Instantons with Quantum Core

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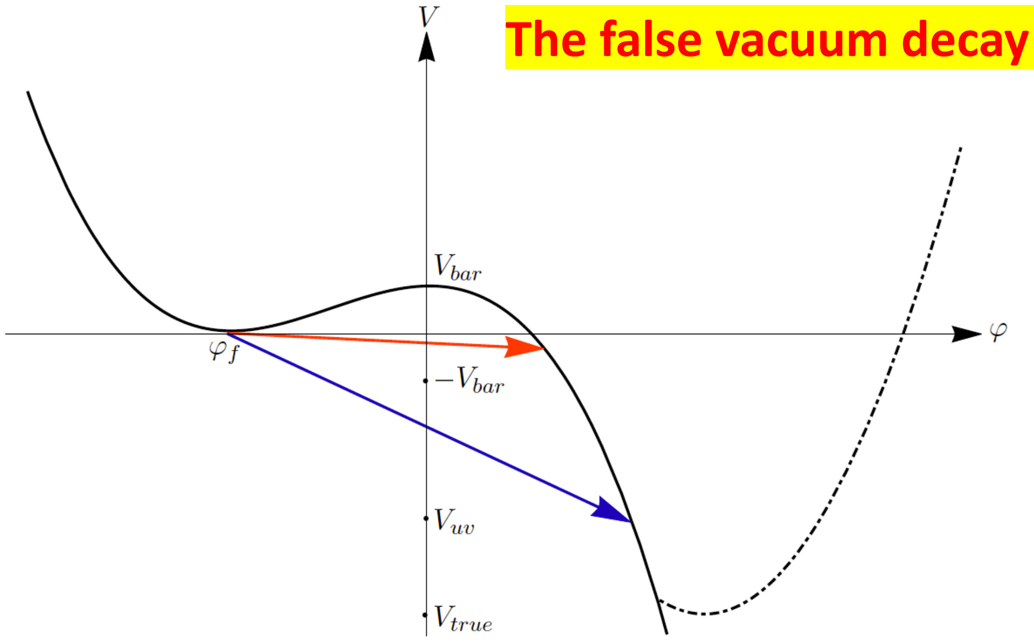
Based on

JCAP (in press), 2105.01996 [hep-th]

JCAP (in press), 2104.12661 [hep-th]

BLTP JINR, Dubna, 14 October 2021



The false vacuum decay (quasiclassical approximation)

$$S = \int (\mathcal{K} - \mathcal{V}) dt\,, \qquad \mathcal{K} \equiv \frac{1}{2}\int (\partial_t \varphi_{\mathbf{x}})^2 d^3x\,, \qquad \mathcal{V} \equiv \int \left(\frac{1}{2}(\partial_i \varphi_{\mathbf{x}})^2 + V(\varphi_{\mathbf{x}})\right) d^3x$$

$$\tau = it$$

$$S_E = \int \left(\frac{1}{2}(\partial_\tau \varphi)^2 + \frac{1}{2}(\partial_i \varphi)^2 + V(\varphi)\right) d^3x d\tau$$

$$\Gamma \simeq \exp(iS) = \exp(-S_E) \rightarrow \partial_\tau^2 \varphi + \Delta\varphi - V' = 0\,,$$

we must find field configurations with $\varphi(\tau \to -\infty, \mathbf{x}) = \varphi_f$, matching the classically allowed state $\varphi(\tau = 0, \mathbf{x})$ with $\partial\varphi/\partial\tau = 0$ and $\mathcal{V}(\varphi(\mathbf{x})) = 0$

The Coleman instanton *(Phys.Rev.D* 15 (1977) 2929)

Coleman proposed to consider $O(4)$ - invariant solutions for which φ depends only on $\varrho = \sqrt{\tau^2 + \mathbf{x}^2}$,

