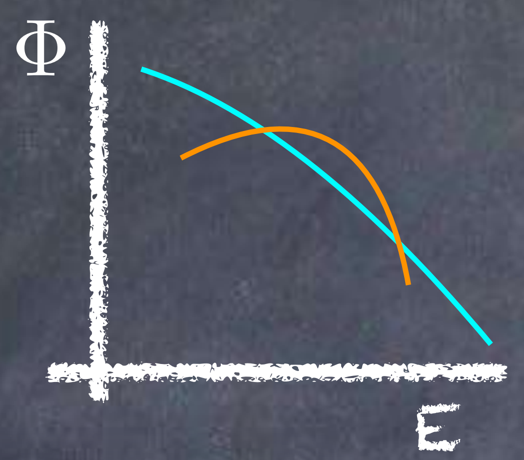
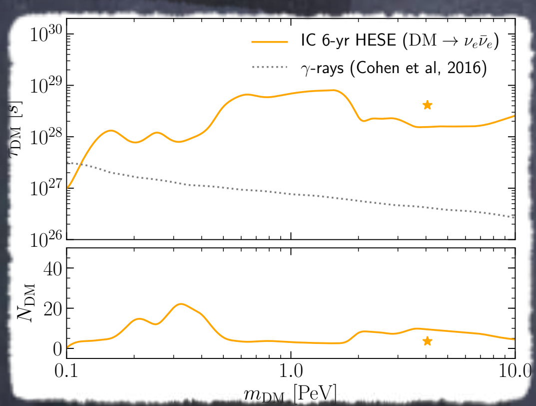
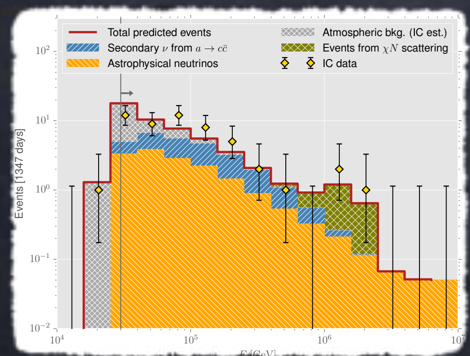


Heavy dark matter decays



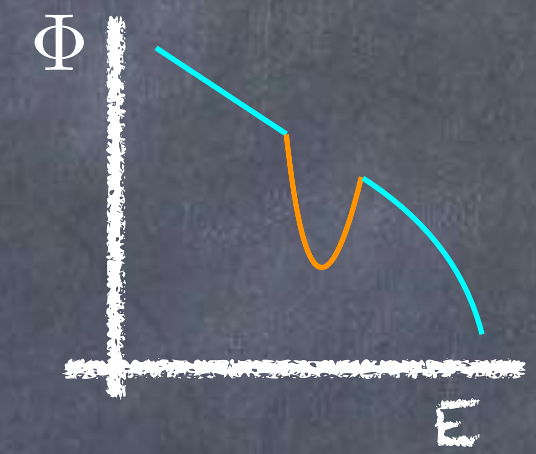
A. Bhattacharya, SPR et al., JCAP 05:051, 2019

Boosted dark matter



A. Bhattacharya et al., JCAP 1705:002, 2017

Features on spectrum



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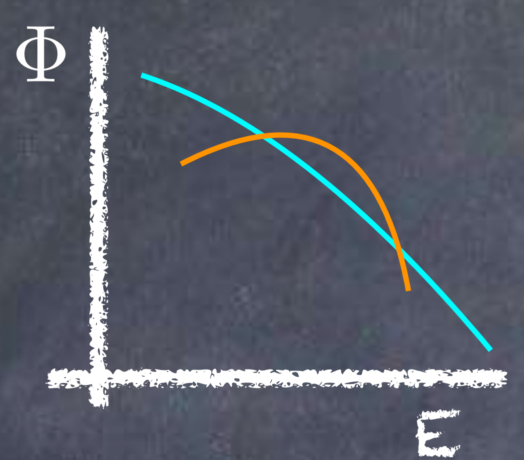

