Bounce, genesis:

how did the Universe start?

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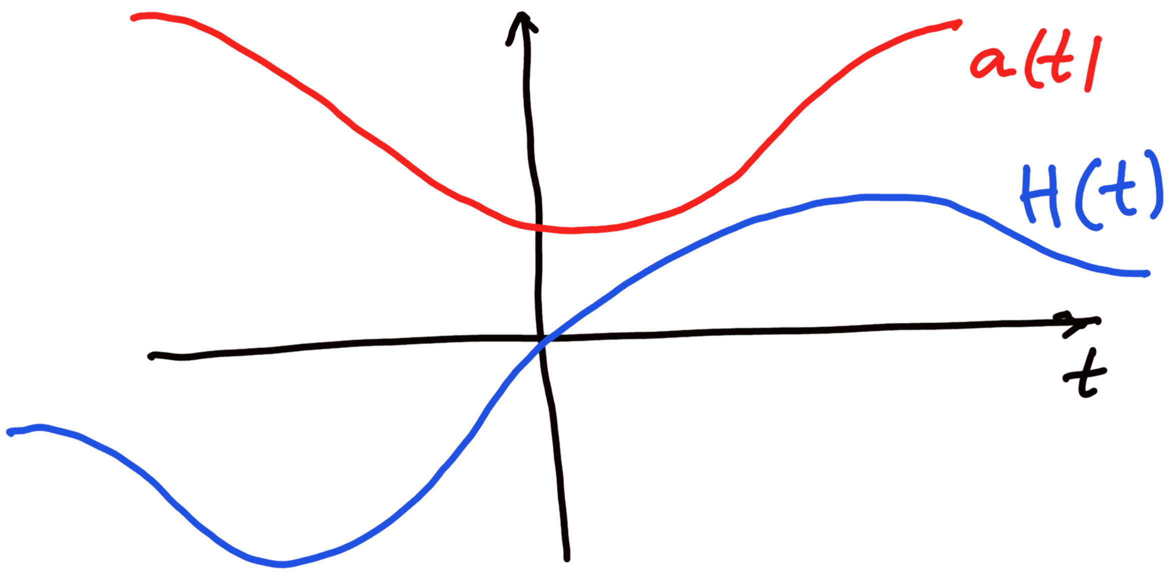
Introduction

Focus of this talk: homogeneous isotropic, spatially flat Universe.

Non-singular models:

Bounce

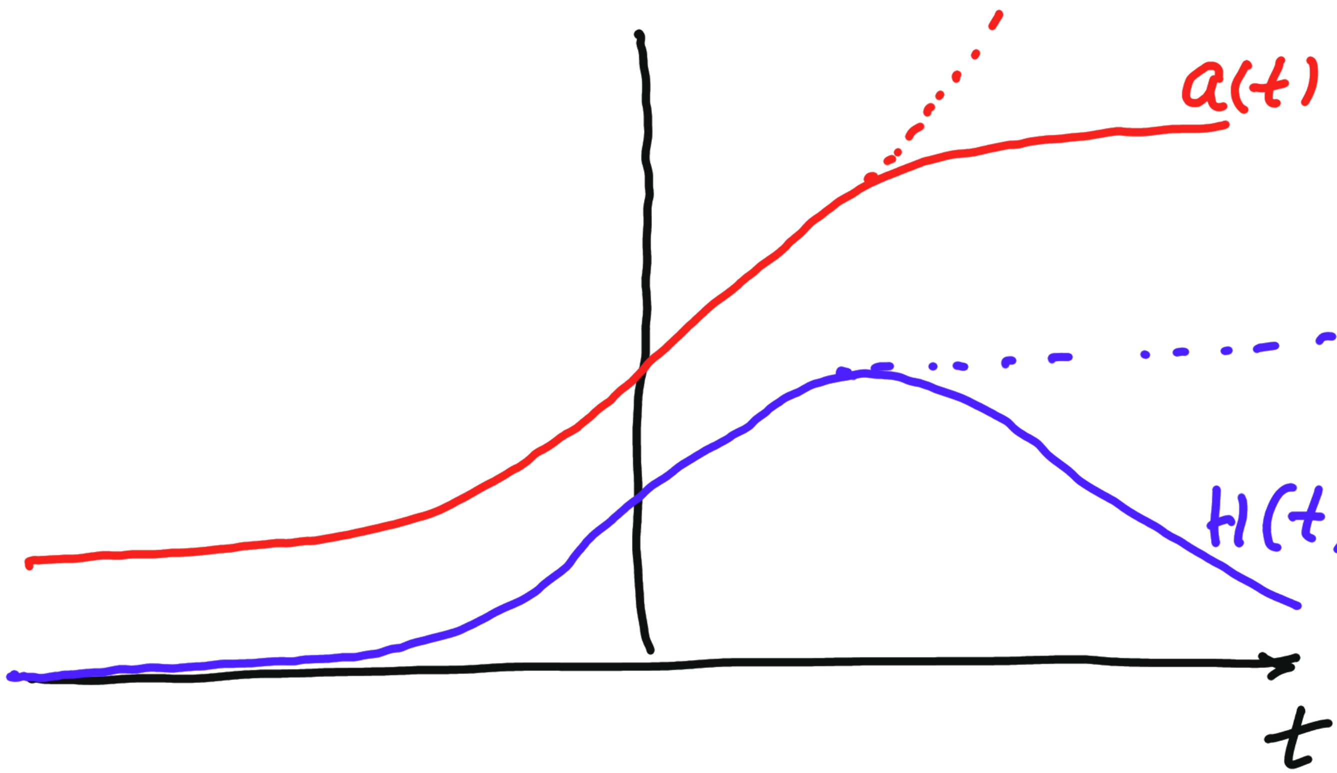
Start from slowly contracting Universe ($H < 0$), then contraction rate increases, energy density builds up, at some moment of time contraction terminates (bounce), Universe starts to expand ($H > 0$). At some point conventional hot epoch (or inflation) begins.



