Predicting the sparticle mass scale in SUGRA models with degenerate vacua

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Ad QFT-2021, JINR

Outline

Based on:

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C. D. Froggatt, H. B. Nielsen, R. Nevzorov and A. W. Thomas, Universe **5** (2019) 214;

C. D. Froggatt, R. Nevzorov, H. B. Nielsen and A. W. Thomas, Phys. Lett. B **737** (2014) 167;

C. D. Froggatt, R. Nevzorov and H. B. Nielsen, Int. J. Mod. Phys. A **27** (2012) 1250063;

C. D. Froggatt, R. Nevzorov and H. B. Nielsen, Nucl. Phys. B 743 (2006) 133;

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Introduction

- The discovery of the Higgs boson with mass $m_h \simeq 125\,\mathrm{GeV}$ is an important step towards our understanding of the mechanism of the EW symmetry breaking.
- Further exploration of the TeV scale physics at the LHC may lead to the discovery of new phenomena beyond the SM.
- New physics (low–scale SUSY, theories with extra dimensions, etc) is expected to shed light on the stabilisation of the EW scale and cancellation of quadratic divergences, nature of dark matter as well as origin of baryon asymmetry.
- Despite the compelling arguments for physics beyond the SM no indication of its presence has been detected so far.
- Moreover there are some reasons to believe that SM is extremely fine-tuned.

- Indeed, astrophysical and cosmological observations indicate that there is a dark energy spread all over the Universe which constitutes about 70% of its energy density

$$\rho_\Lambda \sim 10^{-123} M_{Pl}^4 \sim 10^{-55} M_Z^4 \sim (10^{-3}\,\mathrm{eV})^4\,.$$

- At the same time the presence of QCD condensates in the vacuum is expected to contribute an energy density

$$\rho_{QCD} \sim \Lambda_{QCD}^4 \simeq 10^{-74} M_{Pl}^4\,.$$

- In the SM a much larger contribution must come from the EW symmetry breaking

$$\rho_{EW} \sim v^4 \simeq 10^{-62} M_{Pl}^4\,.$$

- The contribution of zero–modes is expected to push total vacuum energy density even higher up to M_{Pl}^4, i.e.

$$\begin{aligned}\rho_\Lambda &\simeq \sum_b \frac{\omega_b}{2} - \sum_f \frac{\omega_f}{2} = \\ &= \int_0^\Lambda \left[\sum_b \sqrt{|\vec{k}|^2 + m_b^2} - \sum_f \sqrt{|\vec{k}|^2 + m_f^2}\right] \frac{d^3\vec{k}}{2(2\pi)^3} \simeq -\Lambda^4.\end{aligned}$$

- Because of the enormous cancellation between different contributions to ρ_Λ the smallness of the cosmological constant should be regarded as a fine–tuning problem.
- Here, instead of trying to alleviate fine-tuning we postulate the exact degeneracy of different vacua (inspired by the multiple point principle (MPP)).
- MPP postulates the existence of many phases with the same energy density which are allowed by a given theory.
- The MPP applied to the SM implies that the Higgs effective potential

$$V_{eff}(H) = m^2(\phi)H^\dagger H + \lambda(\phi)(H^\dagger H)^2$$

possesses two degenerate minima taken to be at the EW and Planck scales.

- The degeneracy of these vacua can be achieved only if

$$\lambda(M_{Pl}) \simeq 0\,, \qquad \beta_\lambda(M_{Pl}) \simeq 0\,.$$

- Using these conditions one can compute M_t and M_H [C.D.Froggatt, H.B.Nielsen, Phys.Lett. **B368** (1996) 96]

$$M_t = 173 \pm 4\,\text{GeV}\,, \qquad M_H = 135 \pm 9\,\text{GeV}\,.$$

- Recently, using the extrapolation of the SM parameters up to M_{Pl} with full 3–loop RGE precision it was shown [D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, A. Strumia, JHEP **1312** (2013) 089 [arXiv:1307.3536]]

$$\begin{aligned}\lambda(M_{Pl}) &= -0.0128 - 0.0065\left(\frac{M_t}{\text{GeV}} - 173.35\right)\\ &+ 0.0018\left(\frac{\alpha_3(M_Z) - 0.1184}{0.0007}\right) + 0.0029\left(\frac{M_H}{\text{GeV}} - 125.66\right).\end{aligned}$$

- Hence there is still a chance that the MPP (criticality) conditions are fulfilled near the Planck scale [A. V. Bednyakov, B. A. Kniehl, A. F. Pikelner, O. L. Veretin, Phys. Rev. Lett. **115** (2015) 201802 [arXiv:1507.08833]].
- Here the MPP assumption is adapted to models based on $(N = 1)$ local supersymmetry, in order to provide an explanation for the small deviation of the cosmological constant from zero.

