Islands and Quantum Explosions of Black Holes

I. Aref'eva

Steklov Mathematical Institute, RAS

International Conference "Advances in Quantum Field Theory"
Dedicated to 490-th Anniversary of Vladimir Belokurov,
Konstantin Chetyrkin, Dmitry Kazakov, Nikolay Krasnikov,
Anatoly Radyushkin, Vladimir Smirnov, Alexey Vladimirov

October 11–**14, 2021, JINR, Dubna**


RUSSIA, DUBNA
11–14 OCTOBER, 2021
CDXC = LXX x VII
CDXC-th
Anniversary

One more formula: 2121-490 =1631

This formula shows when the magnificent seven was born

Actual events for us at this period

William Oughtred has proposed to use the symbol x for the multiplication.

Thomas Harriot has introduced < and > for less and bigger

1631 in Russia - during the reign of Mikhail Fedorovich Romanov

The broad scientific program of the conference covers the following actual topics:

- **Renormalization Theory**
- **Multiloop Calculations**
- **Amplitudes**
- **Perturbative QCD**
- **Path Integrals**
- **Effective Theories**
- **Physics Beyond the SM**
- **Cosmology and Dark Matter**
- **Gravity**

Information paradox

The black hole information paradox is one of the fundamental problems in physics (quantum mechanics, thermodynamics and the theory of general relativity)

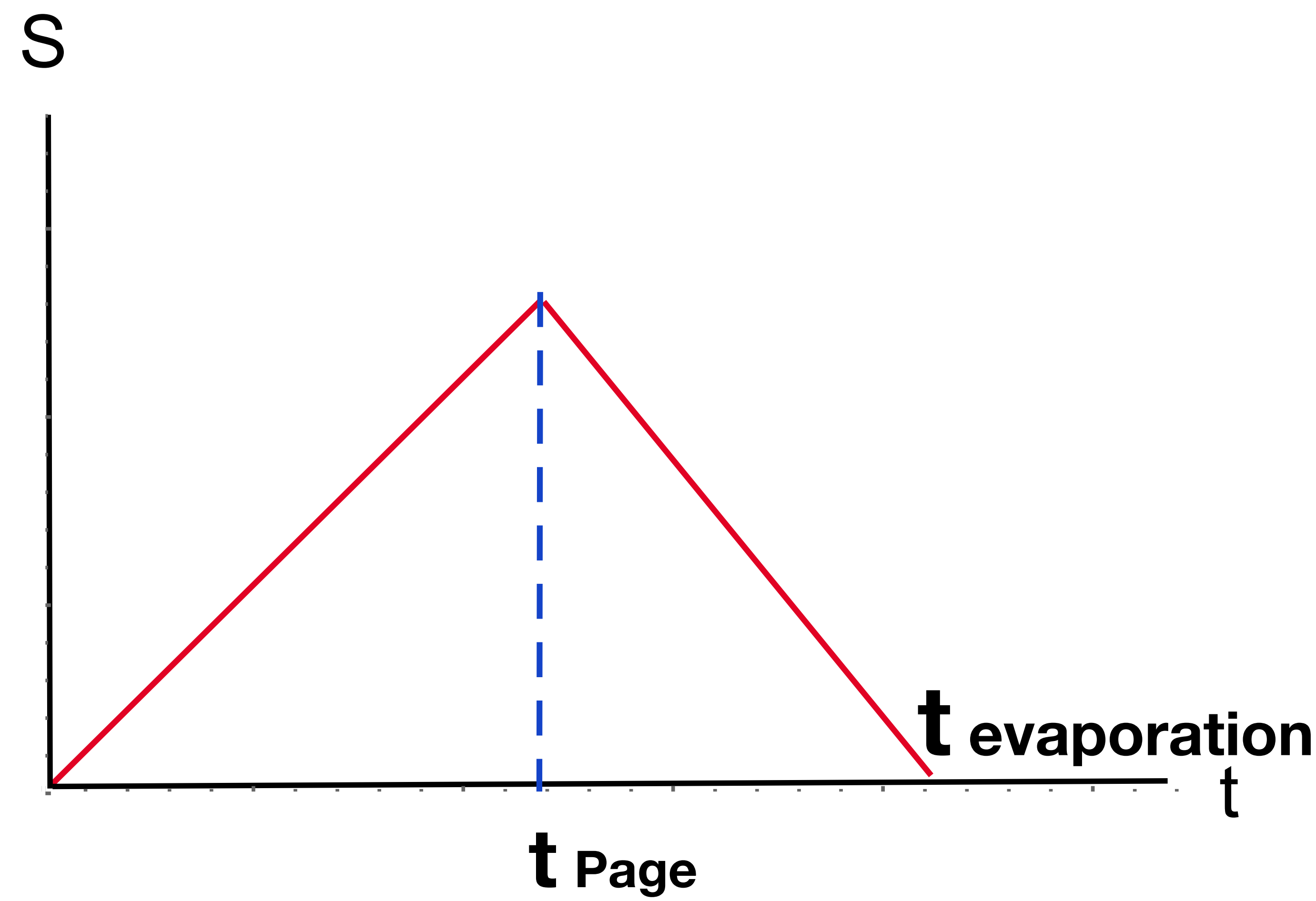
Entropy of Hawking radiation of black holes grows up to infinity during evaporation and it is a manifestation of the information paradox.

This result is contrary to unitarity:
the entanglement entropy has to be zero at the end of the evaporation process since
the final state still must be the pure state.

S.W. Hawking, Particle creation by black holes, CMP 43 (1975) 199.

Information paradox

This increase contrasts with Page's hypothetical behavior, in which entropy decreases after the so-called Page time and which ensures the unitarity of quantum mechanics



D.N.Page, Information in black hole radiation, PRL 71 (1993) 3743

Information paradox

An approach to treating the problem of black hole information was proposed

G. Penington, 1905.08255

A.Almheiri, N.Engelhardt, D.Marolf,H.Maxfield,1905.08762

"Island formula" for the entanglement entropy of Hawking radiation (based on quantum extremal surfaces)

$$S(R) = \min\left\{ \underset{\mathcal{I}}{\mathrm{ext}} \left[\frac{\mathrm{Area}(\partial\mathcal{I})}{4G} + S_{\mathrm{matter}}(R \cup \mathcal{I}) \right] \right\}$$

$\mathcal{I}$ is the island, $\mathrm{Area}[\partial\mathcal{I}]$ its boundary area

S_{matter} is the von Neumann entropy $S_{\mathrm{vN}}(R \cup I)$ of union of the island and the region R.

An extremization on any possible island and then taking the minimum entropy is supposed.

