Why quantum field theory in curved space–time is very far from being well understood

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Quantum fields

- The goal of QFT is to calculate correlation functions, which are building blocks for observables: cross–sections and/or currents;
- In high energy particle physics one uses only Poincaré invariant states:

$$\Big\langle \phi(\underline{x}_1)\dots\phi(\underline{x}_n)\Big\rangle_0 = F_0\left(\left\{\,\left|\Delta\underline{x}_{jk}\right| \pm i\,0\,\mathrm{sign}\Delta t_{jk}\right\}\right),$$

where $\underline{x} = (t,\, \vec{x})$ and $\Delta\underline{x}_{jk} = \underline{x}_j - \underline{x}_k$, $j = \underline{1,\,n}$.

- In condensed matter theory one frequently restricts attention to (*stationary*) thermal states:

$$\Big\langle \phi(\underline{x}_1)\dots\phi(\underline{x}_n)\Big\rangle_T = F_T\left(\left\{\Delta t_{jk}\right\}\Big|\vec{x}_1,\dots,\vec{x}_n\right), \quad \Delta t_{jk} = t_j - t_k.$$

Or slight deviations from them, which are close to equilibrium. The action of the theory can be Poincaré invariant, but the state in such a theory does not have to respect the symmetry.

Quantum fields in early Universe

- There is a good reason to consider correlation functions of such states due to our everyday experience: present day Universe has a very small curvature and we mostly deal with processes which are very close to equilibrium, even if they are out of equilibrium;
- However, the common wisdom is to consider also isometry invariant or very special states in de Sitter space–time;
- Meanwhile in the early Universe, if its metric was truly very close to de Sitter one with a GUT scale curvature, the situation was highly nonstationary. There is no any reason that the rapid expansion has started from a highly symmetric state with isometry invariant correlation functions:

$$\left\langle \phi(\underline{x}_1) \dots \phi(\underline{x}_n) \right\rangle_U = F_U\left(\underline{x}_1, \dots, \underline{x}_n\right).$$

Furthermore, the metric was only approximately equivalent of the de Sitter one.

Observables in early Universe

- After a rapid expansion such correlation functions can become almost spatially homogeneous:

$$\left\langle \phi(\underline{x}_1)\dots\phi(\underline{x}_n)\right\rangle_{HU} \approx F_{HU}\left(t_1,\dots,t_n \middle| \{\Delta\vec{x}_{jk}\}\right).$$

  However, there is no reason for them to become simultaneously stationary, dependent only on Δt_{jk}, in the time–dependent background;
- When the curvature of the Universe is of GUT scale, observables are various stress–energy fluxes. (In–Out scattering cross–sections are not well defined.) Such observables can be measured only indirectly — e.g. via their backreaction on the background geometry.
- Furthermore, there is no reason to assume that stress–energy fluxes in early Universe were separable into anything like particles.

Quantum fields out of equilibrium

- QFT does have a tool to calculate correlation functions even in such unusual conditions;

- To get an unambiguous answer for correlation functions in non–stationary situations one has to specify an initial Cauchy surface, a basis of modes, an initial state and then use the Schwinger-Keldysh rather than the Feynman diagrammatic technique.

- In the described circumstances loop corrections do not just lead to UV renormalization of various coupling constants and masses, they also contain IR secular memory effects, which are totally absent in equilibrium. These loop effects are sensitive to initial and boundary conditions and can provide corrections to correlation functions, which can be even grater than tree–level contributions.

Why all these observations are important for the physics in the early Universe?

- If correlation functions are not isometry invariant, then $\langle T_{\mu\nu} \rangle$ is not proportional to $g_{\mu\nu}$. Furthermore, loop secular memory effects can strongly affect $\langle T_{\mu\nu} \rangle$, which can lead to the screening of the cosmological constant.
- Initial state of QFT in the early Universe (of GUT scale curvature) does not have to be a thermal state of a standart matter (dust or radiation). It can be any quantum state. We don't really know which state.
- States with plankian distribution in curved backgrounds (for exact modes) with generic temperature T lead to singularities in $\langle T_{\mu\nu} \rangle$ on the horizons 2010.10877, 2005.13952, 2106.01791.