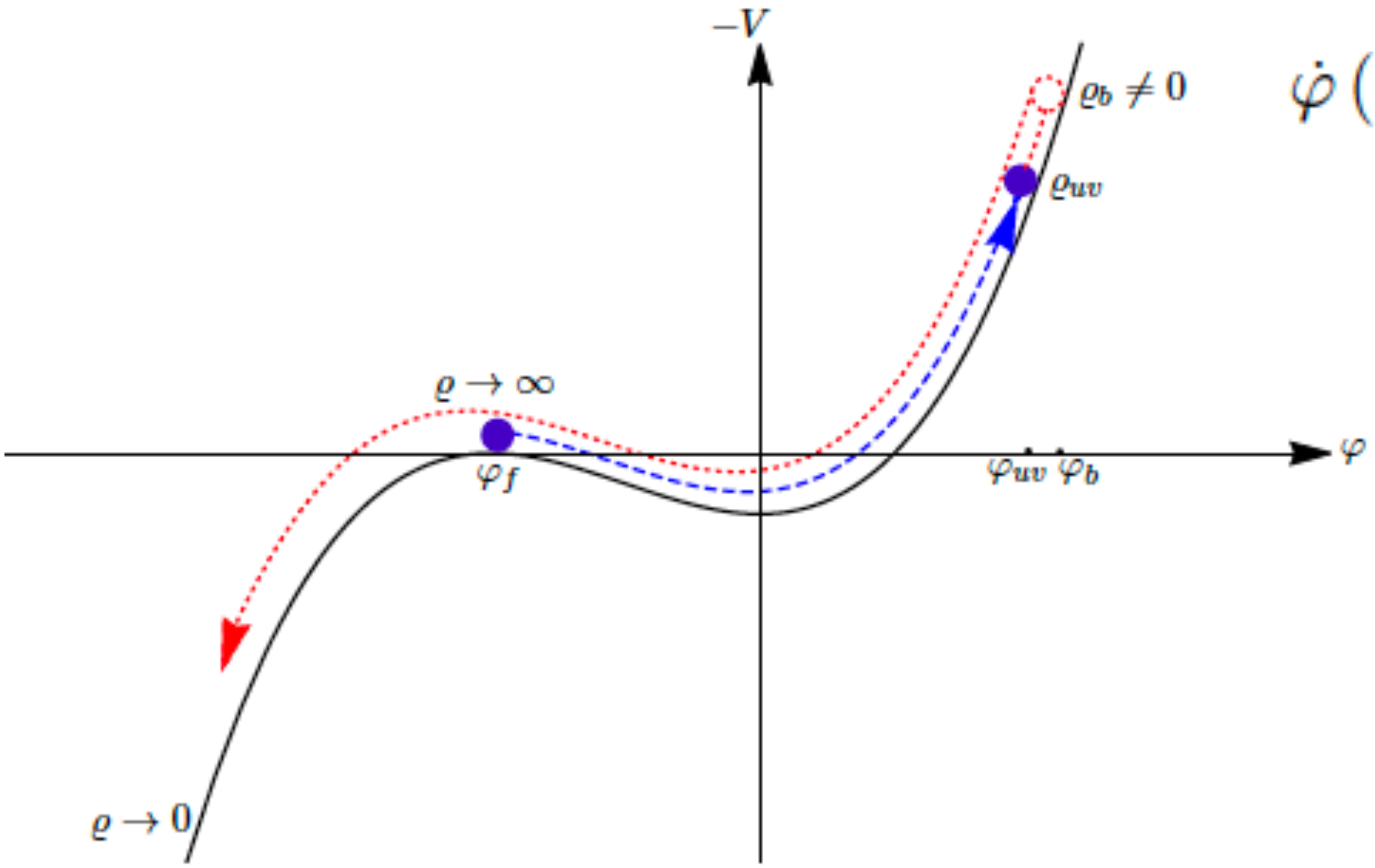
$$\varphi(\tau, \mathbf{x}) = \varphi(\varrho) \rightarrow \ddot{\varphi}(\varrho) + \frac{3}{\varrho}\dot{\varphi}(\varrho) - V' = 0$$

with the two boundary conditions:

$$\varphi(\varrho \to \infty) = \varphi_f \qquad \dot{\varphi}(\varrho = 0) = 0$$

$$\rightarrow \quad S_E = 2\pi^2 \int_0^{+\infty} d\varrho\, \varrho^3 \left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right) \quad \rightarrow \quad \Gamma \simeq \varrho_0^{-4} \exp(-S_E)$$

$$\dot{\varphi}\left(\varrho_b\right) = 0$$



$$\ddot{\varphi}(\varrho) + \frac{3}{\varrho}\dot{\varphi}(\varrho) - V' = 0$$

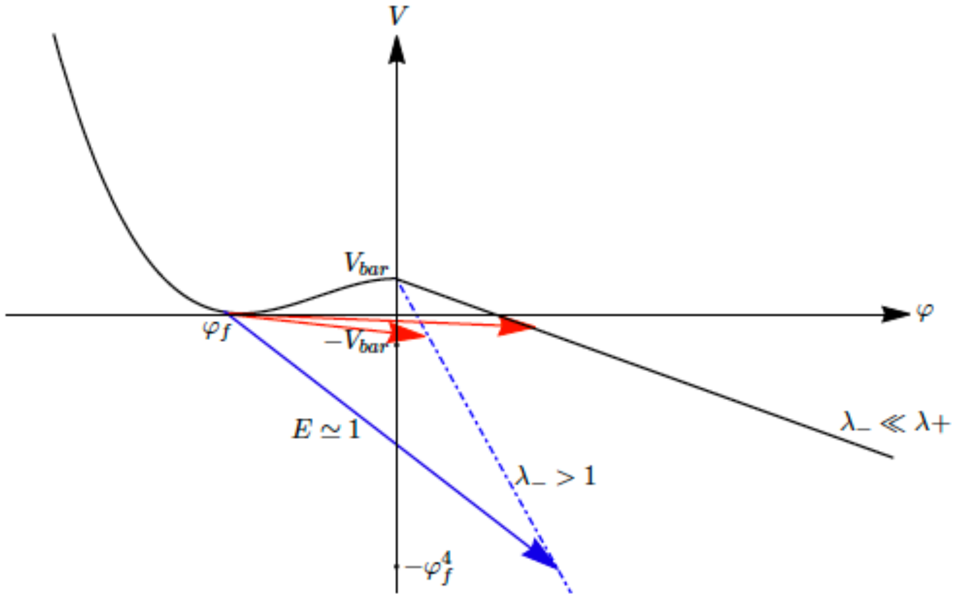
$$\varphi\left(\varrho \to \infty\right) = \varphi_f$$

$$\dot{\varphi}(\varrho = 0) = 0$$

The two puzzles with the Coleman instanton

1. The very fast false vacuum decay

V.F. Mukhanov, E. Rabinovici and A.S.S., Fortsch. Phys. 69 (2021) 2000100 [arXiv:2009.12445]



$\lambda_- \gg 1$ (*zero size instanton problem*)

$\varrho_0 << 1$, $S_E << 1$, $\Gamma >> 1$

The quasiclassical approximation is not trustable!

