Correlators of heavy–light quark currents in HQET: OPE at 3 loops

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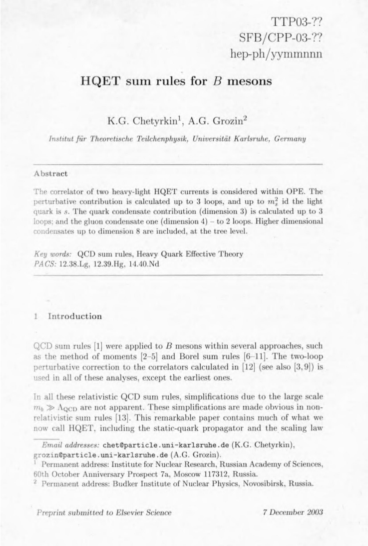

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HQET sum rules for B mesons

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Abstract

The correlator of two heavy-light HQET currents is considered within OPE. The perturbative contribution is calculated up to 3 loops, and up to m_s^2 id the light quark is s. The quark condensate contribution (dimension 3) is calculated up to 3 loops; and the gluon condensate one (dimension 4) – to 2 loops. Higher dimensional condensates up to dimension 8 are included, at the tree level.

Key words: QCD sum rules, Heavy Quark Effective Theory
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1 Introduction

QCD sum rules [1] were applied to B mesons within several approaches, such as the method of moments [2–5] and Borel sum rules [6–11]. The two-loop perturbative correction to the correlators calculated in [12] (see also [3,9]) is used in all of these analyses, except the earliest ones.

In all these relativistic QCD sum rules, simplifications due to the large scale $m_b \gg \Lambda_{QCD}$ are not apparent. These simplifications are made obvious in non-relativistic sum rules [13]. This remarkable paper contains much of what we now call HQET, including the static-quark propagator and the scaling law

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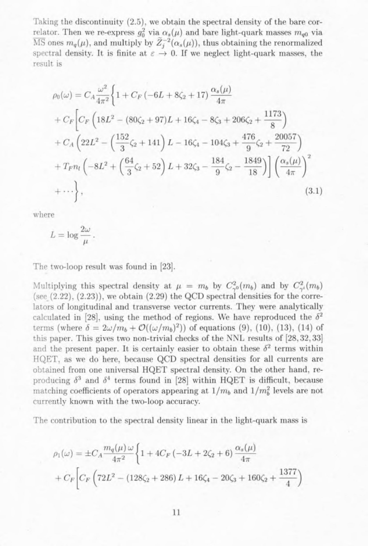


Taking the discontinuity (2.5), we obtain the spectral density of the bare correlator. Then we re-express g_0^2 via $\alpha_s(\mu)$ and bare light-quark masses m_{q0} via $\overline{\rm MS}$ ones $m_q(\mu)$, and multiply by $\tilde{Z}_j^{-2}(\alpha_s(\mu))$, thus obtaining the renormalized spectral density. It is finite at $\varepsilon \to 0$. If we neglect light-quark masses, the result is

$$
\begin{aligned}
\rho_0(\omega) = {} & C_A \frac{\omega^2}{4\pi^2} \biggl\{ 1 + C_F \left(-6L + 8\zeta_2 + 17\right) \frac{\alpha_s(\mu)}{4\pi} \\
& + C_F \biggl[C_F \left(18L^2 - (80\zeta_2 + 97) L + 16\zeta_4 - 8\zeta_3 + 206\zeta_2 + \frac{1173}{8} \right) \\
& + C_A \left(22L^2 - \left(\frac{152}{3}\zeta_2 + 141 \right) L - 16\zeta_4 - 104\zeta_3 + \frac{476}{9}\zeta_2 + \frac{20057}{72} \right) \\
& + T_F n_l \left(-8L^2 + \left(\frac{64}{3}\zeta_2 + 52 \right) L + 32\zeta_3 - \frac{184}{9}\zeta_2 - \frac{1849}{18} \right) \biggr] \left(\frac{\alpha_s(\mu)}{4\pi} \right)^2 \\
& + \cdots \biggr\},
\end{aligned}
\tag{3.1}
$$

where

$$L = \log \frac{2\omega}{\mu}.$$

The two-loop result was found in [23].

Multiplying this spectral density at $\mu = m_b$ by $C_{\gamma^0}^2(m_b)$ and by $C_{\gamma^i}^2(m_b)$ (see (2.22), (2.23)), we obtain (2.29) the QCD spectral densities for the correlators of longitudinal and transverse vector currents. They were analytically calculated in [28], using the method of regions. We have reproduced the δ^2 terms (where $\delta = 2\omega/m_b + \mathcal{O}((\omega/m_b)^2)$) of equations (9), (10), (13), (14) of this paper. This gives two non-trivial checks of the NNL results of [28, 32, 33] and the present paper. It is certainly easier to obtain these δ^2 terms within HQET, as we do here, because QCD spectral densities for all currents are obtained from one universal HQET spectral density. On the other hand, reproducing δ^3 and δ^4 terms found in [28] within HQET is difficult, because matching coefficients of operators appearing at $1/m_b$ and $1/m_b^2$ levels are not currently known with the two-loop accuracy.

