Perturbatively renormalizable quantum gravity

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TRM JHEP 1808 (2018) 024 [1802.04281], Int J Mod Phys D [1804.03834], SciPost Phys. 5 (2018) 4 040 [1806.02206], PRD 103 (2021) 8 [2006.05185];

MP Kellett & TRM, Class.Quant.Grav. 35 (2018) 175002 [1803.00859];

A Mitchell & TRM, JHEP 06 (2020) 138 [2004.06475];

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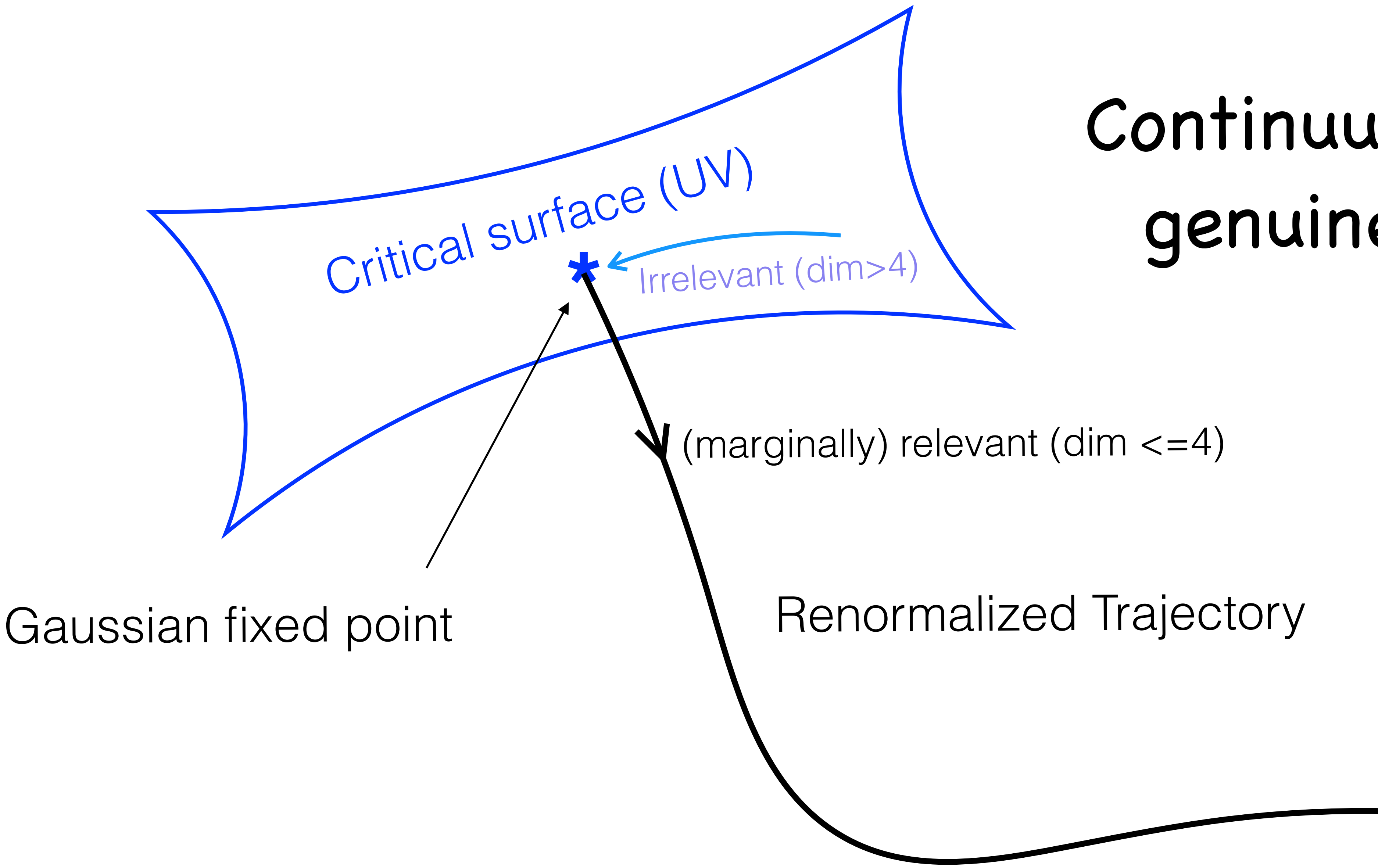
Continuum limit: genuine QFT



