$\mathcal{N} = 2$ supersymmetric higher spins from harmonic superspace

Evgeny Ivanov (BLTP JINR, Dubna)

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Ioseph Buchbinder (Tomsk) and Nikita Zaigraev (Dubna),

arXiv: 2109.07639 [hep-th]

Outline

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Summary and outlook

Preamble: Supersymmetry and higher-spins

- Supersymmetry allowed to construct a lot of new theories with remarkable and surprising features: supergravities, superstrings, superbranes, $\mathcal{N} = 4$ super Yang-Mills theory (the first example of the ultraviolet-finite quantum field theory), etc. It also allowed to establish unexpected relations between these theories, e.g., the AdS/CFT (or "gravity/gauge") correspondence.

- Supersymmetric higher-spin theories are also under intensive development for last decades. One of the basic origins of interest in the higher-spin theory and its superextrensions is that this kind of theories could serve a bridge between superstring theory and low-energy (super)gauge theories.

- The natural tools to deal with supersymmetric theories are the off-shell superfield methods. While the component approach basically yields the on-shell multiplets, with unclosed supersymmetry algebras (open algebras, etc), in the superfield approach the supersymmetry is closed from the very beginning on the off-shell supermultiplets with the correct sets of the auxiliary fields. Unconstrained superfield formulations are most preferable.

- Free massless bosonic and fermionic higher spin field theories have been pioneered in Fronsdal, 1978; Fang, Fronsdal, 1978.

- The component approach to the description of $4D, \mathcal{N} = 1$ supersymmetric free massless higher spin models was initiated in Courtright, 1979; Vasiliev, 1980.

- The complete off-shell Lagrangian formulation of $4D$ free higher spin $\mathcal{N} = 1$ models (including those on the AdS background) has been given in terms of $\mathcal{N} = 1$ superfields in a series of works by S. Kuzenko with collaborators (Kuzenko et al, 1993, 1994).

- Until present, an off-shell superfield Lagrangian formulation for **extended** higher spin supersymmetric theories, with all supersymmetries being manifest, was unknown even for free theories.

- This gap was recently filled in I. Buchbinder, E. Ivanov, N. Zaigraev, arXiv: 2109.07639 [hep-th]. An off-shell manifestly $\mathcal{N} = 2$ supersymmetric unconstrained formulation of $4D, \mathcal{N} = 2$ superextension of the Fronsdal theory for integer spins was constructed for the first time, based on the harmonic superspace approach.

- At present, what concerns four-dimensional theories with extended supersymmetry, the self-consistent off-shell superfield formalism is known only for $\mathcal{N} = 2$ and $\mathcal{N} = 3$ supersymmetries. It is the harmonic superspace approach (Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev, 1984).

- It yields off-shell geometric formulations of the relevant super Yang-Mills and supergravity theories and provides the unique possibility to describe off shell the basic matter multiplet of $\mathcal{N} = 2$ supersymmetry, the hypermultiplet.

- Our paper opens a new area of applications of the harmonic superspace formalism, in $\mathcal{N} = 2$ higher-spin theories.

A brief account of $\mathcal{N}=2$ harmonic formalism

- The standard $\mathcal{N}=2$ superspace

$$z = (x^m, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha} i}), \quad m = 0, \ldots, 3\,,\ \alpha, \dot{\alpha} = 1, 2\,,\ i = 1, 2\,.$$

- Harmonic $\mathcal{N}=2$ superspace

$$Z = (z, u^{\pm i}), \quad u^{\pm i} \in SU(2)/U(1),\ u^{+i} u_i^- = 1\,.$$

- Harmonic superspace in the analytic basis

$$Z = \left(x_A^m, \theta^{+\mu}, \bar{\theta}^{+\dot{\mu}}, u_i^\pm, \theta^{-\mu}, \bar{\theta}^{-\dot{\mu}}\right) \equiv \left(\zeta, \theta^{-\mu}, \bar{\theta}^{-\dot{\mu}}\right),$$
$$\theta^{\pm\alpha,\dot{\alpha}} := \theta^{\alpha,\dot{\alpha} i} u_i^\pm,\ x_A^m := x^m - 2i\theta^{(i}\sigma^m \bar{\theta}^{j)} u_i^+ u_j^+\,.$$

- Analytic harmonic $\mathcal{N}=2$ superspace (true analog of chiral $\mathcal{N}=1$ superspace)

$$\zeta_A = (x_A^m, \theta^{+\alpha}, \bar{\theta}^{+\dot{\alpha}}, u^{\pm i}), \quad \delta_\epsilon x_A^m = -2i\left(\epsilon^- \sigma^m \bar{\theta}^+ + \theta^+ \sigma^m \bar{\epsilon}^-\right),$$
$$\delta_\epsilon \theta^{\pm\hat{\mu}} = \epsilon^{\pm\hat{\mu}}\,, \quad \epsilon^{\pm\hat{\mu}} = \epsilon^{\hat{\mu} i} u_i^\pm\,.$$

All basic $\mathcal{N}=2$ superfields are analytic

- SYM harmonic analytic gauge connection: $V^{++}(\zeta_A)$; Matter hypermultiplets: $q^{+}(\zeta_A)\,,\ \bar{q}^{+}(\zeta_A)$; Supergravity: $H^{++m}(\zeta_A)\,,\quad H^{++5}(\zeta_A)\,,\quad H^{++\alpha,\dot{\alpha}+}(\zeta_A)$.

