QFT and bound muons and electrons

Outline:

- Decay of a muonic atom
  - daughter electron can remain bound
  - decay rate unexpectedly sensitive to components with $E<0$

- Ocean waves vs. de Broglie waves:
  - gravitational time dilation explains the free fall

Advances in Quantum Field Theory

Andrzej Czarnecki University of Alberta

October 12, 2021

Part 1/2: Bound muon decay

PHYSICAL REVIEW D **102**, 073001 (2020)

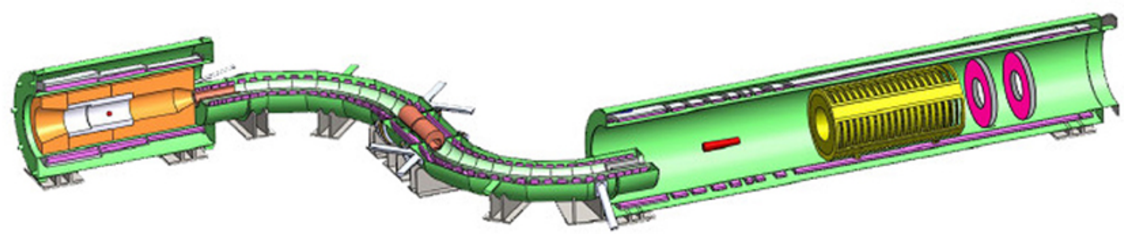
Decay of a bound muon into a bound electron

M. Jamil Aslam,[1,2] Andrzej Czarnecki,[1] Guangpeng Zhang,[1] and Anna Morozova[1]

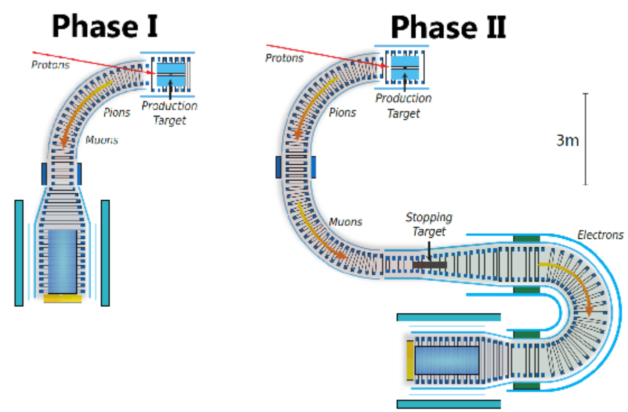
Bound muon decay: why do we care?

Muon-electron conversion searches

Mu2e
Fermilab



COMET
J-PARC



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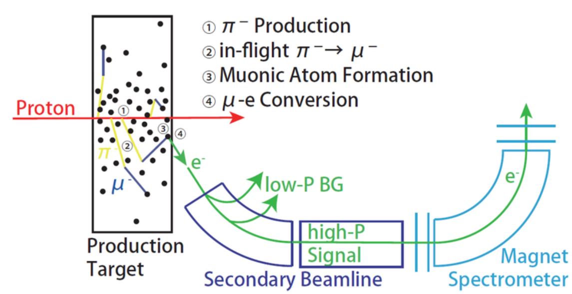

Bound muon decay

Studied in TRIUMF

PHYSICAL REVIEW D **80,** 052012 (2009)