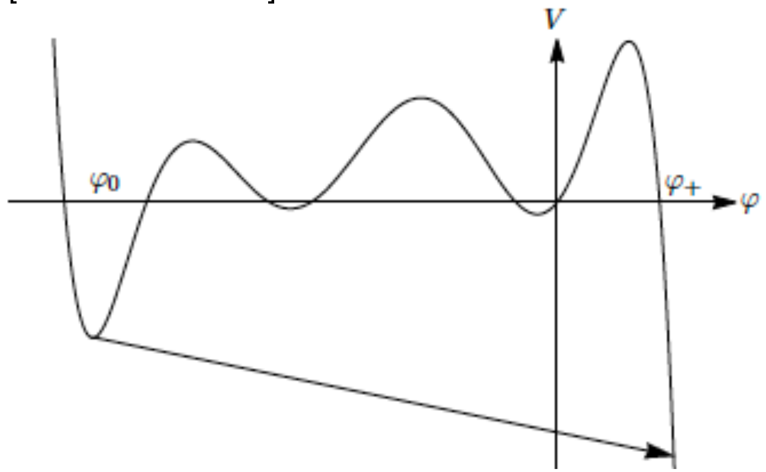
2. There are no Coleman instanton solutions at all

V.F. Mukhanov, E. Rabinovici and A.S.S., Fortsch. Phys. 69 (2021) 2000100 [arXiv:2009.12445]

V.F. Mukhanov and A.S.S., JCAP (in press), [arXiv:2104.12661]

$$V(\varphi) \equiv -\varphi^\alpha v_\alpha(\varphi), \; \alpha \geq 4$$

$$\frac{d v_\alpha(\varphi)}{d\varphi} \geq 0 \text{ at } \varphi > 0$$



$$E(\alpha) = \varrho^{\frac{4}{\alpha-2}} \left(\frac{1}{2} \varrho^2 \dot{\varphi}^2 + \frac{2}{\alpha-2} \varrho \varphi \dot{\varphi} - \varrho^2 V - \frac{2(\alpha-4)}{(\alpha-2)^2} \varphi^2 \right)$$
$$+ \frac{2}{\alpha-2} \int_0^\varrho d\bar{\varrho}\, \bar{\varrho}^{\frac{6-\alpha}{\alpha-2}} \left[(\alpha-4) \left(\bar{\varrho} \dot{\varphi} + \frac{2}{\alpha-2} \varphi \right)^2 + \bar{\varrho}^2 (\alpha V - \varphi V') \right],$$

$$\frac{dE}{d\varrho} = \varrho^{\frac{\alpha+2}{\alpha-2}} \left(\ddot{\varphi}(\varrho) + \frac{3}{\varrho} \dot{\varphi}(\varrho) - V' \right) \left(\varrho \dot{\varphi} + \frac{2}{\alpha-2} \varphi \right),$$

Explicit examples of such potentials:

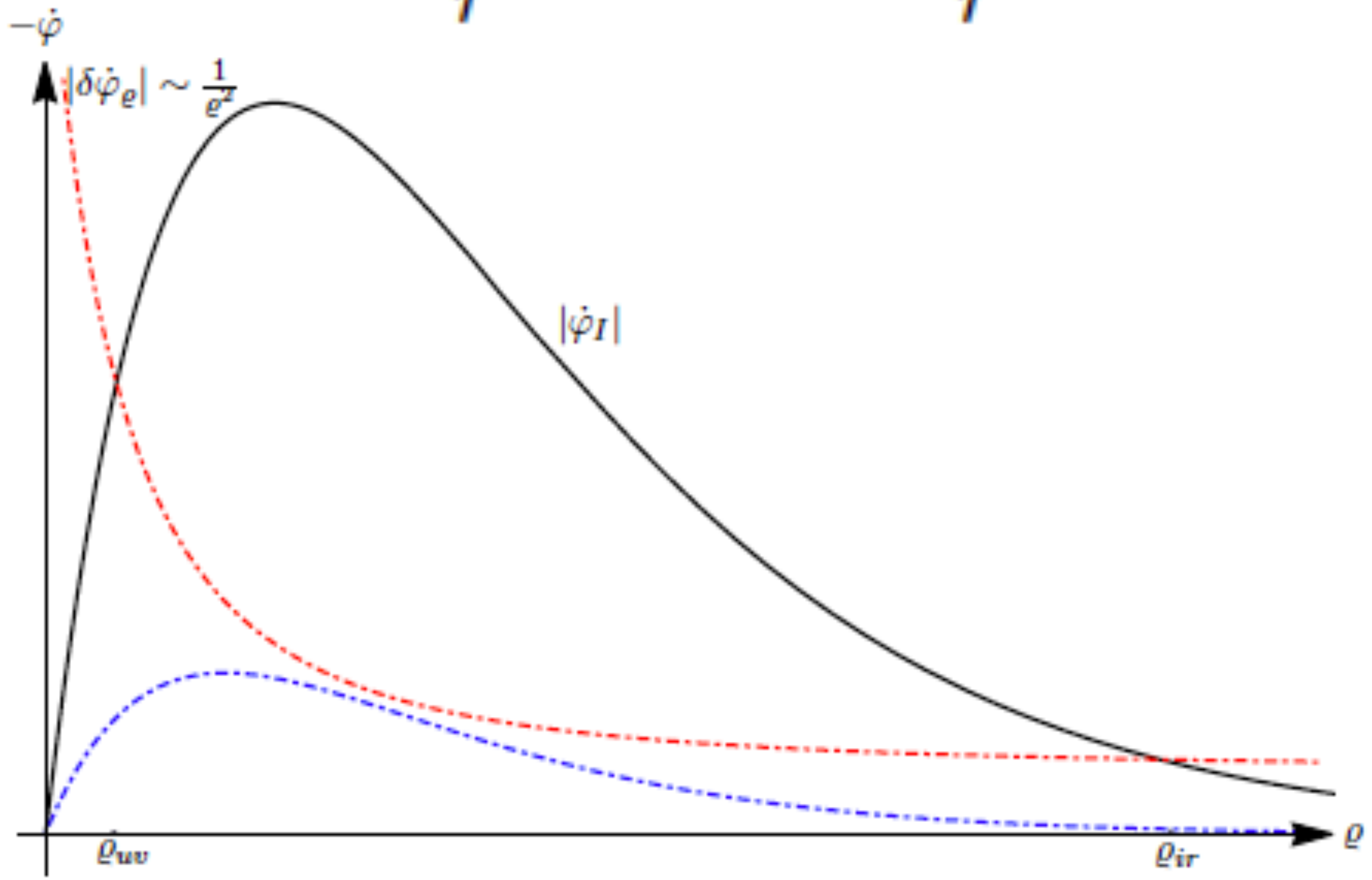
$$v'_\alpha = a\,\varphi^{-\alpha} \prod_{i=1}^{\alpha-2(m+1)} (\varphi - \lambda_i) \prod_{j=1}^{m} \left((\varphi + \beta_j)^2 + \gamma_j^2 \right),$$ where a>0, $\gamma_j, \lambda_i \leq 0$

