Advances in Non-relativistic Quantum Gravity

Eric Bergshoeff

Groningen University

work done in collaboration with

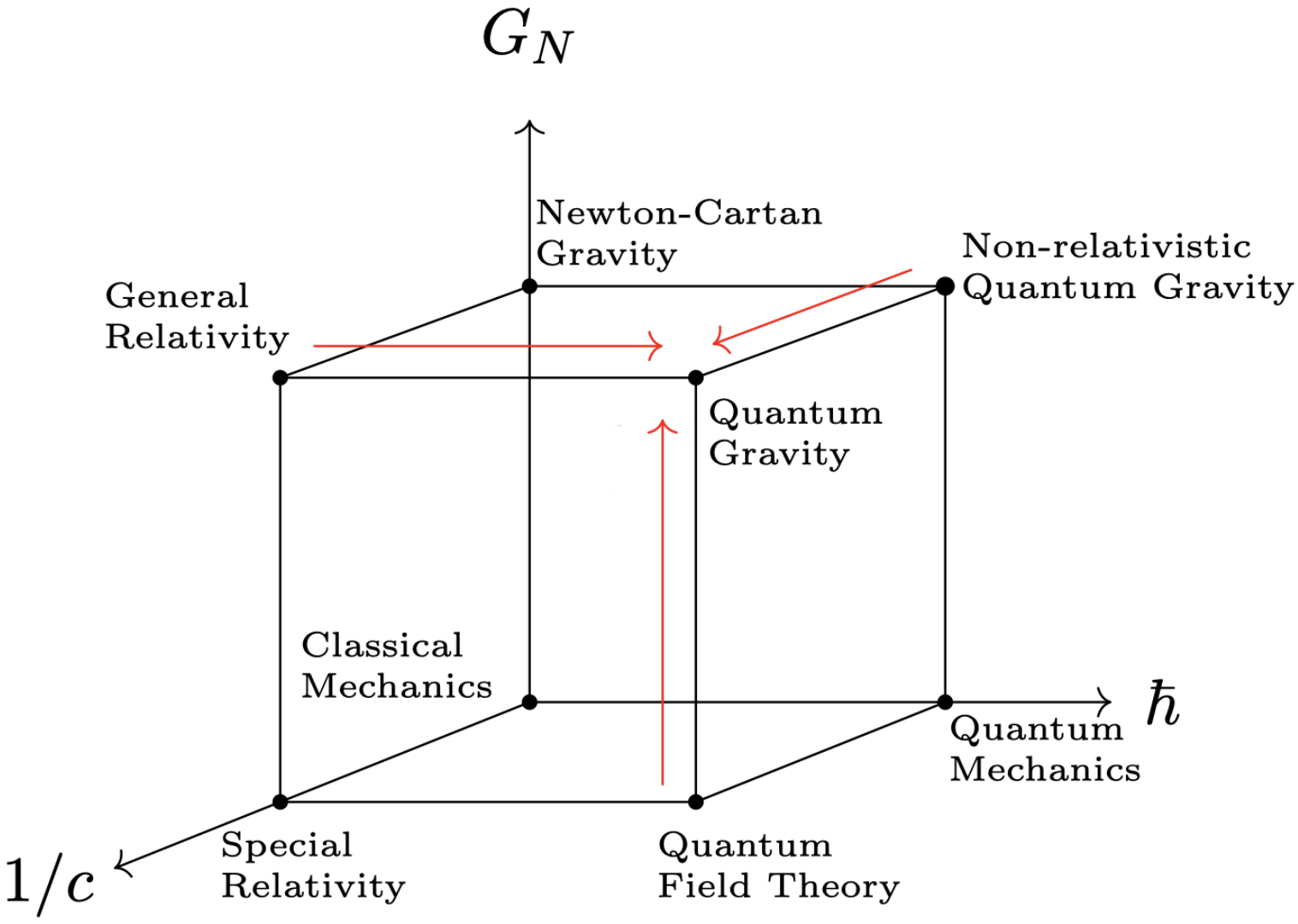
J. Lahnsteiner, L. Romano, J. Rosseel and C. Şimşek

International Conference on
Advances in Quantum Field Theory - 2021

Dubna, October 13 2021

rijksuniversiteit
groningen

Three Roads to Quantum Gravity



NR Quantum Gravity

Does combining gravity with quantum mechanics require relativity?

Does NR string theory define NR quantum gravity?

Does NR gravity has its own holographic principle?

Outline

Outline

Outline

Outline

Defining a NR Limit

STEP 1: decomposing $E_\mu{}^{\hat{A}} = (E_\mu{}^0, E_\mu{}^{A'}) =$ (clock, ruler) and introducing M_μ, perform an invertable field redefinition involving a parameter c:

$$E_\mu{}^0 = c\tau_\mu + c^{-1} m_\mu \,, \qquad E_\mu{}^{A'} = e_\mu{}^{A'} \,, \qquad M_\mu = c\tau_\mu - c^{-1} m_\mu$$

STEP 2: take the limit $c \to \infty$ and take care of possible divergences

Example: Particles and a 'critical' limit

cp. to Seiberg, Susskind, Toumbas (2000); Gopakumar, Maldacena, Minwalla, Strominger (2000); Danielsson, Guijosa, Kruczenski (2000), Gomis, Ooguri (2001)

Starting from a particle coupled to gravity, the red terms in the above field redefinition lead to divergencies in the kinetic and Wess-Zumino term that cancel against each other.

