Present status and the simplest models of the inflationary scenario

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Inflation and two new fundamental observational parameters

The simplest one-parametric inflationary models

The mixed R^2-Higgs model

Post-inflationary behaviour of the mixed R^2-Higgs model

Perspectives of future discoveries

Beyond the slow-roll approximation

Conclusions

Inflation

The inflationary hypothesis :

Some part of the world which includes all its presently observable part was as much symmetric as possible during some period in the past - both with respect to the geometrical background and to the state of all quantum fields (no particles).

Non-universal (due to the specific initial condition) explanation of the cosmological arrow to time - chaos, entropy (in some not well defined sense) can only grow after inflation.

Still this state is an intermediate attractor for a set of pre-inflationary initial conditions with a non-zero measure. Also it is not a unique one, there exists a class of such states leading to the same observable predictions.

Successive realization of this idea is based on the two more detailed and independent assumptions.

1. Existence of a metastable quasi-de Sitter stage in our remote part which preceded the hot Big Bang. During it, the expansion of the Universe was accelerated and close to the exponential one, $|\dot{H}| \ll H^2$.

2. The origin of all inhomogeneities in the present Universe is the effect of gravitational creation of pairs of particles - antiparticles and field fluctuations during inflation from the adiabatic vacuum (no-particle) state for Fourier modes covering all observable range of scales (and possibly somewhat beyond).

Remark regarding these initial conditions for perturbations: they are *not* in the Bunch-Davies state in the rigorous sense, since they may not be imposed for arbitrary large scales. As a consequence, inflationary models typically does *not* predict regular behaviour at spatial infinity both during and after inflation ("multiverse").

Existing analogies in other areas of physics.

1. The present dark energy, though the required degree of metastability for the primordial dark energy is much more than is proved for the present one (more than 60 e-folds vs. ~ 3).
2. Creation of electrons and positrons in an external electric field.

Outcome of inflation

In the super-Hubble regime ($k \ll aH$) in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\mathcal{R}(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0,\ \ g^{(a)}_{,l}\, e_m^{l(a)} = 0,\ \ e_{lm}^{(a)}\, e^{lm(a)} = 1$$

$\mathcal{R}$ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

The most important quantities:

$$n_s(k) - 1 \equiv \frac{d \ln P_{\mathcal{R}}(k)}{d \ln k}, \quad r(k) \equiv \frac{P_g}{P_{\mathcal{R}}}$$

In fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in $\mathcal{R},\ g$).

In particular:

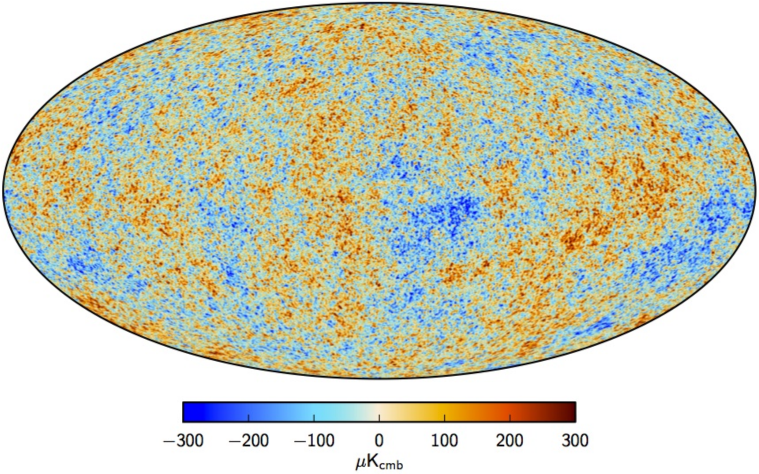
$$\hat{\mathcal{R}}_k = \mathcal{R}_k\, i(\hat{a}_{\mathbf{k}} - \hat{a}^\dagger_{\mathbf{k}}) + \mathcal{O}\left((\hat{a}_{\mathbf{k}} - \hat{a}^\dagger_{\mathbf{k}})^2\right) + ... + \mathcal{O}(10^{-100})(\hat{a}_{\mathbf{k}} + \hat{a}^\dagger_{\mathbf{k}}) + ,,,$$

The last term is time dependent, it is affected by physical decoherence and may become larger, but not as large as the second term.