Dark energy in SUSY/SUGRA models

- An exact global SUSY ensures zero value for ρ_Λ.
- Indeed, SUSY scalar potential is positive definite and can be written as follows

$$V = \frac{1}{2} D^a D^a + F_i^* F_i , \qquad F_i^* = -\frac{\partial \mathcal{W}}{\partial A_i} , \qquad D^a = -g A_i^* T_{ij}^a A_j \, .$$

- Near the global minimum of the scalar potential $\langle F_i \rangle = \langle D^a \rangle = 0$
- However, in the exact SUSY limit, bosons and fermions from one chiral multiplet are degenerate.
- The breakdown of SUSY induces a huge and positive contribution to ρ_Λ

$$\rho_\Lambda \sim \Lambda_S^4 \, ,$$

  where Λ_S is a SUSY breaking scale.
- The non–observation of squarks and sleptons implies that $\Lambda_S >> 100\,\mathrm{GeV}$.

- The full $(N = 1)$ SUGRA Lagrangian is specified in terms of an analytic gauge kinetic functions $f_a(\phi_M)$ and a real gauge-invariant Kähler function $G(\phi_M, \phi^*_M)$.
- The functions $f_a(\phi_M)$ determine the kinetic terms for the fields in the vector supermultiplets and the gauge coupling constants $\mathrm{Re} f_a(\phi_M) = 1/g_a^2$.
- The Kähler function is a combination of two functions

$$G(\phi_M, \phi^*_M) = K(\phi_M, \phi^*_M) + \ln|\mathcal{W}(\phi_M)|^2 \, .$$

where $M_{Pl}/\sqrt{8\pi} = 1$.

- $K(\phi_M, \phi^*_M)$ is the Kähler potential whose second derivatives define the kinetic terms for the fields in the chiral supermultiplets.
- $\mathcal{W}(\phi_M)$ is the complete analytic superpotential.
- The SUGRA scalar potential can be presented as a sum of F and D–terms

$$V(\phi_M, \phi^*_M) = \sum_{M,\bar{N}} e^G \left(G_M G^{M\bar{N}} G_{\bar{N}} - 3 \right) + \frac{1}{2} \sum_a (D^a)^2 \, ,$$

$$G_M \equiv \frac{\partial G}{\partial \phi_M}, \quad G_{\bar{M}} \equiv \frac{\partial G}{\partial \phi^*_M}, \quad G^{M\bar{N}} = G^{-1}_{\bar{N}M}, \quad D^a = g_a \sum_{i,j} \left(G_i T^a_{ij} \phi_j \right)$$

- In order to break supersymmetry in SUGRA models a hidden sector is introduced.
- If hidden sector fields acquire VEVs so that at least one of their auxiliary fields

$$F^M = e^{G/2} G^{M\bar{P}} G_{\bar{P}}$$

is non-vanishing, then local SUSY is spontaneously broken.

- At the same time goldstino is swallowed up by the gravitino which becomes massive, i.e. $m_{3/2} \simeq \langle F^M \rangle / M_{Pl}$.
- In general the vacuum energy density in SUGRA models tends to be negative $\rho_\Lambda \sim -\langle e^G \rangle$.
- The situation changes if supergravity is supplemented by $SU(1,1)$ global symmetry which ensures that $\rho_\Lambda \simeq 0$.
- The Lagrangian of the simplest no–scale SUGRA model is invariant under imaginary translations

$$T \to T + i\beta\,, \qquad \varphi_\sigma \to \varphi_\sigma$$

and dilatations

$$T \to \alpha^2 T\,, \qquad \varphi_\sigma \to \alpha\,\varphi_\sigma\,.$$

- These symmetries constrain Kähler function

$$K = -3\ln\left[T + \overline{T} - \sum_{\sigma} \zeta_{\sigma} |\varphi_{\sigma}|^2\right], \qquad W = \sum_{\sigma,\lambda,\gamma} \frac{1}{6} Y_{\sigma\lambda\gamma}\, \varphi_{\sigma}\varphi_{\lambda}\varphi_{\gamma}\,.$$