Infinities

Using a second-order formulation of general relativity, the NR limit of the spin-connection fields $\Omega_\mu{}^{\hat{A}\hat{B}}(E)$ contains a leading divergence that usually is set to zero by imposing the zero torsion constraint

$$\partial_{[\mu}\tau_{\nu]} = 0$$

Given this constraint the NR limit of the Einstein e.o.m. (no action!) yields the NC gravity e.o.m. where the Newton potential Φ can be identified as the time component of the central charge gauge field m_μ:

$$\Phi \sim \tau^\mu m_\mu$$

This NC gravity theory is a reformulation of Newtonian gravity valid in any frame and including strong gravity effects

The Zero Torsion Constraint

$$\partial_{[\mu}\tau_{\nu]} = 0 \quad \rightarrow \quad \tau_\mu = \partial_\mu \rho \quad \text{with} \quad \tau_\mu \quad \text{clock function}$$

$$\Delta T = \int_{\mathcal{C}} \mathrm{d}x^\mu \tau_\mu = \int_{\mathcal{C}} \mathrm{d}\rho \ \text{ is path-independent} \quad \rightarrow \quad \text{absolute time}$$

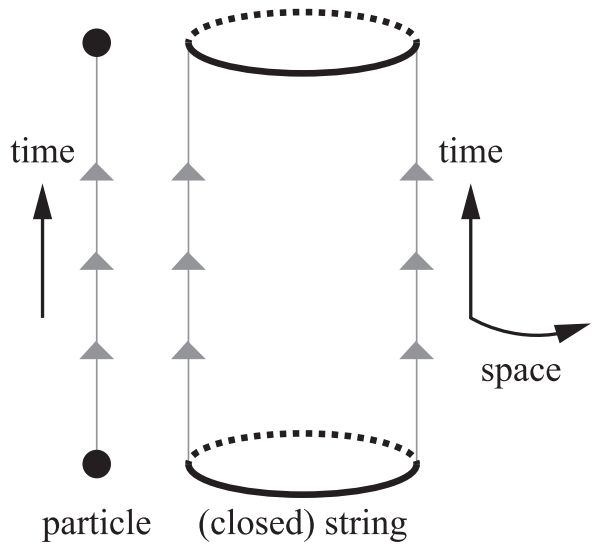
$$\text{Torsional NC gravity}: \ \partial_\mu \tau_\nu - \Gamma_{\mu\nu}{}^\rho \tau_\rho = 0 \quad \rightarrow \quad \Gamma_{[\mu\nu]}{}^\rho \tau_\rho = \partial_{[\mu}\tau_{\nu]}$$

Geometry with Co-dimension 2 Foliation

The string should be coupled to a 2-form gauge field $B_{\mu\nu}$ with

$$B_{\mu\nu} = -c^2 \epsilon_{AB} \tau_\mu{}^A \tau_\nu{}^B + b_{\mu\nu}$$

defining a geometry with a co-dimension 2 foliation where $\tau_\mu \rightarrow \tau_\mu{}^A$ with $\hat{A} = (A, A') = (0, 1, A')$



The Basic Variables

The decomposition leading to NC gravity

$$\{E_\mu{}^{\hat{A}}, M_\mu\} \quad \rightarrow \quad \{\tau_\mu, e_\mu{}^{A'}, m_\mu\}$$

gets replaced by the following redefinition:

$$\{E_\mu{}^{\hat{A}}, B_{\mu\nu}, \Phi\} \quad \rightarrow \quad \{\tau_\mu{}^{A}, e_\mu{}^{A'}, b_{\mu\nu}, \phi\}$$

The Newton potential Φ can be identified with the time-space component $\epsilon^{AB}\tau^\mu{}_A\tau^\nu{}_B b_{\mu\nu}$ of the 2-form gauge field $b_{\mu\nu}$

Outline

The bosonic case

- Due to the presence of matter, one can take the NR of the action avoiding divergencies without imposing any geometric constraints
- the action has an emergent dilatation symmetry and therefore has one 'missing field' and one 'missing e.o.m.'
- The 'missing' e.o.m. follows from taking the NR limit of the e.o.m. and is precisely the Poisson equation of the Newton potential. Note: the Poisson equation needs the action!
- The e.o.m. of the Newton potential itself gives the following non-linear geometric constraint: $\tau_{B'C'A}\tau^{B'C'A} = 0$ thereby preventing an overdetermined set of equations

Minimal Supergravity

- the action has one emergent dilatation and two emergent superconformal symmetries. It therefore has one 'missing' bosonic and two 'missing' fermionic fields plus corresponding 'missing' e.o.m. The fields form a shortened supermultiplet

- The 'missing' e.o.m. follow from taking the NR limit of the e.o.m. and are precisely the Poisson equation of the Newton potential plus two fermionic partner equations

- The minimal supergravity action is a pseudo action in the sense that it is only invariant under supersymmetry if one uses a what is called twistless torsional constraint $\underline{\text{after}}$ varying the action. Due to this the e.o.m. that follow from the action transform under supersymmetry to the 'missing' e.o.m.: they belong to the same supermultiplet

cp. to Vanhecke, Van Proeyen (2017)

Outline

Outlook

- how general is our limit technique?
  - invertable field redefinition, cancellation of divergences, local dilatation symmetry
- including Yang-Mills to obtain heterotic gravity
- open strings

  see lectures by Z. Yan at 1st School on NR QFT, Gravity and Geometry
- extension to IIA/IIB supergravity

Take-Home Message

Our results pave the way for a target space approach to NR string theory:

(supersymmetric) brane solutions, compactifications, NR holography etc.