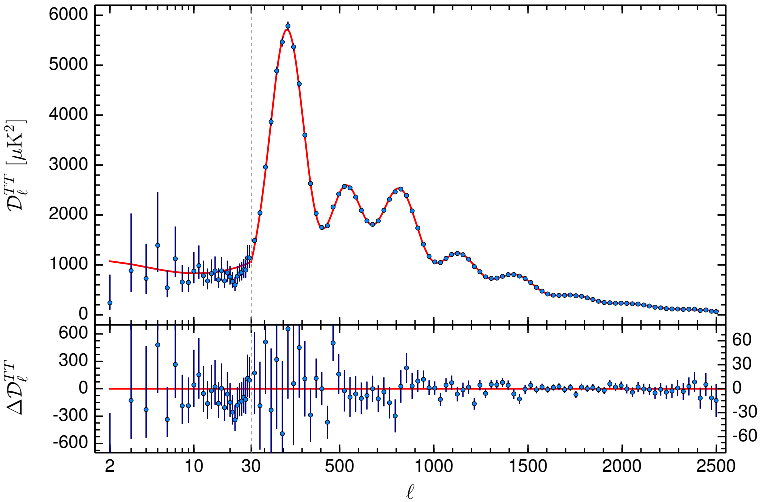
Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).

CMB temperature anisotropy

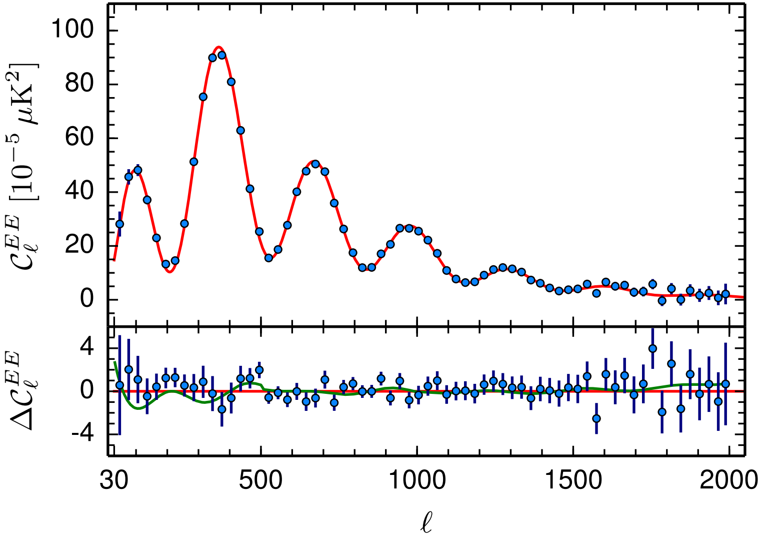
Planck-2015: P. A. R. Ade et al., arXiv:1502.01589



CMB temperature anisotropy multipoles



CMB E-mode polarization multipoles



New cosmological parameters relevant to inflation

Now we have numbers: N. Agranim et al., arXiv:1807.06209

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N_H^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$< \mathcal{R}^2(\mathbf{r}) >= \int \frac{P_{\mathcal{R}}(k)}{k}\, dk, \;\; P_{\mathcal{R}}(k) = (2.10 \pm 0.03) \cdot 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \ \mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.035 \pm 0.004$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$, relating it finally to $N_H = \ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$. (note that $(1 - n_s) N_H \sim 2$).

The most recent upper limit on r

BICEP/Keck Collaboration: P. A. R. Ade et al., Phys. Rev. Lett. 127, 151301 (2021); arXiv:2110.00483:

$r_{\,0.05} < 0.036$ at the 95% C.L.

For comparison, in the chaotic inflationary model $V(\varphi) \propto |\varphi|^n$, $r = \frac{4n}{N}$, $1 - n_s = \frac{n+2}{2N}$. The r upper bound gives $n \lesssim 0.5$ for $N_{0.05} = (55 - 60)$, but then $1 - n_s \leq 0.023$. Thus, this model is disfavoured by observational data.