Genesis

Creminelli, Nicolis, Trincherini' 2010

Start from Minkowski, empty space ($H = 0$), then energy density builds up, Universe starts to expand ($H > 0$), expansion accelerates. At some point conventional hot epoch (or inflation) begins.



Motivation

- Curiuosity. Always good to have alternatives even to compelling scenarios like inflation.

- No initial singularity.

- Horizon, curvature problems “solved” by assumption about initial state.
  Especially with ekpyrotic (slow) contraction

  Ijjas, Pretorius, Steinhardt et. al. ’2020-21

- Very long prehistory without matter energy density $\Longrightarrow$ useful for relaxing the cosmological constant

  V.R. ’99;

  Mukohyama, Randall ’2003

DRAWBACK

Generation of (nearly) flat power power spectrum of scalar perturbations not so automatic as compared to in inflation

Obstacle in classical GR (if spatial curvature negligible): both bounce and genesis need exotic matter which violates the Null Energy Condition,
i.e. has $p < -\rho$; where $\rho = T^0_0$, energy density; $p = T^1_1 = T^2_2 = T^3_s$, effective pressure.

- If the NEC holds: a combination of Einstein equations (spatially flat):

$$\frac{dH}{dt} = -4\pi G(\rho + p)$$

Hubble parameter always decreases. No bounce, no genesis.

Penrose theorem for expanding Universe: there was a singularity in the past, $H = \infty$.

NEC is not violated in conventional field theories

with Lagrangians involving first derivatives only.

Dubovsky, Gregoire, Nicolis, Rattazzi' 2006

Buniy, Hsu, Murray' 2006

NEC-violation in theories of this sort:

- Either ghosts: both kinetic an gradient terms have wrong sign. Hyperbolic equation of motion, but negative energies $\Longleftrightarrow$ ghosts: $E = -\sqrt{p^2 + m^2}$ Catastrophic vacuum instability

- Or gradient instabilities: only gradient term has wrong sign. Elliptic equation of motion $\Longrightarrow$ gradient instability

$$E^2 = -(p^2 + m^2) \quad \Longrightarrow \quad \delta\pi \propto \mathrm{e}^{|E|t}$$

Also catastrophic

Horndeski and $p < -\rho$

Twist: scalar-tensor theories with second derivatives in the Lagrangian.

Danger: higher order equations of motion $\Longrightarrow$ extra degrees of freedom = Ostrogradsky ghosts

Not necessarily!

- Way out # 1: Horndeski Horndeski' 1974
  aka Euler hierarchies, aka generalized Galileons, aka KGB, aka generalized Fab Four

  - Second derivatives in Lagrangian, second order field equations
  - Simplest case: Creminelli, Nicolis, Trincherini' 10, Deffayet, Pujolas, Sawicki, Vikman' 10, Kobayashi, Yamaguchi, Yokoyama' 10

    $$L = -\frac{1}{16\pi G}R + F(\pi, X) - K(\pi, X)\Box\pi$$

    where again $X = (\partial\pi)^2$.
  - Explicit examples of stable NEC-violation.

Early stages of genesis/period around bounce can be made healthy.

However, things are not so simple. NO-GO.

"Complete cosmologies": $-\infty < t < +\infty$

Explicit examples of genesis (or bounce) with Horndeski:
either Big Rip singularity in future, $\pi = \infty$, $H = \infty$ at $t < \infty$

Creminelli, Nicolis, Trincherini '2010

or gradient/ghost instability

Cai, Easson, Brandenberger '2012; Koehn, Lehners, Ovrut '2014;
Pirtskhalava, Santoni, Trincherini, Uttayarat '2014; Qiu, Wang '2015;
Kobayashi, Yamaguchi, Yokoyama '2015; Sosnovikov '2015

Can one avoid instability?

No-go in Horndeski! Libanov, Mironov, V.R.' 16; Kobayashi' 16

General Horndeski theory

Require: both "Einstein" equations and π-field equation second order

Four arbitrary functions of π and X: $F \equiv G_2$; $K \equiv G_3$; G_4; G_5

Horndeski' 1974; Deffayet, Esposito-Farese, Vikman' 09

$$L = F(\pi, X) - K(\pi, X)\Box\pi$$
$$+ G_4(\pi, X)R + G_{4,X} \cdot \left[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)^2\right]$$
$$+ G_5 \cdot G^{\mu\nu}\nabla_\mu\nabla_\nu\pi - \frac{1}{6}G_{5,X} \cdot \left[(\Box\pi)^3 - 3\Box\pi \cdot (\nabla_\mu\nabla_\nu\pi)^2 + 2(\nabla_\mu\nabla_\nu\pi)^3\right]$$

- Modified gravity (scalar-tensor).
- NB: always in Jordan frame.

