ADVANCES IN QUANTUM FIELD THEORY

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Mikhail Shifman

William I. Fine Theoretical Physics Institute,
University of Minnesota

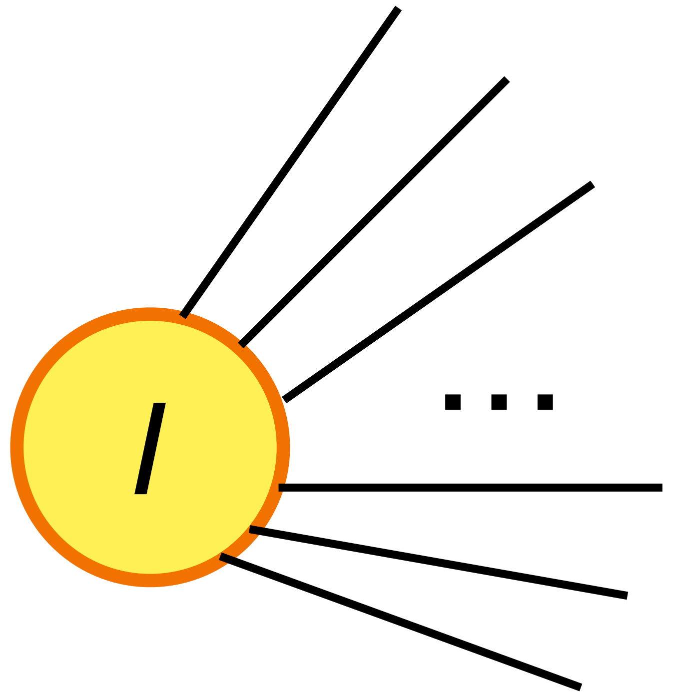
Bringing Yang-Mills Theory Closer to Quasiclassics

In super-Yang-Mills the full β functions is completely determined by quasi classics, e.g.

$$\beta(\alpha) = -\left(n_b - \frac{n_f}{2}\right)\frac{\alpha^2}{2\pi}\left[1 - \frac{(n_b - n_f)\alpha}{4\pi}\right]^{-1}$$

n_b and n_f are the numbers of the instanton zero modes

$$\left.\begin{array}{l} \mathcal{N} = 1 \to n_b = 2n_f = 4N \\ \mathcal{N} = 2 \to n_b = n_f = 4N \\ \mathcal{N} = 4 \to n_b = \dfrac{n_f}{2} = 4N \end{array}\right\} \quad n_f = \mathcal{N} \times 2N$$



$$\boxed{\mathcal{N}=1, \quad S_{\rm inst}=\frac{2\pi}{\alpha_o}}$$

$$d\mu_{\rm inst} \propto \underbrace{S_{\rm inst}^{2N}\, M_{\rm uv}^{4N}}\ \underbrace{\frac{1}{S_{\rm inst}^{N} M_{\rm uv}^{N}}}\ e^{-S_{\rm inst}} \propto \overbrace{M_{\rm uv}^{n_b-\frac{n_f}{2}}\left(\frac{1}{\alpha_0}\right)^{\frac{n_b-n_f}{2}}}^{\text{EXACT}} e^{-S_{\rm inst}}$$

boson : $\frac{M_{\rm uv}^2}{\alpha_0}$ per pair of modes

fermion : $\frac{\alpha_0}{M_{\rm uv}}$ per pair of modes

Why EXACT? All bosonic and fermionic NON-zero modes exactly CANCEL each other!

Let us try to do the same in pure (non)-SUSY Yang-Mills

$$d\mu_{\rm inst} = {\rm const} \times \int \frac{d^4x_0\, d\rho}{\rho^5} (M_{\rm uv}\rho)^{4N} \left(\frac{8\pi^2}{g^2}\right)^{2N} \exp\left(-\frac{8\pi^2}{g^2} + \underline{\Delta_{\rm gl} + \Delta_{\rm gh}}\right)$$

Quantum part

First ignore

$$\boxed{\frac{d}{d\log M_{\rm uv}}\left(4N\log M_{\rm uv} - 2N\log g^2 - \frac{8\pi^2}{g^2}\right) = 0}$$

$$\beta(\alpha) = \frac{d}{d\log M_{\rm uv}}\alpha = -\beta_1\frac{\alpha^2}{2\pi} - \beta_2\frac{\alpha^3}{4\pi^2} + \ldots$$

$$\beta_1 = 4N\,, \quad \beta_2 = 8N^2\,.$$

What is to be added

To cancel this???