- The invariance under the dilatations and imaginary translations leads to the vanishing of the scalar potential of the hidden sector

$$V_{hid} = e^{G}\left(G_T G^{T\bar{T}} G_{\bar{T}} - 3\right) = 0\,.$$

- The full scalar potential takes a form

$$V = \frac{1}{3} e^{2K/3} \sum_{\sigma} \left|\frac{\partial W(\varphi_{\lambda})}{\partial \varphi_{\sigma}}\right|^2 + \frac{1}{2}\sum_{a}(D^a)^2\,.$$

- Global symmetries ensure the vanishing of $\langle F_i\rangle$, $\langle D^a\rangle$ and vacuum energy density along some directions preserving local SUSY.
- In order to get a vacuum, where local SUSY is broken, the global $SU(1,1)$ symmetry should be violated.

MPP inspired SUGRA models

- Being applied to supergravity MPP implies the existence of a phase with global SUSY in flat Minkowski space.
- Such vacuum is realised only if SUGRA scalar potential has a minimum where the following conditions are satisfied

$$\left\langle \mathcal{W}(z_i^0) \right\rangle = \left\langle \frac{\partial \mathcal{W}(z_i)}{\partial z_j} \right\rangle_{z_i = z_i^0} = 0\,,$$

  that requires an extra fine-tuning in general.
- The simplest Kähler potential and superpotential that satisfy these conditions can be written as

$$K(z,\, z^*) = |z|^2\,, \qquad\qquad W(z) = m_0 (z + a_0)^2\,.$$

- If $a_0 = -\sqrt{3} + 2\sqrt{2}$, SUGRA scalar potential possesses two degenerate minima with zero energy density.

- One of them is a supersymmetric Minkowski minimum that corresponds to $z^{(2)} = -a_0$.
- In the other minimum of the SUGRA scalar potential ($z^{(1)} = \sqrt{3} - \sqrt{2}$) local supersymmetry is broken, so that it can be associated with the physical vacuum.
- Varying a_0 around $-\sqrt{3} + 2\sqrt{2}$ one can obtain a positive or a negative contribution from the hidden sector to the total energy density of the physical vacuum.
- Thus a_0 can be fine-tuned so that the physical and second vacua are degenerate.
- Extra fine-tuning can be alleviated in the no–scale inspired SUGRA models with broken dilatation invariance.
- Let us consider no–scale inspired SUGRA model with two hidden sector fields that transform differently under the dilatations

$$T \to \alpha^2 T\,, \qquad\qquad z \to \alpha\, z$$

and imaginary translations

$$T \to T + i\beta\,, \qquad\qquad z \to z\,.$$

- The hidden sector superfield z transforms similarly to φ_α and can appear in the superpotential of the model:

$$\mathcal{W}(z,\,\varphi_\alpha) = \varkappa\left(z^3 + \mu_0 z^2 + \sum_{n=4}^{\infty} c_n z^n\right) + \sum_{\sigma,\beta,\gamma} \frac{1}{6} Y_{\sigma\beta\gamma}\varphi_\sigma\varphi_\beta\varphi_\gamma\,.$$

- The terms $\mu_0 z^2$ and $c_n z^n$ spoil dilatation invariance.
- At the same time we do not allow the breakdown of dilatation invariance in the superpotential of the observable sector to avoid the appearance of potentially dangerous terms.
- We also assume that the dilatation invariance is broken in the Kähler potential of the observable sector

$$\begin{aligned} K &= -3\ln\Big[T + \overline{T} - |z|^2 - \textstyle\sum_\sigma \zeta_\sigma|\varphi_\sigma|^2\Big] + \\ & \textstyle\sum_{\sigma,\lambda}\left(\dfrac{\eta_{\sigma\lambda}}{2}\varphi_\sigma\,\varphi_\lambda + h.c.\right) + \sum_\sigma \xi_\sigma|\varphi_\sigma|^2\,. \end{aligned}$$

- But we do not allow the breakdown of dilatation invariance in the Kähler potential of the hidden sector because it may spoil the vanishing of the vacuum energy density.

