Spontaneous symmetry breaking and Coleman-Weinberg mechanism

Uljana Voznaya

Dubna State University

2021

Plan

- Description of spontaneous symmetry breaking
- Brout-Englert-Higgs mechanism
- Hierarchy and Naturalness problems
- Effective theories approach
- Coleman-Weinberg mechanism
- Model with a scalar and a spinor fields
- Effective low-energy model
- Scales hierarchy
- M parameter
- Conclusions
- Possible development

Description of Spontaneous Symmetry breaking

In quantum theory:

- Brout-Englert-Higgs mechanism
- Coleman-Weinberg mechanism

Brout-Englert-Higgs mechanism

- Suggested in 1964 by Brout, Englert, Higgs and Guralnik, Kibble and Hagen
- Goldstone bosons
- Degrees of freedom rearrangement
- Potential with tachion mass term

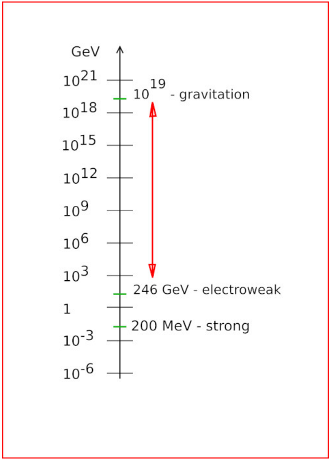
$$V(\varphi) = \frac{m^2}{2}\varphi^2 + \lambda\varphi^4, m^2 < 0 \tag{1}$$

F. Englert and R. Brout, Phys. Rev. Lett. 13 (1964) 321.

P. Higgs.Phys.Ref.D. V.13, N. 16. 1964.

Guralnik, G. S., Hagen, C. R., Kibble, T. W. B. (1964). Physical Review Letters, 13(20), 585–587.

Hierarchy and Naturalness problem



Hierarchy problem:
big difference between energy scales of different interactions

What
is the source of this hierarchy?

Is there any new
physics between the scales?

Hierarchy and Naturalness problem

Naturalness (fine-tuning) problem: corrections to Higgs mass must cancel each other

't Hooft naturalness principle

The quantity is small, if the theory becomes more symmetric, when the quantity tends to zero

Custodial symmetries:

- Dirac fermion mass $m_\Psi \to 0 \longrightarrow$ chyral symmetry
- Mass terms of gauge bosons $m^2 A_\mu A_\mu \to 0 \longrightarrow$ gauge symmetry.

What is such a symmetry in Higgs sector?

Conformal (scale) symmetry

We study breaking of conformal (scale) symmetry

Effective theories approach

- There is a fundamental theory describing physics on all energy scales.
- Effective theories describe physics to certain energy scales.
- Two scales are introduced: regularizaton scale (upper bound) and renormalization scale (the scale of the physics of interest).

Rivat S. Renormalization scrutinized. //Stud.Hist.Phil.Sci.B 68 (2019) 23-39

Coleman-Weinberg mechanism

- Microscopic approach
- Radiative corrections
- Effective potential formalism
- Summarize graphs with loops
- Dynamical symmetry breaking

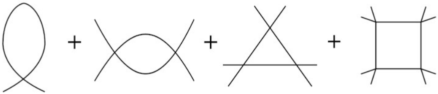
S.Coleman, E.Weinberg. Radiative Corrections as the Origin of Spontaneous Symmetry Breaking. Phys. Ref. D., V.6, N.7. 1973.

Coleman-Weinberg mechanism

For $\lambda\varphi^4$ theory

$$V_{eff}^{tree} = \lambda\varphi^4 \tag{2}$$

$$V_{eff} = \lambda\varphi^4 + \Delta V_{sc} \tag{3}$$



Regularizaton - cut off UV divergence, introduce Λ
Renormalizaton - get rid of IR (logarithmic) divergence, introduce M

Coleman-Weinberg mechanism

$$\Delta V_{sc} = i \int \frac{d^4 k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{1}{2n} \left(\frac{\frac{1}{2}\lambda\varphi_c^2}{k^2 + i\epsilon} \right)^n = \tag{4}$$

$$= \frac{\lambda\Lambda^2}{64\pi^2}\varphi_c^2 + \frac{\lambda^2\varphi_c^4}{256\pi^2}\left(\ln \frac{\lambda\varphi_c^2}{2\Lambda^2} - \frac{1}{2} \right) = \tag{5}$$

$$= \frac{\lambda\varphi_c^4}{4!} + \frac{\lambda^2\varphi_c^4}{256\pi^2}\left(\ln \frac{\varphi_c^2}{M^2} - \frac{25}{6} \right)$$