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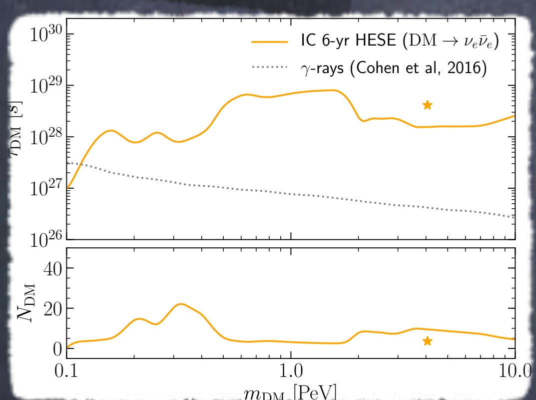

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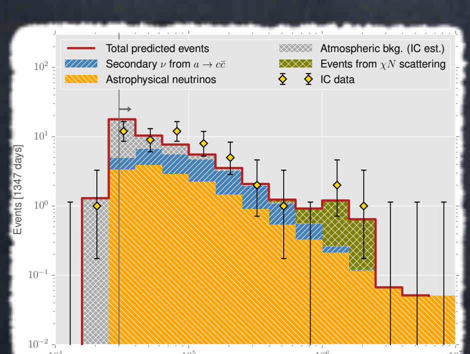

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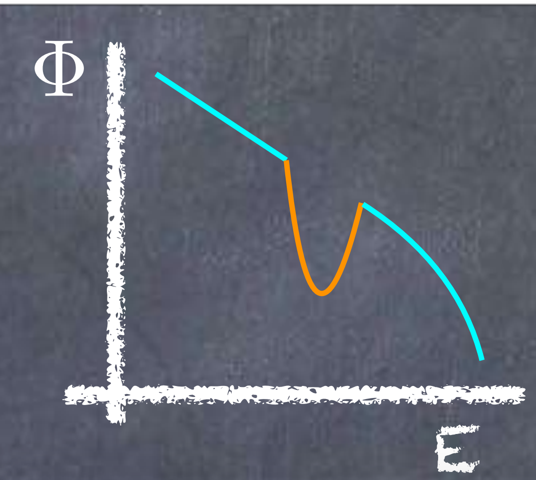
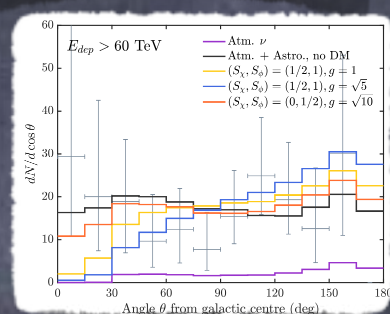
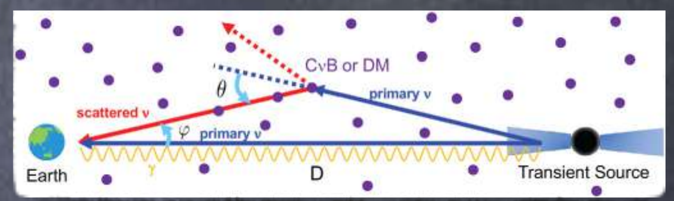


Absorptive effects



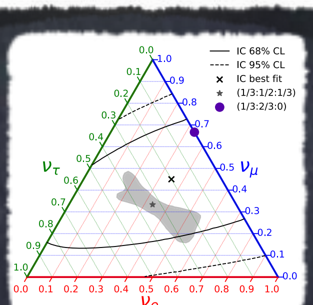
C. A. Argüelles, A. Kheirandish and A. C. Vincent, Phys. Rev. Lett. 119:201801, 2017

neutrino scattering
(energy, direction, flavor, time dependent)



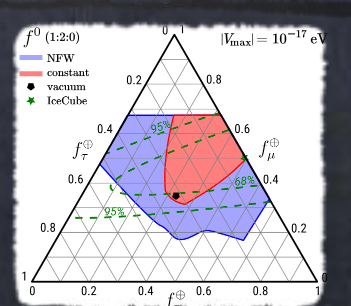
K. Murase and I. M. Shoemaker, Phys. Rev. Lett. 123:24, 2019