Equilibrium vs. non–stationary

- Time evolution of a correlation function:

$$\left\langle \hat{O} \right\rangle (t) \equiv \left\langle \psi_0 \left| \overline{T} e^{i \int_{t_0}^{t} dt' \, \hat{H}(t')} \, \hat{O} \, T e^{-i \int_{t_0}^{t} dt' \, \hat{H}(t')} \right| \psi_0 \right\rangle ,$$

where $\hat{H}(t) = \hat{H}_0(t) + \hat{V}(t)$. True both in Srodinger and Heisenberg representations.

- In the interaction representation:

$$\left\langle \hat{O} \right\rangle (t) = \left\langle \psi_0 \right| \hat{S}^{+}(t, t_0) \, \hat{O}_0(t) \, \hat{S}(t, t_0) \left| \psi_0 \right\rangle = \left\langle \psi_0 \right| \hat{S}^{+}(+\infty, t_0) \, T \left[\hat{O}_0(t) \, \hat{S}(+\infty, t_0) \right] \left| \psi_0 \right\rangle ,$$

where $\hat{S}(t, t_0) = T e^{-i \int_{t_0}^{t} dt' \, \hat{V}_0(t')}$. The dependence on t_0 is of crucial importance here.

Equilibrium situation

- Equilibrium situation is when:
  1. The normal ordered free Hamiltonian $\hat{H}_0$ is time independent and bounded from below;
  2. The expectation value should be taken over the ground state of $\hat{H}_0$: $|\psi_0\rangle = |0\rangle$, $\hat{H}_0\ |0\rangle = 0$;
  3. Interaction term, $\hat{V}$, is turned on adiabatically after t_0 and then switched off adiabatically after t. In effect we have to make the substitution as follows:
     $\hat{S}(+\infty, t_0) \to \hat{S}_{tt_0}(+\infty, -\infty)$.
- Then $\left|\left\langle 0\left|\hat{S}\right|0\right\rangle\right| = 1$ and $\left\langle n \neq 0\left|\hat{S}\right|0\right\rangle = 0$, where $\hat{S} \equiv \hat{S}_{tt_0}(+\infty, -\infty)$, and

$$\left\langle \hat{O} \right\rangle(t) = \sum_n \left\langle 0\right| \hat{S}^+ \left|n\right\rangle \left\langle n\right| T\left[\hat{O}_0(t)\,\hat{S}\right] \left|0\right\rangle =$$

$$= \frac{\left\langle 0\right| T\left[\hat{O}_0(t)\,\hat{S}\right] \left|0\right\rangle}{\left\langle 0\right| \hat{S} \left|0\right\rangle}.$$

- Consider e.g.

$$S = \int d^D x \sqrt{|g|} \left[\frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + \frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4\right],$$

in the background: $ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$.

- The mode expansion of the field operator is as follows:

$$\hat{\phi}(t, \vec{x}) = \int \frac{d^{D-1}\vec{p}}{(2\,\pi)^{D-1}} \left[\hat{a}_{\vec{p}}\, f_p(t)\, e^{-i\,\vec{p}\,\vec{x}} + \text{h.c.}\right].$$

- In Schwinger–Keldysh technique every field is essentially characterized by two propagators:

$$i\, D_0^R\left(t_1,\, t_2\middle|\vec{p}\right) \equiv i\,\theta(t_2 - t_1)\left[\hat{\phi}(t_1, \vec{p}),\, \hat{\phi}(t_2, -\vec{p})\right] =$$

$$= \theta(t_2 - t_1)\, \text{Im}\Big(f_p(t_1)\, f_p^*(t_2)\Big).$$

The propagator is state independent.

The Keldysh propagator

- The Keldysh propagator:

$$i\, D_0^K\left(t_1,\, t_2 \middle| \vec{p}\right) \equiv \frac{1}{2}\left\langle \left\{ \hat{\phi}(t_1, \vec{p}),\, \hat{\phi}(t_2, -\vec{p}) \right\} \right\rangle =$$

$$= \left(\frac{1}{2} + n_p^0\right) f_p(t_1)\, f_p^*(t_2) + \kappa_p^0\, f_p(t_1)\, f_{-p}(t_2) + \text{c.c.}\,.$$

- It characterizes the state of the theory:

$$\left\langle \hat{a}^+_{\vec{p}}\, \hat{a}_{\vec{q}} \right\rangle \equiv \sum_n \rho_n^0 \left\langle n \middle| \hat{a}^+_{\vec{p}}\, \hat{a}_{\vec{q}} \middle| n \right\rangle = n_p^0\, \delta\left(\vec{p} - \vec{q}\right),$$

$$\left\langle \hat{a}^+_{\vec{p}}\, \hat{a}^+_{\vec{q}} \right\rangle^* = \left\langle \hat{a}_{\vec{p}}\, \hat{a}_{\vec{q}} \right\rangle = \kappa_p^0\, \delta\left(\vec{p} + \vec{q}\right),$$

If the state is spatially homogeneous.