Model with a scalar and a spinor fields

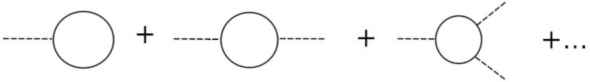
$$L_c = \frac{1}{2}(\partial_\mu \varphi_c)^2 - \frac{\lambda}{4!}\varphi_c^4 + i\overline{\Psi_c}\gamma_\mu\partial_\mu\Psi_c - y\varphi_c\overline{\Psi_c}\Psi_c \quad (6)$$

Effective potential:

$$V_{eff} = V_{tree} + \Delta V_{sc} + \Delta V_f \quad (7)$$

Model with a scalar and a spinor fields

Fermion loops contribution:



$$\Delta V_f = -N_c \int \frac{d^4k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{n!}{n} \frac{1}{n!} \frac{(y\varphi_c)^n Tr(\hat{k}+m)^n}{(k^2-m^2+i\epsilon)^n} \to \tag{8}$$

$$\to -N_c \int \frac{d^4k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{(y\varphi_c)^n}{n} \frac{Tr(\hat{k}+m)^n}{(k^2-m^2)^n} \to$$

$$\to -\frac{N_c}{8\pi^2}\left[(y\varphi_c)^2\Lambda^2 + \frac{(y\varphi_c)^4}{2}\left(\ln\frac{(y\varphi_c)^2}{\Lambda^2} - \frac{1}{2}\right)\right]$$

Model with a scalar and a spinor fields

Renormalization conditions:

$$m^2 = \frac{d^2 V}{d\varphi_c^2}\Big|_{\varphi_c=0} \tag{9}$$

$$\lambda = \frac{d^4 V}{d\varphi_c^4}\Big|_{\varphi_c=M} \tag{10}$$

Renormalized 1-loop effective potential:

$$V_{eff}^{ren} = \frac{\lambda\varphi_c^4}{4!} + \frac{\varphi_c^4}{256\pi^2}\left(\ln\frac{\varphi_c^2}{M^2} - \frac{25}{6}\right)\left(\lambda^2 - 16N_c y^4\right) \tag{11}$$

Model with a scalar and a spinor fields

Minimum of the potential:

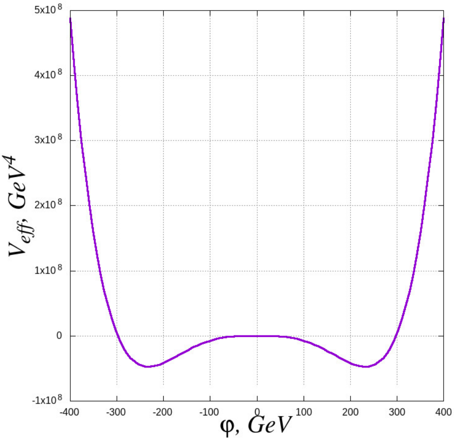
$$\left.\frac{dV_{eff}^{ren}(\varphi_c)}{d\varphi_c}\right|_{\varphi_c=\langle\varphi_c\rangle} = 0 \tag{12}$$

Non-trivial minimum is moved from 0 to the point $\varphi_c = \langle\varphi_c\rangle \equiv v$, derived from the equation

$$\ln\frac{\langle\varphi_c\rangle^2}{M^2} = \frac{11}{3} - \frac{32\pi^2\lambda}{3[\lambda^2 - 16N_c y^4]} \tag{13}$$

Model with a scalar and a spinor fields

Effective potential



Model with a scalar and a spinor fields

Shift of the field:

$$\varphi_c = v + \varphi \tag{14}$$

Scalar field mass:

$$m_\varphi^2 = \frac{\partial^2 V(\varphi + v)}{\partial \varphi^2}\Big|_{\varphi=0} = \frac{\lambda}{2} v^2 + \frac{3v^2}{64\pi^2}\left(\lambda^2 - 16 N_c y^4\right)\left(\ln \frac{v^2}{M^2}\right) \tag{15}$$