Information paradox

$$S(R) = \min\left\{\underset{\mathcal{I}}{\mathrm{ext}}\left[\frac{\mathrm{Area}(\partial\mathcal{I})}{4G} + S_{\mathrm{matter}}(R\cup\mathcal{I})\right]\right\}$$

$$S_{gen} = S_{gr} + S_{vN}$$

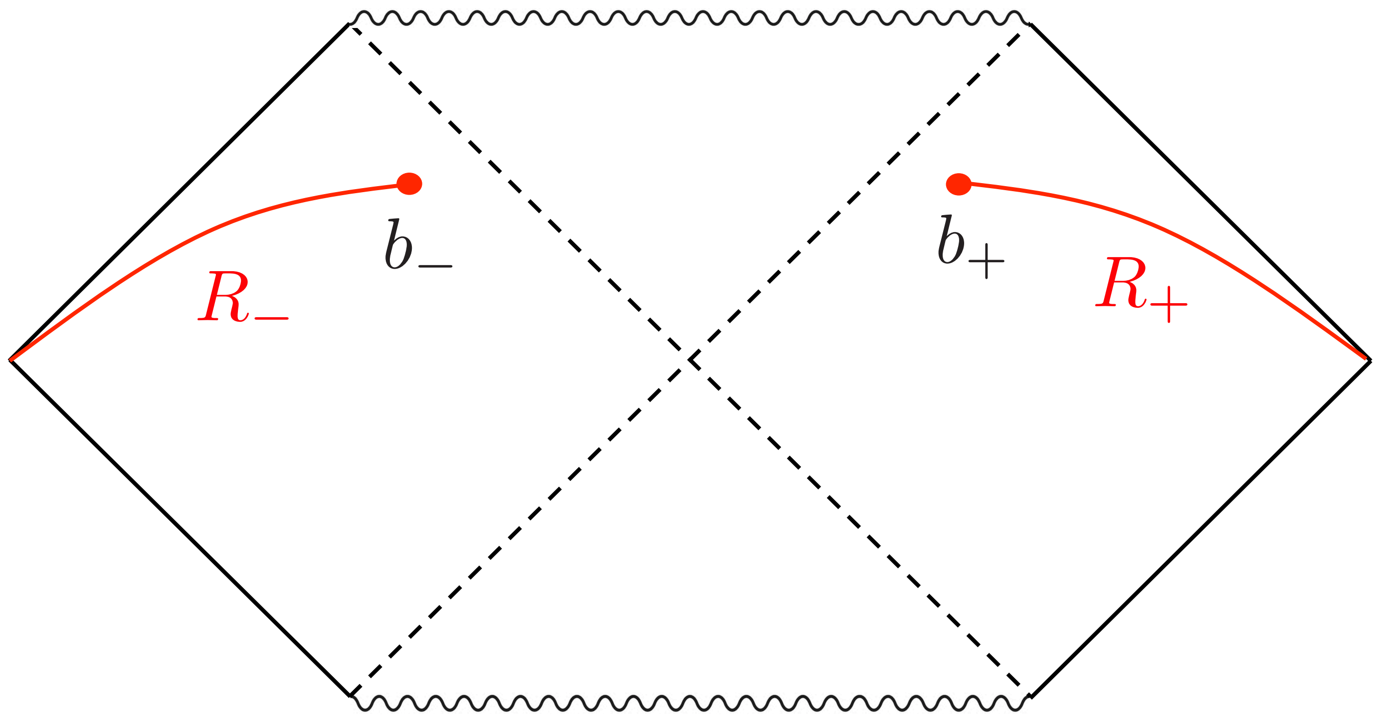
The entanglement entropy of Hawking radiation S(R) is identified with the generalized entropy S_{gen} giving the minimum value over the choice of location of the islands.

For 2 dim gravity the island rule has been derived by making use of replica trick

Information paradox

One of simple example explicitly demonstrated how an island can help to make bounded entanglement entropy of the Hawking radiation is two sided black hole

K.Hashimoto, N.Iizuka, Y.Matsuo, 2004.05863

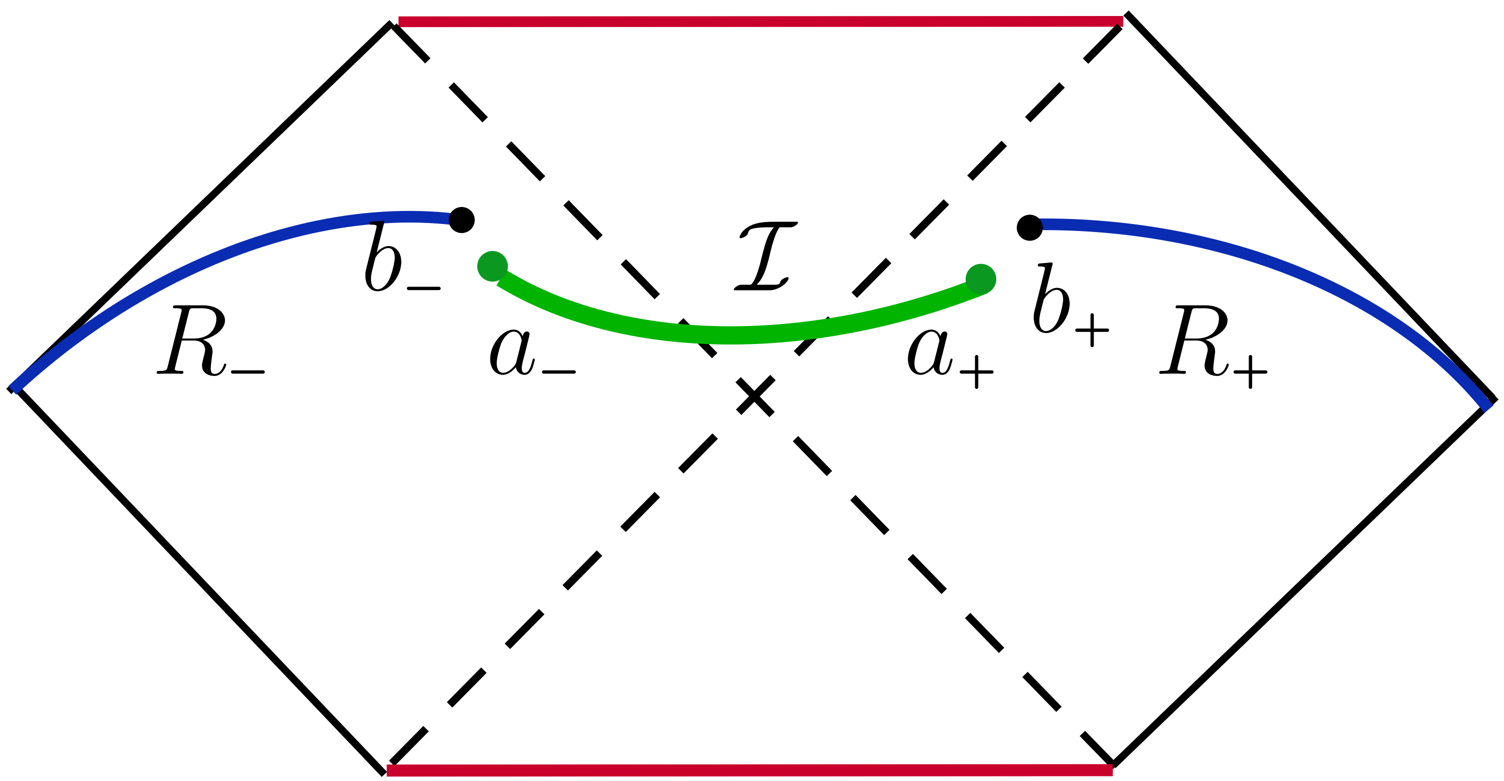


Penrose diagram of the static Schwarzschild spacetime

$$S_{n\mathcal{I}} = \frac{2\pi b^2}{G_N} + \frac{c}{6}\log\left[\frac{16 r_h^2 (b - r_h)}{b}\cosh^2\frac{t_b}{2r_h}\right]$$

