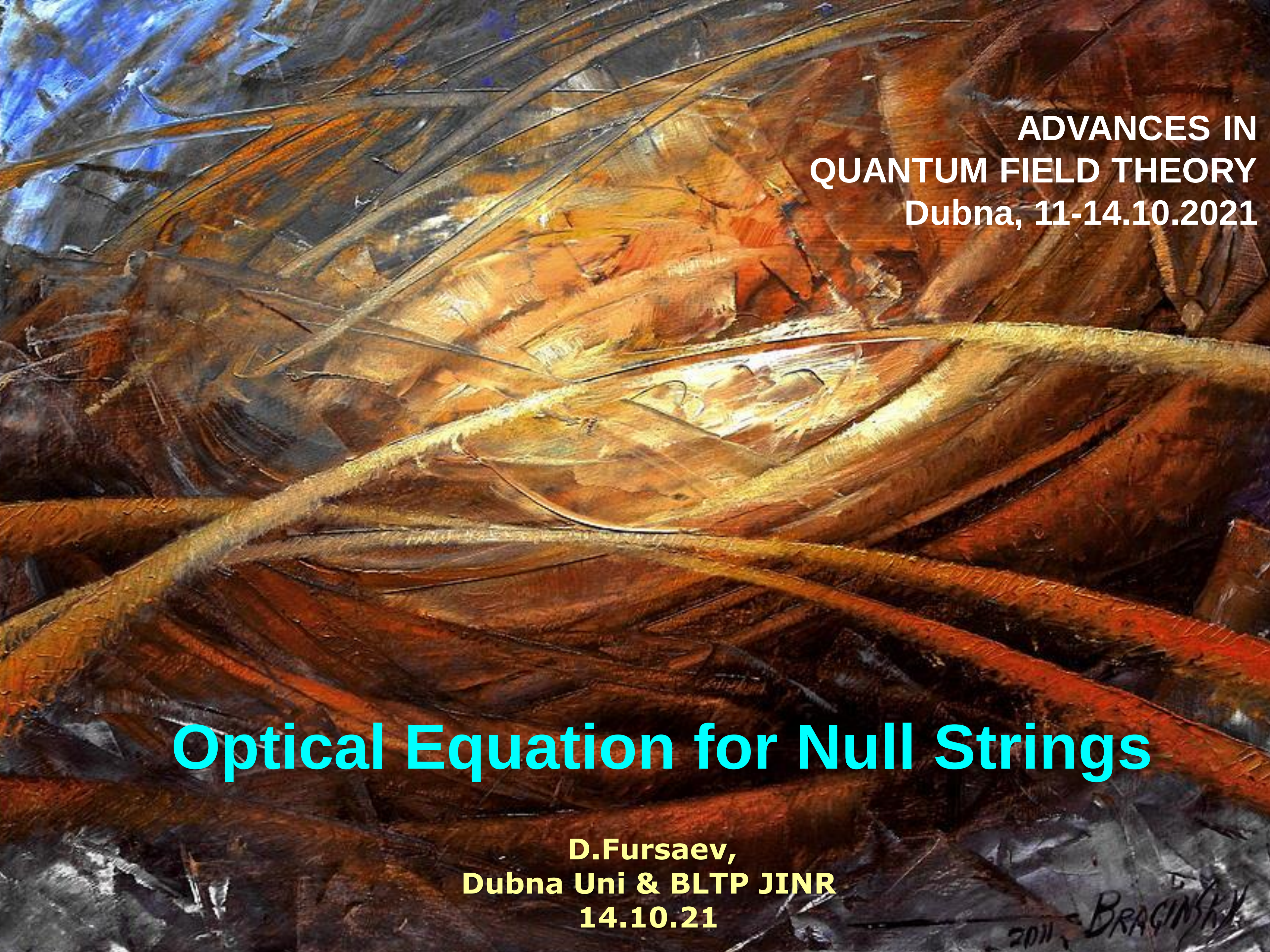


ADVANCES IN
QUANTUM FIELD THEORY
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# Optical Equation for Null Strings

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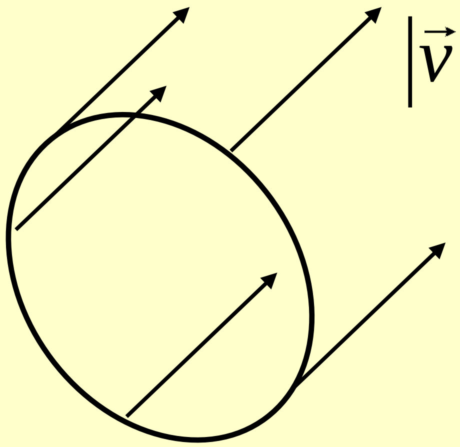
# Null Strings - 1D objects moving with the velocity of light



$$\left|\vec{v}\right| = c$$

First discussed in 1977 by A. Schild (as microscopic objects in early models of strong interactions)