- An instructive example is supplied by Abelian $\mathcal{N}=2$ gauge theory,

$$V^{++}(\zeta_A)\,,\quad \delta V^{++} = D^{++}\Lambda(\zeta_A)\,,\ D^{++} = \partial^{++} - 2i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\,.$$

- Wess-Zumino gauge:

$$V^{++}(\zeta_A) = (\theta^{+})^2\phi + (\bar{\theta}^{+})^2\bar{\phi} + 2i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}A_{\alpha\dot{\alpha}}$$
$$+ (\bar{\theta}^{+})^2\theta^{+\alpha}\psi^i_\alpha u^-_i + (\theta^{+})^2\bar{\theta}^{+}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}i}u^-_i + (\theta^{+})^2(\bar{\theta}^{+})^2 D^{(ik)}u^-_i u^-_k\,.$$

- $4D$ fields ϕ, $\bar{\phi}$, $A_{\alpha\dot{\alpha}}$, ψ^i_α, $\bar{\psi}^i_{\dot{\alpha}}$, $D^{(ik)}$ constitute an Abelian gauge $\mathcal{N}=2$ off-shell multiplet ($8+8$ off-shell degrees of freedom).

- To construct the invariant action, one needs to define the second, non-analytic gauge connection

$$V^{--}(Z), \quad D^{++}V^{--} - D^{--}V^{++} = 0\,,\; \delta V^{--} = D^{--}\Lambda\,,$$
$$D^{--} = \partial^{--} - 2i\theta^{-\alpha}\bar{\theta}^{-\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + \theta^{-\hat{\mu}}\partial_{+\hat{\mu}}\,, \quad \hat{\mu} := (\mu, \dot{\mu})\,.$$

- The action reads

$$S \sim \int d^{12}Z\,(V^{++}V^{--})\,.$$

- To find the component action, one should restore V^{--} by V^{++} in WZ gauge from the harmonic flatness condition (this is straightforward albeit boring) and then do integrals over Grrassmann and harmonic variables.

Minimal $\mathcal{N}=2$ Einstein supergravity in HSS

- For geometric HSS formulation of the “mninimal” off-shell Einstein supergravity (in the component approach pioneered in Fradkin, Vasiliev, 1979) one needs to extend $4D$ HSS by a fifth coordinate, $Z \Rightarrow (Z, x^5)$,

$$\delta x^5 = 2i\big(\epsilon^-\theta^+ - \bar{\epsilon}^-\bar{\theta}^+\big)$$

and to define “flat” analyticity-preserving harmonic derivatives as

$$D^{++} = \partial^{++} - 2i\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}} + \theta^{+\hat{\mu}}\partial^{+}_{\hat{\mu}} + i(\hat{\theta^{+}})^2\partial_5\,,$$

$$D^{--} = \partial^{--} - 2i\theta^{-\rho}\bar{\theta}^{-\dot{\rho}}\partial_{\rho\dot{\rho}} + \theta^{-\hat{\mu}}\partial^{-}_{\hat{\mu}} + i(\hat{\theta^{-}})^2\partial_5\,,$$

$$[D^{++}, D^{--}] = D^0,\; D^0 = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}} + \theta^{+\hat{\mu}}\partial^{-}_{\hat{\mu}} - \theta^{-\hat{\mu}}\partial^{+}_{\hat{\mu}}\,.$$

- The fundamental group of the resulting Einstein $\mathcal{N}=2$ SG is defined as the following analyticity-preserving superdiffeomorphisms

$$\delta_\lambda x^m = \lambda^m(x, \theta^+, u), \quad \delta_\lambda x^5 = \lambda^5(x, \theta^+, u)\,,$$

$$\delta_\lambda \theta^{+\mu} = \lambda^{+\mu}(x, \theta^+, u), \qquad \delta_\lambda \bar{\theta}^{+\dot{\mu}} = \lambda^{+\dot{\mu}}(x, \theta^+, u),$$

$$\delta_\lambda \theta^{-\mu} = \lambda^{-\mu}(x, \theta^+, \theta^-, u), \qquad \delta_\lambda \bar{\theta}^{-\dot{\mu}} = \lambda^{-\dot{\mu}}(x, \theta^+, \theta^-, u),$$

$$\delta_\lambda u^{\pm}_i = 0.$$

- We require that the properly covariantized harmonic derivatives are invariant under these transformations

$$D^{\pm\pm} \Rightarrow \mathfrak{D}^{\pm\pm}, \quad \delta\mathfrak{D}^{\pm\pm} = 0,$$
$$\mathfrak{D}^{++} = D^{++} + h^{++m}\partial_m + h^{++\hat{\mu}+}\partial^-_{\hat{\mu}} + h^{++5}\partial_5,$$
$$\mathfrak{D}^{--} = D^{--} + h^{--m}\partial_m + h^{--\hat{\mu}+}\partial^-_{\hat{\mu}} + h^{--\hat{\mu}-}\partial^+_{\hat{\mu}} + h^{--5}\partial_5.$$