Linear in time at large time

Information paradox



Penrose diagram of the static Schwarzschild spacetime with an island

Hawking radiation has two parts R_+ and R_-

The boundaries of I are located at a_+ and a_-

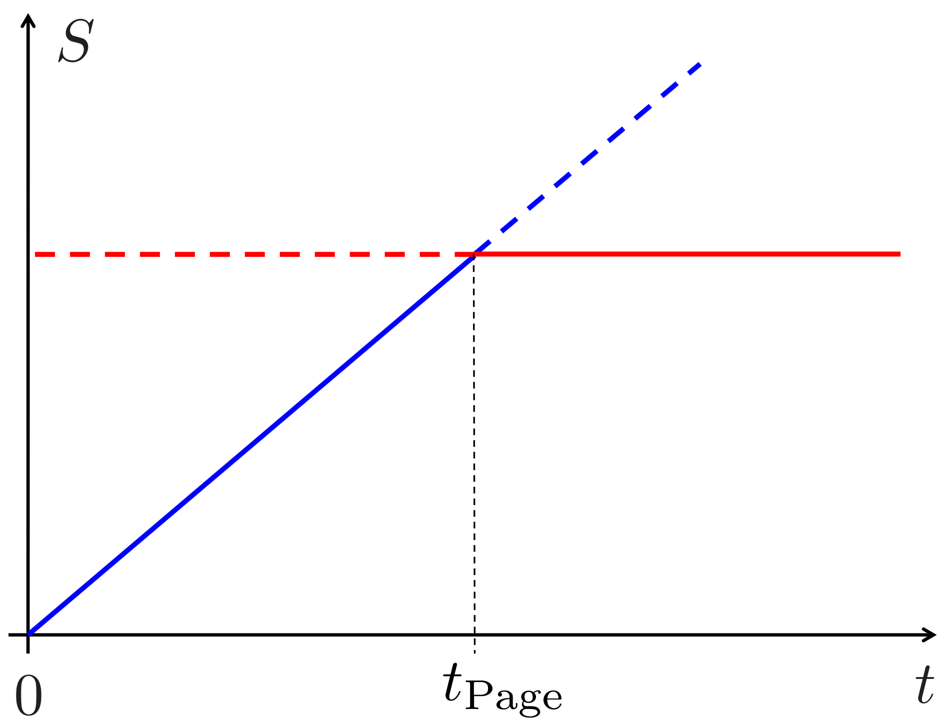
$$S_{\mathcal{I}} = \frac{2\pi r_h^2}{G} + \frac{c}{6}\frac{b - r_h}{r_h} + \frac{c}{6}\log\frac{16 r_h^3 (b - r_h)^2}{G^2 b}$$