$$V(\varphi) = \Lambda \varphi^4 + a \left(\varphi^3 + \beta \varphi^2 + \frac{\beta^2 + \gamma^2}{3} \varphi \right), \; \Lambda < 0 ;$$

$$V = -\frac{\lambda}{4} \varphi^4 \ln\left(\frac{\varphi}{\mu}\right), \; \lambda > 0$$

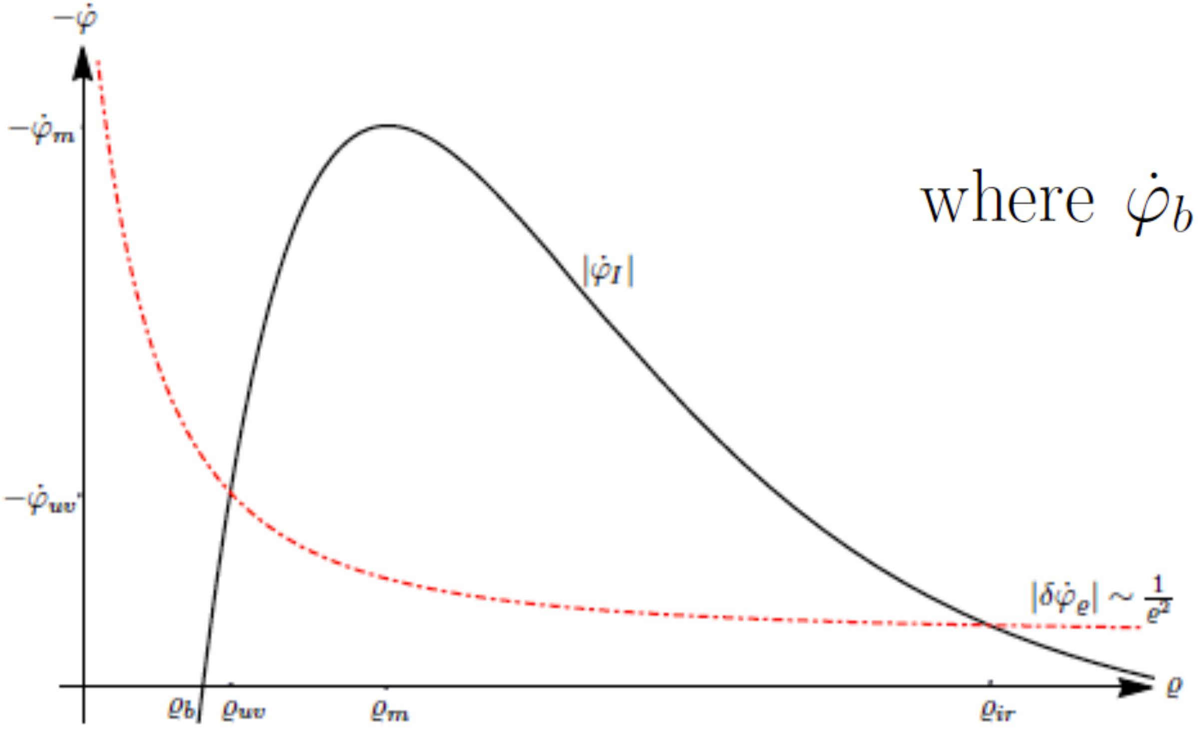
The resolution of the puzzles: quantum fluctuations

$$|\delta\varphi_r| \simeq \frac{\sigma}{r}, \quad |\delta\dot{\varphi}_r| \simeq \frac{\sigma}{r^2}$$



ϱ_{uv} and ϱ_{ir} are two different solutions of the equation: $\dot{\varphi}_I(\varrho) \simeq \frac{\sigma}{\varrho^2}$

Quantum fluctuations and new instantons



$$\mathcal{V}(0) = \frac{2\pi}{3}\dot{\varphi}_b^2\varrho_b^3 - \frac{4\pi}{3}V_b\varrho_b^3 + \mathcal{V}_{\varrho_b}(0) = 0,$$

where $\dot{\varphi}_b \equiv \dot{\varphi}(\varrho_b)$, $V_b \equiv V(\varphi(\varrho_b))$, $\mathcal{V}_{\varrho_b}(0) = \frac{4\pi}{3}V_b\varrho_b^3$

The instanton is trustable only in the range $\varrho_{uv} < \varrho < \varrho_{ir}$

$$\mathcal{V}(0) = \frac{2\pi}{3}\left(\dot{\varphi}_{uv}^2\varrho_{uv}^3 - \dot{\varphi}_{ir}^2\varrho_{ir}^3\right) \simeq \frac{2\pi\sigma}{3}\left(\frac{1}{\varrho_{uv}} - \frac{1}{\varrho_{ir}}\right)$$

With the precision allowed by the time-energy uncertainty relation, the potential energy vanishes and the bubble with the quantum core emerges from under the barrier.