- The harmonic flatness conditions follow from

$$[\mathfrak{D}^{++}, \mathfrak{D}^{--}] = D^0.$$

- We will need only the linearized form of these conditions

$$D^{++}h^{--\alpha\dot{\alpha}} - D^{--}h^{++\alpha\dot{\alpha}} + 4i\big(h^{--\alpha+}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}h^{--\dot{\alpha}+}\big) = 0,$$
$$D^{++}h^{--5} - D^{--}h^{++5} - 2i\big(h^{--\alpha+}\theta^+_\alpha - \bar{\theta}^+_{\dot{\alpha}}h^{--\dot{\alpha}+}\big) = 0,$$
$$D^{++}h^{--\alpha+} - D^{--}h^{++\alpha+} = 0, \quad D^{++}h^{--\dot{\alpha}+} - D^{--}h^{++\dot{\alpha}+} = 0,$$
$$D^{++}h^{--\alpha-} - h^{--\alpha+} = 0, \quad D^{++}h^{--\dot{\alpha}-} - h^{--\dot{\alpha}+} = 0,$$

as well as the linearized form of the gauge transformations of the analytic vielbein coefficients

$$\delta_\lambda h^{++m} = D^{++}\lambda^m + 2i\big(\lambda^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}\big),$$
$$\delta_\lambda h^{++5} = D^{++}\lambda^5 - 2i\big(\lambda^{+\alpha}\theta^+_\alpha - \bar{\theta}^+_{\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}\big), \delta_\lambda h^{++\hat{\mu}+} = D^{++}\lambda^{+\hat{\mu}}.$$

- Using the above gauge transformations, one can fix the WZ gauge as

$$h^{++m} = -2i\theta^+\sigma^a\bar\theta^+\Phi^m_a + \left[(\bar\theta^+)^2\theta^+\psi^{m\,i}u^-_i + c.c.\right] + (\theta^+)^4 V^{m(ij)}u^-_i u^-_j\,,$$
$$h^{++5} = -2i\theta^+\sigma^a\bar\theta^+ C_a + \left[(\bar\theta^+)^2\theta^+\rho^i u^-_i + c.c.\right] + (\theta^+)^4 S^{(ij)}u^-_i u^-_j\,,$$
$$h^{++\mu+} = (\theta^+)^2\bar\theta^+_{\dot\mu}P^{\mu\dot\mu} + (\bar\theta^+)^2\,\theta^+_\nu\left[\varepsilon^{\mu\nu}M + T^{(\mu\nu)}\right] + (\theta^+)^4\chi^{\mu i}u^-_i\,.$$

- The residual gauge transformations are spanned by the parameters

$$\lambda^m \;\Rightarrow\; a^m(x)\,, \quad \lambda^5 \;\Rightarrow\; b(x)\,,$$
$$\lambda^{\mu+} \;\Rightarrow\; \epsilon^{\mu i}(x)u^+_i + \theta^{+\nu}l_{(\nu}{}^{\mu)}(x)\,,\; \bar\lambda^{\dot\mu+} \;\Rightarrow\; \bar\epsilon^{\dot\mu i}(x)u^+_i + \bar\theta^{+\dot\nu}l_{(\dot\nu}{}^{\dot\mu)}(x)\,.$$

- Taking account of this residual gauge freedom, we end just with the standard field content $40+40$ of minimal $\mathcal{N}=2$ Einstein supergravity. The physical gauge fields are $\Phi^m_a, \psi^{m\,i}_\mu, C_a$, the remaining ones are auxiliary. The spin 1 parts of the gauge field Φ^m_a can be gauged away by the local “Lorentz” parameters $l_{(\nu}{}^{\mu)}(x), l_{(\dot\nu}{}^{\dot\mu)}(x)$. In this gauge

$$\Phi_{\beta\dot\beta\alpha\dot\alpha} = \Phi_{(\beta\alpha)(\dot\beta\dot\alpha)} + \varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta}\Phi\,,$$
$$\delta\Phi_{\beta\dot\beta\alpha\dot\alpha} = \frac{1}{2}\left(\partial_{\alpha\dot\alpha}a_{\beta\dot\beta} + \partial_{\beta\dot\beta}a_{\alpha\dot\alpha}\right),\; \delta\Phi = \frac{1}{4}\partial_{\alpha\dot\alpha}a^{\alpha\dot\alpha},\; \delta C_{\alpha\dot\alpha} = -2\partial_{\alpha\dot\alpha}\,b\,.$$

Invariant off-shell action

- The linearized spin 2 $\mathcal{N}=2$ theory is built on two analytic gauge superfields $h^{++m,5}$ and complex spinorial analytic superfield $h^{++\alpha+}$ (and c.c.). Under the standard rigid $\mathcal{N}=2$ supersymmetry they have non-standard transformation properties

$$\delta h^{++m} = -2i\big(h^{++\mu+}\sigma^m_{\mu\dot\mu}\bar\epsilon^{-\dot\mu} + \epsilon^{-\rho}\sigma^m_{\rho\dot\mu}h^{++\dot\mu+}\big)\,,$$
$$\delta_\epsilon h^{++5} = 2i\big(h^{++\mu+}\epsilon^-_\mu - \bar\epsilon^-_{\dot\mu}h^{++\dot\mu+}\big)$$

(and similarly for the negatively charged objects).