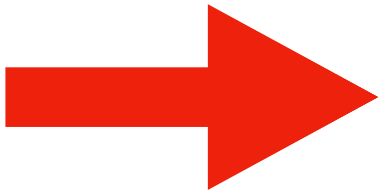
No time dependence

Information paradox

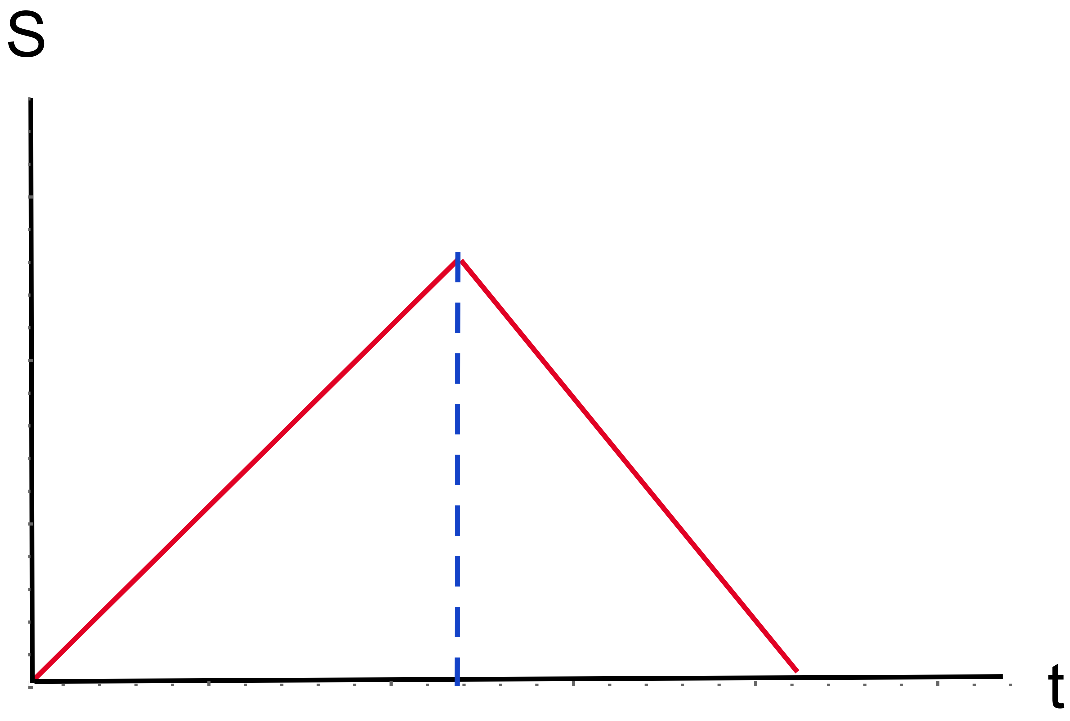
The Page curve for the

eternal Schwarzschild black hole





evaporating black hole

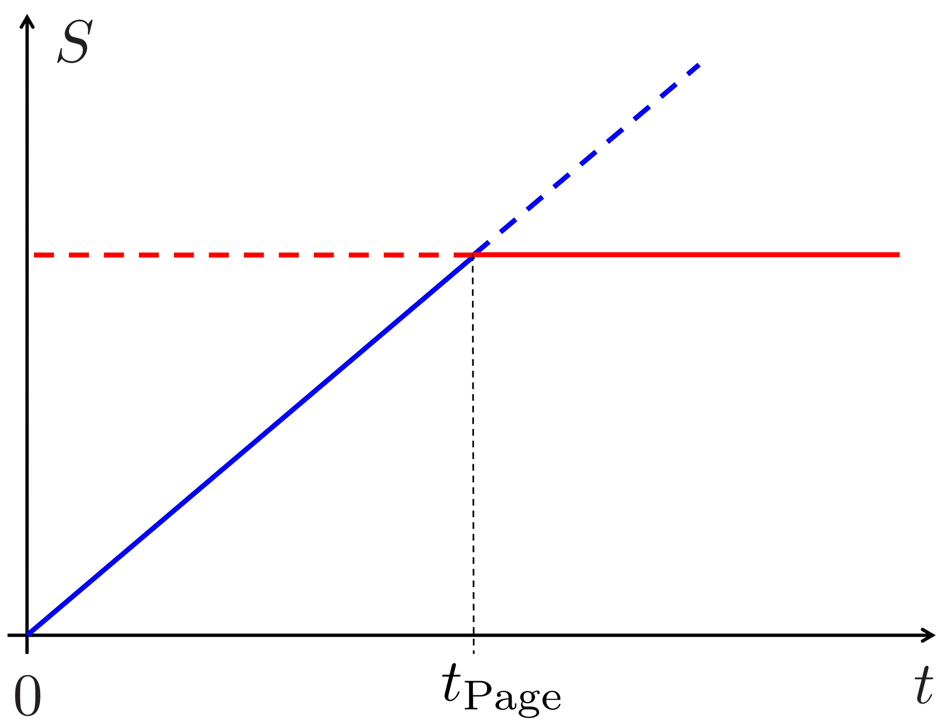


K.Hashimoto, N.Iizuka, Y.Matsuo, 2004.05863

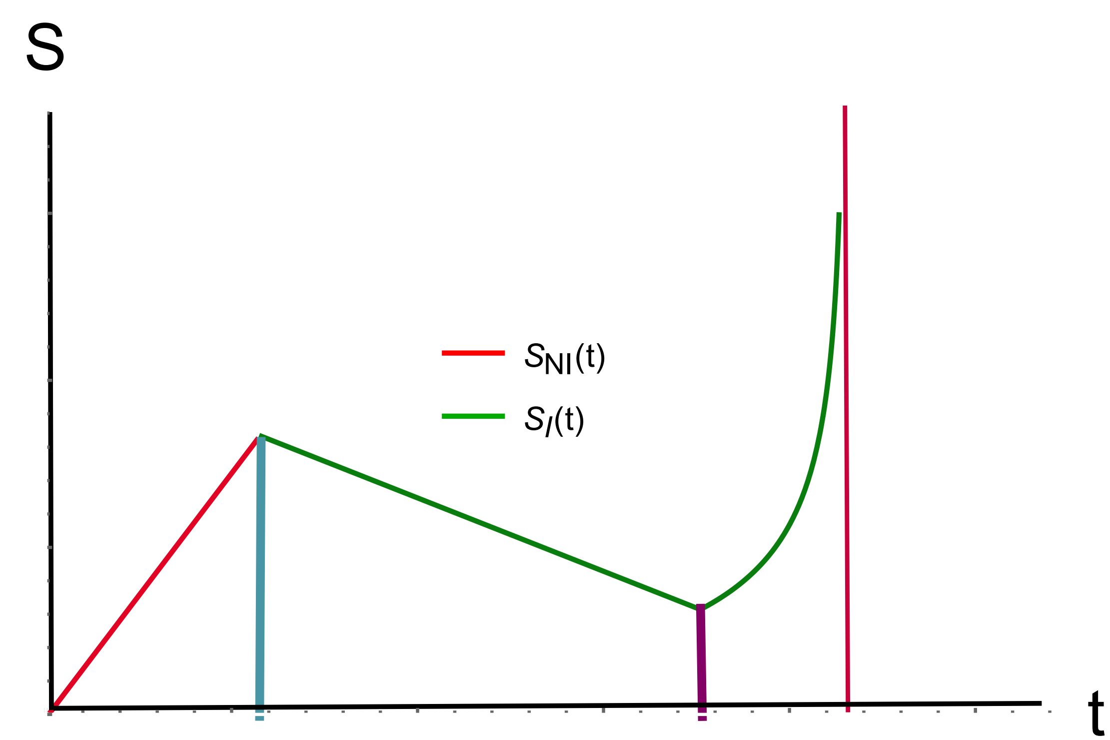
Time dependence of entanglement entropy of the Hawking radiation of evaporating black hole

I.A, I.Volovich
2110.04233

The Page curve for the eternal Schwarzschild black hole



Island entropy increas in the end of evaporation.

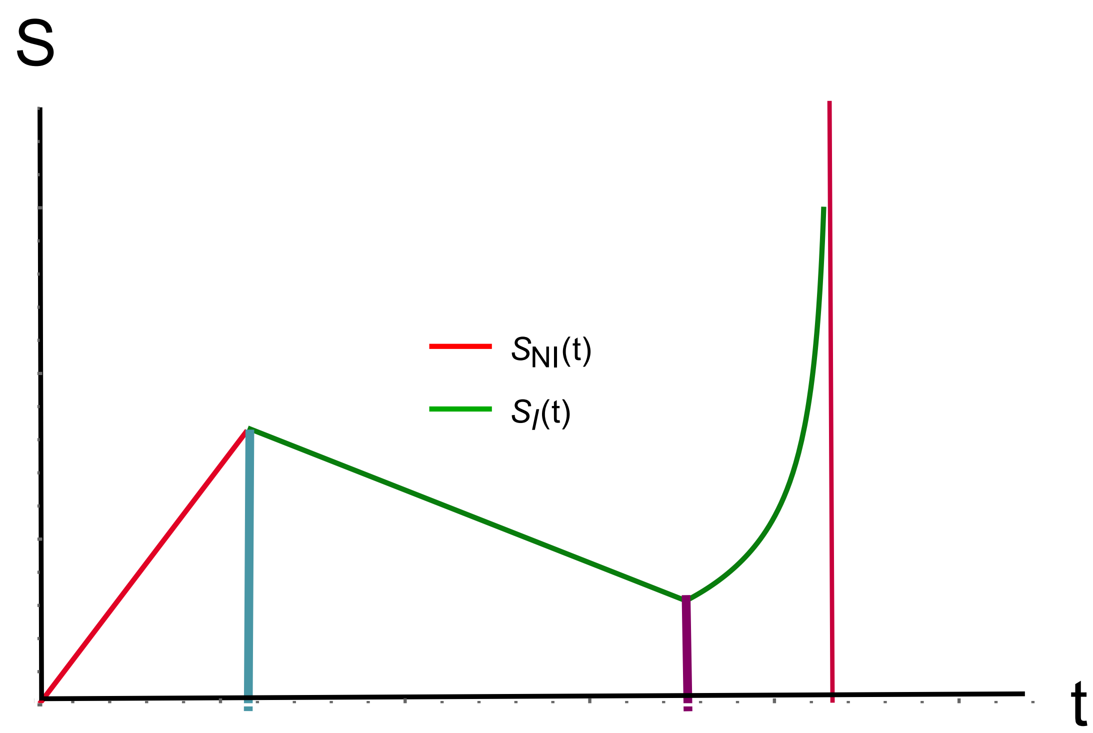


Increasing starts after some decreasing time.

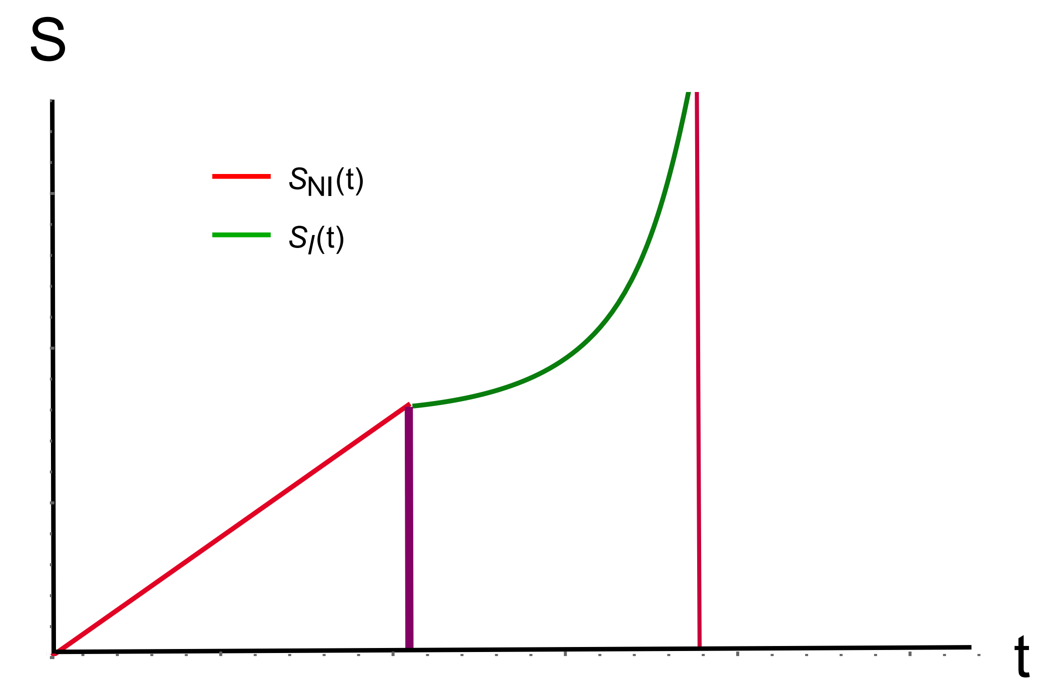
Time dependence of entanglement entropy of the Hawking radiation of evaporating black hole

With I.Volovich

Island entropy increas in the end of evaporation.