Decay of negative muons bound in ^{27}Al

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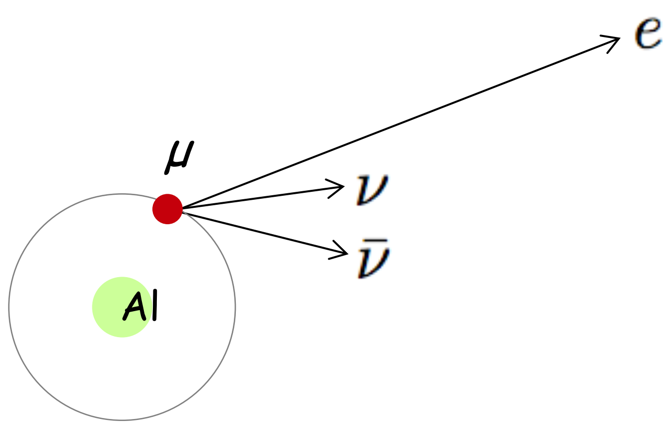
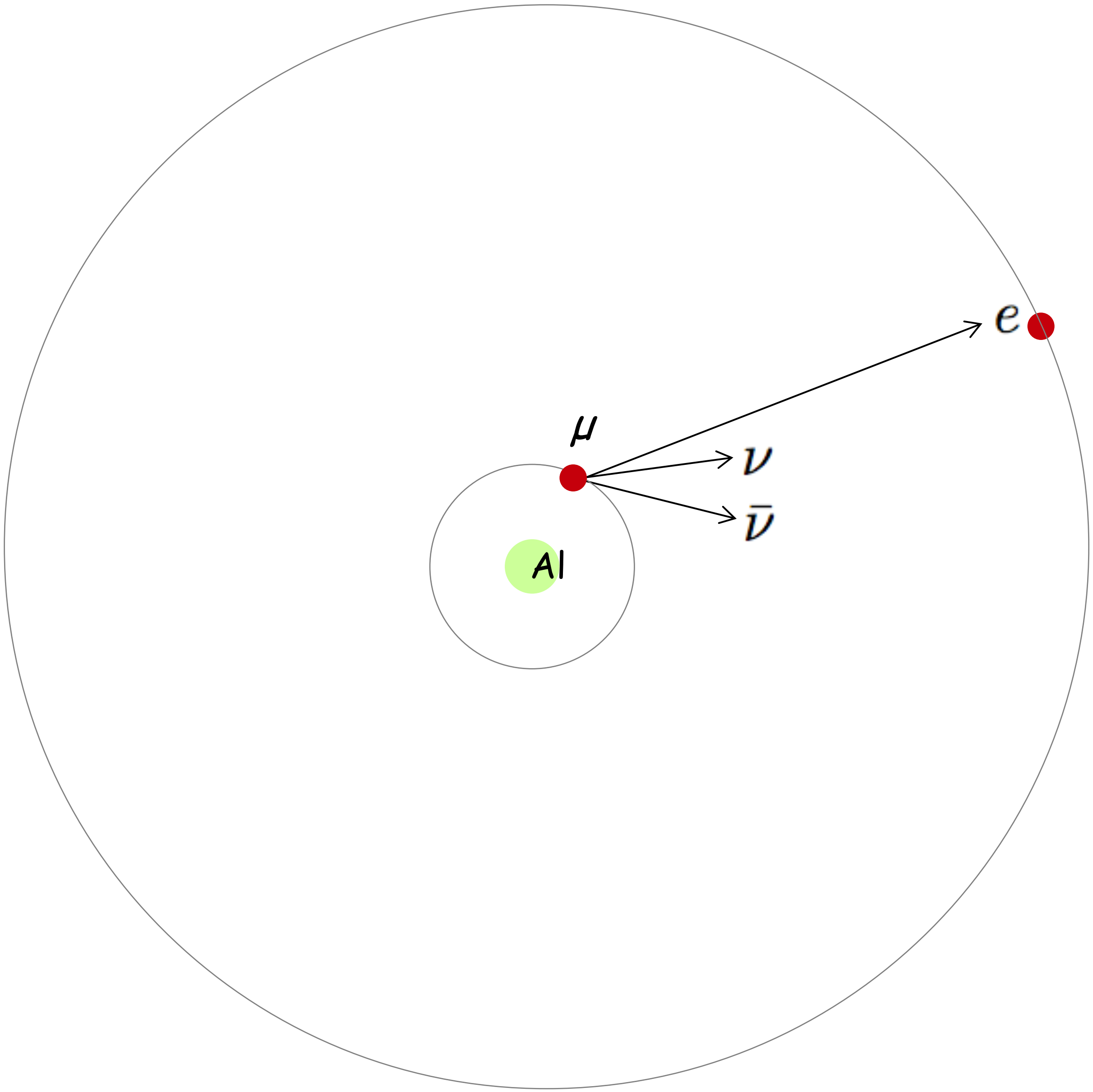


Bound muon decay into a bound electron



What is the probability of binding of the daughter electron?