- Now one defines new $\mathcal{N}=2$ SUSY singlet non-analytic superfields

$$G^{++m} := h^{++m} + 2i\big(h^{++\mu+}\sigma^m_{\mu\dot\mu}\bar\theta^{-\dot\mu} + \theta^{-\mu}\sigma^m_{\mu\dot\mu}h^{++\dot\mu+}\big)\,,$$
$$G^{++5} := h^{++5} - 2i\big(h^{++\mu+}\theta^-_\mu - \bar\theta^-_{\dot\mu}h^{++\dot\mu+}\big)\,,$$
$$G^{--m} := h^{--m} + 2i\big(h^{--\mu+}\sigma^m_{\mu\dot\mu}\bar\theta^{-\dot\mu} + \theta^{-\mu}\sigma^m_{\mu\dot\mu}h^{++\dot\mu+}\big)\,,$$
$$G^{--5} := h^{--5} - 2i\big(h^{--\mu+}\theta^-_\mu - \bar\theta^-_{\dot\mu}h^{--\dot\mu+}\big),$$

which transform under the supergauge transformations as

$$\delta_\lambda G^{\pm\pm m} = D^{\pm\pm}\Lambda^m\,,\quad \delta_\lambda G^{\pm\pm 5} = D^{\pm\pm}\Lambda^5\,,$$
$$\Lambda^m = \lambda^m + 2i\big(\lambda^+\sigma^m\bar\theta^- + \theta^-\sigma^m\bar\lambda^+\big)\,,\quad \Lambda^5 = \lambda^5 - 2i\big(\lambda^+\theta^- - \bar\theta^-\bar\lambda^+\big),$$

and satisfy the flatness conditions

$$D^{++}G^{--m} = D^{--}G^{++m}\,,\quad D^{++}G^{--5} = D^{--}G^{++5}\,.$$

- These superfields are just the building blocks for the invariant superfield action. It is constructed as follows

$$S \sim S_1 + 4S_2,$$

$$S_1 = \int d^4xd^8\theta du\, G^{++\alpha\dot{\alpha}}G^{--}_{\alpha\dot{\alpha}}\,, \quad S_2 = \int d^4xd^8\theta du\, G^{++5}G^{--5},$$

$$\delta S_1 = 8i\int d\zeta^{-4}du(\partial_{\beta\dot{\beta}}\lambda^{+\beta}h^{++\dot{\beta}+} - \partial_{\beta\dot{\beta}}\bar{\lambda}^{+\dot{\beta}}h^{++\beta+}),$$

$$\delta S_2 = -2i\int d\zeta^{-4}du(\partial_{\beta\dot{\beta}}\lambda^{+\beta}h^{++\dot{\beta}+} - \partial_{\beta\dot{\beta}}\bar{\lambda}^{+\dot{\beta}}h^{++\beta+}).$$

Thus $\delta S = 0$.

- This action was earlier derived by Zupnik (Zupnik, 1998) by reduction from the full HSS action of $\mathcal{N} = 2$ Einstein supergravity (Galperin, Nguen Ahn Ky, Sokatchev, 1987).

Component Lagrangians

- Most bulky calculations are related to restoring the negatively charged quantities $G^{--m,5}$ by the gauge analytic potentials.
- Example: spin 1 Lagrangian

$$
\begin{aligned}
&G^{++5}_{(C)} = i\theta^{+\rho}\bar\theta^{+\dot\rho}C_{\rho\dot\rho} \quad \Rightarrow \\
&G^{--5}_{(C)} = i\theta^{-\beta}\bar\theta^{-\dot\beta}C_{\beta\dot\beta} - (\theta^-)^2\bar\theta^{-(\dot\rho}\bar\theta^{+\dot\beta)}\partial^{\beta}_{\dot\rho}C_{\beta\dot\beta} + (\bar\theta^-)^2\theta^{-(\rho}\theta^{+\beta)}\partial^{\dot\beta}_{\rho}C_{\beta\dot\beta} \\
&-i(\theta^-)^2(\bar\theta^-)^2\theta^{+\rho}\bar\theta^{+\dot\rho}\left[\Box C_{\rho\dot\rho} - \partial_{\rho\dot\rho}\partial^m C_m\right], \\
&G^{++5}_{(T)} = -2i(\bar\theta^+)^2\theta^+_\nu\theta^-_\mu T^{(\mu\nu)} - 2i(\theta^+)^2\bar\theta^+_{\dot\nu}\bar\theta^-_{\dot\mu}\bar T^{(\dot\mu\dot\nu)} \quad \Rightarrow \\
&G^{--5}_{(T)} = -2i(\bar\theta^-)^2\theta^+_\nu\theta^-_\mu T^{(\mu\nu)} - 2i(\theta^-)^2\bar\theta^+_{\dot\nu}\bar\theta^-_{\dot\mu}\bar T^{(\dot\mu\dot\nu)} \\
&+2(\bar\theta^-)^2(\theta^-)^2\bar\theta^{+\dot\rho}\theta^+_\mu\partial_{\rho\dot\rho}T^{(\mu\rho)} + 2(\theta^-)^2(\bar\theta^-)^2\theta^{+\rho}\bar\theta^+_{\dot\nu}\partial_{\rho\dot\nu}\bar T^{(\dot\mu\dot\nu)}.
\end{aligned}
$$