Perturbatively renormalizable quantum gravity

Quantum gravity does not have a perturbative continuum limit

$$S_{EH} = \int d^4x \, \mathcal{L}_{EH} \,, \qquad \mathcal{L}_{EH} = -2\sqrt{g}R/\kappa^2$$

$$\kappa = 2/M_{\text{Planck}} \,, \quad \kappa^2 = 32\pi G$$

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa H_{\mu\nu}$$

$$\mathcal{L}_{EH} = \partial H \partial H + \sum_{n=1}^{\infty} \kappa^n H^n \partial H \partial H$$

$\mathcal{L}_{\text{free}}$

only continuum limit

irrelevant operators dim^n n+4

But it also has another problem ...

$$S_{EH} = \int d^4x \, \mathcal{L}_{EH} \,, \qquad \mathcal{L}_{EH} = -2\sqrt{g}R/\kappa^2$$

$$\mathcal{Z} = \int \mathcal{D}g_{\mu\nu} \, \mathrm{e}^{-S_{EH}}$$ does **not** converge

Gibbons, Hawking, Perry '78

Problem is in the conformal factor $g_{\mu\nu} = \varphi^2 \hat{g}_{\mu\nu}$

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa H_{\mu\nu}$$

$$H_{\mu\nu} = h_{\mu\nu} + \tfrac{1}{2}\varphi \, \delta_{\mu\nu}$$

traceless

$$\mathcal{L}_{\text{free}} = \tfrac{1}{2}\left(\partial_\lambda h_{\mu\nu}\right)^2 - \tfrac{1}{2}\left(\partial_\lambda \varphi\right)^2$$

(Feynman - De Donder)

... key to solving the first problem

But it also has another problem ...

$$S_{EH} = \int d^4x\, \mathcal{L}_{EH}\,, \qquad \mathcal{L}_{EH} = -2\sqrt{g}R/\kappa^2$$

$$\mathcal{Z} = \int \mathcal{D}g_{\mu\nu}\; \mathrm{e}^{-S_{EH}}$$ does not converge

Gibbons, Hawking, Perry '78

Problem is in the conformal factor $g_{\mu\nu} = \varphi^2 \hat{g}_{\mu\nu}$

- Although partition function does not make sense, Wilsonian RG still can make sense
- Wilsonian RG anyway more meaningful for defining the continuum limit.

Wilsonian RG with right sign kinetic term necessarily has polynomial interactions

$$\mathcal{L}_\Lambda = \tfrac{1}{2}(\partial_\mu \varphi)^2 + \epsilon\, V_\Lambda(\varphi)$$

$$\Omega_\Lambda = \langle \varphi(x)\varphi(x)\rangle = \frac{\hbar\Lambda^2}{2a^2}$$

$$\Lambda\partial_\Lambda V_\Lambda(\varphi) = -\Omega_\Lambda \partial^2_\varphi V_\Lambda(\varphi)$$

$$V_\Lambda(\varphi) = \Lambda^4 \tilde{V}_\Lambda(\tilde{\varphi} = \varphi/\Lambda)$$

$$\Lambda\partial_\Lambda \tilde{V}_\Lambda - \tilde{\varphi}\partial_{\tilde{\varphi}}\tilde{V}_\Lambda + 4\tilde{V}_\Lambda = -\tfrac{1}{2a^2}\partial^2_{\tilde{\varphi}}\tilde{V}_\Lambda(\tilde{\varphi})$$