Increasing starts after some decreasing time.



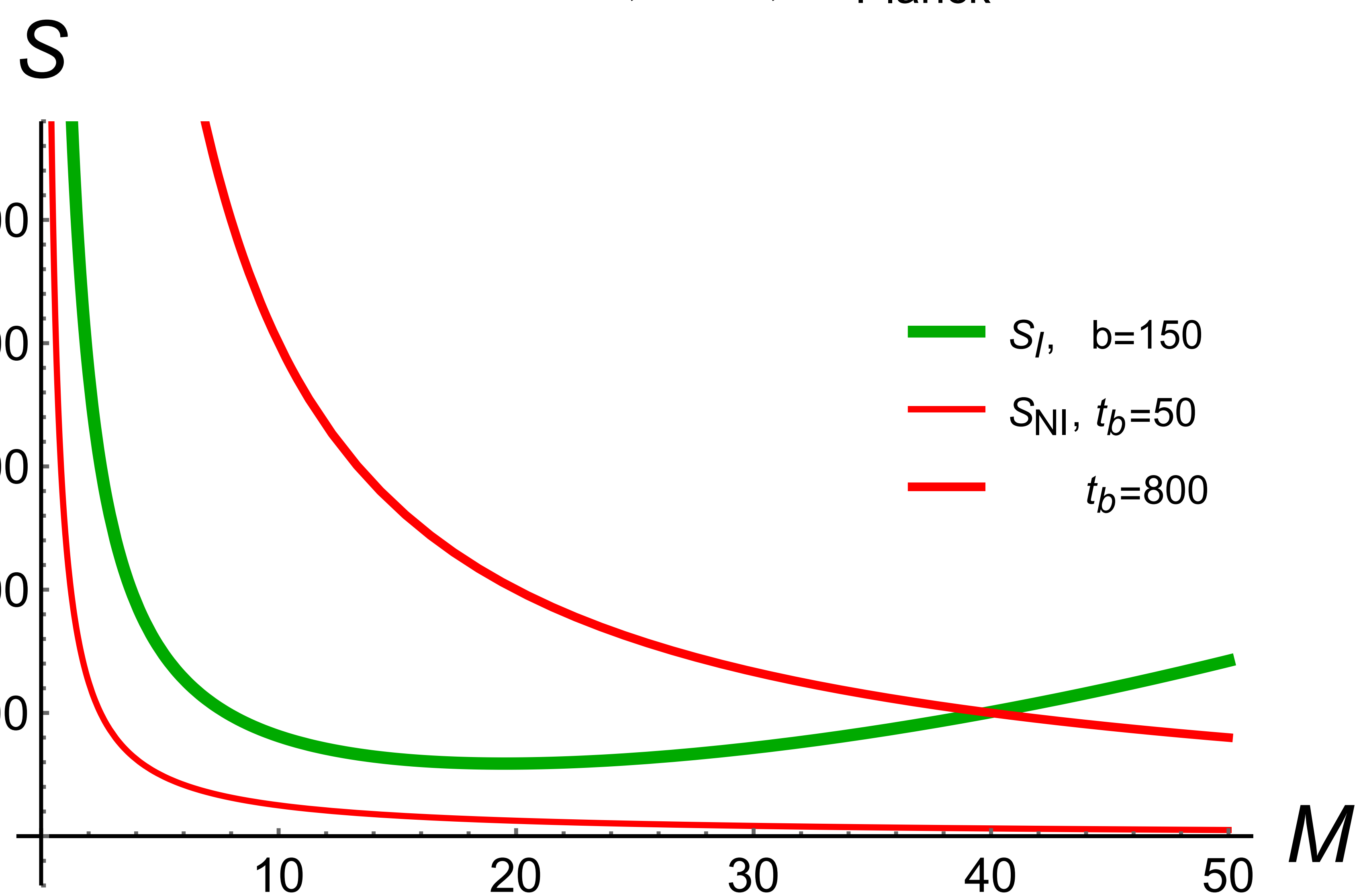
There is no decreasing period.

Entanglement entropy of the Hawking radiation

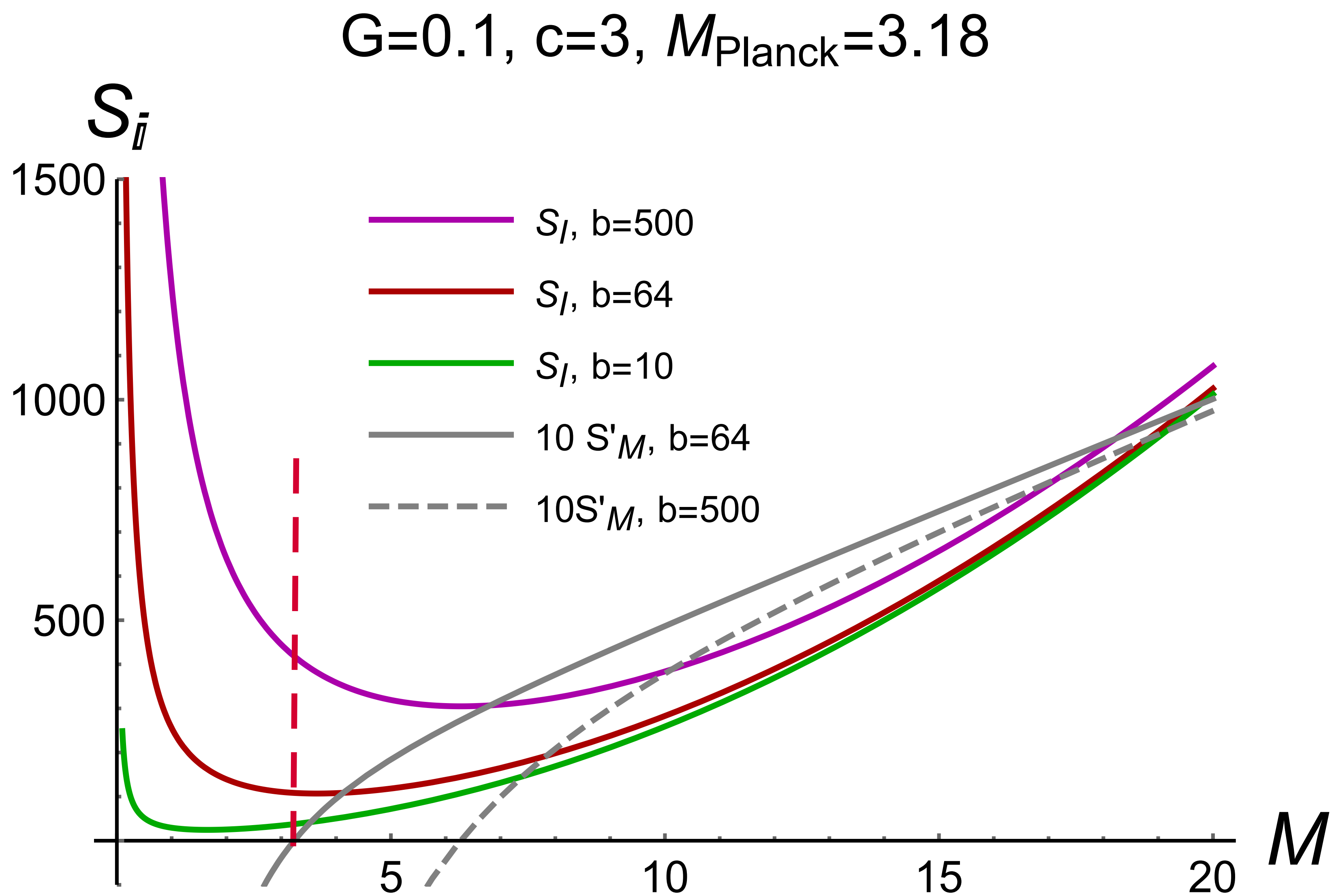
$$S_{n\mathcal{I}} \simeq \frac{c}{6}\frac{t_b}{r_h} \qquad\qquad r_h = 2GM$$

$$S_{\mathcal{I}} = \frac{2\pi r_h^2}{G} + \frac{c}{6}\frac{b-r_h}{r_h} + \frac{c}{6}\log\frac{16 r_h^3 (b-r_h)^2}{G^2 b}$$