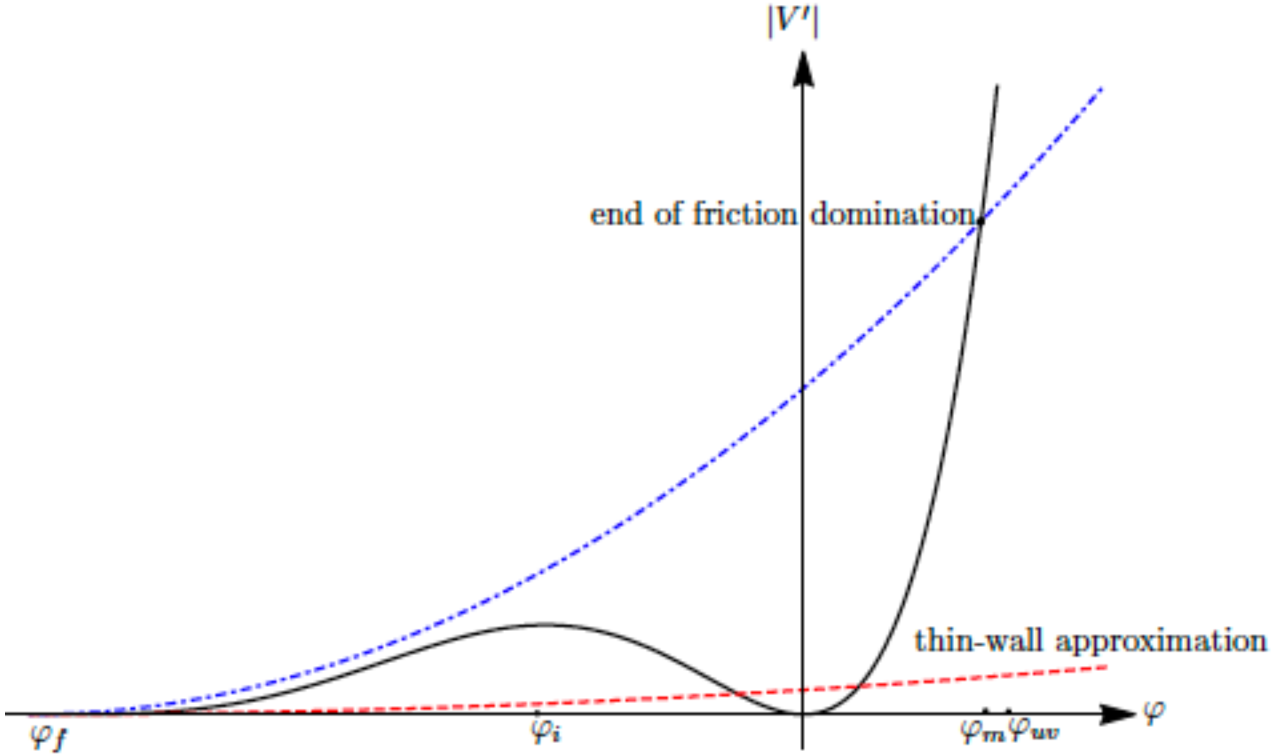
New instantons

$\ddot{\varphi}(\varrho) + \frac{3}{\varrho}\dot{\varphi}(\varrho) - V' = 0$ with the boundary conditions: $\varphi(\varrho \to \infty) = \varphi_f$ and $\dot{\varphi}(\varrho_b) = 0$, ϱ_b is an arbitrary free parameter.

$$S_E = \frac{\pi^2}{2}\int_{\varrho_{uv}}^{\varrho_{ir}}\dot{\varphi}^2\varrho^3 d\varrho + \frac{\pi^2}{4}\left(2V_{ir}\varrho_{ir}^4 + \dot{\varphi}_{uv}^2\varrho_{uv}^4 - \dot{\varphi}_{ir}^2\varrho_{ir}^4\right) \simeq \frac{\pi^2}{2}\int_{\varrho_b}^{\infty}\dot{\varphi}^2\varrho^3 d\varrho$$ ← Good approximation with the accuracy of one quantum: $\int_{\varrho_b}^{\varrho_{uv}}\dot{\varphi}^2\varrho^3 d\varrho < \dot{\varphi}_{uv}^2\varrho_{uv}^4 \simeq O(1)$

The false vacuum decay rate $\Gamma \simeq \varrho_0^{-4}\exp(-S_E)$, where ϱ_0 is the size of the bubble, i.e. $\varphi(\varrho_0) = 0$

