Strings and SUSY Breaking

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[Two recent reviews with J. Mourad : 2107.04064 (Erice 2017), 1711.11494 (LHEP, to appear)]

(DUBNA, OCTOBER 11-14, 2021)
(ONLINE)

Broken SUSY in D=10

The (SUSY) 10D-11D Zoo

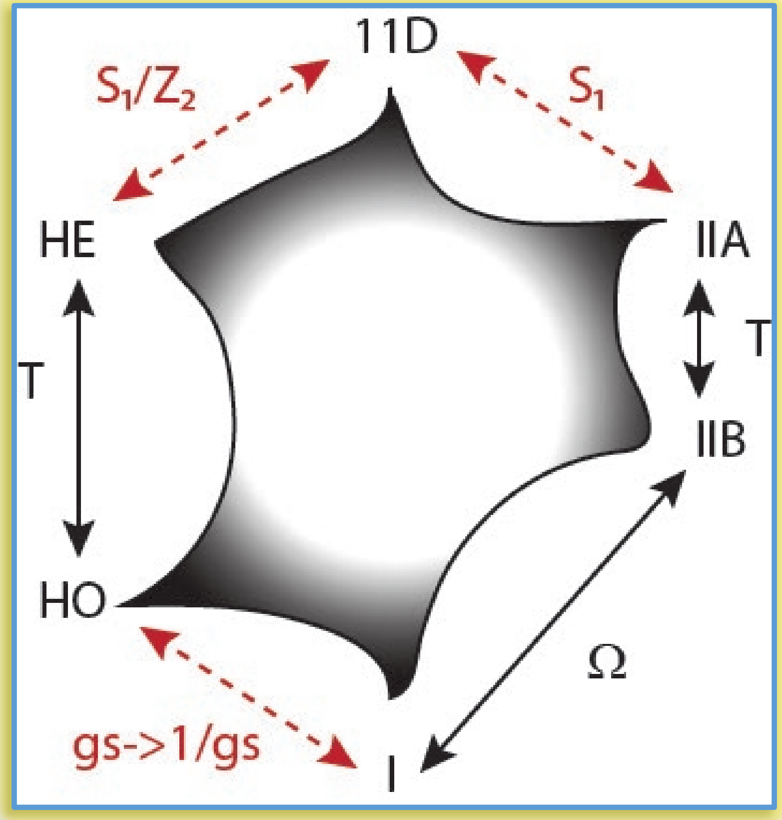
- Highest point of (SUSY) String Theory

BUT:

- Exhibits dramatically our limitarions
- Perturbative → Solid arrows
- [10&11D supergravity → red dashed arrows]