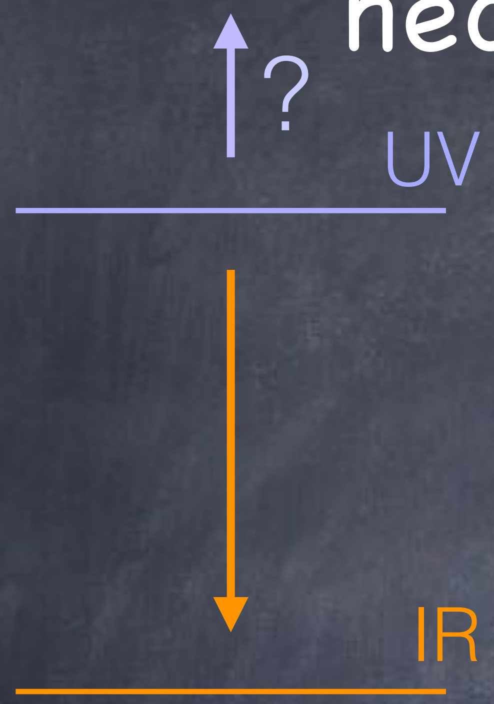
$$\tilde{V}_\Lambda(\tilde{\varphi}) = (\tfrac{\mu}{\Lambda})^\lambda \tilde{V}(\tilde{\varphi})$$

$$-\lambda\tilde{V}(\tilde{\varphi}) - \tilde{\varphi}\partial_{\tilde{\varphi}}\tilde{V} + 4\tilde{V} = -\tfrac{1}{2a^2}\partial^2_{\tilde{\varphi}}\tilde{V}(\tilde{\varphi})$$

Sturm-Liouville $\tilde{V} = \tilde{\mathcal{O}}_n(\tilde{\varphi}) = \frac{H_n(a\tilde{\varphi})}{(2a)^n} = \tilde{\varphi}^n - \frac{n(n-1)}{4a^2}\tilde{\varphi}^{n-2} + \cdots$

$$[\tilde{\mathcal{O}}_n] = n = [\varphi^n]$$

Wilsonian RG with right sign kinetic term necessarily has polynomial interactions



$$\Omega_\Lambda = \langle \varphi(x)\varphi(x)\rangle = \frac{\hbar\Lambda^2}{2a^2}$$

$$\Lambda\partial_\Lambda V_\Lambda(\varphi) = -\Omega_\Lambda \partial_\varphi^2 V_\Lambda(\varphi)$$

$$\int_{-\infty}^{\infty} d\tilde{\varphi}\ \mathrm{e}^{-a^2\tilde{\varphi}^2}\tilde{\mathcal{O}}_n(\tilde{\varphi})\tilde{\mathcal{O}}_m(\tilde{\varphi}) \propto \delta_{nm}$$

$$\int_{-\infty}^{\infty} d\tilde{\varphi}\ \mathrm{e}^{-a^2\tilde{\varphi}^2}\left(\tilde{V}(\tilde{\varphi}) - \sum_{n=0}^{N}\tilde{g}_n\tilde{\mathcal{O}}_n(\tilde{\varphi})\right)^2 \to 0 \quad \text{as} \quad N \to \infty\,.$$

Sturm-Liouville $\tilde{V} = \tilde{\mathcal{O}}_n(\tilde{\varphi}) = \frac{H_n(a\tilde{\varphi})}{(2a)^n} = \tilde{\varphi}^n - \frac{n(n-1)}{4a^2}\tilde{\varphi}^{n-2} + \cdots$