Null Strings (dynamics, Schild) =
Tensionless Strings (origin, string theory) =
Massless Strings (gravitational backreaction effects, D.F)

# Tensile strings vs tensionless strings

## tensile cosmic strings

- possible origin (I) - phase transition in the early Universe
- possible origin (II) - fundamental tensile strings, Universe at the Planck epoch
- physical effects: overdensities of matter, lensing effects,

leave specific imprints on CMB maps

- dynamics: Nambu-Goto equations, string WS is extremal surface

## tensionless (null) cosmic strings

- possible origin - tensionless limit fundamental strings, Universe at the Planck epoch
- physical effects: overdensities of matter, lensing effects,

leave specific imprints on CMB maps

- dynamics: WS is a null (degenerate) surface

## Aim of the work:

- a technique to understand physical features of string evolution in invariant terms;
- relation to studies of null geodesic congruencies in GR (Sachs' eqs);
- a new "language": string diagrams

applications:

- null strings in asymptotically flat spacetimes (freeze-out effects) and cosmological models;
- null strings and gravity waves

the talk is based on: D.V.Fursaev, PRD 103 (2021) 123526

# Schild equations

string world sheet (WS) is a null 2 surface

$$x^\mu = x^\mu(\lambda, \tau)$$

$l^\mu = x^\mu{}_{,\lambda}$ - null tangent vector, generates null geodesics on WS

$\eta^\mu = x^\mu{}_{,\tau}$ - spatial tangent vector (connecting vector)

$$\boxed{l^2 = 0\,, \quad \eta^2 > 0\,, \quad (l \cdot \eta) = 0\,, \quad \nabla_l l \sim l}$$

induced metric on WS is degenerate:

$$\det x^\mu{}_{,a}\, g_{\mu\nu}\, x^\nu{}_{,b} = 0$$

# string optical scalars

define, at the string WS, a tetrade $l, n, p, q :$

$$n^2 = 0, \ (n \cdot l) = -2, \quad p^2 = q^2 = 1,$$

$$p = N^{-1}\eta \, , \quad (p \cdot l) = (q \cdot l) = (p \cdot n) = (q \cdot n) = (p \cdot q) = 0$$

there are only two scalars on WS

$$\boxed{\begin{aligned} \theta_s &= \partial_\lambda \ln|\eta| \\ \kappa &= (q \cdot \nabla_p l) \end{aligned}}$$

- expansion, measures how fast string shirnks or expands
- rotation, measures the rate of rotation of NS

transform as boost-weighted scalars,

$$\theta_s' = g_{,\lambda}\, \theta_s \quad , \quad \kappa' = g_{,\lambda}\, \kappa \, ,$$

with respect to null rotation of $(q, n)$ and reparametrizations

$$\lambda = g(\lambda', \tau') \ , \ \ \tau = \varphi(\tau')$$

# Optical equation for a null string

define :

$$Z = \theta_s + i\kappa,$$

by analysing components of the Riemann tensor one finds :

$$\boxed{\partial_\lambda Z + Z^2 = -\Psi_0 - \Phi_{00}}$$

an analogue of Sachs' optical equation for null geodesic congruences

$\lambda$ - is affine parameter ,

$\Psi_0 = -C_{mlml}$ - Weyl tensor, $m = \frac{1}{\sqrt{2}}(p + iq)$

$\Phi_{00} = -\frac{1}{2}R_{ll}$ - Ricci tensor

solution for $\Psi_0 = \Phi_{00} = 0$: $Z = \dfrac{1}{\lambda + z(\tau)}$

# a new "language": string diagrams

physical aspects of the string evolution are captured by $Z$:

strings trajectories at fixed values of $\lambda$ are curves in complex $Z$-plane