The friction-dominated new instantons



$$\ddot{\varphi}(\varrho) + \frac{3}{\varrho}\dot{\varphi}(\varrho) - V' = 0$$

$$|V'| \ll \left|\frac{3}{\varrho}\dot{\varphi}\right|$$

↓

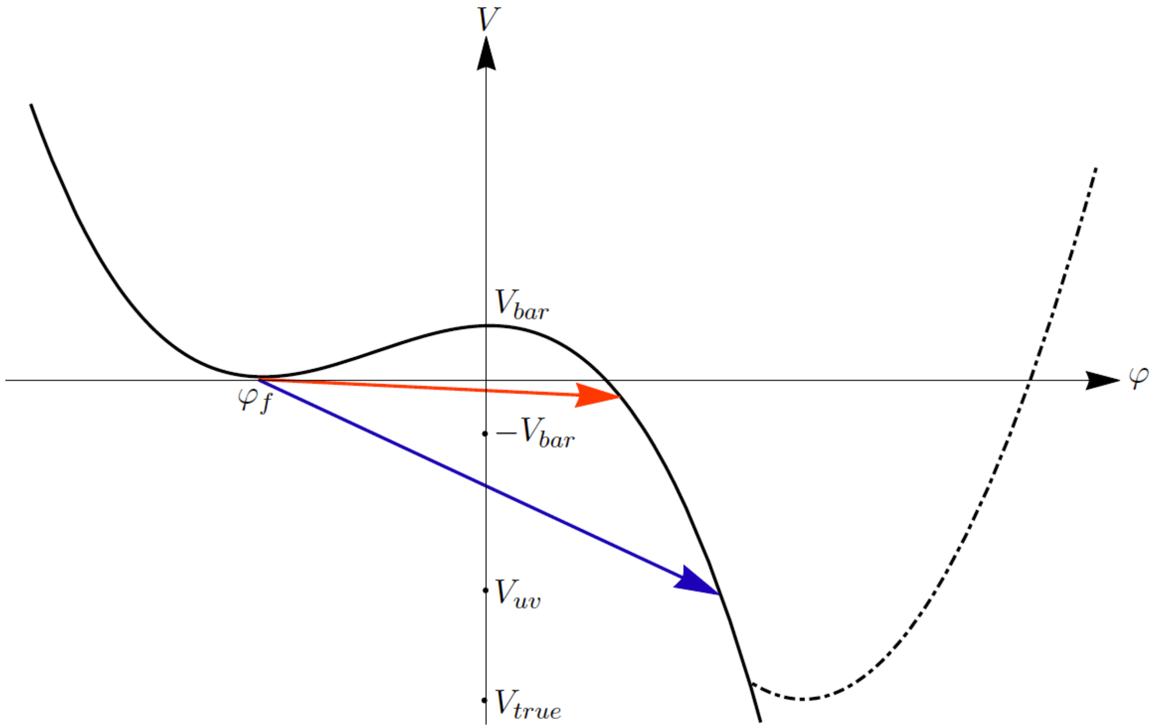
$$\frac{24\,|\varphi_f|}{E}(\varphi_f - \varphi_m)^2 \simeq \alpha\,|V'_m|\ ,$$

$$\varphi\,(\varrho) \simeq \varphi_f - \frac{E}{4\,\varphi_f\,\varrho^2}$$ → $$\left|\frac{3}{\varrho}\dot{\varphi}\right| \simeq \frac{24\,|\varphi_f|}{E}(\varphi_f - \varphi)^2\,,$$ $$E = E(\varrho_0)$$

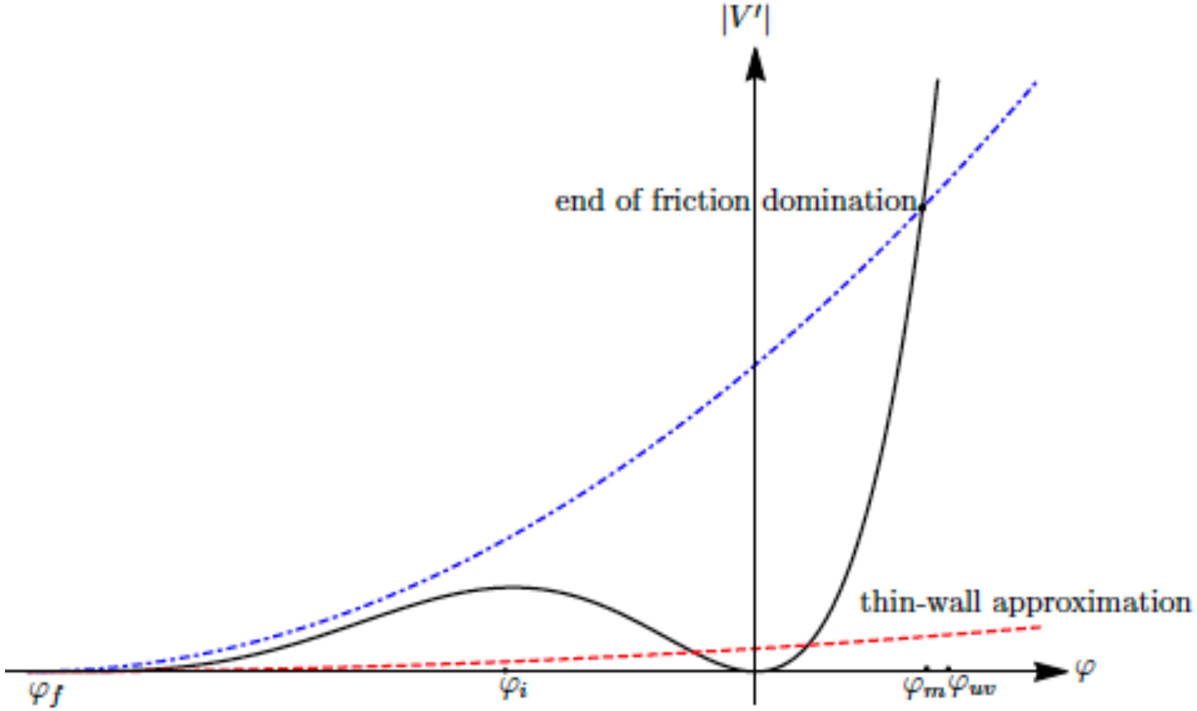
↓

$$\varrho_0^2 \simeq \frac{E}{4\,\varphi_f^2}$$ → $$\dot{\varphi}_0^2\,\varrho_0^4 \simeq \frac{E^2}{4\,\varphi_f^2\,\varrho_0^2} \simeq E$$ **>>1 (**number of quantum)