Coherent effects



Y. Farzan and SPR, Phys. Rev. D99:051702(R), 2019

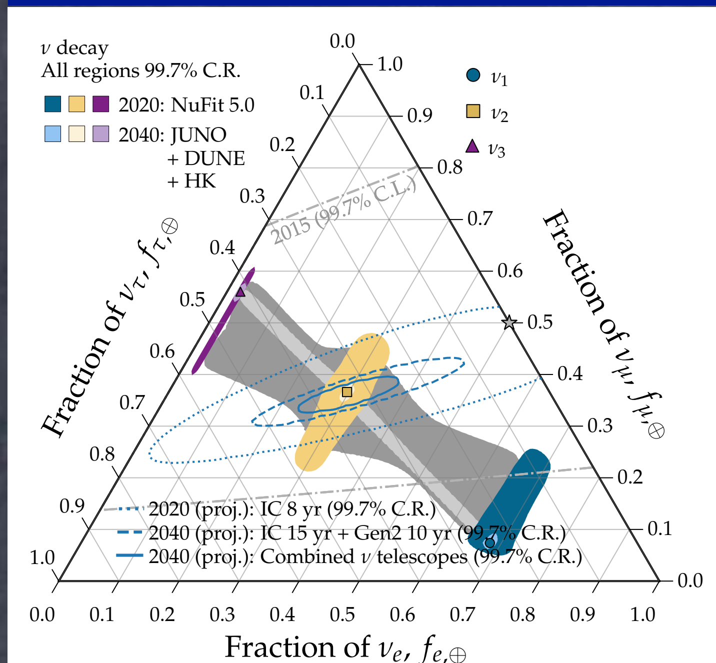
matter potential in neutrino propagation