The simplest models producing the observed scalar slope

1. The $R + R^2$ model (Starobinsky 1980):

$$\mathcal{L} = \frac{f(R)}{16\pi G}, \quad f(R) = R + \frac{R^2}{6M^2}, \quad M_{\rm Pl}^2 = G^{-1}$$

$$M = 2.6 \times 10^{-6} \left(\frac{55}{N}\right) M_{\rm Pl} \approx 3.1 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \;\; r = \frac{12}{N^2} = 3(1-n_s)^2 \approx 0.004, \;\; n_t = -\frac{r}{8}$$

$$N = \ln\frac{k_f}{k} = \ln\frac{T_\gamma}{k} - \mathcal{O}(10), \quad H_{dS}(N = 55) = 1.4 \times 10^{14}\,\mathrm{GeV}$$

2. The same prediction from a scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0, \; |\xi| \gg 1$ (Spokoiny 1984), including the Higgs inflationary model (Bezrukov & Shaposhnikov 2008).

The simplest purely geometrical inflationary model

$$\mathcal{L} = \frac{R}{16\pi G} + \frac{N^2}{288\pi^2 P_{\mathcal{R}}(k)} R^2 + \text{(small rad. corr.)}$$

$$= \frac{R}{16\pi G} + 5.1 \times 10^8\, R^2 + \text{(small rad. corr.)}$$

The quantum effect of creation of particles and field fluctuations works twice in this model:

a) at super-Hubble scales during inflation, to generate space-time metric fluctuations;

b) at small scales after inflation, to provide scalaron decay into pairs of matter particles and antiparticles (AS, 1980, 1981).

Weak dependence of the time t_r when the radiation dominated stage begins:

$$N(k) \approx N_H + \ln \frac{a_0 H_0}{k} - \frac{1}{3} \ln \frac{M_{\rm Pl}}{M} - \frac{1}{6} \ln(M_{\rm Pl} t_r)$$

Evolution of the $R + R^2$ model

1. During inflation ($H \gg M$):

$$H = \frac{M^2}{6}(t_f - t) + \frac{1}{6(t_f - t)} + ..., \quad |\dot{H}| \ll H^2$$

(for the derivation of the second term in the rhs - see A. S. Koshelev et al., JHEP 1611 (2016) 067).

2. After inflation ($H \ll M$):

$$a(t) \propto t^{2/3} \left(1 + \frac{2}{3Mt} \sin M(t - t_1)\right)$$

The most effective decay channel: into minimally coupled scalars and the longitudinal mode of vector bosons with $m \ll M$. In the first case the formula

$$\frac{1}{\sqrt{-g}}\frac{d}{dt}(\sqrt{-g}\,n_s) = \frac{R^2}{576\pi}$$

(Ya. B. Zeldovich and A. A. Starobinsky, JETP Lett. 26, 252 (1977)) can be used for simplicity, but the full integral-differential system of equations for the Bogoliubov α_k, β_k coefficients and the average EMT was in fact solved in AS (1981). For this channel of the scalaron decay:

$$\Gamma = \frac{M^3}{192\,M_{\rm Pl}^2}, \quad N(k) \approx N_H + \ln\frac{a_0 H_0}{k} - \frac{5}{6}\ln\frac{M_{\rm Pl}}{M}$$

that gives $N(k = 0.002\,{\rm Mpc}^{-1}) \approx 54$. For the Higgs and the mixed R^2-Higgs models, $N(k = 0.002\,{\rm Mpc}^{-1}) \approx 58$, the increase is mainly due to the large Higgs non-minimal coupling. Scalaron decay into graviton pairs is suppressed (A. A. Starobinsky, JETP Lett. 34, 438 (1981)).

Possible microscopic origins of this phenomenological model.