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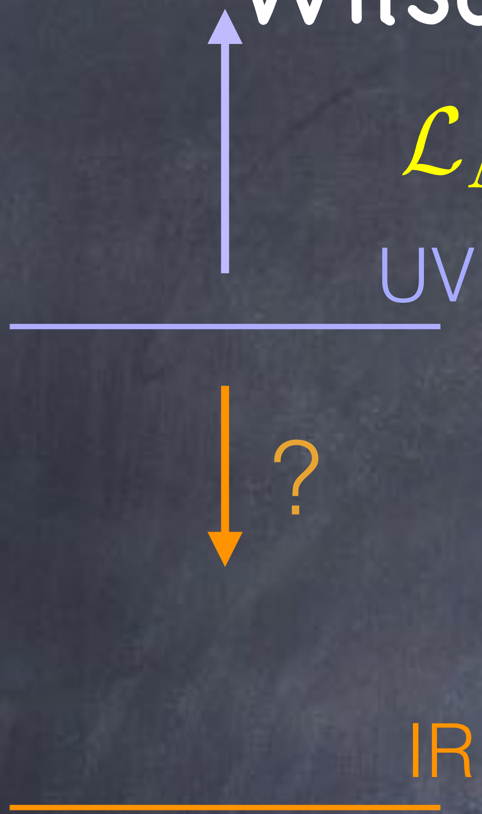
Wilsonian RG with wrong sign kinetic term



$$\mathcal{L}_\Lambda = -\tfrac{1}{2}(\partial_\mu \varphi)^2 + \epsilon\, V_\Lambda(\varphi)$$

$$\Omega_\Lambda = |\langle \varphi(x)\varphi(x)\rangle| = \frac{\hbar\Lambda^2}{2a^2}$$

$$\Lambda\partial_\Lambda V_\Lambda(\varphi) = +\Omega_\Lambda \partial^2_\varphi V_\Lambda(\varphi)$$

$$-\lambda \tilde{V}(\tilde{\varphi}) - \tilde{\varphi}\partial_{\tilde{\varphi}}\tilde{V} + 4\tilde{V} = +\frac{1}{2a^2}\partial^2_{\tilde{\varphi}}\tilde{V}(\tilde{\varphi})$$

Still Sturm-Liouville but $\displaystyle\int_{-\infty}^{\infty} d\tilde{\varphi}\ \mathrm{e}^{a^2\tilde{\varphi}^2}\,\delta_n(\tilde{\varphi})\,\delta_m(\tilde{\varphi}) \propto \delta_{nm}$

$$\int_{-\infty}^{\infty} d\tilde{\varphi}\ \mathrm{e}^{a^2\tilde{\varphi}^2}\left(\tilde{V}(\tilde{\varphi}) - \sum_{n=0}^{N}\tilde{g}_n\,\delta_n(\tilde{\varphi})\right)^2 \to 0 \quad \text{as} \quad N\to\infty\,.$$

$$\delta^{(n)}_\Lambda(\varphi) = \frac{\partial^n}{\partial\varphi^n}\,\delta^{(0)}_\Lambda(\varphi)\,, \qquad \delta^{(0)}_\Lambda(\varphi) = \frac{1}{\sqrt{2\pi\Omega_\Lambda}}\exp\left(-\frac{\varphi^2}{2\Omega_\Lambda}\right)$$

$$[\delta^{(n)}_\Lambda(\varphi)] = -1-n$$

∞ tower super-relevant

Wilsonian RG with wrong sign kinetic term

Quantisation condition $\int_{-\infty}^{\infty} d\varphi \; \mathrm{e}^{\varphi^2/2\Omega_\Lambda} V_\Lambda^2(\varphi) < \infty \quad \forall \Lambda > \Lambda_0$

$$\Omega_\Lambda = |\langle \varphi(x)\varphi(x)\rangle| = \frac{\hbar\Lambda^2}{2a^2}$$