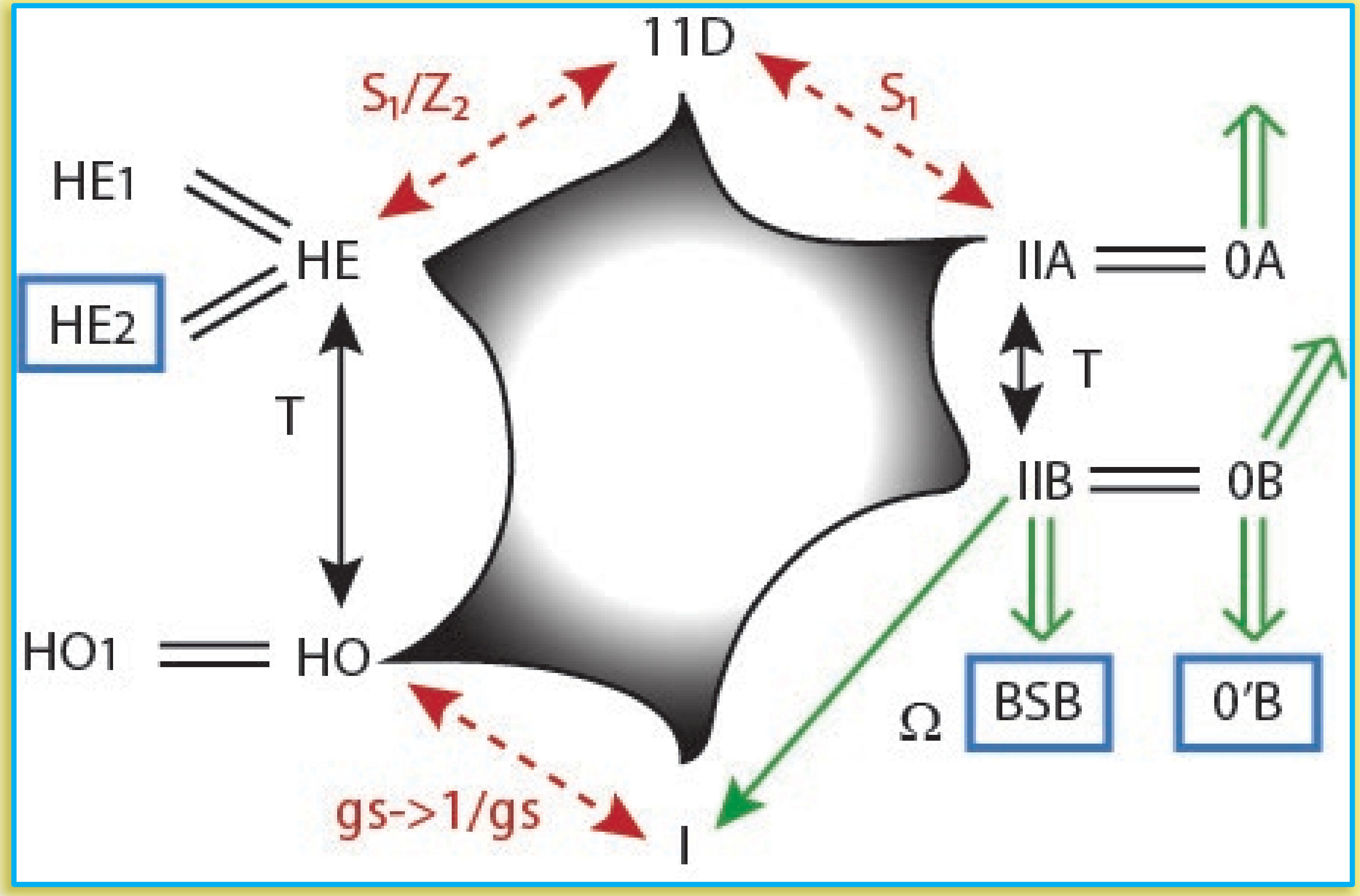
(Witten, 1995)

- SUSY: stabilizes these 10D Minkowski vacua



BROKEN SUSY ?

The 10D-11D Zoo



- 3 D=10 non-SUSY non-tachyonic strings
- SO(16)xSO(16) (Dixon, Harvey, 1987) (Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)
- O'B U(32) (AS, 1995)
- [BSB Usp(32)] (Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)
- String consistency rules OK
- BUT: vacuum modified (Tadpole potential)
- QUESTIONS:
  - NON-PERTURBATIVE LINKS?
  - Compactifications? Stability?

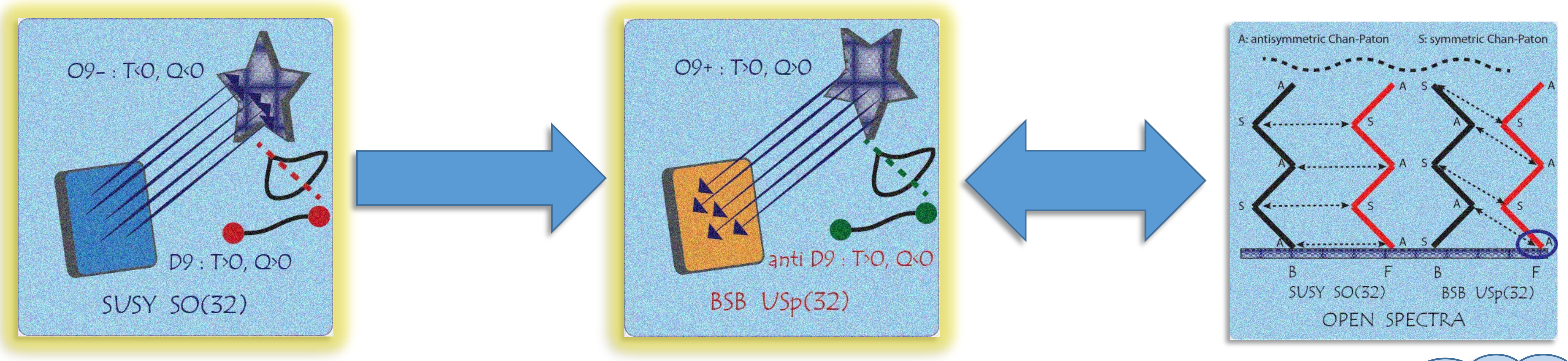
$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2-T\,e^{-\phi}+\ \ldots\right\}$$