- Then

$$
\begin{aligned}
&G^{++5}_{(C)}G^{--5}_{(C)} + G^{++5}_{(T)}G^{--5}_{(T)} + G^{++5}_{(C)}G^{--5}_{(T)} + G^{++5}_{(T)}G^{--5}_{(C)} \quad \Rightarrow \\
&L_{(C,T)} = -\frac{1}{4}F^{mn}F_{mn} - \left[\tilde T^{(\dot\alpha\dot\gamma)}\tilde T_{(\dot\alpha\dot\gamma)} + \tilde T^{(\alpha\gamma)}\tilde T_{(\alpha\gamma)}\right], \\
&T_{(\alpha\beta)} = \tilde T_{(\alpha\beta)} + \frac{i}{2}\partial^{\dot\beta}_{(\alpha}C_{\beta)\dot\beta}, \quad T_{(\dot\alpha\dot\beta)} = \tilde T_{(\alpha\beta)} - \frac{i}{2}\partial^{\beta}_{(\dot\alpha}C_{\beta\dot\beta)}.
\end{aligned}
$$

- An analogous calculation leads to the kinetic term of the spin 2 field

$$G^{++\alpha\dot\alpha}_{(\Phi)}G^{--\alpha\dot\alpha}_{(\Phi)} + 4G^{++5}_{(\Phi)}G^{--5}_{(\Phi)} \quad \Rightarrow$$

$$\mathcal{L}_{(\Phi)} = -\frac{1}{4}\Big[\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}\Box\Phi_{(\alpha\beta)(\dot\alpha\dot\beta)} - \Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}\partial_{\alpha\dot\alpha}\partial^{\rho\dot\rho}\Phi_{(\rho\beta)(\dot\rho\dot\beta)}$$
$$+2\,\Phi\partial^{\alpha\dot\alpha}\partial^{\beta\dot\beta}\Phi_{(\alpha\beta)(\dot\alpha\dot\beta)} - 6\Phi\Box\Phi\Big].$$

- It is invariant under $\delta\Phi_{\beta\dot\beta\alpha\dot\alpha} = \frac{1}{2}\left(\partial_{\alpha\dot\alpha}a_{\beta\dot\beta} + \partial_{\beta\dot\beta}a_{\alpha\dot\alpha}\right)$, $\delta\Phi = \frac{1}{4}\partial_{\alpha\dot\alpha}a^{\alpha\dot\alpha}$, as expected.

Spin 3 model

- While in the spin 2 case we had the clear geometric input ($\mathcal{N} = 2$ SG), no such a hint exists in the case of spin 3. Nevertheless, it surprisingly turns out that the relevant construction can be performed in a close analogy with the spin 2 case.
- We postulate the existence of the triad of analytic unconstrained superfields, two bosonic and one complex fermionic,

$$h^{++(\alpha\beta)(\dot\alpha\dot\beta)}(\zeta)\,,\; h^{++\alpha\dot\alpha}(\zeta),\; h^{++(\alpha\beta)\dot\alpha+}(\zeta),\; h^{++(\dot\alpha\dot\beta)\alpha+}(\zeta)\,,$$

with the following transformation laws

$$\delta h^{++(\alpha\beta)(\dot\alpha\dot\beta)} = D^{++}\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} + 4i\big[\lambda^{+(\alpha\beta)(\dot\alpha}\bar\theta^{+\dot\beta)} + \theta^{+(\alpha}\bar\lambda^{+\beta)(\dot\alpha\dot\beta)}\big],$$
$$\delta h^{++\alpha\dot\alpha} = D^{++}\lambda^{\alpha\dot\alpha} - 2i\big[\lambda^{+(\alpha\beta)\dot\alpha}\theta^{+}_{\beta} + \bar\lambda^{+(\dot\alpha\dot\beta)\alpha}\bar\theta^{+}_{\dot\beta}\big],$$
$$\delta h^{++(\alpha\beta)\dot\alpha+} = D^{++}\lambda^{+(\alpha\beta)\dot\alpha}\,,\; \delta h^{++(\dot\alpha\dot\beta)\alpha+} = D^{++}\lambda^{+(\dot\alpha\dot\beta)\alpha}\,.$$

- Like the spin 2 case, we can maximally exploit this gauge freedom to put the gauge superfields in the WZ gauge form

$$
\begin{aligned}
h^{++(\alpha\beta)(\dot\alpha\dot\beta)} &= -2i\theta^{+\rho}\bar\theta^{+\dot\rho}\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho} + (\bar\theta^{+})^2\theta^{+}\psi^{(\alpha\beta)(\dot\alpha\dot\beta)i}u^-_i \\
&\quad + (\theta^{+})^2\bar\theta^{+}\bar\psi^{(\alpha\beta)(\dot\alpha\dot\beta)i}u^-_i + (\theta^{+})^4 V^{(\alpha\beta)(\dot\alpha\dot\beta)(ij)}u^-_i u^-_j \,, \\
h^{++\alpha\dot\alpha} &= -2i\theta^{+\rho}\bar\theta^{+\dot\rho}C^{\alpha\dot\alpha}_{\rho\dot\rho} + (\bar\theta^{+})^2\theta^{+}\rho^{\alpha\dot\alpha i}u^-_i + (\theta^{+})^2\bar\theta^{+}\bar\rho^{\alpha\dot\alpha i}u^-_i \\
&\quad + (\theta^{+})^4 S^{\alpha\dot\alpha(ij)}u^-_i u^-_j \,, \\
h^{++(\alpha\mu)\dot\alpha+} &= (\theta^{+})^2\bar\theta^{+}_{\dot\mu}P^{(\alpha\mu)\dot\alpha\dot\mu} + (\bar\theta^{+})^2\,\theta^{+}_{\nu}\left[\varepsilon^{\nu(\alpha}M^{\mu)\dot\alpha} + T^{\dot\alpha(\alpha\mu\nu)}\right] \\
&\quad + (\theta^{+})^4\chi^{(\alpha\mu)\dot\alpha i}u^-_i \,, \\
h^{++\alpha(\dot\alpha\dot\mu)+} &= (\widetilde{h^{++(\alpha\mu)\dot\alpha+}}) \,.
\end{aligned}
$$