Bound muon decay into a bound electron

PHYSICAL REVIEW D VOLUME 52, NUMBER 7 1 OCTOBER 1995

Atomic alchemy: Weak decays of muonic and pionic atoms into other atoms

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Bound particle wave function can be expanded in plane waves

$$\tilde{\Phi}(\vec{k}) = \sum_r \left[A_r(\vec{k}) \frac{u_r(\vec{k})}{\sqrt{2k^0}} + B_r^*(-\vec{k}) \frac{v_r(-\vec{k})}{\sqrt{2k^0}} \right]$$

negative energy states:
neglected in the "alchemy" paper:

The integral $\int[d^3k/(2\pi)^3]\sum_r |B_r(\vec{k})|^2$ gives the probability to find a three particle Fock state ($e^+e^-e^-$) in the atom. Even for $Z = 80$ this fraction is tiny ($\approx 0.2\%$), so we only consider the one-Fock contribution characterized by $A_r(\vec{k})$.

Bound muon decay: our study

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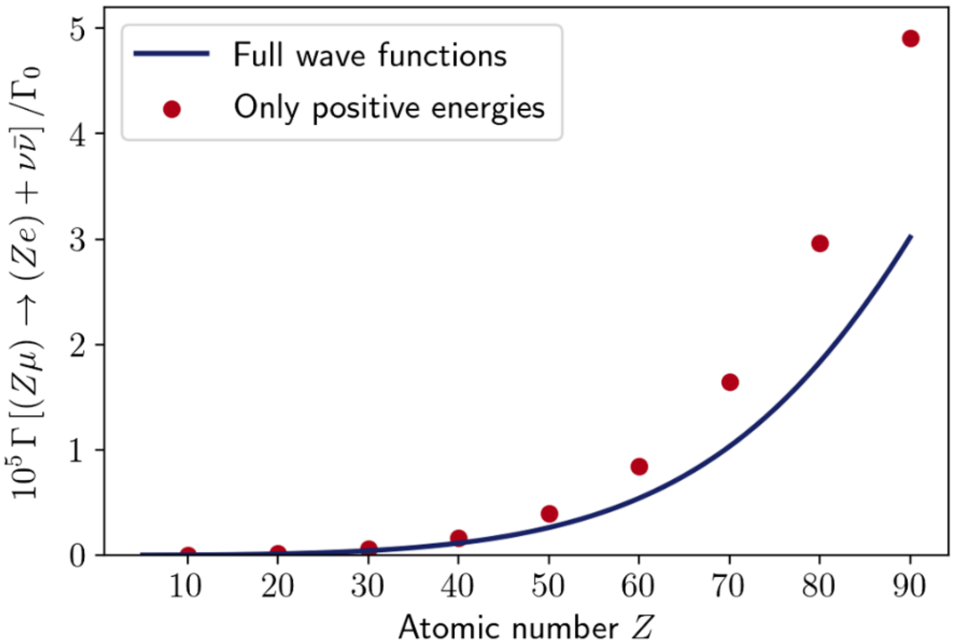
Decay of a bound muon into a bound electron

M. Jamil Aslam,[1,2] Andrzej Czarnecki,[1] Guangpeng Zhang,[1] and Anna Morozova[1]



Why such large effect of negative energies?

Tentative explanation:

The decay happens where the muon and the electron wave functions overlap.
This is a tiny fraction of the electron's range, close to the nucleus.
In that region, $E < 0$ is relatively likely.

Check: position space calculation

$$\mathcal{M} = \frac{g}{\sqrt{2}} \int \mathrm{d}^3\boldsymbol{r}\ \exp\left(i\boldsymbol{q}\cdot\boldsymbol{r}\right)\bar{\Phi}_\mu(\boldsymbol{r})\not{\epsilon}^{\lambda_A\star} L \Phi_e(\boldsymbol{r})$$

$$\frac{1}{\Gamma_0}\Gamma[(Z\mu^-) \to (Ze^-) + \nu\bar{\nu}] = 128 \int_0^{z_{\max}} (N_a^2 + N_b^2 + F_a^2 + F_b^2) k_A z^3 \mathrm{d}z.$$

Our position space and momentum space evaluations agree (if both A and B included).

Part 2/2: Why does everything fall?