P. F. de Salas, R. A. Lineros and M. Tórtola, Phys. Rev. D94:123001, 2016

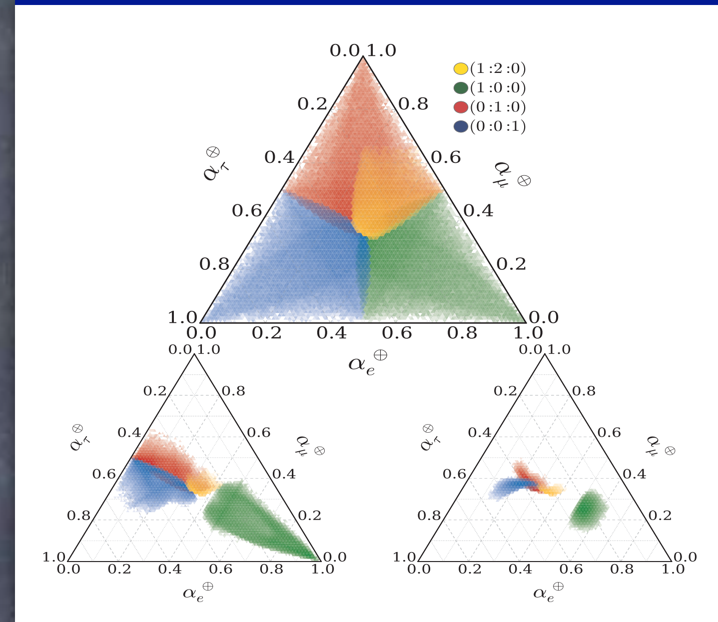
Other exotic searches

Neutrino decay



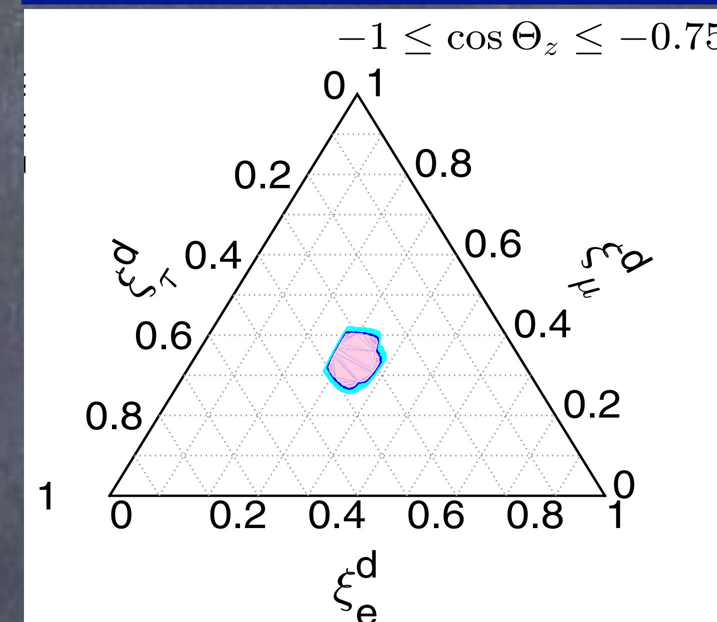
N. Song et al., JCAP 04:054, 2021

Lorentz violation



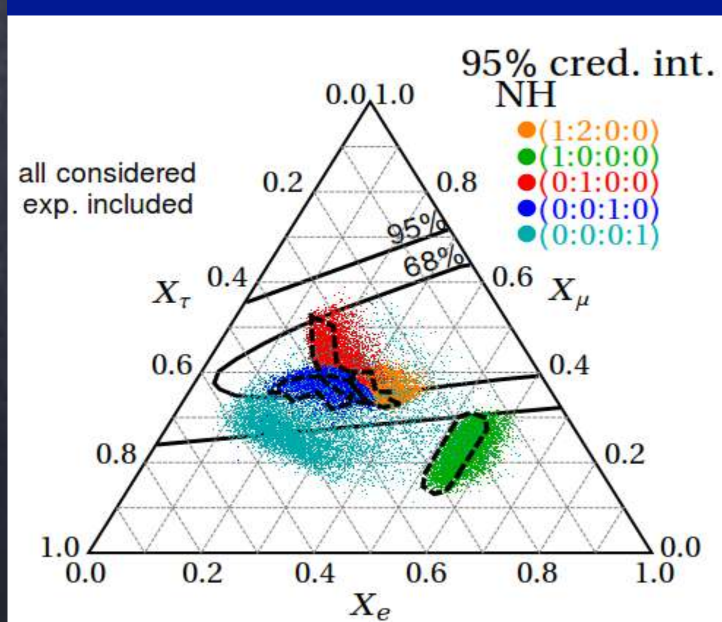
C. A. Argüelles, T. Katori and J. Salvadó, Phys. Rev. Lett. 115:161303, 2015

Non-standard interactions



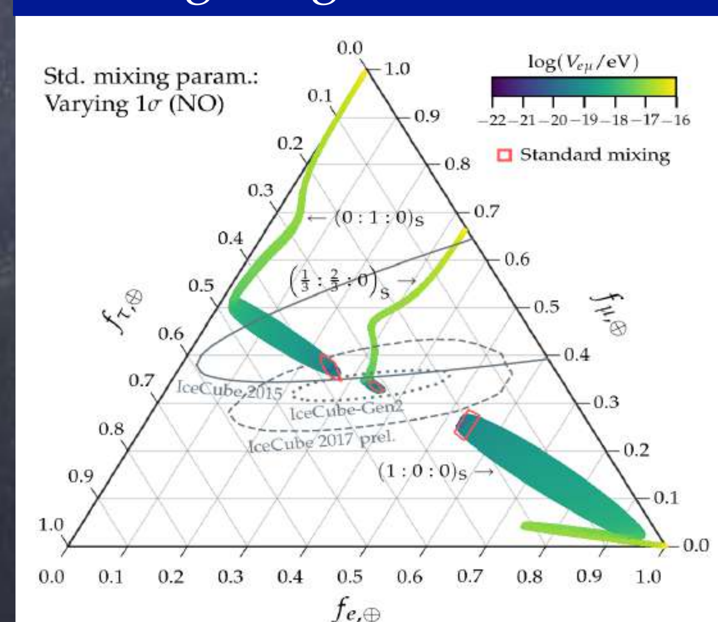
M. C. González-García et al., Astropart. Phys. 84:15, 2016