- The scalar potential of the hidden sector takes a form

$$V(T,\, z) = \frac{1}{3(T+\overline{T}-|z|^2)^2}\left|\frac{\partial W(z)}{\partial z}\right|^2 .$$

- When $c_n = 0$ this SUGRA scalar potential has two minima with zero vacuum energy density

$$z = 0\,, \qquad\qquad z = -\frac{2\mu_0}{3}\,.$$

- In the vacuum where $z = -2\mu_0/3$ local supersymmetry is broken so that gravitino and all scalar particles get non–zero masses:

$$m_{3/2} = \frac{4\varkappa\mu_0^3}{27\left\langle\left(T+\overline{T}-\dfrac{4\mu_0^2}{9}\right)^{3/2}\right\rangle}\,, \quad m_\sigma = m_{3/2}\frac{x_\sigma}{(1+x_\sigma)} \sim \frac{m_{3/2}\xi_\sigma}{\zeta_\sigma}\,.$$

- The effective scalar potential of the observable sector in this vacuum is given by

$$V_{eff} \simeq \sum_\alpha \left|\frac{\partial W_{eff}(y_\beta)}{\partial y_\alpha} + m_\alpha y_\alpha^*\right|^2 + \frac{1}{2}\sum_a (D^a)^2\,.$$

- At low energies the interactions of observable superfields $\hat{y}_\alpha$ in such vacuum are described by the effective superpotential

$$\mathcal{W}_{eff} = \sum_{\alpha,\,\beta} \frac{\mu_{\alpha\beta}}{2}\,\hat{y}_\alpha\,\hat{y}_\beta + \sum_{\alpha,\,\beta,\,\gamma} \frac{h_{\alpha\beta\gamma}}{6}\,\hat{y}_\alpha\,\hat{y}_\beta\,\hat{y}_\gamma\,,$$

$$\mu_{\alpha\beta} = \frac{m_{3/2}\eta_{\alpha\beta}}{(C_\alpha C_\beta)^{1/2}}\,, \qquad h_{\alpha\beta\gamma} = \frac{Y_{\alpha\beta\gamma}(C_\alpha C_\beta C_\gamma)^{-1/2}}{<(T+\overline{T}-|z|^2)^{3/2}>}\,,$$

$$C_\alpha = \xi_\alpha\left(1+\frac{1}{x_\alpha}\right), \qquad x_\alpha = \frac{\xi_\alpha <(T+\overline{T}-|z|^2)>}{3\zeta_\alpha}\,.$$

- Thus due to the presence of extra terms in the Kähler potential that spoil dilatation invariance the bilinear terms in $\mathcal{W}_{eff}(\hat{y}_\alpha)$ and soft SUSY breaking terms are generated.
- In the vacuum with $z=0$ local SUSY remains intact and the low–energy limit of this theory is described by a pure SUSY model in flat Minkowski space.
- If the high order terms $c_n z^n$ are present in the superpotential, $V(T,\,z)$ may have many degenerate vacua with broken and unbroken SUSY which energy density vanishes.

- The presence of degenerate vacua with broken and unbroken local supersymmetry leads to the natural realisation of the multiple point principle (MPP).
- Then the vanishing of ρ_Λ can be considered as a result of degeneracy of all possible vacua, one of which is supersymmetric with $< \mathcal{W} >= 0$.
- The inclusion of perturbative and non–perturbative corrections to the Lagrangian of the no–scale inspired SUGRA model is expected to spoil the degeneracy of vacua inducing a huge energy density in the vacuum where SUSY is broken.
- Therefore this model should be considered as a toy example only.
- It demonstrates that, in $(N = 1)$ supergravity, there might be a mechanism which ensures the vanishing of vacuum energy density in the physical vacuum.
- This mechanism may also lead to a set of degenerate vacua with broken and unbroken supersymmetry, resulting in the realization of the multiple point principle.