G=0.01, c=3, M_{Planck}=10



Entanglement entropy of the Hawking radiation



$$b > r_h$$

$$M_{min} = \frac{1}{4}\left(\frac{bc}{3\pi G^2}\right)^{1/3}$$

$$M_{min} > M_{Planck} \simeq 1/\sqrt{G},$$

Entanglement entropy of the Hawking radiation of evaporating black hole

In four dimensions, due to radiation the mass M of the black hole is reduced as

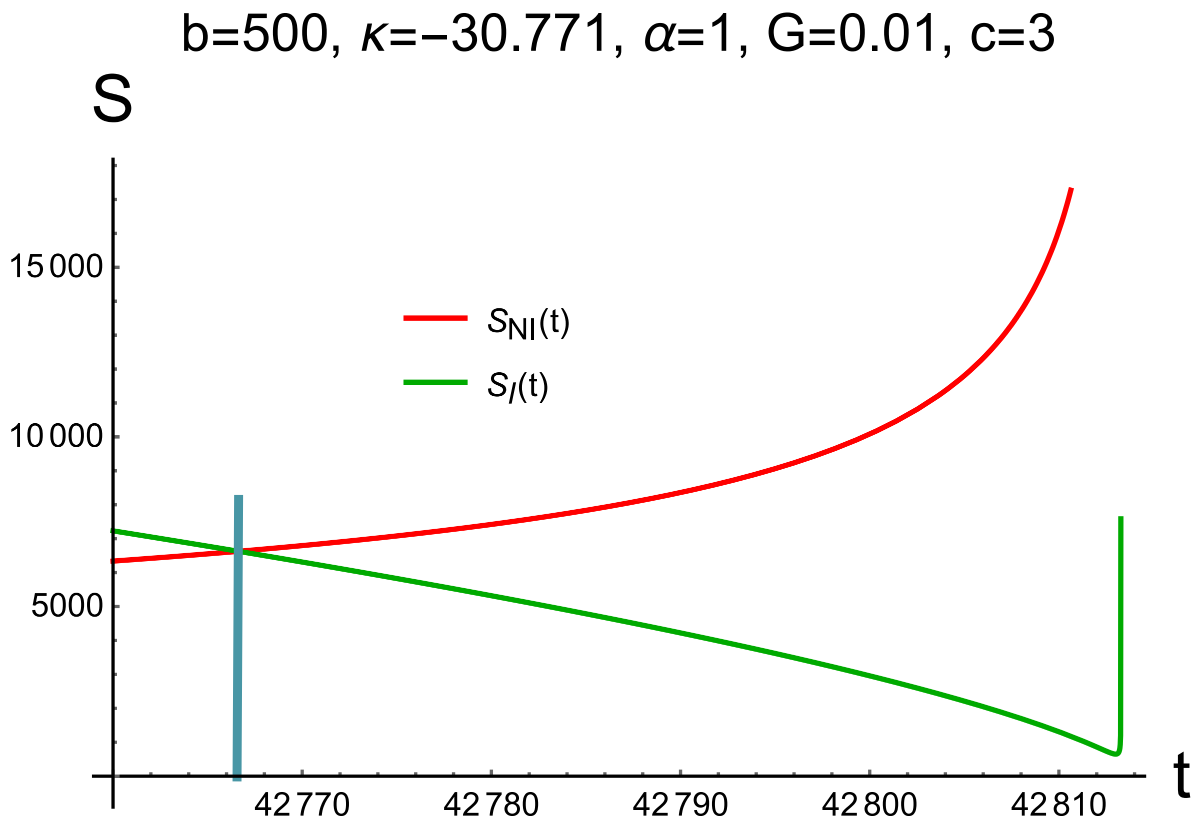
Page

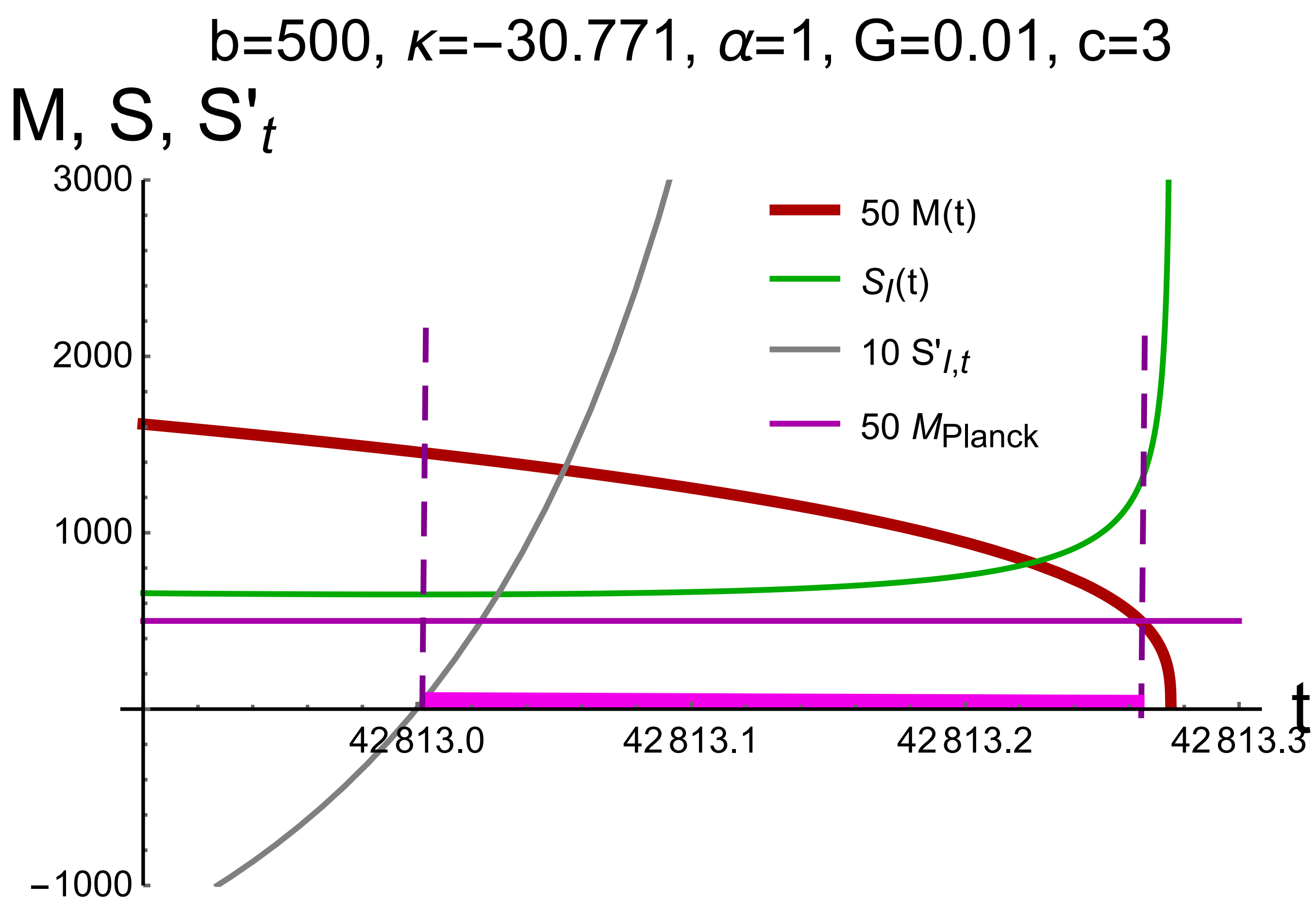
$$M(t) = \frac{r_0}{2G}\left(1 - \frac{24\alpha\, c\, Gt}{r_0^3}\right)^{1/3}$$