Gravitational time dilation, free fall, and matter waves

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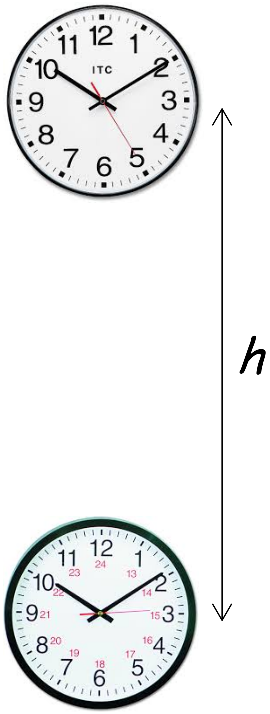
Am. J. Phys. **89** (6), June 2021

Free fall and time dilation

Gravitational time dilation:



$$\Delta t = \frac{gh}{c^2} t$$

Example: a weekend trip from Seattle to Mount Rainier,
t = 40 hours, elevation *h* = 1340 metres,

$$\Delta t = \frac{gh}{c^2} t = \frac{9.8 \cdot 1340}{9 \cdot 10^{16}} 40 \cdot 3600 \text{ s} = 21 \text{ ns}.$$

How does this tiny difference cause falling?

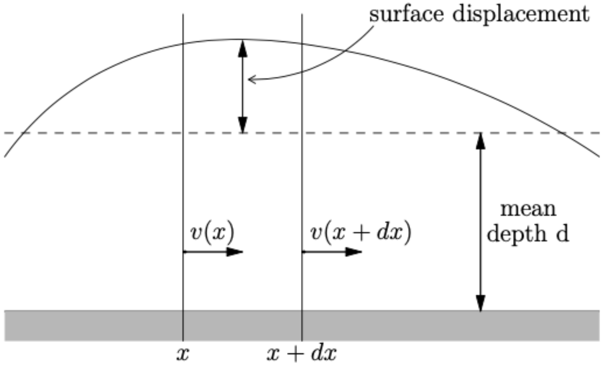
Matter waves evolve faster at a higher elevation.

Intuitive example: ocean waves; why do they always approach the beach?

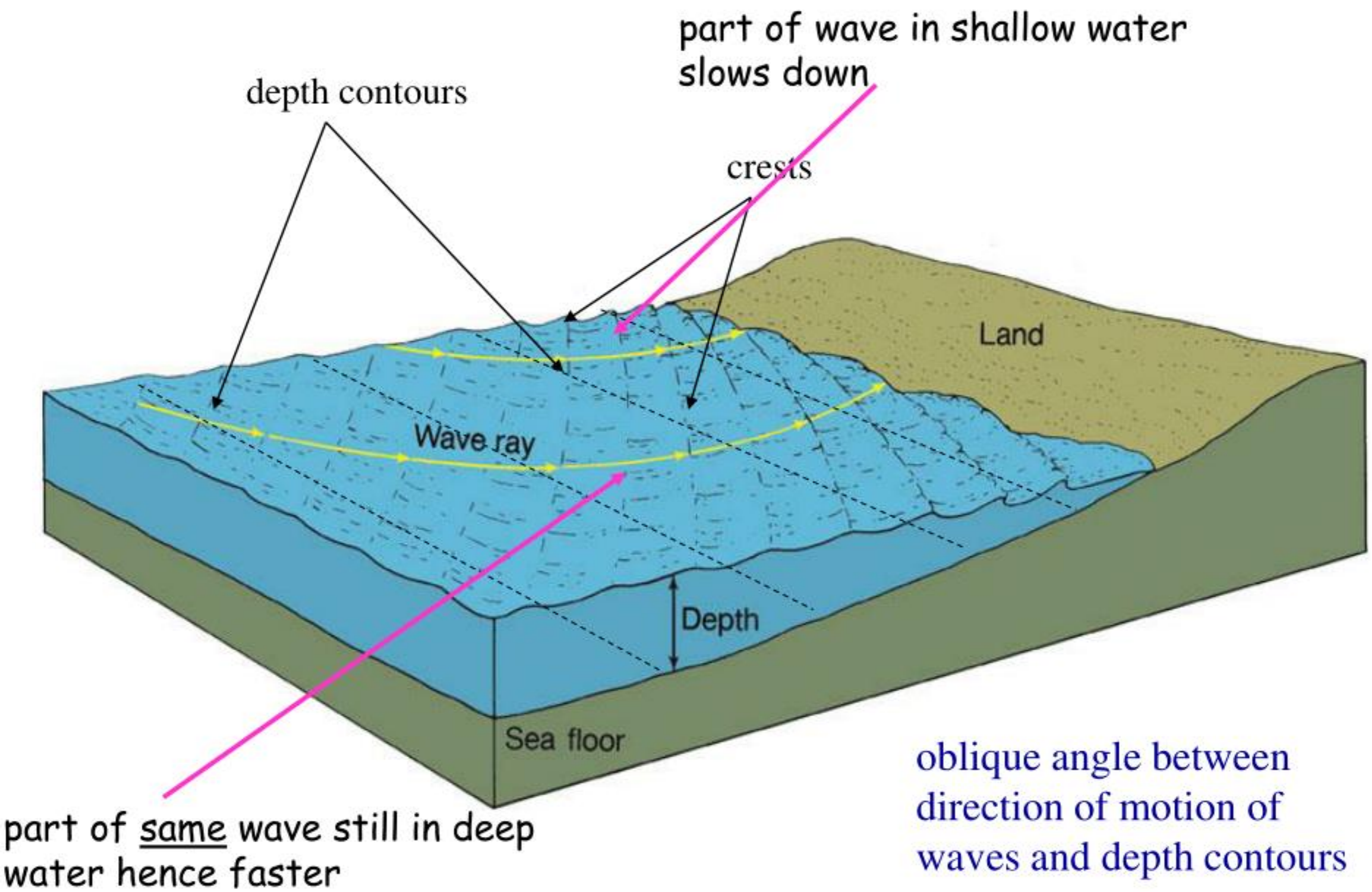
Reason: waves are slower in shallower water,