Cosmological constant

- According to MPP the physical and supersymmetric vacua have the same energy density.
- Since the vacuum energy density of supersymmetric states in flat Minkowski space is zero ρ_Λ in the physical vacuum vanishes in the leading approximation.
- However the cosmological constant in the MPP inspired SUGRA models may differ from zero.
- This occurs if non–perturbative effects in the observable sector give rise to the breakdown of SUSY in the second vacuum (supersymmetric phase) [C. D. Froggatt, RN, H. B. Nielsen, Int. J. Mod. Phys. A **27** (2012) 1250063; C. D. Froggatt, RN, H. B. Nielsen, Nucl. Phys. B **743** (2006) 133].
- The MPP philosophy then requires that the physical phase in which local supersymmetry is broken in the hidden sector has the same energy density as a second phase.

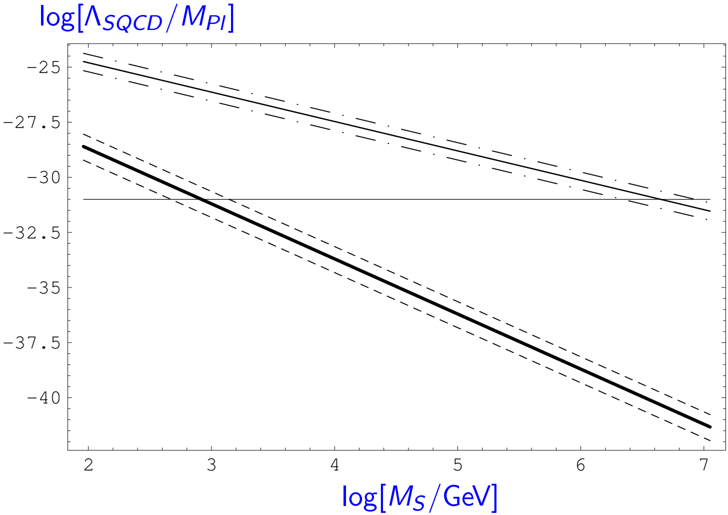
- If supersymmetry breaking takes place in the second vacuum, it is caused by the strong interactions.
- Strong gauge coupling $\alpha_3(Q)$ is the only gauge coupling in the SUSY phase that increases in the infrared region enhancing a role of non–perturbative effects.
- Moreover top quark Yukawa coupling grows with increasing of $\alpha_3(Q)$.
- It may lead to the formation of the top quark condensate that breaks supersymmetry.
- This can happen at the scale Λ_{SQCD}, where the QCD interactions become strong in the second vacuum, resulting in $\rho_\Lambda \simeq \Lambda_{SQCD}^4$.
- We assume that the gauge couplings at the high energy scale and their running down to the sparticle mass scale M_S are the same in both vacua.
- The RG flow of gauge couplings becomes different only at $Q < M_S$ because all superparticles in the physical vacuum decouple and all β functions change.

- In the one–loop approximation the $SU(3)_C$ beta function ($b_3 = -3$) for $Q > M_S$ while below M_S it coincides with the SM beta function, i.e. ($\tilde{b}_3 = -7$).
- Using matching condition $\alpha_3^{(2)}(M_S) = \alpha_3^{(1)}(M_S)$, one finds in the one–loop approximation that the QCD interaction becomes strong in the second vacuum at

$$\Lambda_{SQCD} = M_S \exp\left[\frac{2\pi}{b_3 \alpha_3^{(2)}(M_S)}\right], \quad \frac{1}{\alpha_3^{(2)}(M_S)} = \frac{1}{\alpha_3^{(1)}(M_Z)} - \frac{\tilde{b}_3}{4\pi} \ln \frac{M_S^2}{M_Z^2}.$$

- Thus in the simplest scenario Λ_{SQCD} and ρ_Λ are determined by M_S only.
- Λ_{SQCD} is much lower than the QCD scale in the SM and diminishes with increasing M_S.
- When M_S is of the order of $1\,\mathrm{TeV}$ and $\alpha_3^{(1)}(M_Z) \simeq 0.118$, one obtains $\Lambda_{SQCD} = 10^{-26} M_{Pl} \simeq 100\,\mathrm{eV}$ that results in an enormous suppression of $\rho_\Lambda \simeq 10^{-104} M_{Pl}^4$.