Brane SUSY Breaking

(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)

- Non-linear SUSY: ∃ goldstino!
- NO TACHYONS

(Dudas, Mourad, 2000)
(Pradisi, Riccioni, 2001)



VACUUM ENERGY → POTENTIAL

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2-T\,e^{-\phi}+\ \ldots\right\}$$

SUMMARIZING

NO SUSY → Typically tachyonic modes

(True also for Scherk-Schwarz reductions below R_{min}: SHIFTS?)

- BUT: 3 D=10 non-SUSY non-tachyonic strings

- SO(16)xSO(16) heterotic — (Dixon, Harvey, 1987) (Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)
- 0'B U(32) orientifold (no SUSY) — (AS, 1995)
- Usp(32) orientifold (non-linear SUSY) — (Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)

Vacuum modified (Tadpole potential)

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[R + 4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\right\}$$

NOTE:
- Expansion in powers of $\alpha' R$
- Expansion in powers of $g_s = e^{\phi}$

The "Climbing" Scalar

Cosmological Potentials

What potentials lead to slow-roll, and where ?

$$ds^2 = -dt^2 + e^{2A(t)}\, d\mathbf{x}\cdot d\mathbf{x} \quad\longrightarrow\quad \ddot\phi + 3\dot\phi\sqrt{\frac{1}{3}\dot\phi^2 + \frac{2}{3}V(\phi)} + V' = 0$$

Driving force from V′ *vs* friction from V

- **If V does not vanish :** convenient gauge "makes the damping term neater"

$$ds^2 = e^{2\mathcal{B}(t)}\,dt^2 - e^{\frac{2\mathcal{A}(t)}{d-1}}\,d\mathbf{x}\cdot d\mathbf{x}$$

$$V\,e^{2\mathcal{B}} = V_0$$

$$\tau = t\sqrt{\frac{d-1}{d-2}}\,, \qquad \varphi = \phi\sqrt{\frac{d-1}{2(d-2)}}$$

$$\dot{\mathcal{A}}^2 - \dot\varphi^2 = 1$$

$$\ddot\varphi + \dot\varphi\sqrt{1+\dot\varphi^2} + \frac{V_\varphi}{2V}\left(1+\dot\varphi^2\right) = 0$$

$$m\ddot{x} + b\dot{x} = f$$

- Now driving from logV *vs* O(1) damping

$$V = \varphi^n \longrightarrow \frac{V'}{2V} = \frac{n}{2\varphi}$$

❖ **Quadratic potential?** Far away from origin *(Linde, 1983)*

❖ **Exponential potential?** YES or NO

$$V(\varphi) = V_0\, e^{2\gamma\varphi} \longrightarrow \frac{V'}{2V} = \gamma$$

$V = e^{3\gamma\varphi/2}$: Climbing & Descending Scalars

(Halliwell, 1987;…, Dudas and Mourad, 1999; Russo, 2004; Dudas, Kitazawa, AS, 2010)

- $\gamma < 1$? Both signs of speed

a. "Climbing" solution (φ climbs, then descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