Sterile neutrinos



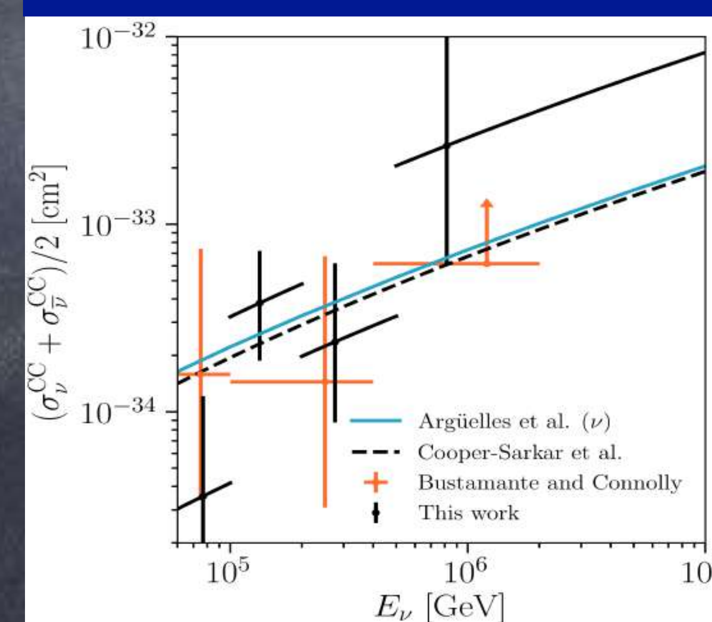
V. Brdar, J. Kopp and X.-P. Wang, JCAP 01:026, 2017

Long-range interactions



M. Bustamante and S. K. Agarwalla, Phys. Rev. Lett. 112:061103, 2019

Neutrino cross section



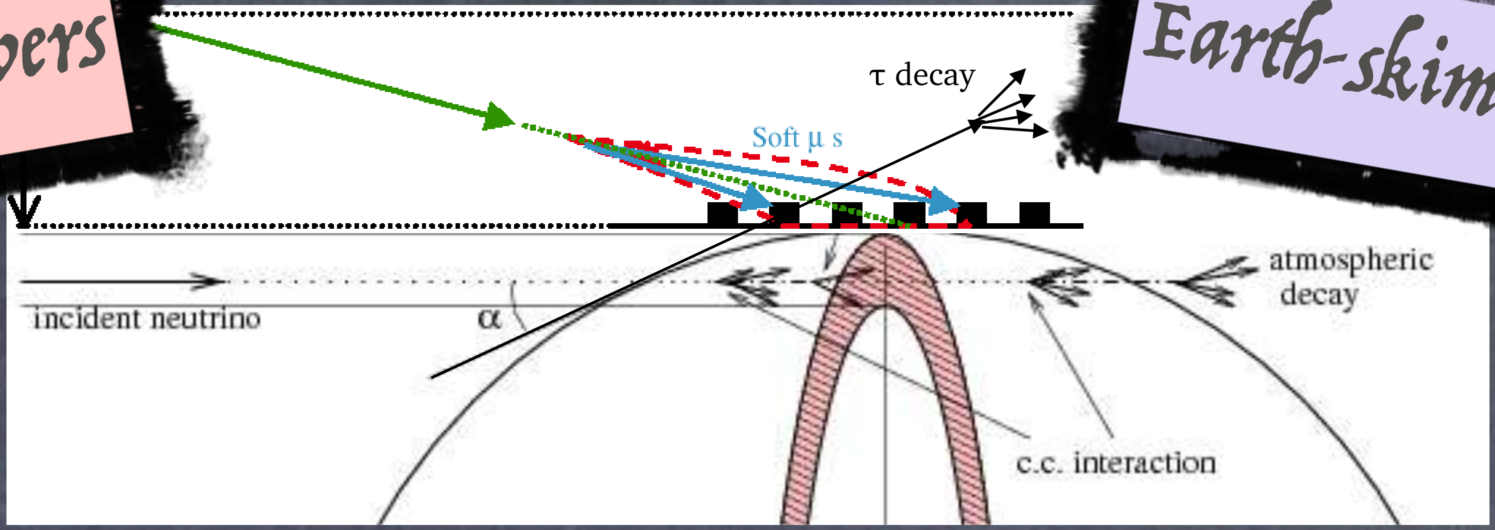
R. Abbasi et al. [IceCube Coll.], Phys. Rev. D104:022001, 2021