- The physical gauge fields are $\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho}$ (spin 3 gauge field), $C^{\alpha\dot\alpha}_{\rho\dot\rho}$ (spin 2 gauge field) and $\psi^{(\alpha\beta)(\dot\alpha\dot\beta)i}_{\gamma}$ (spin 5/2 gauge field). The rest of fields are auxiliary, so we have the set $104 + 104$ off shell. On shell, we end up with the multiplet $(\mathbf{3}, \mathbf{5/2}, \mathbf{5/2}, \mathbf{2})$.

- The residual gauge freedom is spanned by the superparameters

$$\begin{cases} \lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} \;\Rightarrow\; a^{(\alpha\beta)(\dot\alpha\dot\beta)}(x)\,, \\ \lambda^{\alpha\dot\alpha} \;\Rightarrow\; b^{\alpha\dot\alpha}(x)\,, \\ \lambda^{(\mu\alpha)\dot\alpha+} \;\Rightarrow\; \epsilon^{(\mu\alpha)\dot\alpha i}(x)u_i^+ + \bar\theta^{+\dot\alpha}n^{(\mu\alpha)} + \theta^{+\nu}l_{(\nu}{}^{\mu\alpha)\dot\alpha}(x)\,, \\ \bar\lambda^{\alpha(\dot\alpha\dot\mu)+} \;\Rightarrow\; \bar\epsilon^{\alpha(\dot\alpha\dot\mu)i}(x)u_i^+ + \theta^{+\alpha}n^{(\dot\alpha\dot\mu)} + \bar\theta^{+\dot\nu}l_{(\dot\nu}{}^{\alpha\dot\alpha\dot\mu)}(x)\,. \end{cases}$$

- The meaning of the component parameters is as follows

$a^{(\alpha_1\alpha_2)(\dot\alpha_1\dot\alpha_2)}(x)$, spin 3 gauge transformations;

$b^{\alpha\dot\alpha}(x)$ spin 2 gauge transformations;

$\epsilon^{(\mu\alpha)\dot\alpha i}(x), \bar\epsilon^{\alpha(\dot\alpha\dot\mu)i}(x)$ spin $5/2$ fermionic gauge symmetry;

$n^{(\mu\alpha)}, n^{(\dot\alpha\dot\mu)}$ local "Lorentz rotations";

$l^{(\nu\mu\alpha)\dot\alpha}(x), l^{\alpha(\dot\nu\dot\alpha\dot\mu)}(x)$ new spin 3 analogs of local "Lorentz rotations".

- The latter two types of the parameters can be used to put the physical bosonic gauge fields into the irreducible form

$$\Phi_{\gamma\dot\gamma(\alpha\beta)(\dot\alpha\dot\beta)} = \Phi_{(\alpha\beta\gamma)(\dot\alpha\dot\beta\dot\gamma)} + \varepsilon_{\dot\gamma(\dot\alpha}\varepsilon_{\gamma(\beta}\Phi_{\alpha)\dot\beta)}\,,$$
$$C_{\gamma\dot\gamma\alpha\dot\alpha} = C_{(\gamma\alpha)(\dot\gamma\dot\alpha)} + \varepsilon_{\gamma\alpha}\varepsilon_{\dot\gamma\dot\alpha}C\,,$$

with the following residual gauge transformations

$$\delta\Phi_{(\alpha\gamma\beta)(\dot\alpha\dot\gamma\dot\beta)} = \partial_{(\beta(\dot\beta}a_{\alpha\gamma)\dot\alpha\dot\gamma)}\,, \quad \delta\Phi_{\alpha\dot\beta} = \frac{4}{9}\partial^{\gamma\dot\gamma}a_{(\alpha\gamma)(\dot\beta\dot\gamma)}\,,$$
$$\delta C_{(\alpha\beta)(\dot\alpha\dot\beta)} = \partial_{(\beta(\dot\beta}b_{\alpha)\dot\alpha)}, \quad \delta C = \frac{1}{4}\partial_{\alpha\dot\alpha}b^{\alpha\dot\alpha}\,.$$

These are the correct gauge transformations for the Fronsdal spin 3 fields ($\Phi_{(\alpha\beta\gamma)(\dot\alpha\dot\beta\dot\gamma)}, \Phi_{\alpha\dot\beta}$) and spin 2 fields ($C_{(\alpha\beta)(\dot\alpha\dot\beta)}, C$).

Invariant Lagrangians

- The Lagrangian is constructed by analogy with the spin 2 case. One defines the negatively charged potentials

$$h^{--(\alpha\beta)(\dot\alpha\dot\beta)}, \quad h^{--\alpha\dot\alpha}, \quad h^{--(\alpha\beta)\dot\alpha+}, \; h^{--(\dot\alpha\dot\beta)\alpha+}, \quad h^{--(\alpha\beta)\dot\alpha-}, \quad h^{--(\dot\alpha\dot\beta)\alpha-},$$

which are related to the basic analytic potentials by the proper harmonic equations. They have non-standard $\mathcal{N}=2$ SUSY transformation laws.