No-go theorem for genesis in Horndeski: gradient/ghost instability at some stage (which may be quite late)

Libanov, Mironov, V.R.' 16; Kobayashi' 16

Choose unitary gauge $\delta\pi = 0$.

$$ds^2 = N^2 dt^2 - a^2 e^{2\zeta} (\delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj})(N^i dt + dx^i)(N^j dt + dx^j)$$

Dynamical variables: transverse traceless h_{ij} and ζ (lapse δN and shift N^i are not dynamical, as usual).

Upon solving for constraints, find quadratic Lagrangians for perturbations

$$L_S = \mathscr{G}_{\mathscr{S}} \dot{\zeta}^2 - a^{-2} \mathscr{F}_{\mathscr{S}} (\partial_i \zeta)^2 \, , \quad L_T = \mathscr{G}_{\mathscr{T}} \dot{h_{ij}}^2 - a^{-2} \mathscr{F}_{\mathscr{T}} (\partial_k h_{ij})^2$$

NB: $\mathscr{G}_{\mathscr{T}}$, $\mathscr{F}_{\mathscr{T}}$ = effective M_{Pl}^2.

Stable background $\Longleftrightarrow$ $\mathscr{G}_{\mathscr{T}}, \mathscr{F}_{\mathscr{T}}, \mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} > 0$.

To simplify formulas (but not outcome): $G_5 = 0$. Tensor sector:

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4G_{4X}X,$$
$$\mathcal{F}_{\mathcal{T}} = 2G_4$$

Scalar sector:

$$\mathcal{G}_{\mathcal{S}} = \frac{\Sigma \mathcal{G}_{\mathcal{T}}^{\,2}}{\Theta^2} + 3\mathcal{G}_{\mathcal{T}},$$
$$\mathcal{F}_{\mathcal{S}} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_{\mathcal{T}},$$
$$\xi = \frac{a\mathcal{G}_{\mathcal{T}}^{\,2}}{\Theta}.$$

Where

$$\Theta = -K_X X\dot{\pi} + 2G_4 H - 8HG_{4X}X - 8HG_{4XX}X^2 + G_{4\pi}\dot{\pi} + 2G_{4\pi X}X\dot{\pi}$$
$$\Sigma = F_X X + 2F_{XX}X^2 + 12HK_X X\dot{\pi} + 6HK_{XX}X^2\dot{\pi} - K_\pi X - K_{\pi X}X^2$$
$$-6H^2G_4 + 42H^2G_{4X}X + 96H^2G_{4XX}X^2 + 24H^2G_{4XXX}X^3 - 6HG_{4\pi}\dot{\pi}$$
$$-30HG_{4\pi X}X\dot{\pi} - 12HG_{4\pi XX}X^2\dot{\pi}$$

Key relation (by explicit calculation in general Horndeski)

$$\frac{d\xi}{dt} = a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}})$$

$$\xi = -\frac{a(t)\mathscr{G}_{\mathscr{T}}{}^2(t)}{\Theta(t)}$$

where $\Theta(t) = -2HG_4 + \dot{\pi}XK_X + \dots$, a complicated expression.

Main property: ξ never crosses zero ($\Theta = \infty$ is a singularity).

$$\xi(t_f) - \xi(t_i) = \int_{t_i}^{t_f} dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}})$$

Impossible for $\mathscr{F}_{\mathscr{S}} > 0$, $\mathscr{F}_{\mathscr{T}} > 0$, and

$$\int_{-\infty}^{t_f} dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) = \infty\,, \quad \int_{t_i}^{+\infty} dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) = \infty$$

Recall that $a(t) \to \infty/\text{const}$ as $t \to -\infty$ and $a(t) \to \infty$ as $t \to +\infty$ for bounce/genesis No-go

$$\xi(t) - \xi(0) = \int_0^t dt \; a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) \Longrightarrow \xi(t) \to +\infty \text{ as } t \to +\infty$$

$$\xi(0) - \xi(t) = \int_t^0 dt \; a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) \Longrightarrow \xi(t) \to -\infty \text{ as } t \to -\infty$$

Thus, $\xi(t)$ crosses zero, QED.

- Argument intact in presence of extra matter (obeying NEC) which interacts with Horndeski sector only gravitationally
- Extends to Horndeski theory with multiple (Horndeski or conventional) scalars

Kolevatov, Mironov '2016
Akama, Kobayashi '2017

Way out # 1: strong gravity in the past

Within Horndeski theory, classical stability (absence of gradient instabilities and ghosts) requires

$$\int_{-\infty}^{t} dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) < \infty\,.$$

$\mathscr{G}_{\mathscr{T}}, \mathscr{F}_{\mathscr{T}}, \mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} \to 0$ as $t \to -\infty$,

Kobayashi '2016; Ijjas, Steinhardt '2016

No-go theorem does not work.

But gravity tricky as $t \to -\infty$: effective Planck mass vanishes.

Strong coupling?