Thermalization in flat space–time

- In Minkowski space–time: $f_p(t) = \frac{e^{-i\,\omega_p\,t}}{\sqrt{2\,\omega_p}}$, where $\omega_p = \sqrt{\vec{p}^2 + m^2}$.
- If $\kappa_p^0 = 0$, but $n_p^0 \neq 0$ the two–loop correction to the Keldysh propagator in the limit $t \equiv \frac{t_1+t_2}{2} \gg |t_1 - t_2|$ is as follows:

$$n_p^{0+2}(t) \approx n_p^0 + \lambda^2 \cdot (t - t_0) \cdot \frac{I\left[n^0\right]}{\omega_p},$$

where

$$I\left[n^0\right] \propto \int \frac{d^{D-1}\vec{q}_1\, d^{D-1}\vec{q}_2\, d^{D-1}\vec{q}_3}{\omega_1\,\omega_2\,\omega_3}\,\delta^{(D)}\left(\underline{p} + \underline{q}_1 - \underline{q}_2 - \underline{q}_3\right) \times$$
$$\times \Big[\left(1+n_p^0\right)\,\left(1+n_1^0\right)\,n_2^0\,n_3^0 - n_p^0\,n_1^0\,\left(1+n_2^0\right)\,\left(1+n_3^0\right)\Big].$$

Thermalization in flat space–time

- Violation of the time–translational symmetry.
- IR catastrophe: Unlike the stationary situation, the parameter t_0 cannot be taken to the past infinity.
- For the plankian level–population, $n_p^T = \frac{1}{e^{\omega_p/T}-1}$, the collision integral vanishes $I\left[n^T\right] = 0$. Energy conservation!
- Even if λ is small, after a long enough evolution time, $t - t_0 \to \infty$, the secular loop correction becomes of the same order as the tree–level contribution $\lambda^2 (t - t_0) I/\omega \sim 1$.
- Unlike the UV renormalization, in this IR case there is a clear grading between contributing diagrams.
- Classical kinetic equation resums IR loop corrections.

Thermalization with anomalous quantum expectation values

- Whats if we choose a state with modes as follows: $f_p(t) = \alpha_p \frac{e^{-i\,\omega_p\,t}}{\sqrt{2\,\omega_p}} + \beta_p \frac{e^{i\,\omega_p\,t}}{\sqrt{2\,\omega_p}}$?
- Conditions:
  – $|\alpha_p|^2 - |\beta_p|^2 = 1$, to have the proper commutation relations for $\hat{a}_{\vec{p}}$ and $\hat{a}^+_{\vec{p}}$ and for the field and its conjugate momentum;
  – One should also demand that $\beta_p \to 0$, as $\omega_p \to \infty$, to have the proper (Hadamard) UV behaviour.
- To describe dynamics close to equilibrium one needs a system of kinetic equations for level populations and anomalous expectation values
- If β_p is small, then thermalization does happen (see arXiv:2110.00454).

Peculiarities in curved backgrounds

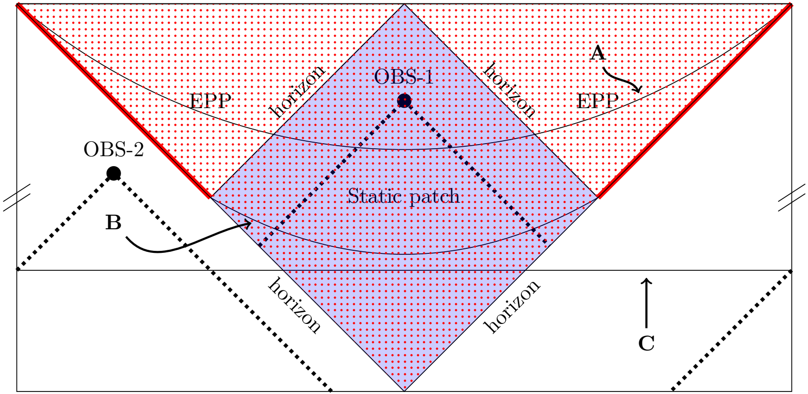

Figure: Penrose diagram of the global de Sitter space–time. Red dotted region is the expanding Poincaré patch, velvet coloured region is the static patch. "A", "B" and "C" are Cauchy surfaces in different patches.