1. Follow the purely geometrical approach and consider it as the specific case of the fourth order gravity in 4D

$$\mathcal{L} = \frac{R}{16\pi G} + AR^2 + BC_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} + (\text{small rad. corr.})$$

for which $A \gg 1, \ A \gg |B|$. Approximate scale (dilaton) invariance and absence of ghosts in the curvature regime $A^{-2} \ll (RR)/M_P^4 \ll B^{-2}$.

One-loop quantum-gravitational corrections are small (their imaginary parts are just the predicted spectra of scalar and tensor perturbations), non-local and qualitatively have the same structure modulo logarithmic dependence on curvature.

2. Another, completely different way:

consider the $R + R^2$ model as an approximate description of GR + a non-minimally coupled scalar field with a large negative coupling ξ ($\xi_{conf} = \frac{1}{6}$) in the gravity sector:

$$\mathcal{L} = \frac{R}{16\pi G} - \frac{\xi R \phi^2}{2} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi), \quad \xi < 0, \quad |\xi| \gg 1 \ .$$

Geometrization of the scalar:

for a generic family of solutions during inflation, the scalar kinetic term can be neglected, so

$$\xi R \phi = -V'(\phi) + \mathcal{O}(|\xi|^{-1}) \ .$$

No conformal transformation, we remain in the the physical (Jordan) frame!

These solutions are the same as for $f(R)$ gravity with

$$\mathcal{L} = \frac{f(R)}{16\pi G}, \ f(R) = R - \frac{\xi R \phi^2(R)}{2} - V(\phi(R)).$$

For $V(\phi) = \frac{\lambda(\phi^2 - \phi_0^2)^2}{4}$, this just produces $f(R) = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right)$ with $M^2 = \lambda/24\pi\xi^2 G$ and $\phi^2 = |\xi|R/\lambda$.

The same theorem is valid for a multi-component scalar field.

More generally, R^2 inflation (with an arbitrary n_s, r) serves as an intermediate dynamical attractor for a large class of scalar-tensor gravity models.

Inflation in the mixed R^2-Higgs model

M. He, A. A. Starobinsky and J. Yokoyama, JCAP 1805, 064 (2018).

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right) - \frac{\xi R \chi^2}{2} + \frac{1}{2}\chi_{,\mu}\chi^{,\mu} - \frac{\lambda\chi^4}{4}, \ \ \xi < 0, \ \ |\xi| \gg 1$$

Can be conformally transformed to GR with two interacting scalar fields in the Einstein frame. The effective two scalar field potential for the dual model:

$$U = e^{-2\alpha\phi}\left(\frac{\lambda}{4}\chi^4 + \frac{M^2}{2\alpha^2}\left(e^{\alpha\phi} - 1 + \xi\kappa^2\chi^2\right)^2\right)$$

$$\alpha = \sqrt{\frac{2}{3}}\,\kappa, \quad \kappa^2 = 8\pi G, \quad R = 3M^2\left(e^{\alpha\phi} - 1 + \xi\kappa^2\chi^2\right)$$

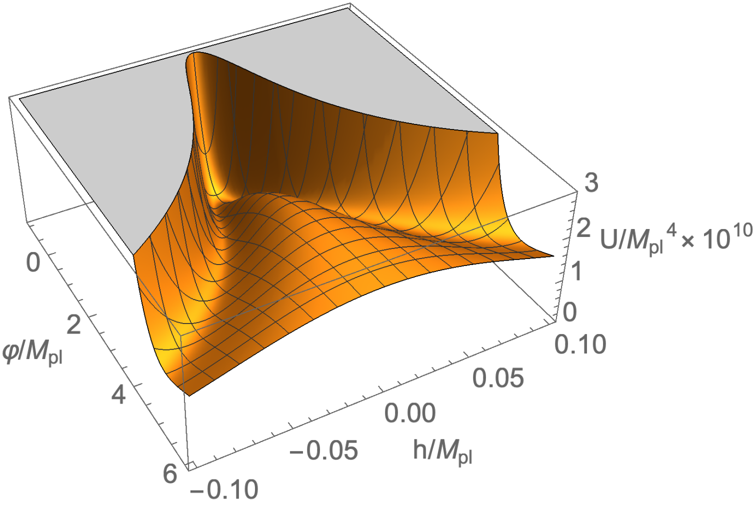
Attractiveness of the model from the quantum field theory point of view

The mixed R^2-Higgs model helps to remove some UV problems of the Higgs inflationary model and may be considered as its UV-completion up to the Planck energy if

$$\sqrt{\lambda} \lesssim \frac{|\xi| M}{M_{\rm Pl}} \lesssim 1$$

(see D. Gorbunov and A. Tokareva, Phys. Lett. B 788, 37 (2019)).