The contribution to the spectral density linear in the light-quark mass is

$$
\begin{aligned}
\rho_1(\omega) = {} & \pm C_A \frac{m_q(\mu)\,\omega}{4\pi^2} \biggl\{ 1 + 4C_F \left(-3L + 2\zeta_2 + 6\right) \frac{\alpha_s(\mu)}{4\pi} \\
& + C_F \biggl[C_F \left(72L^2 - (128\zeta_2 + 286) L + 16\zeta_4 - 20\zeta_3 + 160\zeta_2 + \frac{1377}{4} \right)
\end{aligned}
$$

11

HQET

Single heavy antiquark $\bar{Q}$ momentum $P = Mv + p$
M — on-shell mass, $p \ll M$, light fields $p_i \ll M$
static field $\bar{h} = -\bar{h}\not{v}$
Leading order in $1/M$: heavy-quark spin does not interact, can be rotated (heavy-quark spin symmetry)
It can even be switched off (superflavor symmetry), scalar heavy antiquark φ^*
v rest frame: free propagator $\delta(\vec{x}\,)S_0(x^0)$

$$S_0(t) = -i\theta(t) \qquad S_0(p) = \frac{1}{p^0 + i0}$$

no loops

Heavy–light current

$$j_0 = \varphi_0^* q_0 = Z_j(\alpha_s(\mu)) j(\mu)$$

$$\langle T j_0(x) \bar{\jmath}_0(0) \rangle = \delta(\vec{x}\,) \Pi_0(x^0)$$

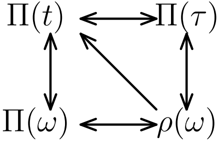
$$i \int d^d x \, \langle T j_0(x) \bar{\jmath}_0(0) \rangle e^{ip\cdot x} = \Pi_0(p^0)$$

$$\Pi_0(\omega) = \int_0^\infty dt \, \Pi_0(t) e^{i\omega t} \qquad \Pi_0(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \Pi_0(\omega) e^{-i\omega t}$$

$$\Pi_0 = A + B \not{v} \qquad j_{\pm 0} = \frac{1 \pm \gamma^0}{2} j_0 \qquad \Pi_\pm = \pm A + B$$

Renormalization $\rho_0(\omega) = Z_j^2(\alpha_s(\mu)) \rho(\omega; \mu)$

Correlator



$$\rho_0(\omega) = \frac{1}{2\pi i}\left[\Pi_0(\omega - i0) - \Pi_0(\omega + i0)\right]$$

$$\Pi_0(\omega) = i\int_0^\infty \frac{d\varepsilon\,\rho_0(\varepsilon)}{\omega - \varepsilon + i0} + \sum c_n \omega^n$$

$$\Pi_0(t) = \theta(t)\int_0^\infty d\omega\,\rho_0(\omega) e^{-i\omega t} + \sum c_n i^n \delta^{(n)}(t)$$

$$t = -i\tau \qquad \Pi_0(\tau) = \int_0^\infty d\omega\,\rho_0(\omega) e^{-\omega\tau}$$

$$\rho_0(\omega) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} d\tau\,\Pi_0(\tau) e^{\omega\tau}$$

Borel transform $\hat{B}_E \Pi(\omega) = -i\Pi(\tau = 1/E)$

OPE

$-\omega \gg \Lambda$

$$\Pi(\omega;\mu) = \sum_i C_i(\omega;\mu)\langle O_i(\mu)\rangle$$

O_i include light-quark masses

- even dimensions: $\not{\psi}$
- odd dimensions: 1

dim	0	1	2	3	4
pert	1	m	$m^2, \sum m_i^2$	$m^3, m \sum m_i^2$	$m^4, m^2 \sum m_i^2, \sum m_i^4$
quark				$\bar{q}q$	$m\bar{q}q, \sum m_i \bar{q}_i q_i$
gluon					$G^a_{\mu\nu} G^a_{\mu\nu}$

D. Broadhurst, A. G. (1992): perturbative terms up to dimension 3 and the $\bar{q}q$ up to 2 loops, gluon condensates of dimension 6 from at 1 loop QCD results of S. Generalis (1984)