example: a string twisted along $y$-axix

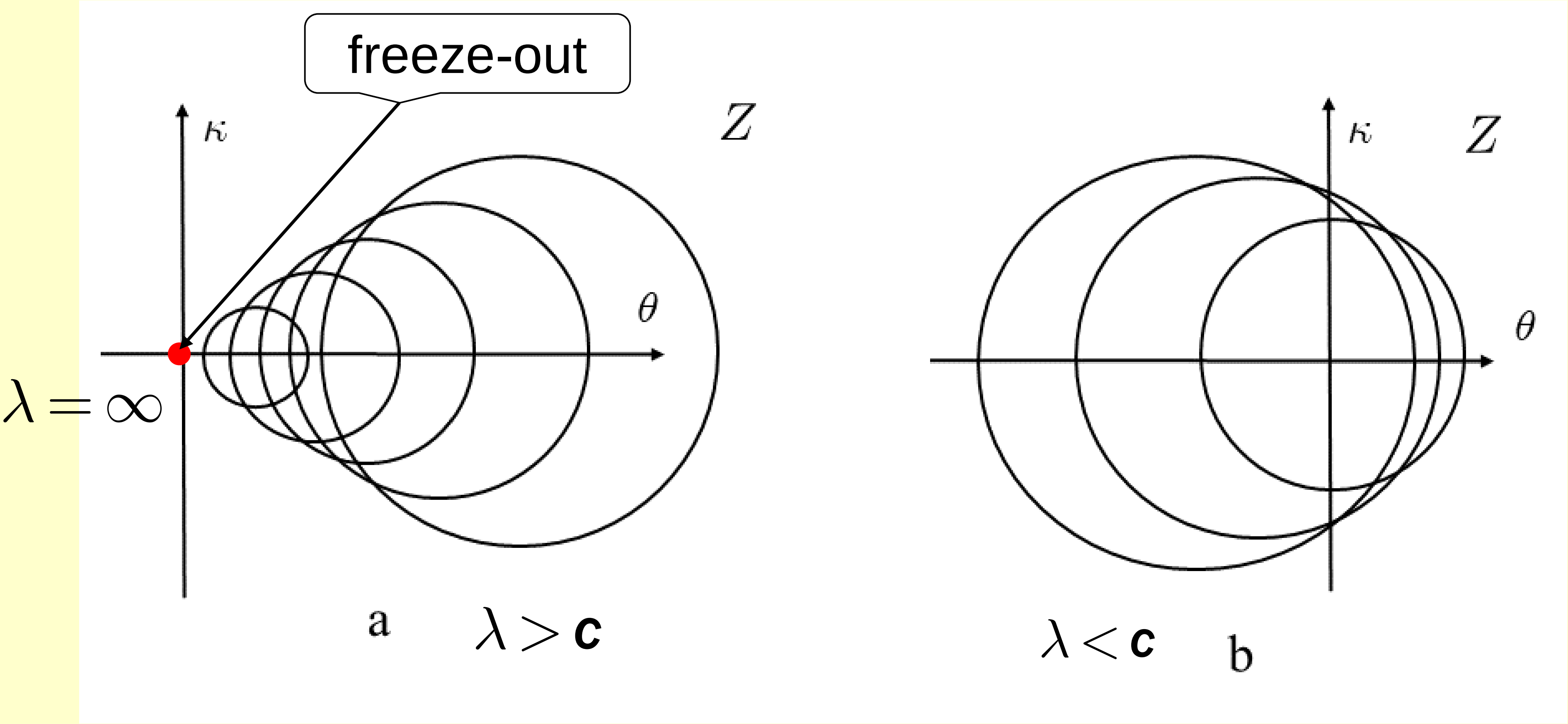
$$t = \lambda,\ x = \lambda \cos\tau + \frac{c}{4}\cos 2\tau,\ y = \lambda \sin\tau + \frac{c}{4}(\sin 2\tau + 2\tau),\ z = \cos\tau$$

one finds:

$$\boxed{(\theta_s - d)^2 + \kappa^2 = R}$$

$$d = d(\lambda) = \frac{\lambda}{\lambda^2 - c^2},\ R = R(\lambda) = \frac{c}{\left|\lambda^2 - c^2\right|}$$

# string diagrams for a twisted null string in Minkowsky space-time




Freeze-out regime: vanishing of expansion and rotation

# strings in asymptotically flat spacetimes

as future null infinity is approached: $\Psi_0 \simeq \dfrac{\psi_0^0}{\lambda^5}, \quad \Phi_{00} \simeq \dfrac{\varphi_0^0}{\lambda^6},$

therefore

$$Z \simeq \frac{1}{\lambda} + \frac{z(\tau)}{\lambda^2} + \frac{z_2(\tau)}{\lambda^3} + \frac{z_3(\tau) + \psi_0^0}{\lambda^4} + \ldots$$

one finds:

$$z(\tau) = -MG + \frac{1}{2}\ C_+ + iC_X \ + \ldots$$

$$z_2(\tau) = z^2(\tau)$$

$M$ - mass of the gravitating source

$C_+/r, C_X/r$ - amplitudes of incoming gravity waves (with different polarizations)

Freeze-out is a generic property of strings in asmptotically flat spacetimes

## Null strings in flat de Sitter universe

$$ds^2 = -dt^2 + e^{2Ht}(dx^2 + dy^2 + dz^2)$$

$$Z(\lambda,\tau) = \frac{H^2 z(\tau)}{Hz(\tau) - e^{-H\lambda}}$$

$\lambda$ - coincides with cosmological time, at large $\lambda$

$$Z(\lambda,\tau) \to H$$

Null strings in asmptotically de Sitter stretch according with cosmological

expansion

## Summary

- the string optical scalars $Z$ and associated diagrams are convenient tools to study basic properties of trajectories of null strings ;

- at future null infinity subleading terms in $Z$ encode information about mass distribution, gravity wave background and etc ;

## Future work

- scattering problem for null strings on massive spinning sources

- understanding if string optical scalars may encode an information about origin of null strings in the early Universe (analogy to relic gravity waves)

# thank you for attention