Neutrino cross section at UHE (ZeV)

Two types of events

horizontal air-showers

Earth-skimming neutrinos



Mainly electron neutrinos

Just above the horizon

Very penetrating $\propto \sigma$

Only tau neutrinos

Just below the horizon

tau decays $\propto 1/\sigma$

A. Kusenko and T. J. Weiler, Phys. Rev. Lett. 88:161101, 2002

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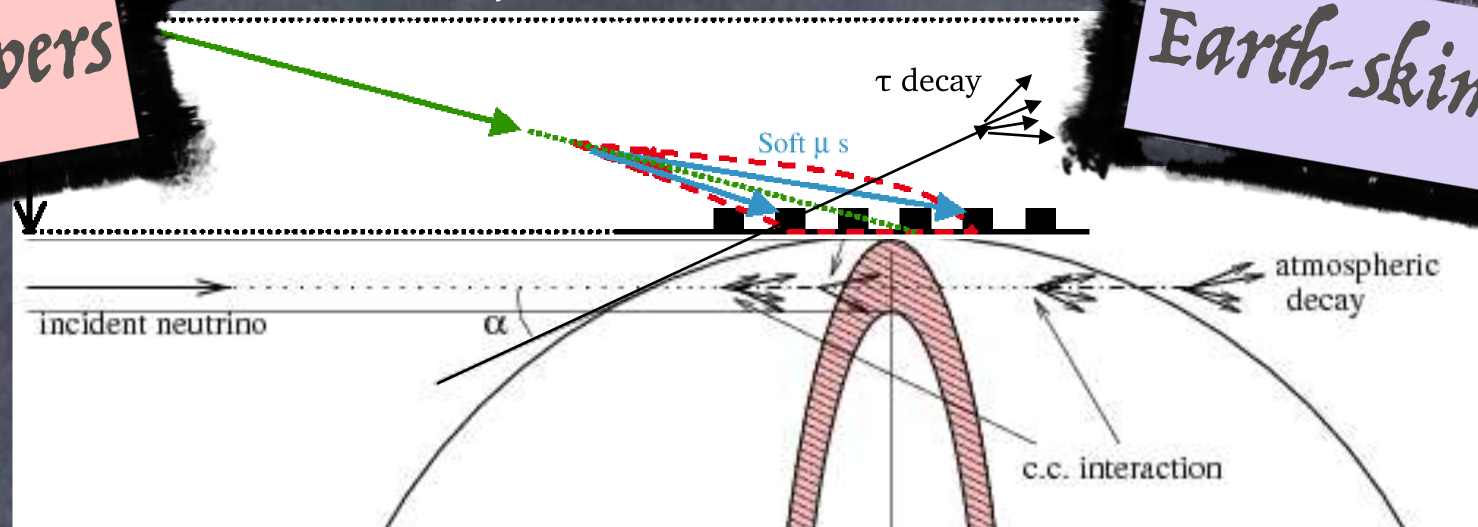