Non-perturbative in ħ: $\exp\left(-\frac{a^2\varphi^2}{\Lambda^2\hbar}\right)$

$$\delta_\Lambda^{(n)}(\varphi) = \frac{\partial^n}{\partial\varphi^n}\,\delta_\Lambda^{(0)}(\varphi)\,, \qquad \delta_\Lambda^{(0)}(\varphi) = \frac{1}{\sqrt{2\pi\Omega_\Lambda}}\exp\left(-\frac{\varphi^2}{2\Omega_\Lambda}\right)$$

$$V_\Lambda(\varphi) = \sum_{n=0}^{\infty} g_n\,\delta_\Lambda^{(n)}(\varphi)$$

amplitude suppression scale Λ_σ

$$V(\varphi) = \lim_{\Lambda\to 0} V_\Lambda(\varphi) \qquad V(\varphi) \sim \mathrm{e}^{-\varphi^2/\Lambda_\sigma^2}$$

Wilsonian RG of perturbative quantum gravity

Non-diff'd fields must be square-integrable under

$$\exp \frac{1}{2\Omega_\Lambda}\left(\varphi^2 - h_{\mu\nu}^2 - 2\,\bar{c}_\mu c_\mu\right)$$

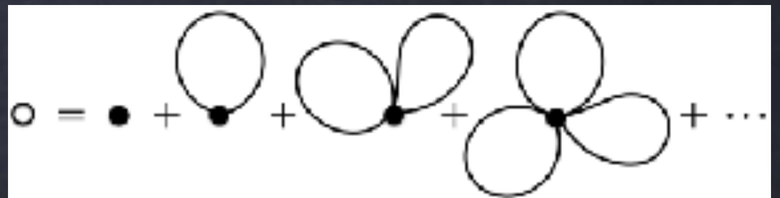
Interactions are $\delta_\Lambda^{(n)}(\varphi)$ polynomials

Operator: $f_\Lambda^\sigma(\varphi)\,\sigma(\partial_\alpha, \partial_\beta\varphi, h_{\gamma\delta}, \bar{c}_\varepsilon, c_\zeta, \Phi_A^*) + \cdots$

Renormalizability: $[\sigma] - 1 - n \leq 4$

Coefficient f^n: $f_\Lambda^\sigma(\varphi) = \sum_{n=n_\sigma}^{\infty} g_n^\sigma \delta_\Lambda^{(n)}(\varphi)$

tadpole corrections



What is the quantum version of diffeomorphism invariance?

Can show that obstruction arises from BRST transformation of coefficient functions:

$$Q_0 f^\sigma_\Lambda(\varphi) = \partial\cdot c\, f^{\sigma\,\prime}_\Lambda(\varphi)$$

Can prove solution if & only if $f^\sigma_\Lambda(\varphi)$ indept of φ

But that can be done by sending $\Lambda_\sigma \to \infty$

For $\sigma \sim H\partial H\partial H$ so $[\sigma] = 5$:

$$g_{2m}^{\sigma} = \frac{\sqrt{\pi}}{m!4^m}\,\kappa\,\Lambda_{\sigma}^{2m+1} \qquad (m = 0, 1, 2, \cdots)$$

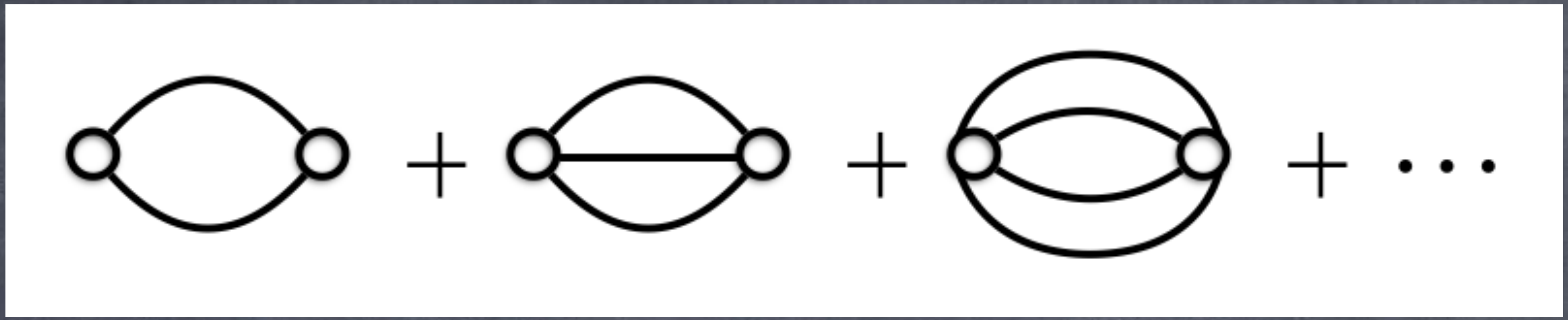
$$f_{\Lambda}^{\sigma}(\varphi) = \frac{\kappa a \Lambda_{\sigma}}{\sqrt{\Lambda^2 + a^2\Lambda_{\sigma}^2}}\,\exp\left(-\frac{a^2\varphi^2}{\Lambda^2 + a^2\Lambda_{\sigma}^2}\right)$$