α is a constant dependent on the spin of the radiating particle

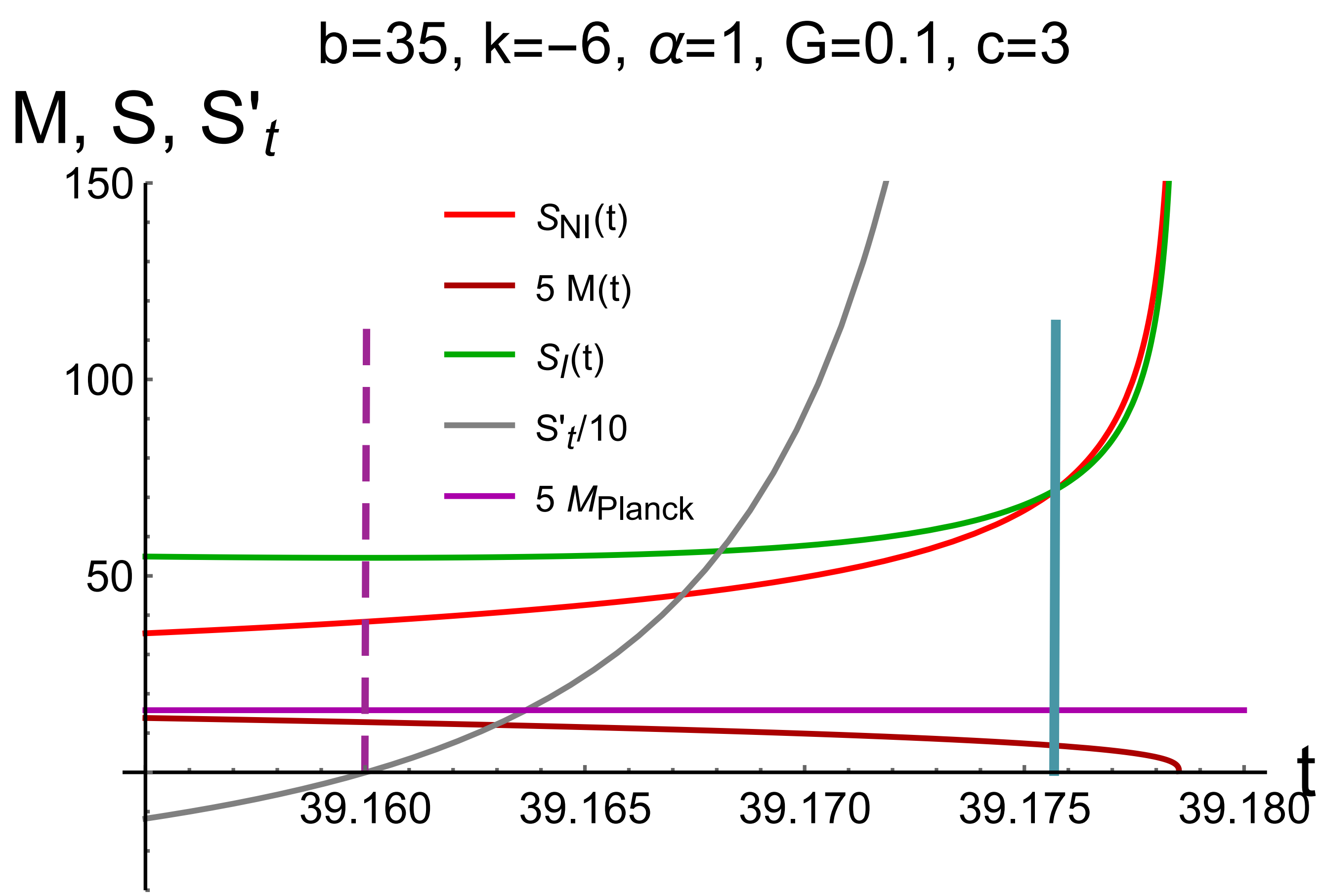
$$t_{evaporate} = \frac{r_0^3}{24c\,\alpha G}$$

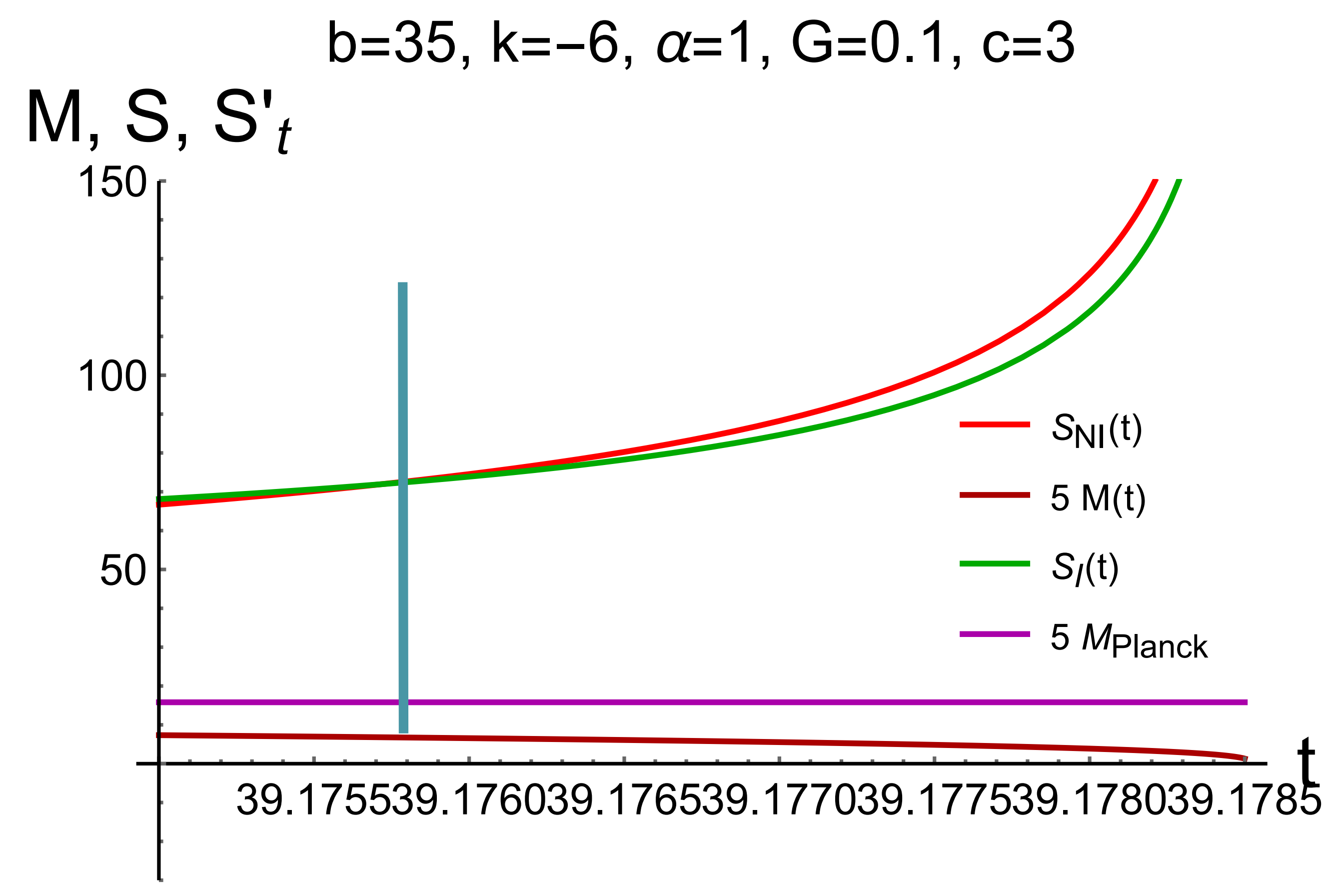
Time dependence of entanglement entropy of the Hawking radiation of evaporating black hole