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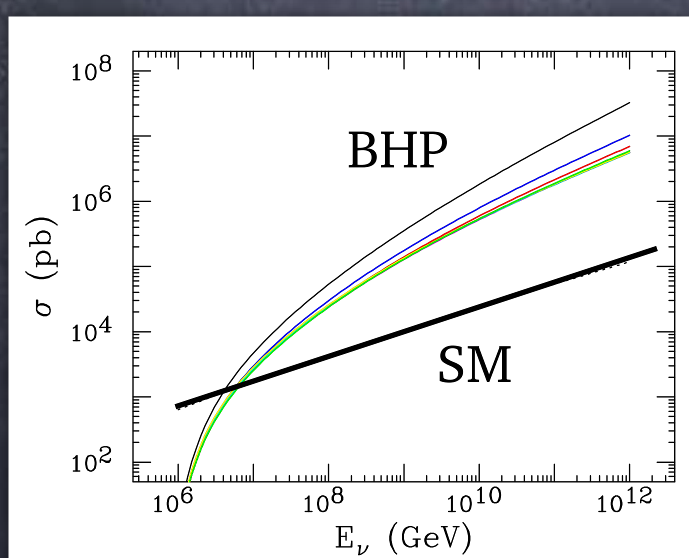
Black hole production



J. L. Feng and A. D. Shapere, Phys. Rev. Lett. 88:021303, 2002

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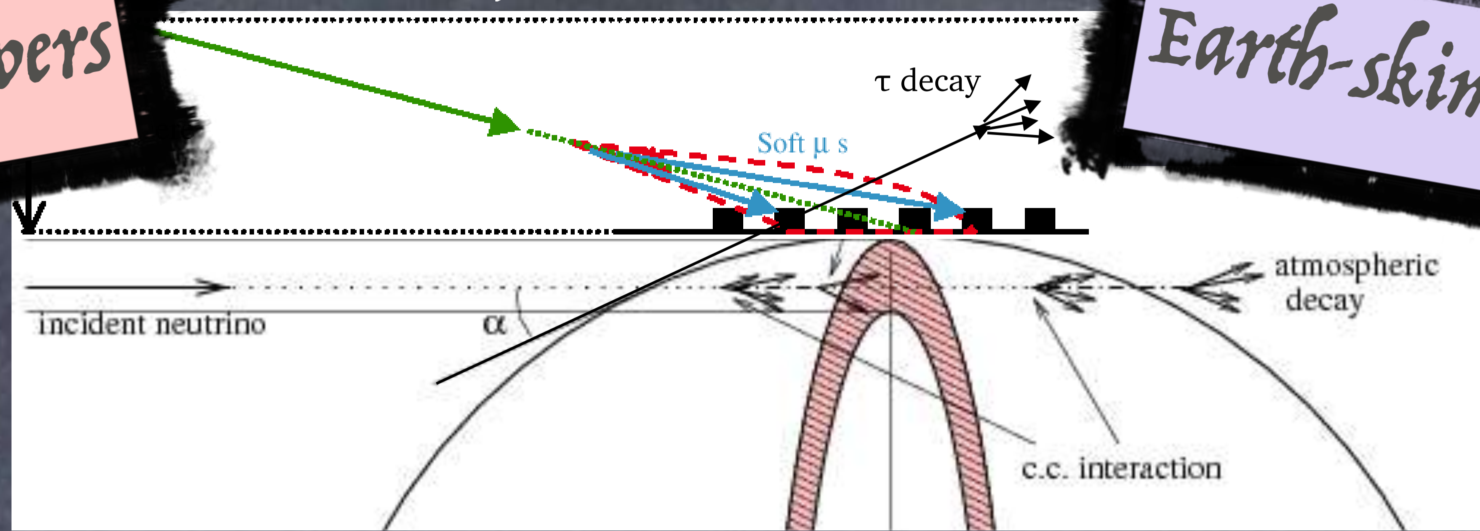

