Analytical approximations of the non-equilibrium neutrino distribution function in core-collapse supernova and their verification

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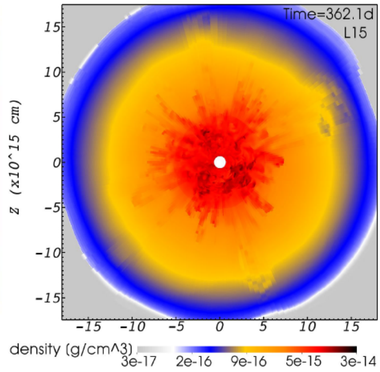
Core-collapse supernova

- The final answer about the nature of a successful supernova explosion has not yet been found
- Neutrino plays a dominant role in supernova explosion, and without its participation, a successful explosion is impossible
- Description of the propagation of neutrino radiation in supernova needs a self-consistent solution of transport equation with hydrodynamics
- Many aspects of neutrino physics in supernova are not included in the simulation of explosion



[Gabler et al, 2021]

Analytical description of neutrino radiation

Neutrino radiation in core-collapse supernova can be described by a the local non-equilibrium distribution function.

As a rule, the collapse of the pre-supernova core is isotropic. The neutrino propagates almost spherically symmetric.

- In this case the neutrino distribution function $f_\nu(r, \theta, \omega)$ can be described by three parameters at each moment of time:

  r is the distance from the center of a proto-neutron star
  θ is the angle between the neutrino momentum and the radial direction
  ω is neutrino energy

- $f_\nu(r, \theta, \omega)$ can be factorized on the angular and energy components:

$$f_\nu(r, \theta, \omega) \simeq N(r)\ \Phi(r, \theta)\ F(r, \omega)$$

The calculations were carried out in the natural system of units $c = k = \hbar = 1$.

Analytic representations of $F(r,\omega)$ and $\Phi(r,\theta)$

Two well-known analytical approximations were suggested for the neutrino energy distribution:

- nominal Fermi-Dirac distribution [Janka H.-T. et al, 1989]:

$$F_F(r,\omega) = \frac{1}{\exp[\omega/T_\nu - a_\nu] + 1}$$

T_ν and a_ν are the nonequilibrium fitting spectral parameters.

- so-called "alpha-fit"[Keil M.T. et al, 2003]:

$$F_\gamma(r,\omega) = \left(\frac{\omega}{\omega_1}\right)^{\gamma-3} e^{-\gamma\ \omega/\omega_1}$$

ω_1 is the neutrino mean energy and γ represents the amount of spectral "pinching"

- Angular neutrino distribution in terms of $y = 1 - cos\ \theta$ can be approximated by simple one-parametric Gaussian function

$$\Phi(r,y) = \exp\left[-\left(A(r)\,y\right)^2\right]$$

[Koptyaeva E. A. 2020; Dobrynina A. A. et al, 2020]

Used numerical data

Analysis of the results of simulations of a supernova explosion. These simulation were performed with the 1D version of the PROMETHEUS-VERTEX code [L. Huedepohl, PhD thesis, Technische Univ. (2014)].

Numerical data features:

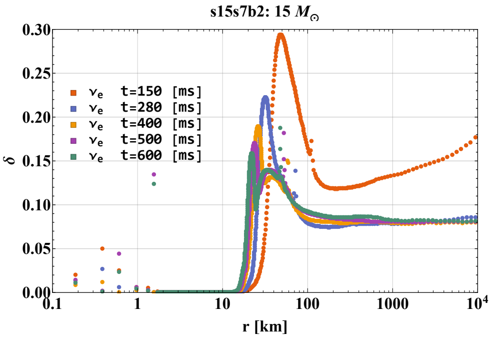
- Specific neutrino intensity $I(\mu, \omega)$ is the main quantity of modeling, where $\mu = cos\,\theta$ and ω is the neutrino energy
- Range of neutrino energy is separated on bins and intensity I is integrated over energy in each bin
- The radial grid is represented in the range from 1 to 10^4 km
- Time evolution is in the interval from 150 to 600 ms
- The data are presented for electron neutrinos ν_e, electron antineutrinos $\bar{\nu_e}$ and all other neutrino types $\nu_x = \nu_\mu,\ \bar{\nu_\mu},\ \nu_\tau,\ \bar{\nu_\tau}$

The data were considered for presupernova masses from 11 to 25 $M_\odot$.

Comparison of analytical approximation with data

Verification of analytical approximation for neutrino angle distribution $\Phi(r, y)$ was based on relative deviations from numeric data $J_{data}(r, y)$:

$$\delta = \left(\int_0^2 \left| \Phi_{data}(r, y) - \Phi(r, y) \right| dy \right) \left(\int_0^2 \Phi_{data}(r, y) dy \right)^{-1}$$



The similar preliminary results for verification of $\Phi(r, y)$ were obtained in [Dobrynina A.A. et al, 2020].

Construction of local neutrino distribution function

Based on proposed analytical approximation for the angular neutrino distribution function, we construct two variants of the complete f_ν.