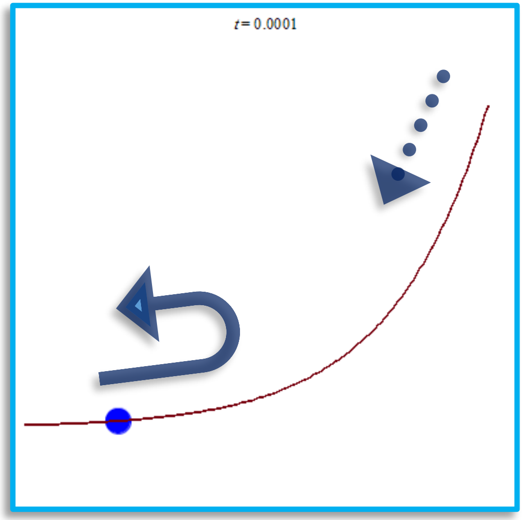
b. "Descending" solution (φ only descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

Limiting τ- speed (LM attractor):

$$v_{lim} = -\frac{\gamma}{\sqrt{1-\gamma^2}}$$

(Lucchin and Matarrese, 1985)



$\gamma = 1$ is "critical": LM attractor & descending solution disappear there and beyond

CLIMBING : in ALL asymptotically exponential potentials with $\gamma \geq 1$!

The U(32) and Usp(32) 10D Orientifolds HAVE PRECISELY $\gamma = 1$ (bounded g_s !)

- $\gamma = 1$:

$$\varphi(\tau) = \varphi_0 + \frac{1}{2}\left[\log|\tau-\tau_0| - \frac{1}{2}(\tau-\tau_0)^2\right]$$

$$\mathcal{A}(\tau) = \mathcal{A}_0 + \frac{1}{2}\left[\log|\tau-\tau_0| + \frac{1}{2}(\tau-\tau_0)^2\right]$$

Cosmology with Tadpoles: The Climbing Scalar

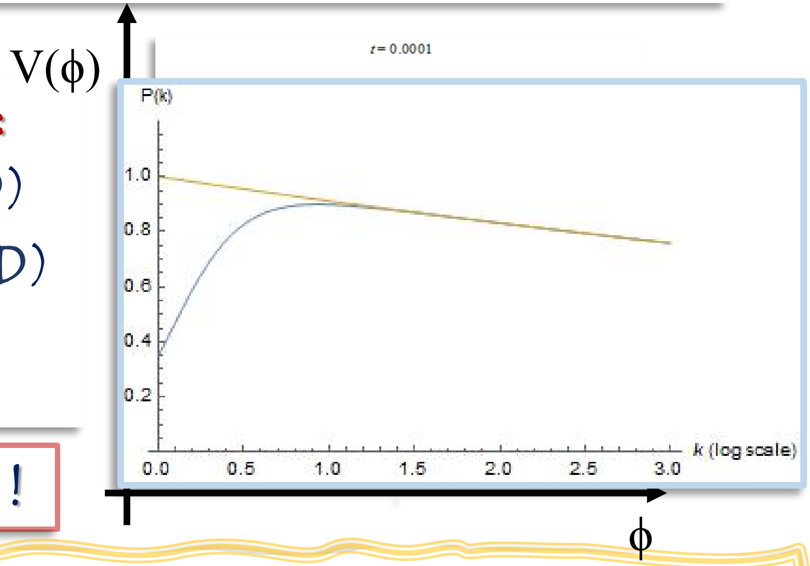
(Halliwell, 1987;…, Dudas and Mourad, 1999; Russo, 2004)
(Dudas, Kitazawa, AS, 2010; Kitazawa, AS, 2014))

$V = T(e^{\gamma\phi}\ [+\ e^{\gamma'\phi}])$ (Einstein frame, [$\gamma' < 3/2$])

For $\gamma < 3/2$ [canonical: beware of different notation in earlier work]:

- "Climbing" solution (ϕ climbs, then descends) → $g_s = e^{\phi}$ BOUNDED)
- "Descending" solution (ϕ only descends) → $g_s = e^{\phi}$ UNBOUNDED)
- Limiting τ- speed (LM attractor) *(Lucchin and Matarrese, 1985)*

LM attractor & descending solution disappear for $\gamma \geq 3/2$!



CLIMBING : BSB (Usp(32)) and U(32) HAVE $\gamma = 3/2$! [SO(16) x SO(16) has $\gamma = 5/2$] !

