Congratulations to Seven Courageous!

Quantum mechanics of radiofrequency-driven coherent beam oscillations in storage rings

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Sequel to

Collective oscillations of a stored deuteron beam close to the quantum limit

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By all evidence the first look into the quantum regime of transvesrsal beam oscillations

Smallest measured coherent oscillation amplitude 1.06 +/- 0.52 micrometer (beam size ~ 1 mm)

Zero-point betatron oscillations in the confining harmonic oscillator potential Q = 41 nanometer

Still in the classical mechanics domain

What if the measured amplitude had been way below Q ?

How to treat sub-quantum beam oscillations in the picometer domain of ultimate pEDM storage rings ?

All-electric frozen-spin proton EDM ring: radial magnetic fields are no go → monitor vertical separation of counter-rotating beams to about 5 pm accuracy

CERN Yellow Reports: Monographs, 3/2021

Emergence of picometers from the beam displacement by the Earth gravity

Relate the oscillator spring constant to the betatron frequency

$$\Delta y \approx \frac{(2\gamma^2 - 1)\,|g_\oplus|}{\gamma^2 \nu_y^2\, \omega_{\rm rev}^2}$$

Electric focusing, identical displacements of CW and CCW beams: $\Delta y_E \approx 13\,\mathrm{pm}$

Hybrid ring with magnetic focusing $\Delta y_B \approx 1.3\,\mathrm{pm}$

Ineliminable splitting of CW and CCW trajectories ~ $2.6\,\mathrm{pm}$

Classical mechanics of RF driven collective betatron oscillations

Oscillator variable $z = y - iv_y/\omega_y$

One-pass kick in the RF WF $\Delta v_y(n) = \dfrac{F_y(n)\Delta t}{\gamma m} = -\zeta\omega_y \cos(n\,\omega_{\mathrm{WF}}T)$

Master equation for stroboscopic evolution $z(n) = z(n-1)\exp(i\omega_y T) - \dfrac{i}{\omega_y}\Delta v_y(n)$

Solution for RF driven oscillations

$$z(n) = -\frac{i}{\omega_y}\exp(i\omega_y nT)\sum_{k=1}^{n}\Delta v_y(k)\exp(-i\omega_y kT) \;=\frac{i\zeta}{2}\,\frac{\exp(in\omega_y T) - \exp(in\,\omega_{\mathrm{WF}}T)}{\exp[i(\omega_y - \omega_{\mathrm{WF}})T] - 1} + \{\omega_{\mathrm{WF}} \to -\omega_{\mathrm{WF}}\}\,.$$

Classical mechanics of RF driven collective betatron oscillations

Isolate the RF driven component
$$y_{\rm WF}(n) = {\rm Re}\, z(n) = -\xi_y \cos(n\,\omega_{\rm WF} T)$$

$$\xi_y = \frac{\zeta}{2} \cdot \frac{\sin(2\pi\nu_y)}{\cos(2\pi\nu_{\rm WF}) - \cos(2\pi\nu_y)}$$

Resonance at
$$\nu_{\rm WF} = \nu_y .$$

$$y^{\rm res}(n) = -\tfrac{\zeta}{2}(n-1)\sin(n\omega_y T)$$

Neither intrabeam scattering nor scattering off the residual gas do affect coherent oscillations

Coherent oscillation amplitudes are identical for all particles in the bunch irrespective of their individual idle betatron oscillations

Quantum mechanics of coherent oscillations

$$i\frac{d}{dt}\Psi(t) = \{H_0 + V(t)\}\,\Psi(t)$$

Perturbative potential in terms of the HO creation and annihilation operators

$$V(t) = -F_y \cdot y \cdot \cos(\omega_{\rm WF} t) \quad = -\frac{F_y}{\sqrt{2m\gamma\omega_y}}(a^\dagger + a)\cos(\omega_{\rm WF} t)$$

Wave function discontinuity equation $i\{\Psi(+;n) - \Psi(-;n)\} = V(nT)\,\Delta t\,\Psi(-;n)$

Stroboscopic master equation

$$\Psi(-;n) = \left\{1 + i\frac{F_y \Delta t}{\sqrt{2m\gamma\omega_y}}\cos(n\omega_{\rm WF}T)\left(a^\dagger e^{-i\omega_y T} + a e^{i\omega_y T}\right)\right\}\Psi(-;n-1)e^{-i\omega_{\rm in}T}$$

Solution

$$|\Psi(+;n)\rangle = \left\{1 + i\frac{F_y \Delta t}{\sqrt{2\gamma m\omega_y}}\left(w(n)a^\dagger + w^*(n)a\right)\right\}|{\rm in}\rangle\,,$$

Quantum oscillations

Deja vue from the classical mechanics analysis

$$w(n) = \sum_{k=1}^{n} \cos(k\omega_{\rm WF}T)\exp\{-i(n-k)\omega_y T\}$$
$$= \frac{1}{2}\cdot\left[\frac{\exp(-in\omega_y T) - \exp(-in\omega_{\rm WF}T)}{\exp(-i(\omega_y - \omega_{\rm WF})T) - 1} + \{\omega_{\rm WF} \to -\omega_{\rm WF}\}\right]$$

Particle displacement as a quantum-mechanical expectation value

$$y(n) = \frac{1}{\sqrt{2\gamma m\omega_y}}\left\langle \Psi^*(+;n)\left|(a^\dagger + a)\right|\Psi(+;n)\right\rangle$$
$$= -i\frac{F_y \Delta t}{2\gamma m\omega_y}(w^*(n) - w(n))\left\langle {\rm in}\left|[a, a^\dagger]\right|{\rm in}\right\rangle \quad = -i\frac{F_y \Delta t}{2\gamma m\omega_y}(w^*(n) - w(n))$$

Independent of the initial quantum state of the particle because of $[a, a^\dagger] \; = \; 1$

Exact replica of the classical mechanics result .

Summary

- Coherent oscillation amplitude is independent of the idle betatron motion of individual particles be it either classical or quantum
- Exemplary case of the Ehrenfest theorem: identical functional form of the RF driven amplitude of coherent oscillations of the bunch from the classical mechanics domain down to the deep quantum domain.
- Neither intrabeam scattering nor scattering off the residual gas do affect coherent oscillations
- Heisenberg uncertainty relation does not preclude observation of subpicometer coherent oscillation amplitudes --- the sole issue is to develope pickups capable of detection of a very weak periodic signal in the noisy environment
- Take advantage of 1 year observation time: accuracy propto the inverse square root of the observation time

JEDI, Phys. Rev. ST Accel. Beams 17, 052803 (2014)

520 nm (96 s) → 1.6 nm (10^7 s)

5 pm sensitivity target: a task of 300-fold improving of the sensitivity of the beam position monitors

There are more things in Heaven and Earth, Horatio, than are dreamt of in your philosophy.