Examples:

$$\mathscr{G}_{\mathscr{T}}, \mathscr{F}_{\mathscr{T}}, \mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} = \frac{1}{(-t)^{2\mu}} \quad \text{as} \quad t \to -\infty.$$

Can one trust classical field theory treatment of cosmological evolution?

Energy scale of classical evolution $E_{class} = H$, $\dot{H}/H = (-t)^{-1} \to 0$

How does it compare with strong coupling scales E_{strong} inferred from interactions of ζ and h_{ij}?

Classical treatment of evolution legitimate for $E_{strong} >> E_{class}$ as $t \to -\infty$.

Example (part of the story): tensor sector up to cubic terms.
At given moment of time rescale spatial cordinates to set $a = 1$ (equivalently, work in terms of physical spatial momenta $\vec{p} = \vec{k}/a$).
Then (note that $\mathscr{G}_{\mathscr{T}} = \mathscr{F}_{\mathscr{T}}$)

$$S_{hh}^{(2,3)} = \int d^4x \left(\mathscr{F}_{\mathscr{T}} \dot{h_{ij}}^2 - \mathscr{F}_{\mathscr{T}} (\partial_k h_{ij})^2 + \frac{\mathscr{F}_{\mathscr{T}}}{4} \left(h_{ik}h_{jl} - \frac{1}{2}h_{ij}h_{kl}\right) \partial_k \partial_l h_{ij} \right)$$

To figure out strong coupling energy scale, canonically normalize

$$h_{ij} = h_{ij}^{(c)} / \sqrt{\mathscr{F}_{\mathscr{T}}}$$

$$S_{hh}^{(2,3)} = \int d^4x \left(\dot{h}_{ij}^{(c)2} - (\partial_k h_{ij}^{(c)})^2 + \frac{1}{4\sqrt{\mathscr{F}_{\mathscr{T}}}} (h_{ik}^{(c)} h_{jl}^{(c)} - \frac{1}{2} h_{ij}^{(c)} h_{kl}^{(c)}) \partial_k \partial_l h_{ij}^{(c)} \right)$$

Dimension-5 operator "suppressed" by $1/\sqrt{\mathscr{F}_{\mathscr{T}}}$ $\Longleftrightarrow$
quantum strong coupling energy scale $E_{strong} = \sqrt{\mathscr{F}_{\mathscr{T}}} \propto (-t)^{-\mu}$

$E_{strong} \to 0$ as $t \to -\infty$, but $E_{strong} \gg E_{class} = (-t)^{-1}$ for $\mu < 1$.
Healthy early bounce stage within classcal field theory at weak coupling.

- This extends to scalar plus tensor sectors and all orders in perturbation theory. Viable scenario.

Ageeva, Evseev, Melichev, V.R.' 18, 20;
Ageeva, V.R., Petrov' 20, 21

Overall picture: Universe starts at very low quantum gravity scale $E_{strong} \propto |t|^{-\alpha}$ but expands so slowly that $E_{class} \ll E_{strong}$. Standard Model scales are above E_{strong}. Gravity is the strongest force.

Similar construction works for genesis

Complete cosmologies

Intelligent design: proof by example

Dubbed "Inverse method" by Ijjas, Steinhardt' 2016

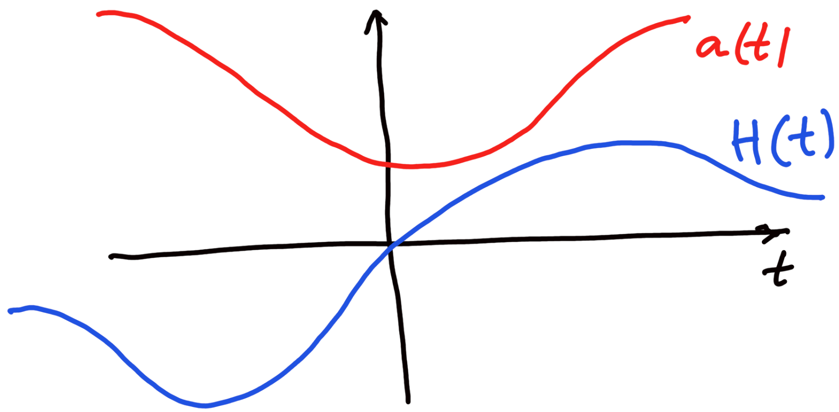
- Choose background $\pi(t) = t$, no loss of generality (field redefinition).

  Then $X = (\partial\pi)^2 = 1$.

  Field equations and stability conditions involve Lagrangian functions F, K, G_4 and their X-derivatives F_X, F_{XX}, etc, all at $\pi(t) = t$, $X = 1$.

  These are yet undetermined independent functions of time $f_0(t) = F(\pi(t), X = 1)$, $f_1(t) = F_X(\pi(t), X = 1)$, etc..

- Choose your favorite $H(t)$.