Background Field Method

I/AI

$$A_\mu^a \equiv \left(A_\mu^a\right)_{\text{ext}} + a_\mu^a$$

Enhanced BRST

$$\mathscr{L}_{\text{quant}} = \frac{1}{g^2}\left\{ -\frac{1}{2}\left(D_\mu^{\text{ext}} a_\nu^a\right)^2 + a^{a\,\mu}\left(G_{\mu\nu}^b\right)_{\text{ext}} f^{abc} a^{c\,\nu} + \left(D_\mu^{\text{ext}}\bar{c}\right)\left(D^{\mu\,\text{ext}} c\right) + \left(D_\mu^{\text{ext}}\bar{\Phi}\right)\left(D^{\mu\,\text{ext}}\Phi\right)\right.$$

Charge interaction of quantum gluons; 4 dof ✓

Magnetic interaction of quantum gluons → zm
THE ONLY UNCANCELED

Ghosts; −2 dof ✓

Phantom = 2nd ghost→ −2 dof ✓

A_μ^a and (c^a plus Φ^a) form two doublets of a global $SU(2)$

The simplest example of exact "supersymmetry"

$$\mathscr{L}_{\varphi\Phi} = \partial_\mu \bar{\varphi} \partial^\mu \varphi + (\bar{\varphi}\varphi)^2 + \partial_\mu \bar{\Phi} \partial^\mu \Phi + 2(\bar{\varphi}\varphi)(\bar{\Phi}\Phi)$$

Regular complex field

Phantom

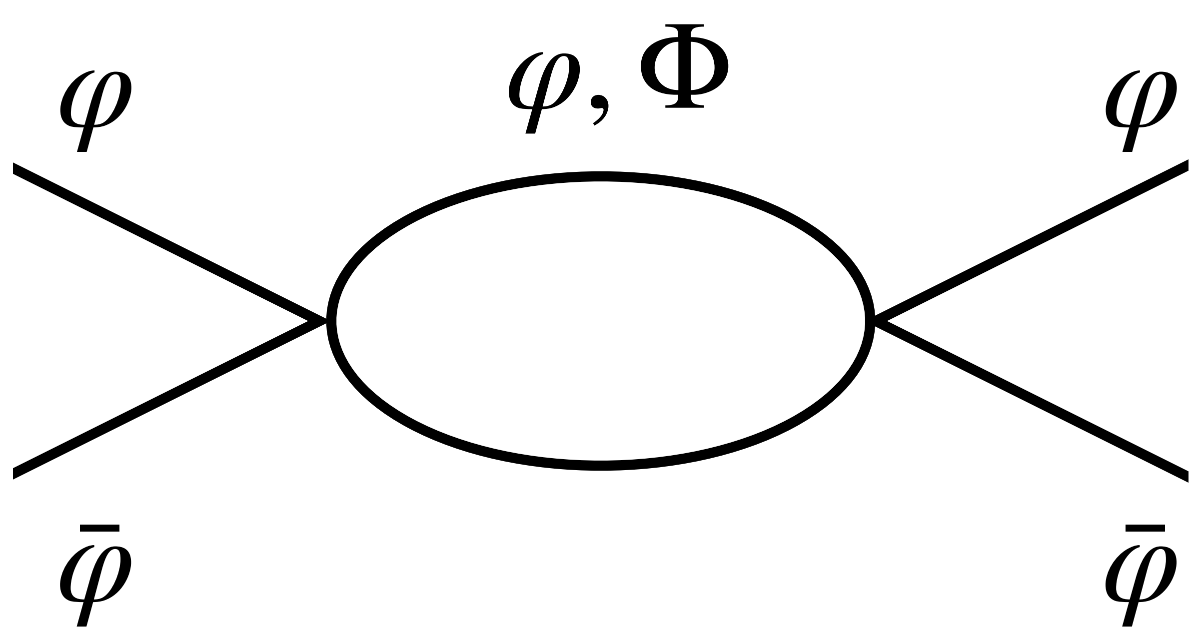
$$\mathcal{J}^\mu = \left(\varphi \overleftrightarrow{\partial^\mu} \Phi + \mathrm{H.c.}\right),$$

$$\partial_\mu \mathcal{J}^\mu = \varphi(\partial^2 \Phi) - (\partial^2 \varphi)\Phi = 2\varphi^2 \bar{\varphi}\,\Phi - 2\varphi^2 \bar{\varphi}\,\Phi + \mathrm{H.c.} = 0\,.$$

$$\{\mathcal{Q}\mathcal{J}^\mu\} = -i \sum_{\phi=\varphi,\,\Phi} \phi\, \partial^\mu\, \bar{\phi} \longleftarrow$$

NOT Hamiltonian, graded global SU(2)

The theory is not empty!



Does NOT vanish; φ dominates!

Conclusion/Conjecture

- In Yang – Mills theory with one phantom the β function is quasiclassical at least in first and second loops;

- Limiting the physical sector to amplitudes in which Φ^a propagate only in loops we get a theory which is probably unitary ????????