- With nominal Fermi-Dirac approximation:

$$f_\nu(y,\omega) \simeq f_\nu^{(F)}(y,\omega) = N_F \; \frac{e^{-(A\,y)^2}}{\exp[\omega/T_\nu - a_\nu] + 1}$$

  Here $N_F(t,r)$, $A(t,r)$, $a_\nu(t,r)$ and $T_\nu(t,r)$ are parameters of approximation

- With "alpha-fit":

$$f_\nu(y,\omega) \simeq f_\nu^{(\gamma)}(y,\omega) = N_\gamma \; e^{-(A\,y)^2} \; \left(\frac{\omega}{\omega_1}\right)^{\gamma-3} \; e^{-\gamma\;\omega/\omega_1}$$

  Here $N_\gamma(t,r)$, $A(t,r)$, $\omega_1(t,r)$ and $\gamma(t,r)$ are parameters of approximation

Both variants of complete neutrino distribution function are 4-parametric. With results of simulations for different types of neutrino, these parameters were found as functions of the distance from SN center and time after a bounce.

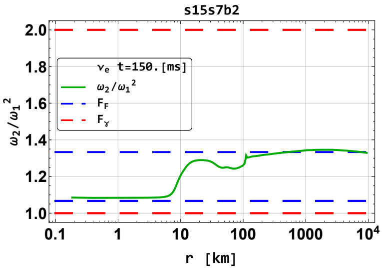
Restriction on used Fermi-Dirac approximation

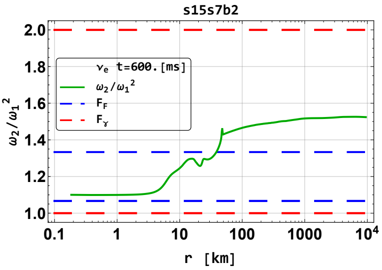
We have obtained spectral parameters from the first two energy moments ω_1 and ω_2:

$$\omega_n = \left(\int_0^\infty \omega^{n+2} f_\nu \, d\omega\right)\left(\int_0^\infty \omega^2 f_\nu \, d\omega\right)^{-1}$$

As analysis shown, the ratio ω_2/ω_1^2 can not be arbitrary $\Rightarrow$

for Fermi-Dirac : $\omega_2/\omega_1^2 \in [16/15, 4/3]$; for alpha-fit : $\omega_2/\omega_1^2 \in [1, 2]$

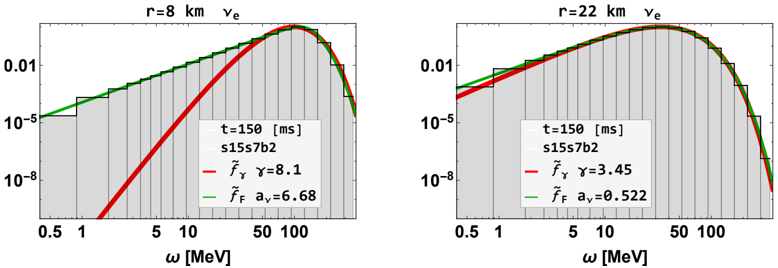




Thus the parameters of nominal Fermi-Dirac approximation can not be obtained in most part of supernova from two first energy moments. However it could be possible using another method for search of parameters.

Comparison of numerical data with spectral approximations at small distances

We normalize the spectrum $\tilde{f}$ of neutrino, which propagate in radial direction $(y = 0)$, to one.



The Fermi-Dirac approximation well describes the numerical data in the inner part of supernova, because of neutrinos are in equilibrium with matter in this area. However alpha-fit differs from numerical data only in low-energy part of spectra, which contain small fraction of all neutrino.

Conclusions

- Results of the 1D numerical simulations of the neutrino angular distribution in core-collapse supernova were analyzed.
- Neutrino angular distribution can be approximated by the one-parametric Gaussian function. It was shown, that the proposed approximation well describes most part of the supernova neutrinos. The accuracy of this approximation varies from a few to tens percent, depending on the part of the supernova and the time after a bounce.
- Using the well-known analytical approximations of the neutrino spectrum in a supernova and proposed analytical angular distribution, two variants of the local nonequilibrium neutrino distribution function were constructed. Parameters, which characterize them, were obtained as functions of the distance from SN center and time after a bounce.
- The nominal Fermi-Dirac approximation well describes the simulation results in the inner part of the supernova, but is less suitable for description of the outer parts.

Analytical approximation of angular distribution

The main goal of our investigation is to find a simple approximation for angular neutrino distribution in supernova.

- To find an approximation we use $J_{data}(y)$, where $y = 1 - \cos\,\theta$. This quantity is the specific intensity $I(y, \omega)$ integrated over energy ω and normalized to one.
- The $J_{data}(y)$ monotonically decreases and has no singularities, but its derivative $J'_{data}(y)$ has a well-defined minimum
- Good approximation for derivative nearby a minimum can be presented as: $J'_{data}(y) \propto -y\ \exp\left[-(A\,y)^2\right]$
- Thus, the Gaussian function could be good approximation for angular neutrino distribution in supernova: $\Phi(y) = \exp\left[-(A\,y)^2\right]$