$$gd\frac{\partial^2 \zeta}{\partial x^2} = \frac{\partial^2 \zeta}{\partial t^2}$$

$$u = \sqrt{gd}$$

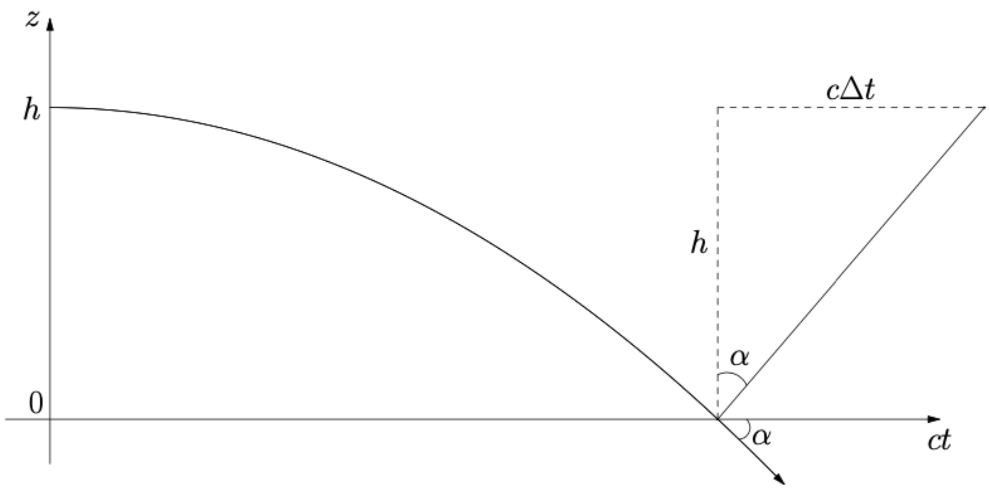


Wave Refraction – slowing and bending of waves as they approach shore at an angle



oblique angle between direction of motion of waves and depth contours

What does this mean for matter waves?



Note: this plot shows a one-dimensional motion (only vertical), not a two-dimensional projectile motion.