The friction-dominated new instantons: the thick-wall approximation



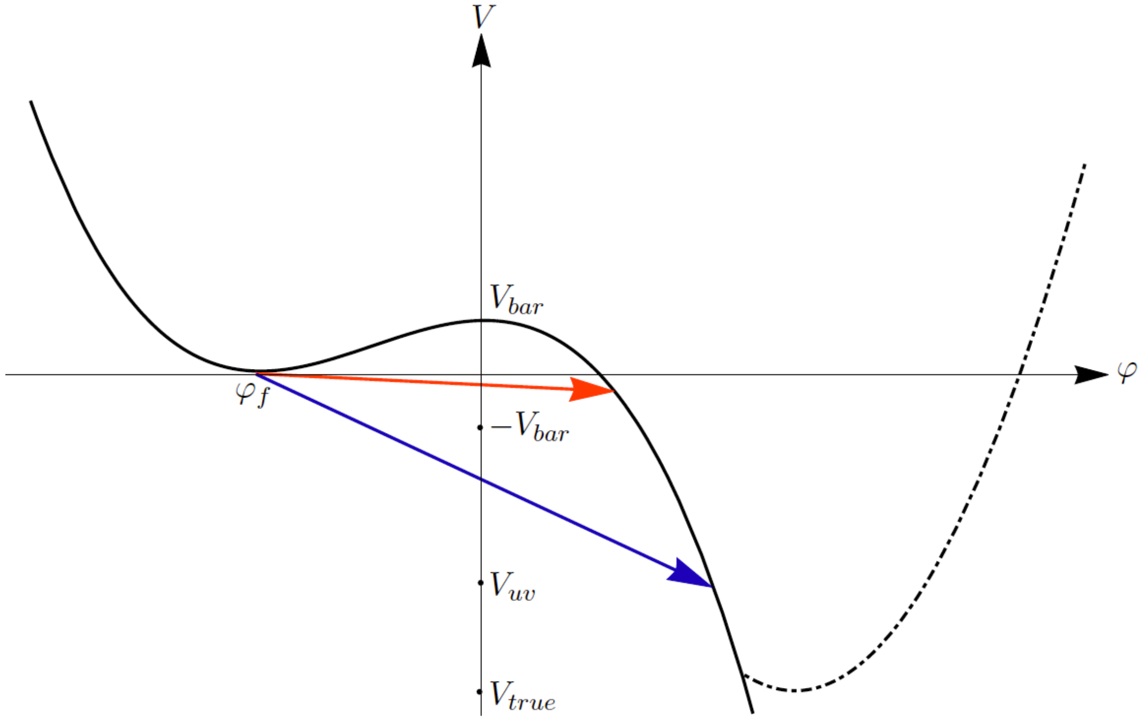
$$|V_{uv}| \gg V_{bar} \;\rightarrow\; 1 \ll E \ll \frac{O(1)\,\varphi_f^4}{V_{bar}}\,,$$



$$\frac{24\,|\varphi_f|}{E}\left(\varphi_f-\varphi_m\right)^2 \simeq \alpha\,|V'_m|\,,$$

$$\varrho_0 \simeq \frac{E^{1/2}}{2\,|\varphi_f|}\,, \qquad V_{uv} \simeq V\left(\varphi_m\right) - \frac{8\,\varphi_f\left(\varphi_f-\varphi_m\right)^3}{E}\,, \qquad S_E \simeq \frac{3\,\pi^2}{8}\frac{E\left(\varphi_f-\varphi_m\right)}{\varphi_f}$$