- In the one–loop approximation the measured value of ρ_Λ, that corresponds to $\Lambda_{SQCD} = 10^{-31} M_{Pl} \simeq 10^{-3}$ eV, is reproduced for $M_S = 10^3 - 10^4$ TeV.



- If the MSSM particle content is supplemented by a pair of $5+\bar{5}$ supermultiplets of $SU(5)$ then the observed value of ρ_Λ can be obtained in the one–loop approximation even for $M_S \simeq 1\,\text{TeV}$.
- In the physical vacuum extra particles gain masses $\sim M_S$ due to the presence of the bilinear terms $\left[\eta(5\cdot\bar{5})+h.c.\right]$ in the Kähler potential.
- In SUSY phase new particles remain massless that leads to $b_3=-2$ and smaller values of Λ_{SQCD}.
- The two–loop corrections to the beta functions change the running of $Y_t(Q)$ and $\alpha_3(Q)$ in the second vacuum considerably.
- In the limit $Y_t(Q)\to 0$ the solutions of the two–loop RG equations of the MSSM are gathered near the infrared fixed point when $Q\to 0$, i.e.

$$\alpha_1(Q)\to 0\,,\quad \alpha_2(Q)\to 0\,,\quad \alpha_3(Q)\simeq\frac{6\pi}{7}\,.$$

- The sufficiently large values of the top quark Yukawa coupling result in the appearance of a Landau pole in the infrared region.

The values of Λ_{SQCD} for $\alpha_3(M_Z) = 0.115 - 0.121$ and $M_t = 171 - 176\,\mathrm{GeV}$

M_S (TeV)	100	50	150
Λ_{SQCD} (eV)	$0.7 - 1.8 \cdot 10^{-3}$ $(0.1 - 0.25)$	$1.9 - 4.7 \cdot 10^{-3}$ $(0.25 - 0.62)$	$3.9 - 9.6 \cdot 10^{-4}$ $(0.058 - 0.143)$

- Nevertheless the value of Λ_{SQCD} evaluated in the two–loop approximation is substantially lower than in the one-loop case.
- In this case $\Lambda_{SQCD} \simeq 0.001 - 0.002\,\mathrm{eV}$ can be obtained for $M_S \simeq 50 - 150\,\mathrm{TeV}$.
- In the MSSM with extra pairs of $5 + \bar{5}$ supermultiplets of $SU(5)$ the Landau pole in the second phase disappears entirely.
- When the MSSM particle spectrum is supplemented by one pair of $5 + \bar{5}$ supermultiplets, the solutions of the two–loop RG equations are gathered near the infrared fixed point for $Q \to 0$

$$\alpha_1(Q) \to 0\,, \qquad \alpha_2(Q) \to 0\,, \qquad \alpha_3(Q) \simeq 1.15\,, \qquad Y_t(Q) \simeq 1.01$$

- Thus it remains unclear whether the top quark condensate can get formed in these scenarios.

Conclusions

- In $N = 1$ supergravity a supersymmetric Minkowski vacuum and a vacuum with broken local supersymmetry (SUSY) can be degenerate.
- In the SUSY Minkowski (second) phase the breakdown of SUSY may be induced by non-perturbative effects in the observable sector that give rise to a tiny positive vacuum energy density.
- Postulating the exact degeneracy of the physical and second vacua as well as assuming that at high energies the couplings in both phases are almost identical, we estimate the dark energy density ρ_Λ in these vacua

$$\rho_\Lambda \ll 10^{-100} M_{Pl}^4 \,.$$

- The measured value of the cosmological constant can be reproduced if M_S varies from 50 TeV to 150 TeV.
- If we allow $\alpha_3^{(2)}(M_X)$ and $Y_t^{(2)}(M_X)$ to be slightly different in the second phase then the interval of variation of M_S enlarges considerably.

Happy Anniversary, Vladimir Belokurov, Konstantin Chetyrkin, Dmitry Kazakov, Nikolay Krasnikov, Anatoly Radyushkin, Vladimir Smirnov and Alexey Vladimirov!!