In particular, theory at late times becomes GR + conventional massless scalar field $\phi = (2/3)^{1/2} \log \pi$ ("kination"),

- Cook up Lagrangian functions in such a way that
  - Field equations are satisfied
  - Stability conditions are satisfied at all times
  - Classical field theory description of background is reliable at all times, including $t \to -\infty$

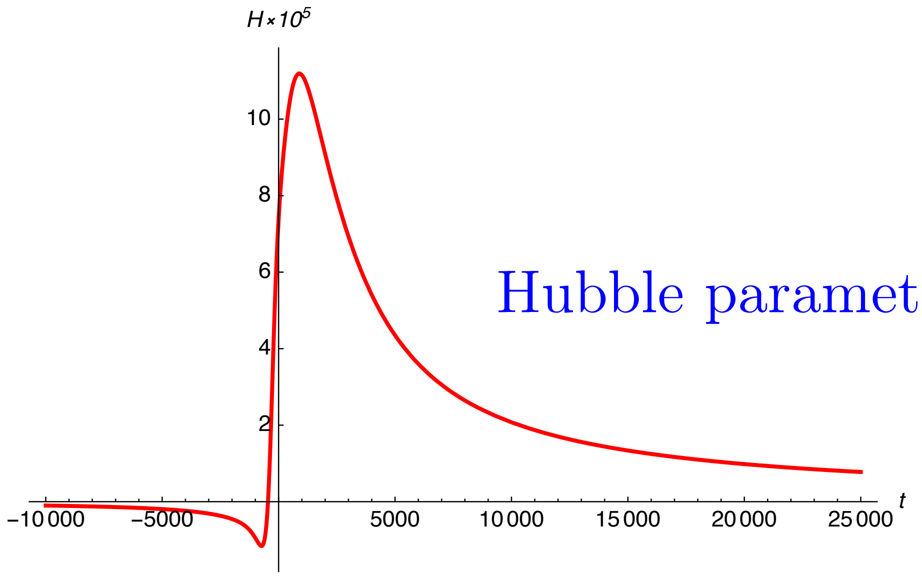
All this can be done for bounce (and also genesis)

Ageeva, Petrov, V.R.' 2021

Moreover, one can design a model in such a way that

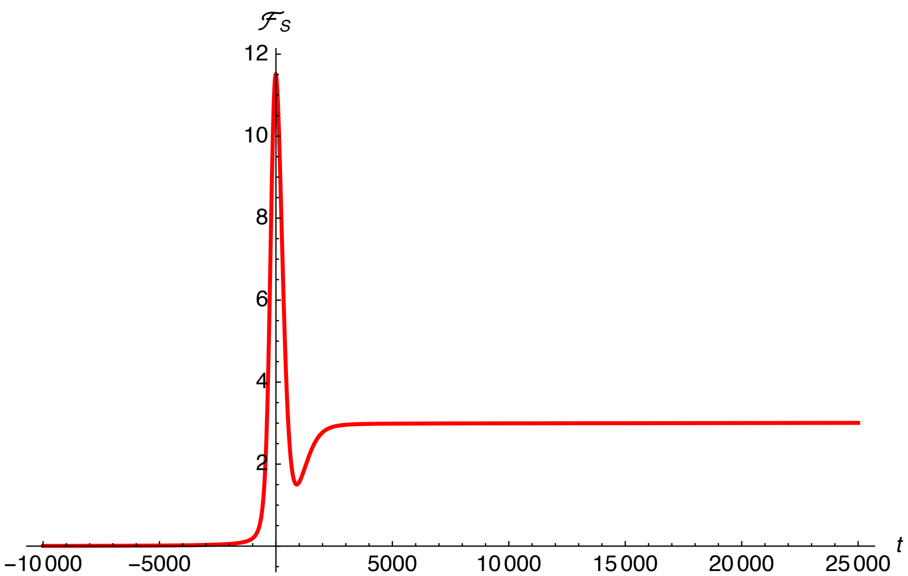
- Tensor and scalar perturbations are subluminal at all times (or luminal, if one wishes so)

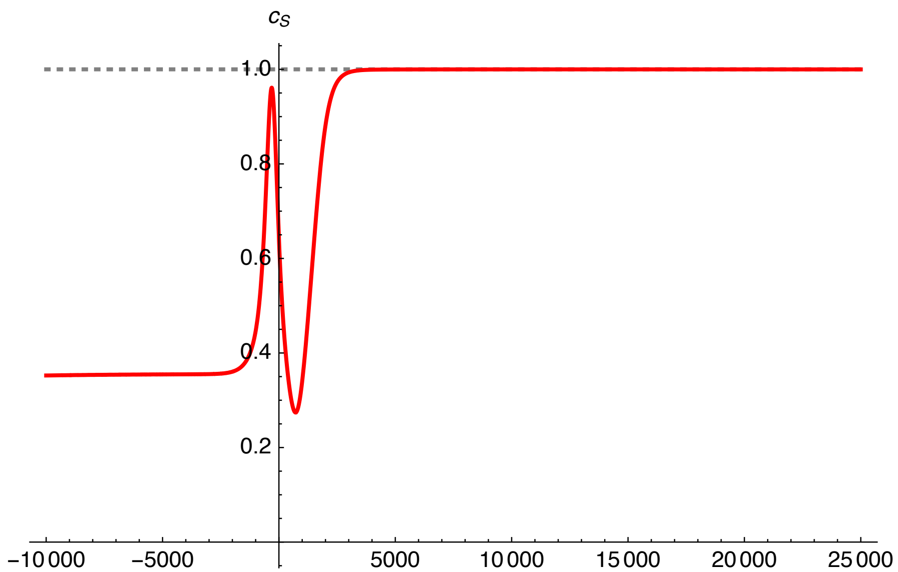
Bounce to kination



Hubble parameter $\boldsymbol{H}$

t





$\mathscr{F}_{\mathscr{S}}$

speed of scalar prturbations

Yet another approach

Horndeski is not the most general scalar-tensor theory with tensor + one scalar modes $\Longrightarrow$ No Ostrogradsky ghost