Effective potential in the Einstein frame



One-field inflation in the attractor regime

In the attractor regime during inflation:

$$\alpha\phi \gg 1, \quad \chi^2 \approx \frac{|\xi|R}{\lambda}, \quad e^{\alpha\phi} \approx \chi^2 \left(|\xi|\kappa^2 + \frac{\lambda}{3|\xi|M^2} \right)$$

that directly follows from the geometrization of the Higgs boson in the physical (Jordan) frame. Thus, we return to the $f(R) = R + \frac{R^2}{6M^2}$ model with the renormalized scalaron mass $M \to \tilde{M}$:

$$\frac{1}{\tilde{M}^2} = \frac{1}{M^2} + \frac{3\xi^2\kappa^2}{\lambda}$$

Double-field inflation reduces to the single $(R + R^2)$ one for the most of trajectories in the phase space. For $\lambda = 0.01$,

$$|\xi| \leq \xi_c \approx 4400$$

Post-inflationary heating in the mixed R^2-Higgs model through particle creation

The most effective channel of reheating though particle creation: creation of longitudinal quanta of vector bosons with $m \ll \min(M, \sqrt{\lambda}M_{\rm Pl}/|\xi|)$. More effective than in the pure R^2 model, but less effective than in the pure Higgs case.

The simplified variant - creation of NG (phase direction) quanta of a complex Higgs-like scalar field: M. He, R. Jinno, K. Kamada, S. C. Park, A. A. Starobinsky and J. Yokoyama, Phys. Lett. B 791, 36 (2019) [arXiv:1812.10099]. Inflaton decay is not instant and occurs after a large number of scalaron oscillations. However, the Higgs field introduces anharmonic effects in these oscillations.

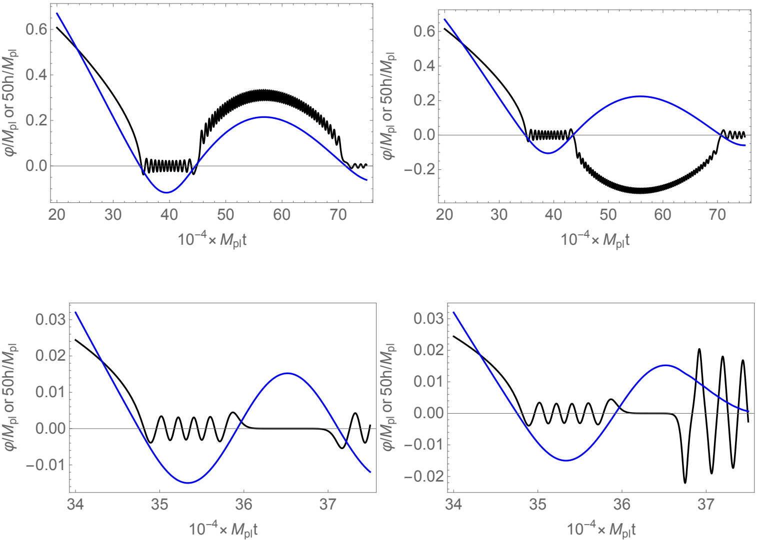
Post-inflationary heating in the mixed R^2-Higgs model through tachyonic preheating

M. He, R. Jinno, K. Kamada, A. A. Starobinsky and J. Yokoyama, JCAP 2101 (2021) 066 [arXiv:2007.10369]

Another mechanism of rapid creation and heating of matter after inflation: tachyonic instability of the Higgs field leading to formation of quasi-classical matter inhomogeneity. It arises when the background Higgs field stays long near $\chi = 0$ in the regime $\phi > 0$.