$$f^{\sigma}(\varphi) = \lim_{\Lambda\to 0} f_{\Lambda}^{\sigma}(\varphi) = \kappa\,\mathrm{e}^{-\varphi^2/\Lambda_{\sigma}^2}$$

$$f_{\Lambda}^{\sigma}(\varphi) \to \kappa \quad \text{as} \quad \Lambda_{\sigma} \to \infty$$

N.B. Newton's constant is a `collective' effect

Second order in κ



$$\dot{\Gamma}_2 = \frac{1}{2}\,\mathrm{Str}\,\dot{\triangle}_\Lambda \Gamma_2^{(2)} - \frac{1}{2}\,\mathrm{Str}\,\dot{\triangle}_\Lambda \Gamma_1^{(2)} \triangle_\Lambda \Gamma_1^{(2)}$$

$$f^2_\Lambda(\varphi) = \sum_{n=0}^{\infty} g^2_n(\Lambda)\,\delta^{(n)}_\Lambda(\varphi) \qquad f^1_\Lambda(\varphi) = \sum_{n=n_\sigma}^{\infty} g^1_n\,\delta^{(n)}_\Lambda(\varphi)$$

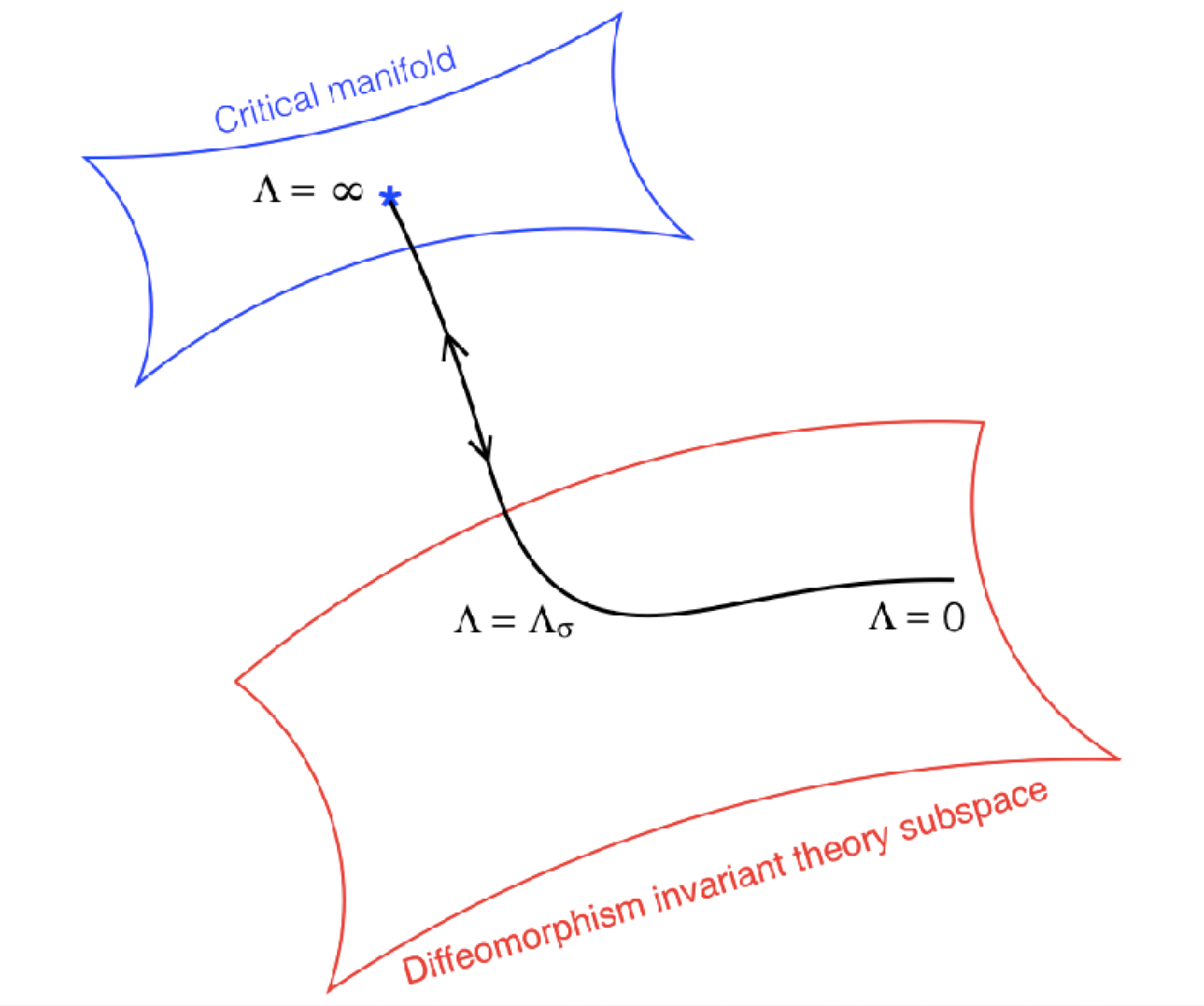
such that modified ST identities

$$0 = s_0\,\Gamma_2 + \frac{1}{2}\,(\Gamma_1, \Gamma_1) + \mathrm{Tr}\,C^\Lambda\,\Gamma^{(2)}_{1*} \triangle_\Lambda \Gamma^{(2)}_1$$

are satisfied at scales much less than Λ_σ

Amplitudes equivalent to standard perturbative QG

Construction crucially different from other QFTs, & other conceptions for QG.