- Variation of action may give higher order field equations, but they may combine into second order equations. Degenerate Higher-Order Scalar Tensor theories, DHOST

Langlois, Noui' 16; Crisostomi, Koyama, Tasinato' 16

- Relatively simple subclass: "beyond Horndeski" theories

Zumalacárregui, Gacia-Bellido' 2014; Gleyzes, Langlois, Piazza, Vernizzi' 2014

Example of additional (to Horndeski) term

$$F_4(\pi,X)\varepsilon^{\mu\nu\lambda\rho}\varepsilon^{\mu'\nu'\lambda'}{}_{\rho}\ \pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\lambda\lambda'}$$

- Way to understand (sometimes): disformal transformation

$$g_{\mu\nu} \to \Omega(\pi,X)g_{\mu\nu} + \Lambda(\pi,X)\partial_\mu\pi\partial_\nu\pi$$

Horndeski $\to$ beyond Horndeski

NB: This is formal trick: Ω, Λ may be singular

No-go theorem no longer holds

Effective field theory: Cai et.al.' 2016, Creminelli et.al.'2016

Covariant formalism: Kolevatov et.al.' 2017, Cai, Piao' 2017

One again has

$$\frac{d\xi}{dt} = a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}})$$

but now

$$\xi = -\frac{a(t)\mathscr{G}_{\mathscr{T}}(\mathscr{G}_{\mathscr{T}} + 2F_4X^2)}{\Theta(t)}$$

can cross zero.

NB: $\Theta = 0$ not a problem, gauge artifact

Ijjas' 17;

Mironov, V.R., Volkova' 18

Concrete models again by intelligent design.

However, there is still an issue to worry about: superluminality.

Theory with superluminal excitations cannot descend from healthy Lorentz-invariant UV-complete theory

Adams et. al.' 2006

Not an issue in DHOST theories per se.

But things change once one allows for extra field(s) ("matter")

one of the modes is always superluminal

unless a special non-linear constraint is imposed on functions in Lagrangian

Mionov, Volkova, VR' 2020

To conclude

- Constructing complete ($-\infty < t < +\infty$) non-singular cosmology (bounce, genesis) is difficult.
  - Within scalar-tensor gravity: non-trivial kinetic/gradient terms
  - bounce epoch, early genesis per se not so prolematic
  - however, almost all complete cosmologies plagued with instability ("No-go")
- Way out #1:

  Strong gravity in the past; effective Planck mass tends to 0 as $t \to -\infty$. "Gravity as the strongest force".
  - Classical field theory treatment of background evolution can be rendered legitimate, nevertheless.
  - Healthy bounce and genesis cosmologies constructed in this framework
  - Whether realistic scalar (and tensor) perturbations may be generated without inflation, remains to be seen.

- Way out # 2:

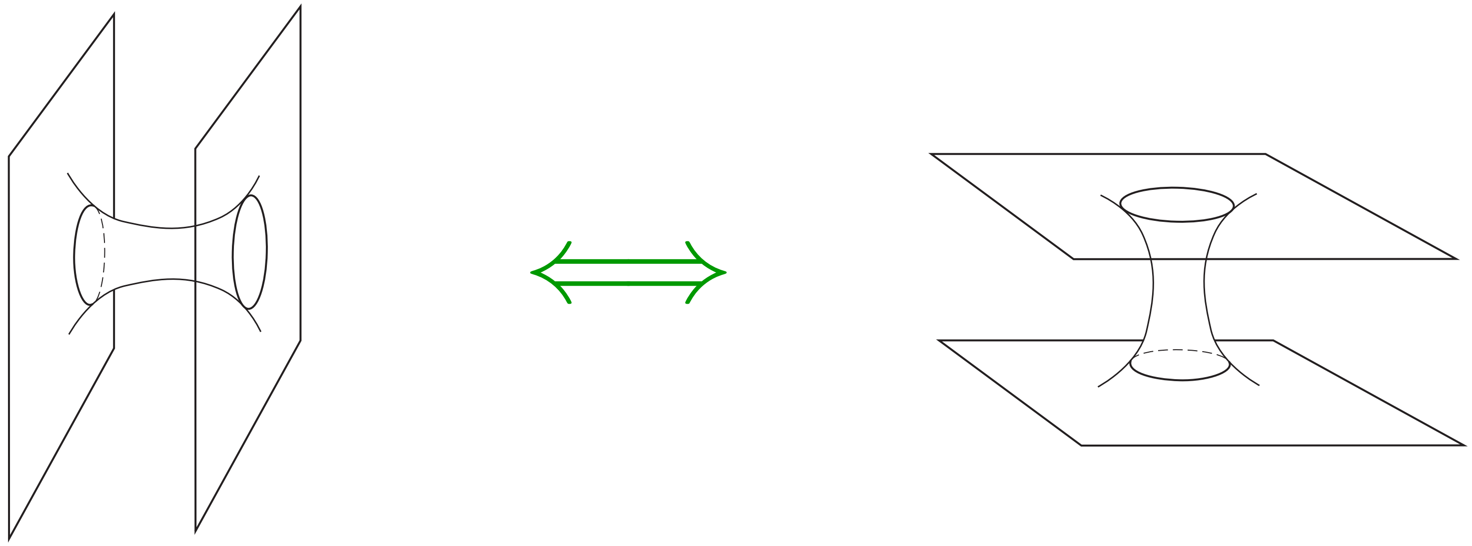
  Theories with even more complicated Lagrangians involving second derivatives: beyond Horndeski, DHOST.