The same sort of problems do also appear in flat spacetime

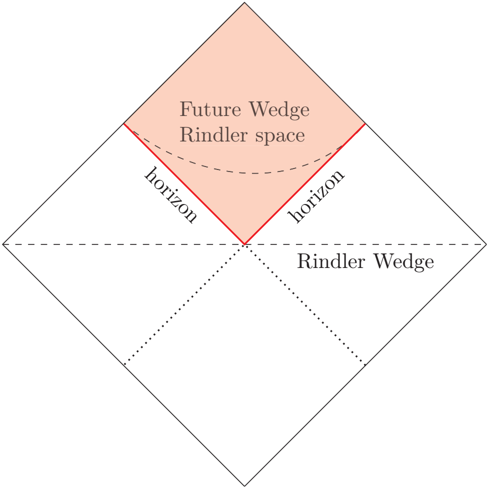

Figure: The dashed lines depict the Cauchy surfaces in various charts arXiv:2106.01791. Only for invariant states the situation does not depend on the patch (paper in preparation).

Analytic continuation

- In symmetric spaces there are invariant states for which propagators are maximally analytic functions of geodesic distances: $G(x, y) = G(l_{xy})$.
- For such states independently of the Patch loop corrections can be mapped to the Euclidian signature, e.g.:

$$\int_{Eucl} dy\, dz\, G(l_{xy})\, G^3(l_{yz})\, G(l_{zw}) =$$
$$= \int_{Patch} dy\, dz\, G(l_{xy})\, G^3(l_{yz})\, G(l_{zw}).$$

These are the states that we deal with in high energy physics.

- For states out of equilibrium that is not true.

E.g. in expanding Poincare patch of de Sitter space–time

- Expanding patch: $ds^2 = -dt^2 + e^{2t}\, d\vec{x}^2$.
- Bunch–Davies modes: $f_p(t) \propto e^{\frac{D-1}{2}\, t}\, H^{(1)}_{i\mu}\left(p\, e^{-t}\right)$, where $\mu = \sqrt{m^2 - \left(\frac{D-1}{2}\right)^2}$.
- In the expanding Poincare patch for the Bunch–Davies state one obtains secular growth rather than secular divergence:

$$n_p^{(2)}(t) \propto \lambda^2 \log\left(\frac{\mu}{p\, e^{-t}}\right) \sim \lambda^2\, t,$$

$$\text{and} \quad \kappa_p^{(2)}(t) \propto \lambda^2 \log\left(\frac{\mu}{p\, e^{-t}}\right) \sim \lambda^2\, t,$$

- In x–space:

$$G_{0+2}(Z_{xy}) \approx \left[1 + \lambda^2\, K\, \log Z_{xy}\right] G_0(Z_{xy}), \quad |Z_{xy}| \to \infty,$$

where K is some constant and Z_{xy} is hyperbolic distance.

In contracting Poincare patch of de Sitter space–time

- Contracting patch: $ds^2 = -dt^2 + e^{-2t}\, d\vec{x}^2$. Time reversal of the expanding patch.
- Now in the loops one sees the secular divergence:

$$n_p^{(2)}(t), \quad \kappa_p^{(2)}(t) \propto \begin{cases} \lambda^2 \log\left(\frac{p\, e^t}{p\, e^{t_0}}\right) \sim \lambda^2 (t - t_0) & p\, e^t < \mu, \\ \lambda^2 \log\left(\frac{\mu}{p\, e^{t_0}}\right) & p\, e^t > \mu. \end{cases}$$

- Loop corrected propagator is not a function of the geodesic distance anymore. For any initial state!
- Global de Sitter contains both expanding and contracting patches. The situation there is similar to the one in contracting patch.

Conclusions

- Secular memory loop effects are generic property of QFT out of equilibrium.
- No energy conservation in curved backgrounds: unlike high energy QFT out of equilibrium one has to work as in open condensed matter system.
- In general the resummation is unsolved problem. Partial solution of the problem is given in 2105.05039, 1901.07293.
- The question is if equilibration happens before the strong backreaction on the background geometry or after?

THANKS !