- Next, one defines $\mathcal{N}=2$ SUSY singlet superfields

$$G^{\pm\pm(\alpha\beta)(\dot\alpha\dot\beta)} = h^{\pm\pm(\alpha\beta)(\dot\alpha\dot\beta)} + 4i\big[h^{\pm\pm(\alpha\beta)(\dot\alpha+}\bar\theta^{-\dot\beta)} - h^{\pm\pm(\dot\alpha\dot\beta)(\alpha+}\theta^{-\beta)}\big],$$
$$G^{\pm\pm\alpha\dot\beta} = h^{\pm\pm\alpha\dot\beta} - 2i\big[h^{\pm\pm(\alpha\beta)\dot\beta+}\theta^-_\beta - \bar\theta^-_{\dot\alpha} h^{\pm\pm(\dot\alpha\dot\beta)\alpha+}\big],$$
$$D^{++}G^{--(\alpha\beta)(\dot\alpha\dot\beta)} - D^{--}G^{++(\alpha\beta)(\dot\alpha\dot\beta)} = 0\,, \; D^{++}G^{--\alpha\dot\beta} - D^{--}G^{++\alpha\dot\beta} = 0,$$

with the simple gauge transformation laws

$$\delta_\lambda G^{\pm\pm(\alpha\beta)(\dot\alpha\dot\beta)} = D^{\pm\pm}\Lambda^{(\alpha\beta)(\dot\alpha\dot\beta)}, \quad \delta_\lambda G^{\pm\pm\alpha\dot\beta} = D^{\pm\pm}\Lambda^{\alpha\dot\beta},$$
$$\Lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} = \lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} + 4i\big[\lambda^{+(\alpha\beta)(\dot\alpha}\bar\theta^{-\dot\beta)} - \bar\lambda^{+(\dot\alpha\dot\beta)(\alpha}\theta^{-\beta)}\big],$$
$$\Lambda^{\alpha\dot\beta} = \lambda^{\alpha\dot\beta} - 2i\big[\lambda^{+(\alpha\beta)\dot\alpha}\theta^-_\beta - \bar\theta^-_{\dot\beta}\bar\lambda^{+(\dot\alpha\dot\beta)\alpha}\big].$$

- The invariant superfield action is constructed literally on the pattern of the spin 2 case

$$S_{s=3} = \int d^4x d^8\theta du \left\{ G^{++(\alpha\beta)(\dot\alpha\dot\beta)} G^{--}_{(\alpha\beta)(\dot\alpha\dot\beta)} + 4 G^{++\alpha\dot\beta} G^{--}_{\alpha\dot\beta} \right\}.$$

Its gauge invariance can be directly checked. It is $\mathcal{N}=2$ supersymmetric by construction. The coefficient before this invariant and its sign can be fixed by those of the spin 3 field component action.

- After some work one can find the spin 3 and spin 2 component actions. They look standard, in particular spin 2 action have the sam form as the corresponding action in $\mathcal{N}=2$ spin 2 model. The spin 3 component action, up to a normalization, reads

$$\begin{aligned}
S_{(s=3)} = \int d^4x \Big\{ & \Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \Box \Phi_{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \\
& - \frac{3}{2} \Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \partial_{\alpha_1\dot\alpha_1} \partial^{\rho\dot\rho} \Phi_{(\rho\alpha_2\alpha_3)(\dot\rho\dot\alpha_2\dot\alpha_3)} \\
& + 3 \Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \partial_{\alpha_1\dot\alpha_1} \partial_{\alpha_2\dot\alpha_2} \Phi_{\alpha_3\dot\alpha_3} - \frac{15}{4} \Phi^{\alpha\dot\alpha} \Box \Phi_{\alpha\dot\alpha} \\
& + \frac{3}{8} \partial_{\alpha_1\dot\alpha_1} \Phi^{\alpha_1\dot\alpha_1} \partial_{\alpha_2\dot\alpha_2} \Phi^{\alpha_2\dot\alpha_2} \Big\}.
\end{aligned}$$

General integer spin s case

- The set of analytic gauge potentials is

$$h^{++\alpha(s-1)\dot\alpha(s-1)}(\zeta), h^{++\alpha(s-2)\dot\alpha(s-2)}(\zeta), h^{++\alpha(s-1)\dot\alpha(s-2)+}(\zeta), h^{++\dot\alpha(s-1)\alpha(s-2)+}(\zeta),$$

where $\alpha(s) := (\alpha_1 \ldots \alpha_s), \dot\alpha(s) := (\dot\alpha_1 \ldots \dot\alpha_s)$.

- The corresponding gauge group is spanned by the transformations

$$\begin{aligned}
\delta h^{++\alpha(s-1)\dot\alpha(s-1)} &= D^{++}\lambda^{++\alpha(s-1)\dot\alpha(s-1)} + 4i\big[\lambda^{\alpha(s-1)(\dot\alpha(s-2)}\bar\theta^{+\dot\alpha_{s-1})} \\
&\quad + \theta^{+(\alpha_{s-1}}\bar\lambda^{+\alpha(s-2))\dot\alpha(s-1)}\big], \\
\delta h^{++\alpha(s-2)\dot\alpha(s-2)} &= D^{++}\lambda^{++\alpha(s-2)\dot\alpha(s-2)} - 2i\,\big[\lambda^{+(\alpha(s-2)\alpha_{s-1})\dot\alpha(s-2)}\theta^{+}_{s-1} \\
&\quad + \bar\lambda^{+(\dot\alpha(s-2)\dot\alpha_{s-1})\dot\alpha(s-2)}\bar\theta^{+}_{\dot\alpha_{s-1}}\big], \\
\delta h^{++\alpha(s-1)\dot\alpha(s-2)+} &= D^{++}\lambda^{+\alpha(s-1)\dot\alpha(s-2)}\,, \\
\delta h^{++\dot\alpha(s-1)\alpha(s-2)+} &= D^{++}\bar\lambda^{+\dot\alpha(s-1)\alpha(s-2)}\,.
\end{aligned}$$