  - Absence of Ostrogradsky ghost, catastrophic instabilities and superluminality imposes strong (non-linear!) constraints on functions in Lagrangian.
  - Is the price too high?

Instead of conclusion: where else DHOST may be instrumental?

- Lorentzian wormholes

Static wormhole $\Longleftrightarrow$ Bouncing Universe



No-go in NEC-preserving theories

No-go in Horndeski: no stable, static, spherically symmetric wormholes: always ghosts.

V.R.’ 16; Evseev, Melichev’ 18

Not obviously impossible in DHOST

Mironov, V.R., Volkova’ 18; Francolini et. al.’ 18

Studying stability HUGELY difficult.

Creation of a universe in the laboratory

- Question raised in mid-80's, right after invention of inflationary theory

  Berezin, Kuzmin, Tkachev' 1984; Guth, Farhi' 1986

  Idea: create, in a finite region of space, inflationary initial conditions $\Longrightarrow$ this region will inflate to enormous size and in the end will look like our Universe.

- Do not need much energy: pour little more than Planckian energy into little more than Planckian volume.

  If NEC holds, no way: initial singularity

  Guth, Farhi' 1986; Berezin, Kuzmin, Tkachev' 1987

  How about DHOST theories?

Amazingly, many questions of principle still not answered. Ahead: more to understand.

Backup

No-go theorem for theories with Lagrangians involving first derivatives of fields only (and minimal coupling to gravity)

Dubovsky, Gregoire, Nicolis, Rattazzi' 2006

Buniy, Hsu, Murray' 2006

$$L = F(X^{IJ}, \pi^I)$$

with $X^{IJ} = \partial_\mu \pi^I \partial^\mu \pi^J \implies$

$$T_{\mu\nu} = 2\frac{\partial F}{\partial X^{IJ}} \partial_\mu \pi^I \partial_\nu \pi^J - g_{\mu\nu} F$$

In homogeneous background

$$T_{00} \equiv \rho = 2\frac{\partial F}{\partial X^{IJ}} X^{IJ} - F$$

$$T_{11} = T_{22} = T_{33} \equiv p = F$$

and

$$\rho + p = 2\frac{\partial F}{\partial X^{IJ}} X^{IJ} = 2\frac{\partial F}{\partial X^{IJ}} \dot{\pi}^I \dot{\pi}^J$$

NEC-violation: matrix $\partial F / \partial X_c^{IJ}$ non-positive definite. But

Lagrangian for perturbations $\pi^I = \pi_c^I + \delta\pi^I$

$$L_{\delta\pi} = A_{IJ}\ \partial_t \delta\pi^I \cdot \partial_t \delta\pi^J - \frac{\partial F}{\partial X_c^{IJ}}\ \partial_i \delta\pi^I \cdot \partial_i \delta\pi^J + \dots$$

Gradient instabilities and/or ghosts

NB. Loophole: $\partial F / \partial X_c^{IJ}$ degenerate.

Higher derivative terms (understood in effective field theory sense) become important and help.
Ghost condensate

Simple NEC-violating Horndeski: scale-invariant model,
$\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln\lambda$

$$L = F(Y) \cdot \mathrm{e}^{4\pi} + K(Y) \cdot \Box\pi \cdot \mathrm{e}^{2\pi}$$

$$\Box\pi \equiv \partial_\mu \partial^\mu \pi \,, \qquad Y = \mathrm{e}^{-2\pi} \cdot (\partial\pi)^2$$

Homogeneous solution in Minkowski space (attractor)

$$\mathrm{e}^{\pi_c} = \frac{1}{\sqrt{Y_*}\,|t|} \,, \qquad t < 0$$

- $Y \equiv \mathrm{e}^{-2\pi_c} \cdot (\partial_\mu \pi_c)^2 = Y_* = \mathrm{const}$, a solution to

$$Z(Y_*) \equiv -F + 2Y_* F_Y - 2Y_* K + 2Y_*^2 K_Y = 0$$

$$F_Y = dF/dY \,.$$

Perturbations about homogeneous Minkowski solution

$$\pi(x^\mu) = \pi_c(t) + \delta\pi(x^\mu)$$

- Quadratic Lagrangian for perturbations:

$$L^{(2)} = \mathrm{e}^{2\pi_c}\mathscr{F}(\partial_t\delta\pi)^2 - \mathscr{G}(\vec{\nabla}\delta\pi)^2 + W(\delta\pi)^2$$

  Absence of ghosts: $\mathscr{F} = Z_Y \equiv dZ/dY > 0$ at $Y = Y_*$, no problem.