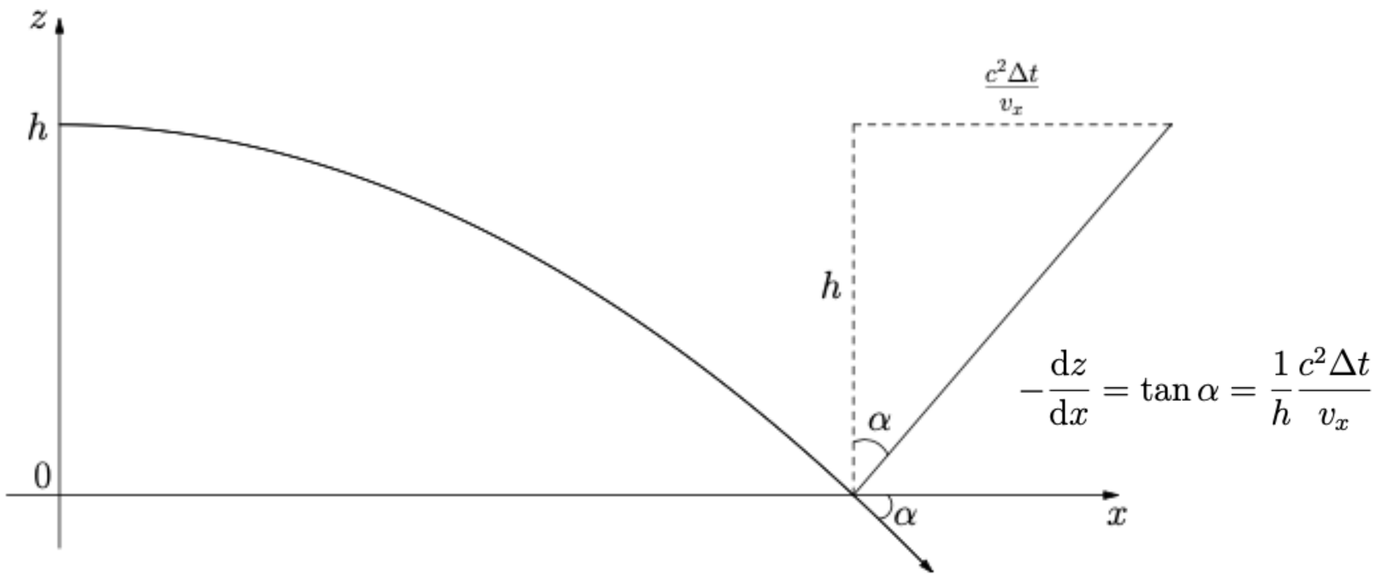
Velocity from the slope: $$\frac{v}{c} = -\tan\alpha$$

$$\frac{v}{c} = -\frac{c\Delta t}{h}$$ *h* cancels with $\Delta t = \frac{gh}{c^2}t$

We reproduce the familiar result,

$$\boxed{v = -gt}$$

Another point of view: projectile motion



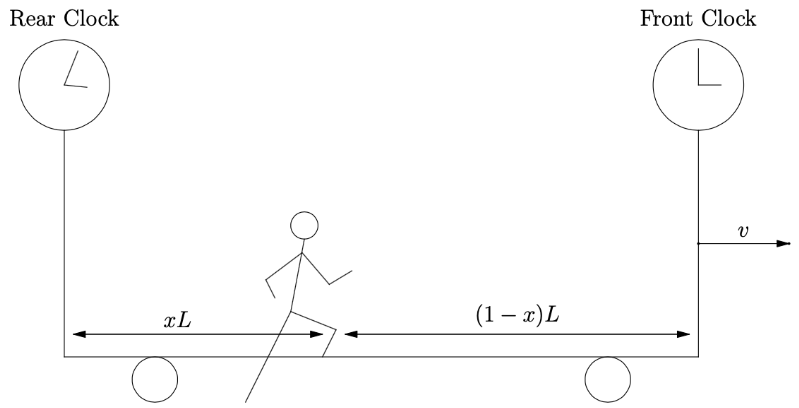
But $\mathrm{d}x = v_x \mathrm{d}t$

$$-\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{c^2\Delta t}{h} = gt$$

Application: twin paradox (local version)

Rear clock is ahead by

$$\Delta t = \frac{Lv}{c^2}$$



duration of the spurt is $t = v/g$ (here "g" is the average acceleration)

$$\Delta t_R = g\,xL\,t/c^2 \qquad \Delta t_F = g(1-x)Lt/c^2$$

$$\Delta t = \Delta t_F + \Delta t_R = \frac{gLt}{c^2} = \frac{Lv}{c^2}$$

When the observer settles on the train, both clocks show the same time.

Summary

Decay of a bound muon into a bound electron:
- unexpected lesson in the Dirac equation
- negative energy components are important

Free fall explained by the evolution rate of matter waves at various altitudes. Analogy to ocean waves near beaches.