CLUES for the onset of INFLATION?

$$P(k) = A k^{n_s - 1} \ \rightarrow \ \frac{k^3}{[k^2 + \Delta^2]^{2 - \frac{n_s}{2}}}$$

Tensor-to-scalar ratio & (squeezed) non-gaussianity : enhanced at transition

(Bartolo, Matarrese, AS, Vanzan, work in progress)

Compactification

9D Dudas-Mourad Vacua

(Dudas, and Mourad, 2000, 2001)

$$S = \frac{1}{2\,k_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[-R + 4(\partial\phi)^2 \right] - \frac{1}{12} \mathcal{H}_3^2 - \frac{1}{4} e^{-\phi} \,\mathrm{tr}\, \mathcal{F}^2 - T\, e^{-\phi} + \ldots \right\}$$

9D solutions → T DRIVES the compactification

[For both Usp(32) and U(32), & similar but more complicated for SO(16) x SO(16)]

- SPONTANEOUS COMPACTIFICATION: intervals of FINITE length $\sim \frac{1}{\sqrt{T}}$
- FINITE 9D Planck mass and gauge coupling

- g_s diverges at one end & curvature at the other
- Internal interval & 9D flat space (with warping)
- QUESTIONS:
  - Fermi fields in an interval ? (Mourad, AS, 2020)
  - Are large values of g_s NEEDED for these types of compactification ?
  - Stability ?

$$e^{\phi} = e^{u+\phi_0}\, u^{\frac{1}{3}}$$

$$ds^2 = e^{-\frac{u}{6}}\, u^{\frac{1}{18}}\, dx^2 + \frac{2}{3\,T\,u^{\frac{3}{2}}}\, e^{-\frac{3}{2}(u+\phi_0)}\, du^2$$

AdS x S Flux Vacua with Tadpoles

(Gubser, Mitra, 2002)
(Mourad, AS, 2016)

$$S = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R + 4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\right\}$$

- Dilaton Eq: constraint from positivity of T (orientifolds NEED H_3 fluxes, SO(16)xSO(16) H_7 fluxes)
- Eqs. determine AdS (in Poincaré coordinates or in other slicings)
- $AdS_3 \times S^7$ (U(32) & Usp(32) orientifolds) ; $AdS_7 \times S^3$ (SO(16)xSO(16) heterotic)

❖ WIDE REGIONS where the two couplings $\alpha' R$ and $g_s = e^{\phi}$ are SMALL

❖ (H_3 or H_7) FLUXES: SUPPORT THESE SYMMETRIC COMPACTIFICATIONS

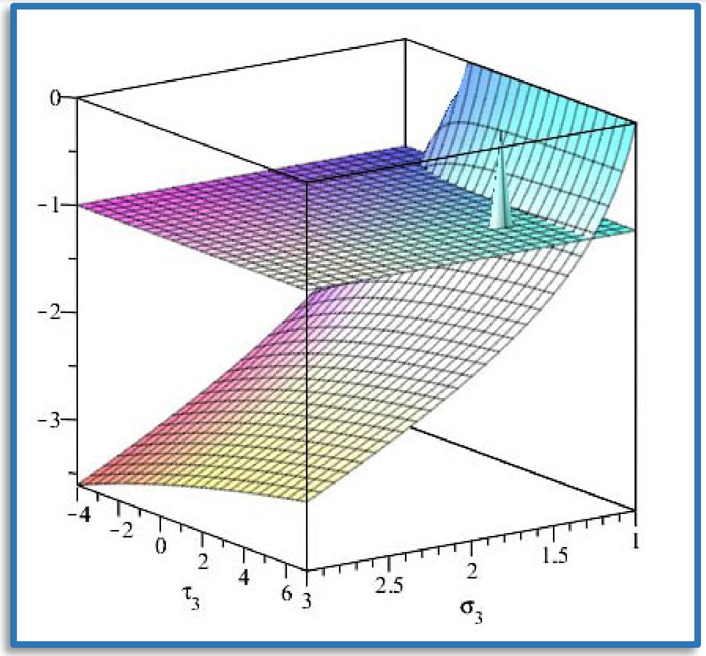
Stability ?