Renormalization: dimension 3

$$\frac{dO_3}{d\log\mu} + \gamma_3 O_3 = 0$$

$$O_3 = \begin{pmatrix} m^3 \\ m\sum m_i^2 \\ \bar{q}q \end{pmatrix} \qquad \gamma_3 = \begin{pmatrix} 3\gamma_m & 0 & 0 \\ 0 & 3\gamma_m & 0 \\ \gamma & \gamma' & -\gamma_m \end{pmatrix}$$

$$\gamma = 1 + \alpha_s + \alpha_s^2 + \cdots \qquad \gamma' = \alpha_s^2 + \cdots$$

V. Spiridonov, K. Chetyrkin (1988), ...
P. Baikov, K. Chetyrkin (2018)

Renormalization: dimension 4

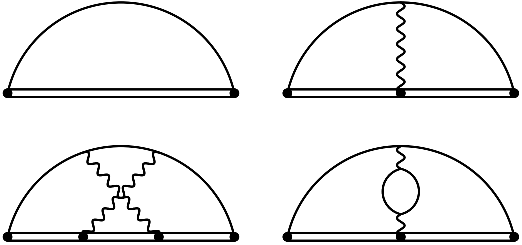
$$\frac{d(mO_3)}{d\log\mu} + (\gamma_3 + \gamma_m)O_3 = 0$$

$$\frac{dO_4}{d\log\mu} + \gamma_4 O_4 = 0$$

$$O_4 = \begin{pmatrix} \sum m_i^4 \\ \left(\sum m_i^2\right)^2 \\ \sum m_i \bar{q}_i q_i \\ G^a_{\mu\nu} G^a_{\mu\nu} \end{pmatrix}$$

$$\gamma_4 = \begin{pmatrix} 4\gamma_m & 0 & 0 & 0 \\ 0 & 4\gamma_m & 0 & 0 \\ \gamma & \gamma' & 0 & 0 \\ -\frac{d\gamma}{d\log\alpha_s} & -\frac{d\gamma'}{d\log\alpha_s} & 4\frac{d\gamma_m}{d\log\alpha_s} & -2\frac{d\beta}{d\log\alpha_s} \end{pmatrix}$$

Perturbative contribution



IBP reduction A. G. (2000) GRINDER (REDUCE);
R. Lee (2012) LITERED (MATHEMATICA)
Non-trivial master integrals M. Beneke, V. Braun (1994);
A. G. (2000); A. Czarnecki, K. Melnikov (2002)

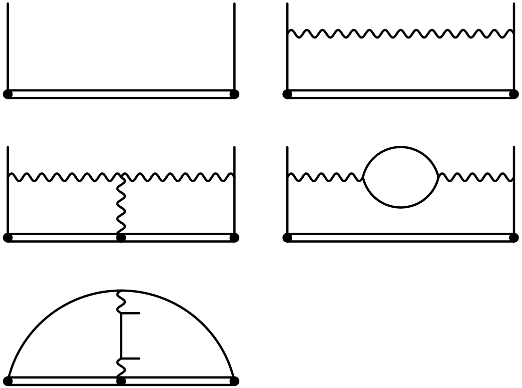
Spectral density

$$
\begin{aligned}
\rho(\omega) = \frac{N_c \omega^2}{4\pi^2} \Bigg\{ & 1 - C_F \frac{\alpha_s}{4\pi} \left(6L - \frac{4}{3}\pi^2 - 17 \right) \\
+ C_F \left(\frac{\alpha_s}{4\pi} \right)^2 & \Bigg\{ C_F \left[18L^2 - \left(\frac{40}{3}\pi^2 + 97 \right) L \right. \\
& \left. - 8\zeta_3 + \frac{8}{45}\pi^4 + \frac{103}{3}\pi^2 + \frac{1173}{8} \right] \\
& + C_A \left[22L^2 - \left(\frac{76}{9}\pi^2 + 141 \right) L \right. \\
& \left. - 104\zeta_3 - \frac{8}{45}\pi^4 + \frac{238}{27}\pi^2 + \frac{20057}{72} \right] \\
& - T_F n_l \left[8L^2 - 4\left(\frac{8}{9}\pi^2 + 13 \right) L - 32\zeta_3 + \frac{92}{27}\pi^2 + \frac{1849}{18} \right] \Bigg\} \\
+ \mathcal{O}(\alpha_s^3) \Bigg\} & \qquad L = \log \frac{2\omega}{\mu}
\end{aligned}
$$

Spectral density

- Up to 2 loops agrees with D. Broadhurst, A. G. (1992)
- This (leading in m) spectral density, when multiplied by the corresponding QCD/HQET matchings, agrees with the leading $1/M$ terms in A. Czarnecki, K. Melnikov (2002)

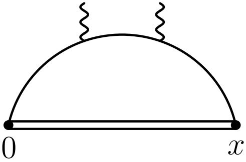
Quark condensate



An *independent* confirmation of $2\gamma_j - \gamma_{\bar{q}q}$ at 3 loops
(it starts from α_s^2)