Perturbatively: $f^{\sigma}_{\Lambda}(\varphi)\,\sigma(\partial_\alpha, \partial_\beta\varphi, h_{\gamma\delta}, \bar{c}_\varepsilon, c_\zeta, \Phi^*_A) + \cdots$

non-polynomial — polynomial

Non-perturbatively in κ: non-polynomial in $h_{\gamma\delta}$

? ?

Schematically:

$$\Lambda\partial_\Lambda V_\Lambda(\varphi, h_{\alpha\beta}) \sim \Omega_\Lambda \left(\partial^2_\varphi - \partial^2_{h_{\alpha\beta}}\right) V_\Lambda(\varphi, h_{\alpha\beta})$$

Only ϕ: parabolic - solutions typically singular towards IR

Only h: parabolic - solutions typically singular towards UV

Only RHS: hyperbolic - continuous infinity of wave-like FP solutions

Can show that if choose parametrisation so that EH term has non-singular flow,
then higher derivative terms will fail.

Construction establishes quantum gravity as a genuine continuum quantum field theory at $O(\kappa^2)$ with all the correct properties.

Appears to work at each order in κ, but with new coupling constants needed order by order, just as in standard perturbative quantum gravity.

However we see hints that these couplings are fixed at the non-perturbative level in order to get acceptable RG flows.