These transformations can be used to choose the appropriate WZ gauge, like in the $s = 2$ and $s = 3$ cases, and then to show that the physical multiplet involves spins $(\mathbf{s}, \mathbf{s}-\mathbf{1/2}, \mathbf{s}-\mathbf{1/2}, \mathbf{s}-\mathbf{1})$.

- The next steps are to define the relevant negatively charged potentials and then to construct $\mathcal{N}=2$ singlet superfields

$$G^{\pm\pm\alpha(s-1)\dot{\alpha}(s-1)} = h^{\pm\pm\alpha(s-1)\dot{\alpha}(s-1)} + 4i\big[h^{\pm\pm\alpha(s-1)(\dot{\alpha}(s-2)+}\bar{\theta}^{-\dot{\alpha}_{s-1})}$$
$$-h^{\pm\pm\dot{\alpha}(s-1)(\alpha(s-2)+}\theta^{-\alpha_{s-1})}\big],$$
$$G^{\pm\pm\alpha(s-2)\dot{\alpha}(s-2)} = h^{\pm\pm\alpha(s-2)\dot{\alpha}(s-2)} - 2i\big[h^{\pm\pm(\alpha(s-2)\alpha_{s-1})\dot{\alpha}(s-2)+}\theta^{-}_{\alpha_{s-1}}$$
$$+h^{\pm\pm\alpha(s-2)(\dot{\alpha}(s-2)\dot{\alpha}_{(s-1)})+}\bar{\theta}^{-}_{\dot{\alpha}_{s-1}}\big],$$

  satisfying the harmonic flatness conditions and possessing some simple gauge transformation laws. Then one construct the $\mathcal{N}=2$ supersymmetric action (up to a normalization)

$$S_{(s)} = (-1)^{s+1}\int d^4x d^8\theta du \Big\{G^{++\alpha(s-1)\dot{\alpha}(s-1)}G^{--}_{\alpha(s-1)\dot{\alpha}(s-1)}$$
$$+4G^{++\alpha(s-2)\dot{\alpha}(s-2)}G^{--}_{\alpha(s-2)\dot{\alpha}(s-2)}\Big\}.$$

- Its $\mathcal{N}=2$ supersymmetry is manifest, while gauge invariance is checked by bringing the gauge variation to the form

$$\delta_\lambda S_{(s)} = 2(-1)^{s+1}\int d^4x d^8\theta du\Big\{D^{--}\Lambda^{\alpha(s-1)\dot{\alpha}(s-1)}G^{++}_{\alpha(s-1)\dot{\alpha}(s-1)}$$
$$+4D^{--}\Lambda^{\alpha(s-2)\dot{\alpha}(s-2)}G^{++}_{\alpha(s-2)\dot{\alpha}(s-2)}\Big\}.$$

  Finally, $\delta S_{(s)} = 0$.

Summary

- We presented, for the first time, an off-shell $\mathcal{N} = 2$ supersymmetric extension of the Fronsdal theory for integer spins. For any spin $s \geq 2$ the relevant multiplet is described by a triad of unconstrained harmonic analytic superfields $h^{++\alpha(s-1)\dot{\alpha}(s-1)}(\zeta)$, $h^{++\alpha(s-2)\dot{\alpha}(s-2)}(\zeta)$ and $h^{++\alpha(s-1)\dot{\alpha}(s-2)+}(\zeta)$ (and c.c.), which are subjected to gauge transformations with the analytic superfield parameters.
- The on-shell content of the spin s multiplet is $(\mathbf{s}, \mathbf{s} - \mathbf{1/2}, \mathbf{s} - \mathbf{1/2}, \mathbf{s} - \mathbf{1})$.
- For these superfields we found the $\mathcal{N} = 2$ supersymmetric and gauge invariant superfield actions which surprisingly have the universal form.
- Thus the harmonic superspace approach proved to be efficient in the new research domain, the theory of $\mathcal{N} = 2$ supersymmetric higher spins. This opens many new directions of research.

Some further problems:

- $\mathcal{N}=2$ supersymmetric half-integer spins?;
- An extension to AdS background;
- $\mathcal{N}=2$ superconformal spins?;
- Higher-spin analogs of other off-shell versions of $\mathcal{N}=2$ Einstein supergravity;
- Interactions. As a first step, covariantize everything with respect to local $\mathcal{N}=2$ SUSY, just via replacement $D^{\pm\pm} \Rightarrow \mathfrak{D}^{\pm\pm}$;
- Coupling $\mathcal{N}=2$ higher-spins to the hypermultiplet matter. Constructing higher-spin analogs of $\mathcal{N}=4$ SYM;
- Towards higher dimensions, e.g., $6D$, and higher $\mathcal{N}>2$;
- ETC ...

THANK YOU FOR YOUR PATIENCE!

WARMEST CONGRATS TO THE MAGNIFICENT SEVEN!