The thin-wall approximation

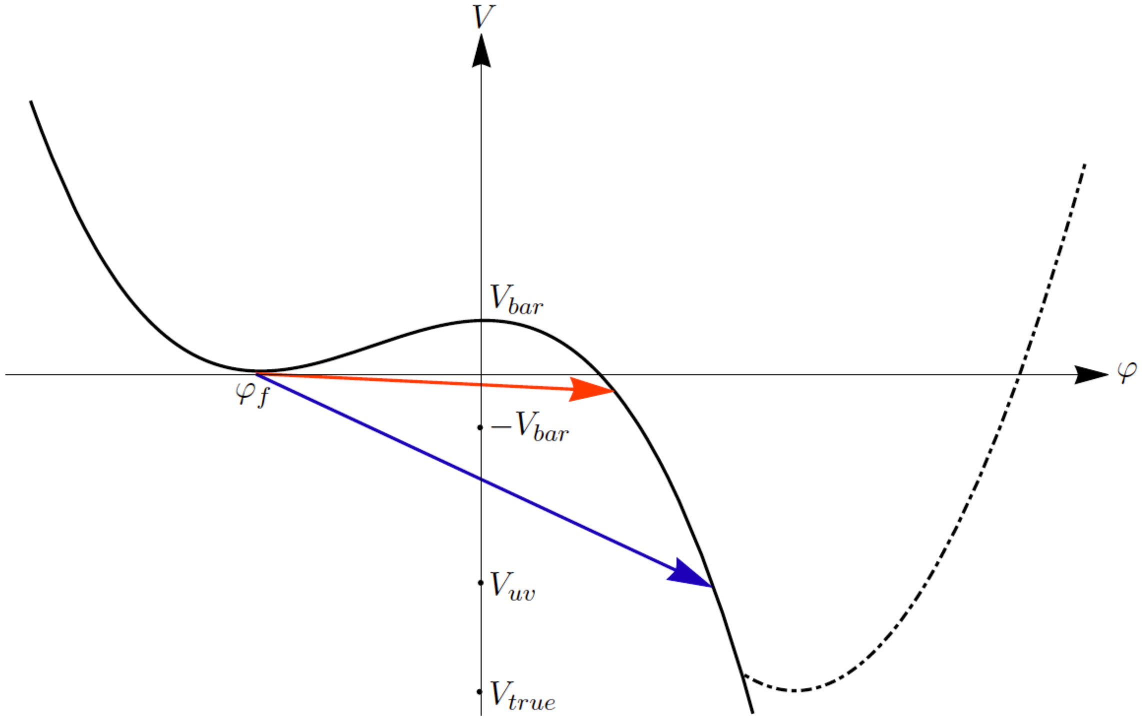


$$|V_{uv}| \ll V_{bar} \quad \rightarrow \quad E \gg \frac{O(1)\,\varphi_f^4}{V_{bar}},$$

$$\varrho_0 \simeq \left(\frac{E}{2\,V_{bar}}\right)^{1/4}, \quad V_{uv} \simeq -\frac{3\,(2\,V_{bar})^{3/4}\,(\varphi_{uv}-\varphi_f)}{E^{1/4}}, \quad S_E \simeq \frac{\pi^2}{2}\frac{E^{3/4}}{(2\,V_{bar})^{1/4}}\,(\varphi_{uv}-\varphi_f)$$

where φ_{uv} is the solution of the equation $V(\varphi_{uv}) \approx 0$

Examples



$$V(\varphi) = \begin{cases} \frac{\lambda_+}{4}(\varphi - \varphi_f)^4 & \text{for } \varphi < \beta\varphi_f \\ -\frac{\lambda_-}{n}\varphi_f^4\left(\frac{\varphi}{\varphi_f}\right)^n + V_{bar} & \text{for } \varphi > \beta\varphi_f \end{cases}$$

$$\lambda_- \ll \lambda_+$$

$$1 \ll E \ll \frac{1}{\lambda_-} \quad \rightarrow \quad \varrho_0 \sim \frac{E^{1/2}}{|\varphi_f|}, \quad V_{uv} \sim -\frac{V_{bar}}{(\lambda_- E)^{n/(n-3)}}, \quad S_E \sim \frac{E}{(\lambda_- E)^{1/(n-3)}}$$

$$E \gg \frac{1}{\lambda_-} \quad \rightarrow \quad \varrho_0 \sim \frac{1}{|\varphi_f|}\left(\frac{E}{\lambda_-}\right)^{1/4}, \quad V_{uv} \sim -\frac{V_{bar}}{(\lambda_- E)^{1/4}}, \quad S_E \sim \frac{(\lambda_- E)^{3/4}}{\lambda_-}$$

Conclusions

The boundary condition $\dot{\varphi}(\varrho = 0) = 0$ *for 0 (4) instantons* *must be abandoned because quantum fluctuations induce ultraviolet cutoff entirely determined by the parameters of the classical solution*

This cutoff regularizes the singular solutions and thus there appears a new class of nonsingular instantons, which all contribute to the false vacuum decay.


Happy
Birthday
The Magnificent Seven!


70
ADVANCES
IN QUANTUM
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Thank you for attention!

Bubbles in Metastable Vacuum
I. Yu. Kobzarev, L. B. Okun, M. B. Voloshin
Sov.J.Nucl.Phys. 20 (1975) 644-646, *Yad.Fiz.* 20 (1974) 1229-1234
414 citations (iNSPIRE hep)

The Fate of the False Vacuum. 1. Semiclassical Theory
Sidney R. Coleman
Phys.Rev.D 15 (1977) 2929-2936, *Phys.Rev.D* 16 (1977) 1248 (erratum)
2181 citations (iNSPIRE hep)

The Fate of the False Vacuum. 2. First Quantum Corrections
Curtis G. Callan, Jr., Sidney R. Coleman
Phys.Rev.D 16 (1977) 1762-1768
1406 citations (iNSPIRE hep)

Action Minima Among Solutions to a Class of Euclidean Scalar Field Equations
Sidney R. Coleman, V. Glaser, Andre Martin
Commun.Math.Phys. 58 (1978) 211-221
283 citations (iNSPIRE hep)

Instantons with Quantum Core
V. F. Mukhanov, A. S. Sorin
JCAP (in press), 2105.01996 [hep-th]

On the Existence of the Coleman Instantons
V. F. Mukhanov, A.S. Sorin
JCAP (in press), 2104.12661 [hep-th]

Quantum Fluctuations and New Instantons II: Quartic Unbounded Potential
V. F. Mukhanov, E. Rabinovicii, A. S. Sorin
Fortsch.Phys. 69 (2021) 2, 2000101, 2009.12444 [hep-th]

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Fortsch.Phys. 69 (2021) 2, 2000100, 2009.12445 [hep-th]