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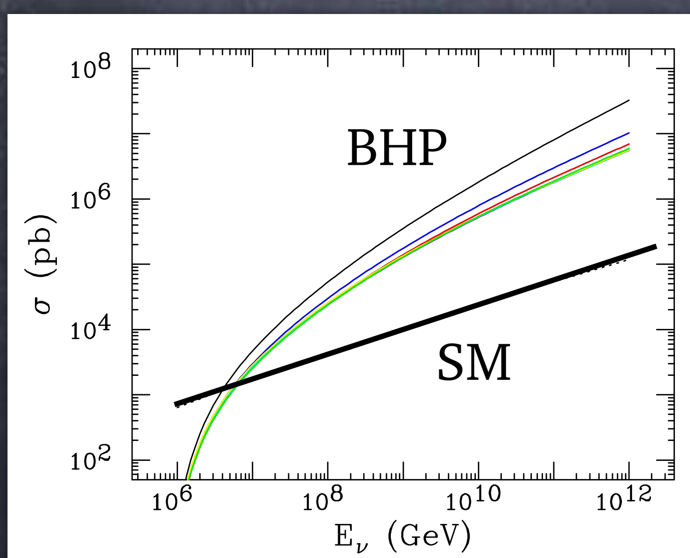
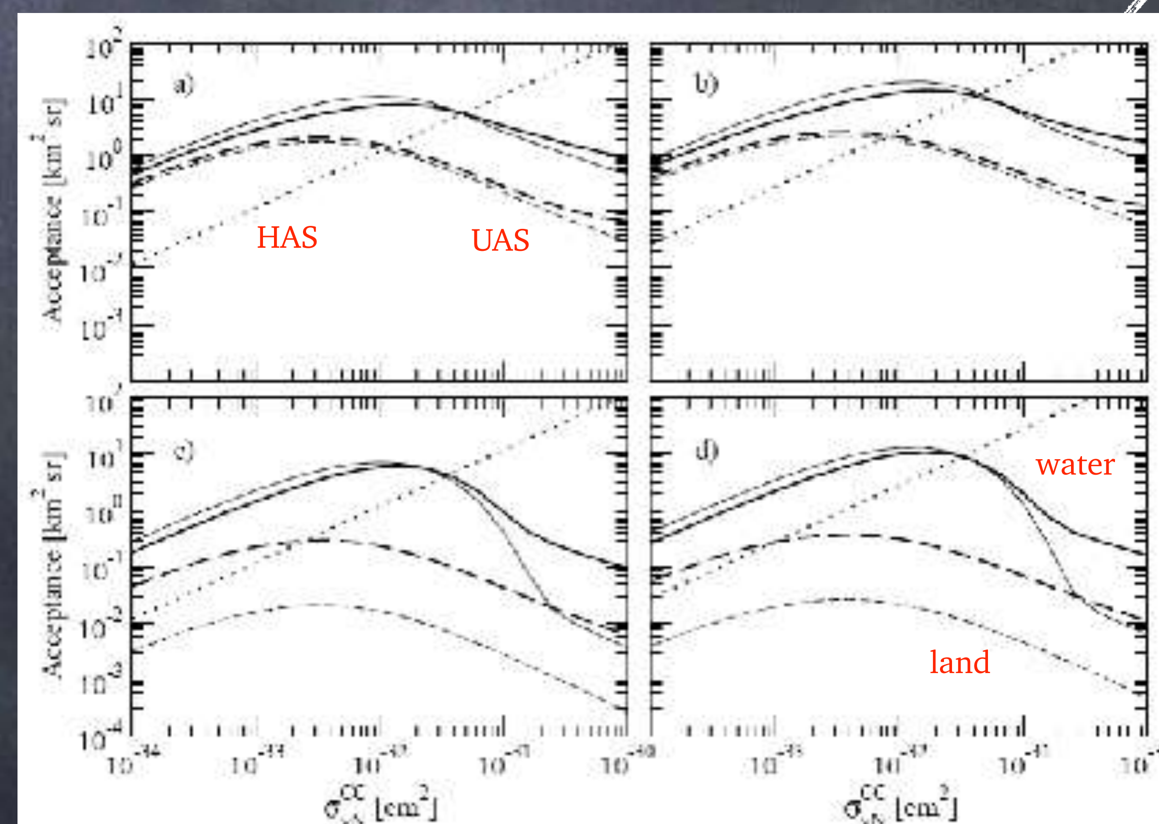


J. L. Feng and A. D. Shapere, Phys. Rev. Lett. 88:021303, 2002

shower development

E_{th}



SPR, A. Irimia and T. J. Weiler, Phys. Rev. D73:083003, 2006

Thanks for your attention!

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