Time dependence of entanglement entropy of the Hawking radiation of evaporating black hole





Entanglement entropy of the Hawking radiation at M-> 0. Regularization

Time dependence of the entanglement entropy of the evaporating black hole in the end of evaporation has singular behaviour.

This behaviour is related with singular behaviour of the Kruskal coordinates in the limit of the black hole mass

We remove this singularity using thermal coordinates, that provide a regularization of the Kruskal coordinates near M=0

I.A, I.Volovich
2104.12724

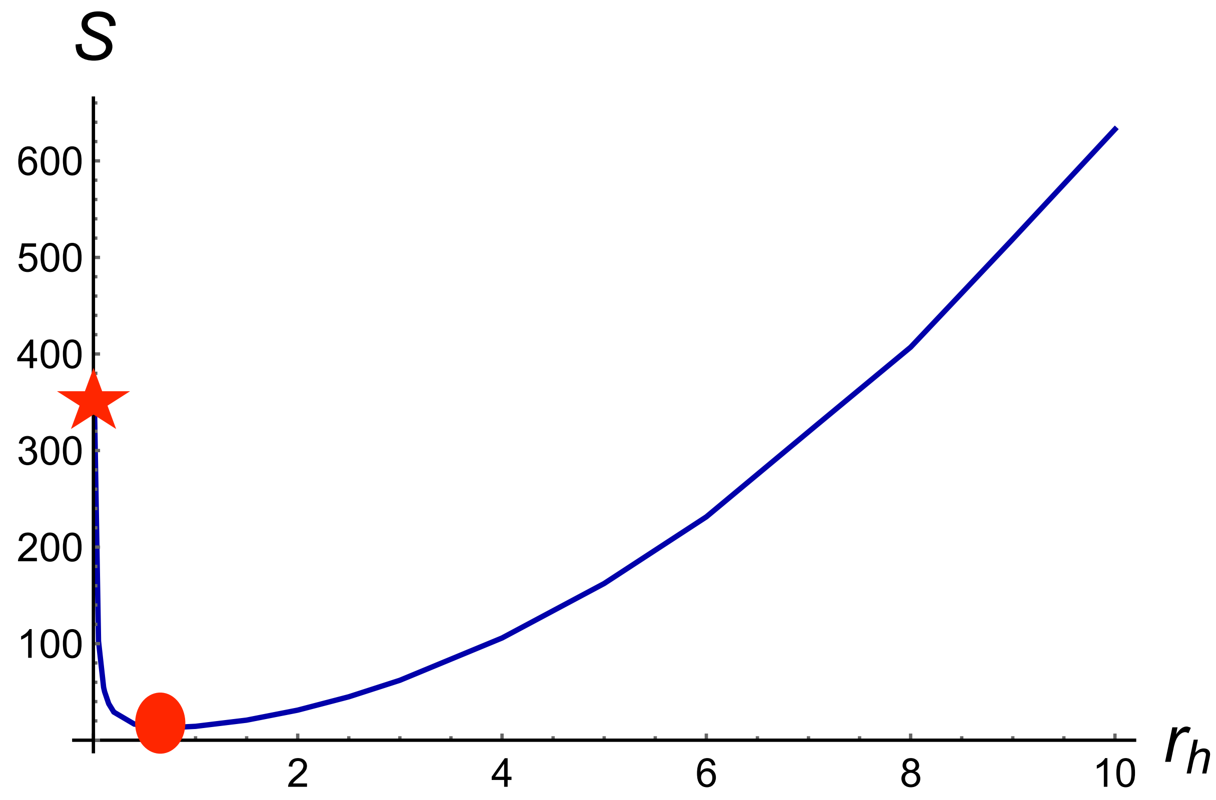
$$\mathscr{U} = -e^{-\frac{t-(r-r_h)}{B}} \left(\frac{r-r_h}{r_h} \right)^{\frac{r_h}{B}} \qquad B = (4M+\mu)G$$

$$\mathscr{V} = e^{\frac{t+(r-r_h)}{B}} \left(\frac{r-r_h}{r_h} \right)^{\frac{r_h}{B}} \qquad T = 1/2\pi B$$

The regularized entanglement entropy

$$S_{\mathcal{I},reg} = \frac{2\pi a^2}{G} + \frac{c}{3}\log\mathcal{L}_{reg}(a,b,t_a,t_,b,r_h,\mu)$$

$$\mathcal{L}_{reg} = \frac{\mathcal{D}(a_+,a_-)\mathcal{D}(b_+,b_-)\mathcal{D}(a_+,b_+)\mathcal{D}(a_-,b_-)}{\mathcal{D}(a_+,b_-)\mathcal{D}(a_-,b_+)} \qquad \mathcal{D}(\ell_1,\ell_2) = \sqrt{\frac{(\mathscr{U}(\ell_2)-\mathscr{U}(\ell_1))(\mathscr{V}(\ell_1)-\mathscr{V}(\ell_2))}{\mathscr{W}(\ell_1)\mathscr{W}(\ell_2)}}$$



$$S_{\mathcal{I},reg}\Big|_{M=0} \approx \frac{2\pi a^2}{G} + \frac{c}{6}\log\left[4G^2\mu^4\cosh\left(\frac{a-b}{\mu G}-1\right)^2\right]$$

Conclusion

We consider evaporation of the Schwarzschild black hole and note that, generally speaking, an island doesn't provide a bounded entanglement entropy in the end of the black hole evaporation.

Entropy increases even beyond the Planck scale

Despite the fact that including the island and the extremization about its location results in saturation of the entropy for the eternal black hole, the evaporating of black hole ends up by unbounded increasing of the entropy.

Conclusion

Possible exits out

Work out of s-mode approximation

Take into account back reaction

Take into account modifications of gravity
(quantum gravity corrections, string corrections)

More pessimistic expectation: island formula works only for non-flat cases
For 2-dim cases

General remarks

The black hole information problem has been considered as a particular example of the fundamental irreversibility problem in statistical physics.

A quantum mechanical explanation for the emergence of the second law of thermo-dynamics in macroscopic systems

N.N. Bogoliubov, Problems of dynamical theory in statistical physics, 1946

The Bogoliubov method of derivation of the Boltzmann kinetic equation has been suggested to generalized to quantum gravity to get a quantum gravity analog of the Boltzmann equation.

Th. Nieuwenhuizen, I.Volovich, 0507272

The Bogoliubov equation for the one-particle distribution function in the kinetic theory of gases is an analog of the extrimization equation for the entanglement entropy with island

Thank you for your attention!

Happy birthday for heroes of the day!