$AdS_3 \times S^7$ ($\&$ $AdS_7 \times S^3$) Vacua

(Gubser, Mitra, 2001)
(Mourad ,AS, 2016)
(Basile, Mourad, AS, 2018)

❖ Orientifold & SO(16)xSO(16) vacua: WEAK coupling but UNSTABLE

- VIOLATIONS of Breitenlohner-Freedman bounds for modes with INTERNAL EXCITATIONS (mixings)
- Wide NEARBY regions of stability. Quantum corrections?
- At least in SO(16)xSO(16): perturbative instabilities can be removed by internal projections on S^3



Dudas-Mourad Vacua

(Basile, Mourad, AS, 2018)

❖ Dudas-Mourad vacua: STRONG COUPLING but STABLE !

- E.g.: Scalar perturbations: $ds^2 \;=\; e^{2\Omega(z)}\left[(1+A)\,dx^\mu\,dx_\mu \;+\; (1-7A)\,dz^2\right] ,$

$$A'' + A'\left(24\,\Omega' \;-\; \frac{2}{\phi'}\,e^{2\Omega}\,V_\phi\right) \;+\; A\left(m^2 \;-\; \frac{7}{4}\,e^{2\Omega}\,V \;-\; 14\,e^{2\Omega}\,\Omega'\,\frac{V_\phi}{\phi'}\right) \;=\; 0$$

❖ Schrödinger-like form

$$m^2\,\Psi \;=\; \left(b \;+\; \mathcal{A}^\dagger\,\mathcal{A}\right)\Psi$$

$$\mathcal{A} \;=\; \frac{d}{dr} \;-\; \alpha(r)\;, \qquad \mathcal{A}^\dagger \;=\; -\,\frac{d}{dr} \;-\; \alpha(r)\;, \qquad b \;=\; \frac{7}{2}\,e^{2\Omega}\,V\,\frac{1}{1\;+\;\frac{9}{4}\,\alpha_O\,y^2} \;>\; 0$$

NO tachyons in 9D : PERTUBATIVE STABILITY

The Climbing Scalar

(Basile, Mourad, AS, 2018)

- COSMOLOGY : the issue is the time evolution of perturbations
- INITIALLY (large η) V is negligible: tensor perturbations evolve as

$$h''_{ij} + \frac{1}{\eta}\, h'_{ij} + \mathbf{k}^2\, h_{ij} = 0$$

$$h_{ij} \sim A_{ij}\, J_0(k\eta) + B_{ij}\, Y_0(k\eta) \qquad (\mathbf{k} \neq 0)$$

$$h_{ij} \sim A_{ij} + B_{ij}\, \log\left(\frac{\eta}{\eta_0}\right) \qquad (\mathbf{k} = 0)$$

- NOTICE: logarithmic growth for k=0 (instability of isotropy) !!
- RESONATES with work on IIB Matrix Model

(Kim, Nishimura, Tsuchiya, 2018)
(Anagnostopoulos, Auma, Ito, Nishimura, Papadoulis, 2018)

Dynamical origin of compactification ?

SUSY Breaking in IIB with a Bounded g_s

D=4 with Fluxes on T^5 x I

(Mourad, AS, to appear)

- Five-form flux in IIB → ϕ CONSTANT, SPATIAL INTERVAL of length ℓ

$$ds^2 = \Lambda^2 \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu}{[\sinh(r)]^{\frac{1}{2}}} + \ell^2\, [\sinh(r)]^{\frac{1}{2}}\, e^{-\frac{5\,r}{\sqrt{10}}}\, dr^2 + \left(\sqrt{2}\,\Phi\,\ell\right)^{\frac{2}{5}} [\sinh(r)]^{\frac{1}{2}}\, e^{-\frac{r}{\sqrt{10}}}\, d\vec{y}^{\,2}$$

$$\mathcal{H}_5^{(0)} = \frac{\Lambda^4}{\sqrt{2}}\, \frac{dx^0 \wedge \ldots \wedge dr}{[\sinh(r)]^2} + \Phi\, dy^1 \wedge \ldots \wedge dy^5$$

$$q_3\, \Phi = N \qquad q_3 = \sqrt{\pi}\, m_{Pl(10)}^4$$

- FINITE gs , BUT STILL CURVATURE SINGULARITY]