From the minimum condition:

$$\left[\lambda^2 - 16 N_c y^4\right] \ln \frac{v^2}{M^2} = \frac{32}{3}\pi^2 + \frac{11}{3}\left[\lambda^2 - 16 N_c y^4\right], \tag{16}$$

we have:

$$m_\varphi^2 = \frac{v^2}{32\pi}\left(\lambda^2 - 16 N_c y^4\right) \tag{17}$$

$$\lambda^2 > 16 N_c y^4 \tag{18}$$

Model with a scalar and a spinor fields

Scalar mass:

$$m_\varphi = \frac{v}{4\pi\sqrt{2}}\sqrt{\lambda^2 - 16N_c y^4} \tag{19}$$

Fermion mass:

$$m_f = yv \tag{20}$$

Effective low-energy model

We build an effective low-energy model applicable near the minimum. Shift of the field:

$$\varphi_c = v + \varphi \tag{21}$$

$$V_{eff}^{1-loop}(v+\varphi) = \frac{\lambda (v+\varphi)^4}{4!} + \\ + \frac{(v+\varphi)^4}{256\pi^2}\left(\ln\frac{(v+\varphi)^2}{M^2} - \frac{25}{6}\right)\left(\lambda^2 - 16 N_c y^4\right) \tag{22}$$

Effective low-energy model

Expanding in series near the minimum:

$$V_0(\varphi) = \frac{m_0^2\varphi^2}{2} + \frac{h_0\varphi^3}{3!} + \frac{\lambda_0\varphi^4}{4!} + \mathcal{O}\left(\varphi^4\frac{\varphi}{M}\right) \qquad (23)$$

Coupling constants and the scalar field mass in the effective model:

$$\lambda_0 = \left.\frac{\partial^4 V_{\mathrm{eff}}^{\mathrm{ren}}}{\partial\varphi^4}\right|_{\varphi=0} = \frac{11}{32\pi^2}\left[\lambda^2 - 16N_c y^4\right], \qquad (24)$$

$$h_0 = \left.\frac{\partial^3 V_{\mathrm{eff}}^{\mathrm{ren}}}{\partial\varphi^3}\right|_{\varphi=0} = \left[\lambda^2 - 16N_c y^4\right]\frac{5v}{32\pi^2} = \lambda_0\frac{5v}{11}, \qquad (25)$$

$$m_0^2 = \left.\frac{\partial^2 V_{\mathrm{eff}}^{\mathrm{ren}}}{\partial\varphi^2}\right|_{\varphi=0} = \frac{\left[\lambda^2 - 16N_c y^4\right]v^2}{32\pi^2} = \frac{\lambda_0 v^2}{11}. \qquad (26)$$

Effective low-energy model

λ in initial and effective models:

$$\lambda_0 \sim \lambda^2 \tag{27}$$

- No continious renormalization group transition between initial and efffective model.
- Now M is the upper bound of the range of applicability of the effective theory.

Relation between v and M:

$$v = M \exp\left\{\frac{11}{6} - \frac{\lambda 16\pi^2}{3[\lambda^2 - 16 N_c y^4]}\right\} \tag{28}$$

Scales hierarchy

From the experiments:

$$m_0 = 125 \text{ GeV}, \quad v = 246 \text{ GeV}, \quad y = 1, \quad N_c = 3. \tag{29}$$

Then the fermion mass:

$$m_f = yv = 246 \text{ GeV} \tag{30}$$

Substituting it into (28), we get

$$M \approx 60 \text{ TeV}. \tag{31}$$

In Coleman-Weinberg method big difference between the scales is generated dynamically

M parameter

- Scale symmetry breaking scale
- Renormalization scale

If M is arbitrary - scale symmetry is unbroken!
If $M = \langle\varphi\rangle$, then

$$\left.\frac{dV}{d\varphi}\right|_{\varphi=\langle\varphi\rangle} = \frac{\lambda\langle\varphi^3\rangle}{3!} + \frac{\lambda^2\langle\varphi^3\rangle}{64\pi^2}\left(\log\frac{\langle\varphi\rangle^2}{\langle\varphi\rangle^2} - \frac{11}{3}\right) = \tag{32}$$

$$= \frac{\lambda\langle\varphi\rangle^3}{3!} - \frac{\lambda^2\langle\varphi\rangle^3}{64\pi^2}\frac{11}{3} = 0,$$

and $\langle\varphi\rangle = 0$ - the case of unbroken symmetry.

Conclusions

Results

- Spontaneous symmetry breaking by Coleman-Weinberg mechanism in the model with a scalar and a spinor field was considered
- Effective low-energy model was constructed
- Regularization parameter M can't be arbitrary in theories with broken conformal (scale) symmetry

Published in: Phys.Part.Nucl.Lett. 18 (2021) 2, 123-127, e-Print: 2007.09750 [hep-ph]

Possible development

Realization of the Coleman-Weinberg mechanism in Standard Model

- Add gauge bosons
- Consider Higgs doublet
- More loops

Thank you for your attention!