Gluon condensate

1-loop term = 0
In fixed-point gauge the static quark does not interact with gluons



This term in massless $S(x, 0)$ vanishes after vacuum averaging

Higher dimensional quark condensates

Nonlocal quark condensate
S. Mikhailov, A. Radyushkin (1986)

$$\langle q(x)[x,0]\bar{q}(0)\rangle = -\frac{\langle \bar{q}q\rangle}{4}\left[f_S(x^2) - \frac{i\not{x}}{d} f_V(x^2)\right]$$

Expansion in local condensates up to dimension 8:
A. G. (1995). Anomalous condensate:

$$A = i\langle \bar{q} D_\alpha D_\beta D_\gamma D_\delta D_\varepsilon \gamma^{[\alpha}\gamma^\beta\gamma^\gamma\gamma^\delta\gamma^{\varepsilon]} q\rangle$$

Higher dimensional gluon condensates

$-\omega \gg m$

$$\Pi(\omega) = \Pi_G(\omega) + \Pi_Q(\omega) = \sum \tilde{a}_n(\omega) G_n$$

$$\Pi_Q(\omega) = \sum b_k(\omega) Q_k \qquad Q_k = \sum c_{kn}(m) G_n$$

$$d_k = d_n : \quad c_{kn} = \gamma_{kn} \left(\frac{1}{\varepsilon} - \log \frac{m^2}{\mu^2} \right) + c'_{kn}$$

$$\Pi_G(\omega) = \sum a_n(\omega) G_n \qquad a_n(\omega) = \tilde{a}_n(\omega) - \sum b_k(\omega, m) c_{kn}(m)$$

Dimensional regularization of IR singularities

D. Broadhurst, S. Generalis (1984–1985)
Hard region: Taylor expand integrand in m

$$\Pi(\omega) = \sum \bar{a}_n(\omega) G_n \qquad Q_k = \frac{1}{\varepsilon} \sum_{d_k = d_n} \gamma_{kn} G_n$$

$$a_n(\omega) = \bar{a}_n(\omega) - \frac{1}{\varepsilon} \sum_{d_k = d_n} b_k(\omega, m) \gamma_{kn}$$

$$\Rightarrow \bar{a}_n(\omega) - \sum_{d_k = d_n} \frac{d b_k(\omega, m)}{d\varepsilon} \gamma_{kn}$$

Dimension 6 agrees with the QCD results of S. Generalis (1984)

Sum rules

$$\Pi(\tau) = \int_0^\infty d\omega\, \rho_{d\leq 2}(\omega) e^{-\omega\tau} + \Pi_{d\geq 3}(\tau)$$

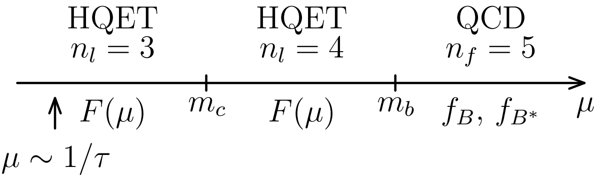
$$= \int_0^\infty d\omega\, \rho(\omega) e^{-\omega\tau}$$

Model: $\rho(\omega) = |F|^2 \delta(\omega - \bar{\Lambda}) + \rho_{d\leq 2}(\omega)\theta(\omega - \omega_c)$

$$|F|^2 e^{-\bar{\Lambda}\tau} = \int_0^{\omega_c} d\omega\, \rho_{d\leq 2}(\omega) e^{-\omega\tau} + \Pi_{d\geq 3}(\tau)$$

$$|F|^2 \bar{\Lambda} e^{-\bar{\Lambda}\tau} = \int_0^{\omega_c} d\omega\, \rho_{\text{pert}}(\omega) \omega e^{-\omega\tau} - \frac{d\Pi_{d\geq 3}(\tau)}{d\tau}$$

Matching and running



- HQET running
  - 2 loops X. Ji, M. Musolf (1991)
    D. Broadhurst, A. G. (1991)
  - 3 loops K. Chetyrkin, A. G. (2003)
- HQET-1/HQET-2 matching
  - 3 loops A. G., A. Smirnov, V. Smirnov (2006)
- QCD/HQET matching
  - 2 loops D. Broadhurst, A. G. (1995), A. G. (1998)
  - 3 loops S. Bekavac, A. G., P. Marquard, J. Piclum, D. Seidel, M. Steinhauser (2010)

Conclusion

- Coefficient functions of all operators up to dimension 4 up to 3 loops
- Coefficient functions of quark condensates up to dimension 8 at the tree level
- Coefficient functions of gluon condensates up to dimension 8 at 1 loop