Used extensively: (Bergshoeff, Kallosh, Ortin, Roest, Van Proeyen, 2001)

- Split perturbations according to SO(1,3)xSO(5) [or SO(4) for internal excitations]
- SUSY BREAKING ~ 1/ℓ
- Tensor eqs:
- (+ Einstein eqs.)

$$\partial_{[\mu}\, b^{(2)}{}_{\nu]}{}^{lm} + \frac{1}{2}\, \epsilon^{lmnpq}\, \partial_n\, b_{\mu\nu pq} = -\frac{e^{-4A-4C}}{2}\, \epsilon_{\mu\nu\rho\sigma}\, \partial_r\, b^{\rho\sigma lm},$$

$$\partial_r\, b^{(2)}{}_\mu{}^{lm} = e^{2A+6C} \left(\partial^{[l}\, b_\mu{}^{m]} + \frac{1}{2}\, \epsilon^{\alpha\beta\gamma}{}_\mu\, \partial_\alpha\, b_{\beta\gamma}{}^{lm}\right),$$

$$\partial_\mu\, b^m - \partial_n\, b^{(2)}{}_\mu{}^{mn} = e^{-2C} \left[\frac{H_5}{2}\, h_\mu{}^m - e^{-6A}\, \partial_r\, b_\mu{}^m\right],$$

$$\partial_r\, b^m = e^{-2C} \left[\frac{H_5}{2}\, h_r{}^m - e^{10C} \left(\partial^m\, b - \partial_\mu\, b^{\mu m}\right)\right],$$

$$\partial_p\, b^p = \frac{H_5}{4} \left(-\, e^{-2A}\, h_\alpha{}^\alpha - e^{-2B}\, h_{rr} + e^{-2C}\, h_i{}^i\right) + e^{-8A}\, \partial_r\, b.$$

Mixing & Stability with Fluxes on T^5 x I

(Mourad, AS, to appear)

$$ds^2 = e^{2A(r)} dx^2 + e^{2B(r)} dr^2 + e^{2C(r)} dy^2$$

- NO instabilities for **k=0, BUT mixings** induce them for **k≠0**
- E.g:

$$M\,Z = m^2\,Z$$

$$\begin{pmatrix} \mathcal{K}^2 + (-\partial + \alpha)_z(\partial + \alpha)_z & \frac{kH_5}{\sqrt{2}}\, e^{2(A-3C)} \\ \frac{kH_5}{\sqrt{2}}\, e^{2(A-3C)} & \mathcal{K}^2 + (-\partial + \beta)_z(\partial + \beta)_z \end{pmatrix} \qquad \alpha = \frac{C-A}{2}, \qquad \beta = -\frac{5A+3C}{2}$$

- PERTURBATIONS → Schrödinger-like systems

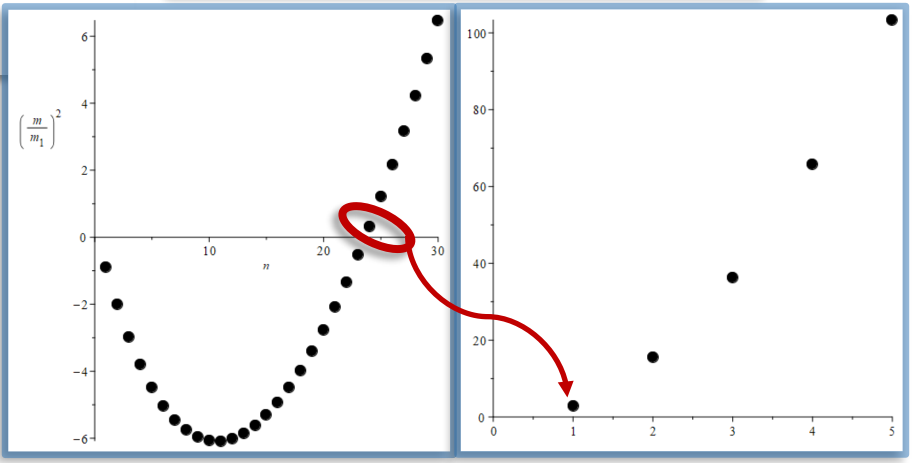
$$m^2\,\Psi = \left(b + \mathcal{A}^\dagger \mathcal{A}\right)\Psi$$