- NEC-violation and absence of gradient instabilities:

$$\rho + p = \mathrm{e}^{4\pi_c}(F_Y - 2K + Y_* K_Y)\cdot 2Y_* \quad < 0$$

$$\mathscr{G} = \mathrm{e}^{2\pi_c}(F_Y - 2K + 4Y_* K_Y) \quad > 0$$

Easy to arrange.

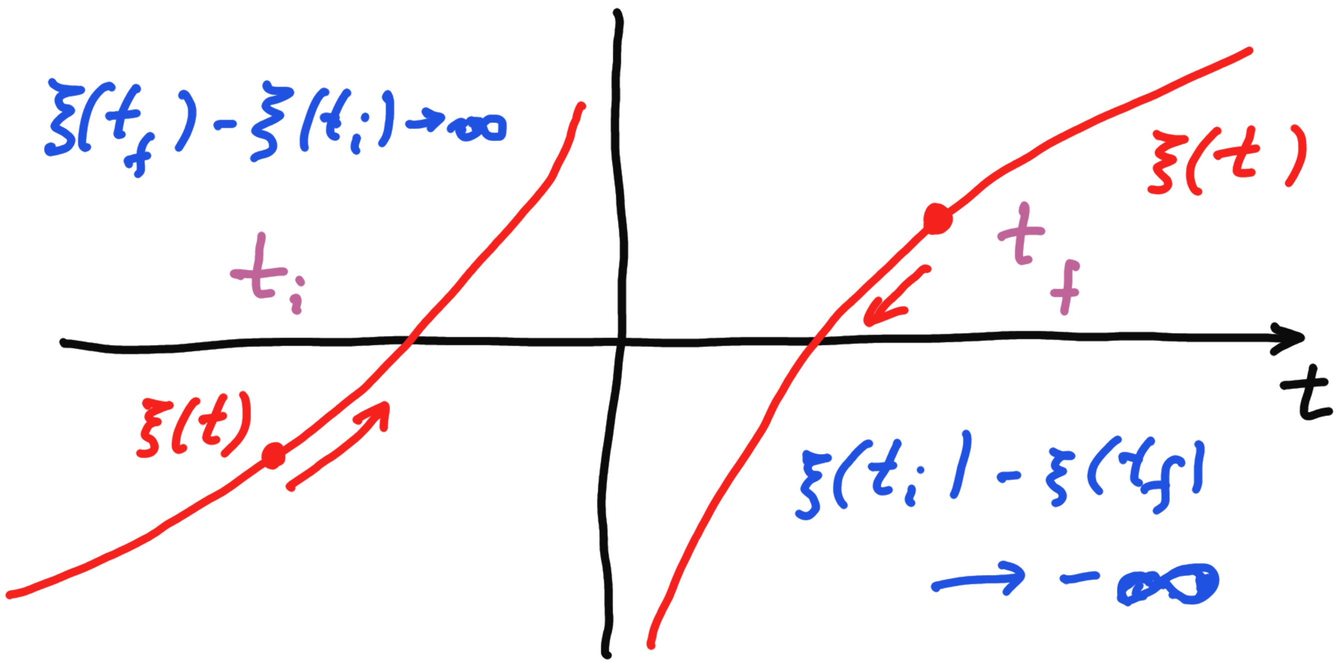
NB: $\rho = 0$, $p < 0$ $\qquad$ $p \to 0$ as $t \to -\infty$

When coupled to gravity $\Longrightarrow$ early stage of Genesis.

Creminelli, Nicolis, Trincherini' 10,

No-Go

Even if $\Theta = 0$ at some time $\Longleftrightarrow$ $\xi = \infty$, there is necessarily ξ-crossing:



Side remark: Θ-crossing $\Theta = 0$ at some t is not a problem by itself. $\mathscr{F}_{\mathscr{S}}, \mathscr{G}_{\mathscr{S}} = \infty$, but solutions for ζ remain finite. Also: no singularity in equations in Newtonian gauge

Ijjas' 17; Mironov, V.R., Volkova' 18

DHOST with additional scalar field

Additional minimally coupled scalar: $L_\chi = (\partial\chi)^2$

New featue: DHOST pertutbations kinetically mix with $\delta\chi$ if $\dot{\chi}_c \neq 0$ in background:
quadratic action reads (modulo terms with less than two derivatives)

$$L^{(2)\,scalar}_{\pi+\chi} = G_{AB}\,\dot{u}^A\dot{u}^B - \frac{1}{a^2}F_{AB}\,\partial_i u^A\partial_i u^B$$

where $A,B = 1,2$, $u^1 = \zeta$, $u^2 = \delta\chi$,

$$G_{AB} = \begin{pmatrix} \mathscr{G}_\mathscr{S} & \dot{\chi}_c g \\ \dot{\chi}_c g & 1 \end{pmatrix}, \qquad F_{AB} = \begin{pmatrix} \mathscr{F}_\mathscr{S} & \dot{\chi}_c f \\ \dot{\chi}_c f & 1 \end{pmatrix},$$

$g, f(\pi, X)$ = combinations of functions in DHOST Lagrangian.

One of the modes superluminal unless $g = f$

$g = f \Longrightarrow$ Very special DHOST theory (not beyond Horndeski), non-linear relations between terms in Lagrangian