Occurs not for all values of parameters M, ξ and requires some fine-tuning of them to be efficient: at least $< \mathcal{O}(0.1)$ in the deep Higgs-like regime with a large scalaron mass, while more severe fine-tuning $\sim \mathcal{O}(10^{-4}) - \mathcal{O}(10^{-5})$ is needed in the R^2-like regime with a small non-minimal coupling.

Stochastic behaviour of ϕ, R and χ in this regime – stochastic reheating.



Upper left: $\xi = 4000$. Upper right: $\xi = 4100$. Lower left: $\xi = 4435.759104801013$. Lower right: $\xi = 4435.7591048$. All the digits shown above are needed.

Perturbative reheating the mixed R^2-Higgs model

If the tachyonic preheating does not occur, the heating stage ends in the perturbative regime, like in the $R + R^2$ model. However, it is still sufficiently fast due to $|\xi| \gg 1$. The scalaron-Higgs decay rate is now

$$\Gamma = \frac{M^3(1-6\xi)^2}{192\, M_{\rm Pl}^2}\, .$$

As a result, numerical calculations in M. He, JCAP 2105 (2021) 021 show that $T_{reh} = (10^{13} - 10^{14})$ GeV weakly depending on ξ for $|\xi| \gg 1$.

Perspectives of future discoveries

- Primordial gravitational waves from inflation: r.
  $r \lesssim 8(1-n_s) \approx 0.3$ (confirmed!) but may be much less. However, under reasonable assumptions one may expect that $r \gtrsim (n_s-1)^2 \approx 10^{-3}$. The target prediction in the simplest (one-parametric) models is
  $r = 3(n_s-1)^2 \approx 0.004$.
- A more precise measurement of $n_s - 1 \Longrightarrow$ duration of transition from inflation to the radiation dominated stage $\Longrightarrow$ information on inflaton (scalaron) couplings to known elementary particles at super-high energies $E \lesssim 10^{13}\ \mathrm{Gev}$.
- Local non-smooth features in the scalar power spectrum at cosmological scales (?).
- Local enhancement of the power spectrum at small scales leading to a significant amount of primordial black holes (?).

Generating peaks and troughs in the primordial scalar spectrum

To obtain large peaks and troughs in $P_{\mathcal{R}}$, temporal breaking of the slow-roll approximation during inflation is needed. The simplest way: fast break in the first derivative of the inflaton potential $V(\phi)$ (A. A. Starobinsky, JETP Lett. 55, 489 (1992)). Leads to a step in $P_{\mathcal{R}}$ with superimposed oscillations. To obtain a peak, two such features with opposite signs, or a fast break in the $V(\phi)$ itself are needed (so that an inflection point appears in between). However, it is not sufficient to have an inflection point only, it should be combined with a strong breaking of the slow-roll conditions.

Let $V(\phi) = V_0 + A_+\phi\,\theta(\phi - \phi_0) + A_-\phi\,\theta(\phi_0 - \phi)$ for ϕ close to ϕ_0. Then

$$\dot{\phi} = -\frac{A_+}{3H_0}\,\theta(-t) - \frac{A_- + (A_+ - A_-)e^{-3H_0 t}}{3H_0}\,\theta(t)$$

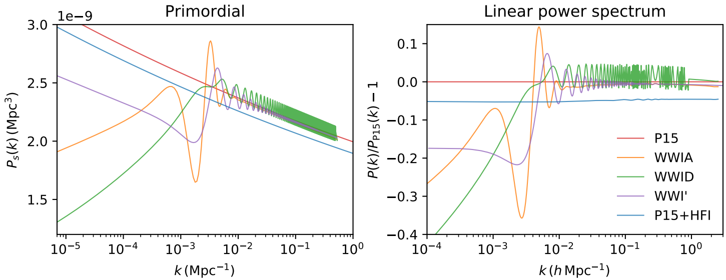
The slow-roll spectrum $P_{\mathcal{R}}$ is modulated by the multiplier