$$\mathcal{A} = \frac{d}{dr} - \alpha(r)$$

  - Variational method: $m_\Psi^2 = \frac{\langle\Psi|\widetilde{M}|\Psi\rangle}{\langle\Psi|\Psi\rangle} \geq m_0^2$

- Mixings → unstable KK excitations
- BUT: NO link between 4D & internal scale

$$m_{Pl(10)}\,\ell > \mathcal{O}\left(10^2\right) N^{\frac{1}{4}}$$



Scalar Perturbations with Fluxes on T^5 x I

(Mourad, AS, to appear)

- **Schrödinger-like** system, but with a **non-symmetric real potential !**
- [NO NATURAL measure for self-dual form] → from Schrödinger-like system ?
- → **Eigenvalues** for **m²** may be **complex**
- BASIC TOOL: (extension of) variational principle

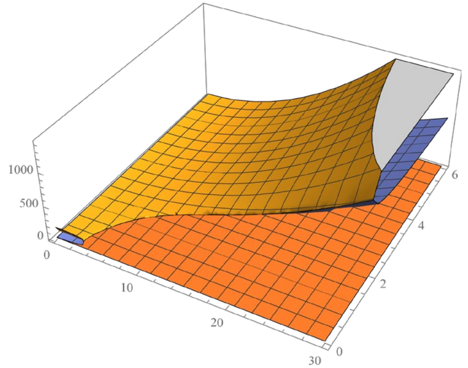
$$m^2_\Psi \;=\; \frac{1}{2}\,\frac{\langle\Psi|\left(\widetilde{M}+\widetilde{M}^\dagger\right)|\Psi\rangle}{\langle\Psi|\Psi\rangle} \;\geq\; m_0^2$$

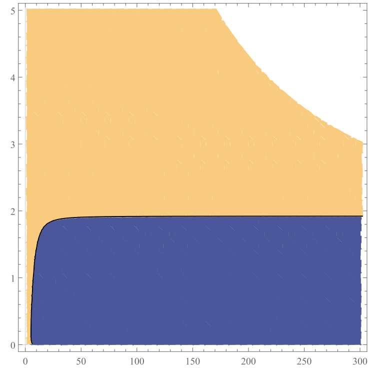
$$m^2 \;=\; Inf\left\{\frac{\int dz\left[\partial_z Y^\dagger H^{-1}\partial_z Y + \frac{\epsilon(z)}{2}Y^\dagger\left(N-N^T\right)\partial_z Y + Y^\dagger H^{-1} U\, Y\right]}{\int dz\, Y^\dagger H^{-1} Y}\right\}$$

$$H^{-1} \;=\; 1-\frac{F(z)}{2}\left(N+N^T\right) + \frac{F(z)^2}{4}N^T N$$

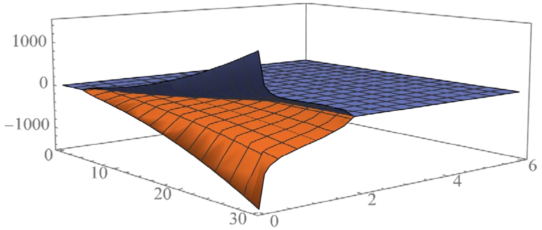
$$F \;=\; -\frac{1}{\sqrt{2}\,h}\int_{\frac{r}{\rho}}^{\infty} dx\, \frac{e^{\frac{x}{2\sqrt{10}}}}{\left[\sinh(x)\right]^{\frac{3}{2}}}$$

$$N \;=\; \begin{pmatrix} 0 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$





SOME EVIDENCE THAT:
NO Instabilities beyond a critical value H_{min} for field strength H_5



There seems to be room for [non-SUSY strings → stable 4D Minkowski vacua]

Thank You