$$D^2 = 1 - 3\left(\frac{A_-}{A_+} - 1\right)\left[\left(1 - \frac{1}{y^2}\right)\sin 2y + \frac{2}{y}\cos 2y\right] +$$

$$\frac{9}{2}\left(\frac{A_-}{A_+} - 1\right)^2 \frac{1}{y^2}\left(1 + \frac{1}{y^2}\right) \times$$

$$\left[1 + \frac{1}{y^2} + \left(1 - \frac{1}{y^2}\right)\cos 2y - \frac{2}{y}\sin 2y\right],$$

$$y = \frac{k}{k_0},\ D(0) = \frac{A_-}{A_+},\ D(\infty) = 1$$



Primordial
1e−9
3.0
2.5
2.0
1.5
$P_s(k)\,(\mathrm{Mpc}^3)$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
$k\,(\mathrm{Mpc}^{-1})$
Linear power spectrum
0.1
0.0
−0.1
−0.2
−0.3
−0.4
$P(k)/P_{\mathrm{P15}}(k) - 1$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
$k\,(h\,\mathrm{Mpc}^{-1})$
P15
WWIA
WWID
WWI'
P15+HFI

Non-scale-free features at cosmological scales

The most recent analysis of this type of spectra with power suppression at large scales (D. K. Hazra, D. Paoletti, I. Debono, A. Shafieloo, G. F. Smoot, A. A. Starobinsky, arXiv:2107.09460) using the CMB temperature and polarization data from the Planck 2018 data release shows marginal (68% C.L.) preference of suppression from the large scale temperature angular power spectrum. However, the large-scale E-mode likelihood does not support this suppression and in the combined data the preference towards the suppression becomes negligible. For models with oscillatory features along with the suppression, unbinned data from the recently released CamSpec 12.5 likelihood was used which updates Planck 2018 results. Comparison of the Bayesian evidences of the feature models with their baseline slow-roll inflaton potentials showed that the latter are moderately preferred against potentials with features.

Conclusions

- The typical inflationary predictions that $|n_s - 1|$ is small and of the order of N_H^{-1}, and that r does not exceed $\sim 8(1 - n_s)$ are confirmed. Typical consequences following without assuming additional small parameters: $H_{55} \sim 10^{14}\,\mathrm{GeV}, \; m_{infl} \sim 10^{13}\,\mathrm{GeV}$.
- In $f(R)$ gravity, the simplest $R + R^2$ model is one-parametric and has the preferred values $n_s - 1 = -\frac{2}{N}$ and $r = \frac{12}{N^2} = 3(n_s - 1)^2 \approx 0.004$. The first value produces the best fit to present observational CMB data. The same prediction follows for the Higgs and the mixed R^2-Higgs models, though actual values of $N(k)$ are slightly different for these 3 cases.
- Inflation in $f(R)$ gravity represents a dynamical attractor for slow-rolling scalar fields strongly coupled to gravity . As a result, double field inflation in the mixed R^2-Higgs model reduces to the single R^2-like inflation for a dense set of the most interesting trajectories in the phase space

- The mixed R^2-Higgs model helps to remove some UV problems of the Higgs inflationary model and may be considered as its UV-completion up to the Planck energy.
- The rate of post-inflationary heating though particle creation in the mixed R^2-Higgs model is intermediate between those in the Higgs and $R + R^2$ models. Generically inflaton (scalaron) decay is not instant and occurs after a large number of its oscillations. However, the reheating temperature is high, $T_{reh} = (10^{13} - 10^{14})$ GeV, mainly due to a large non-minimal Higgs coupling to gravity.
- In some fine-tuned sub-regions of parameters of the mixed R^2-Higgs model, more rapid preheating through tachyonic instability of the Higgs field becomes possible.

- As for local non-scale-free features at cosmological scales, the present CMB temperature anisotropy and polarization data do not favor them, but are not able to exclude them completely.

CONGRATULATIONS AND BEST WISHES

TO THE MAGNIFICENT SEVEN !