Loops and Expansions

Advances in Quantum Field Theory – Dubna, October 11-14, 2021 – via Zoom

Matthias Steinhauser | 14. October 2021

TTP KIT

Loops and Expansions

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Russian Loops


ADVANCES
IN QUANTUM
FIELD
THEORY
2021

Kostja: 39 publications
Volodya: 43 publications

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Volodya: 43 publications

1 paper where all three of us are co-authors

Kostja

Loops – conventions – automatization

Karlsruhe Institute of Technology

- 1992,1993: inofficial PhD supervisor
- “most correct QCD Feynman rules”

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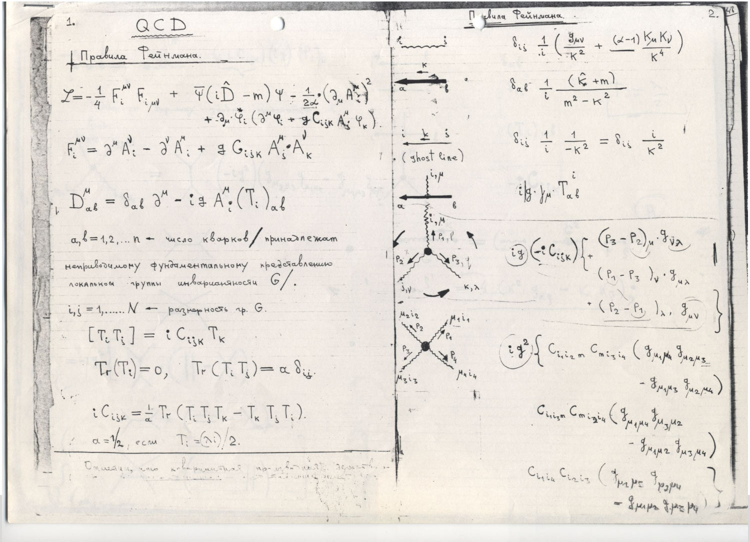

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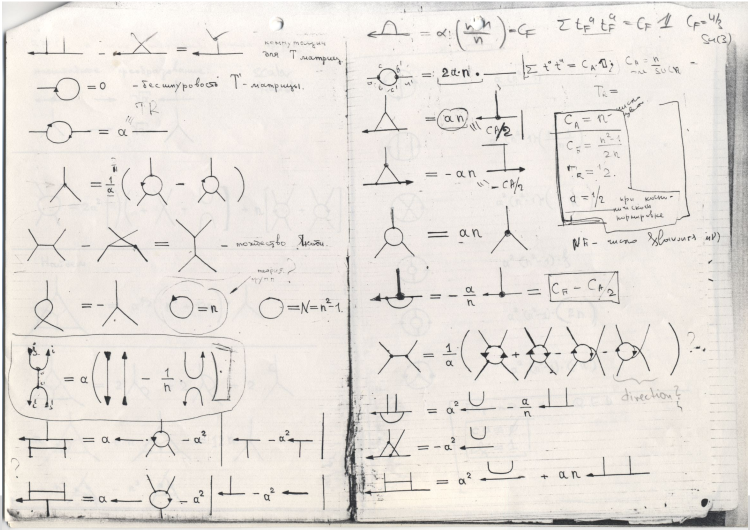

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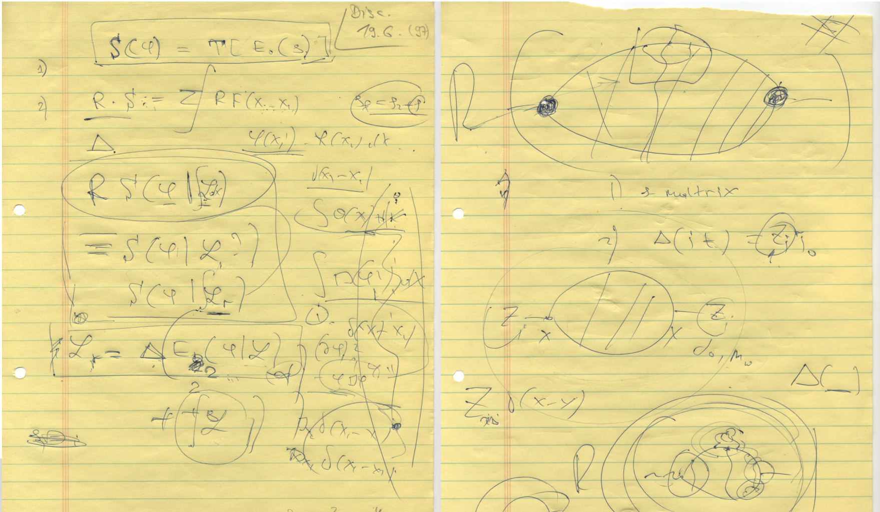

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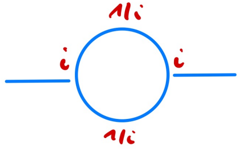
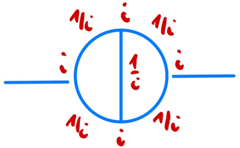


$$i \underbrace{\frac{1}{i}\int dp}_{\to\, dp^E} \longrightarrow i \int dp^E$$



$$\underbrace{i \frac{1}{i}\frac{1}{i}\int dp_1\, dp_2} \longrightarrow i \int dp_1^E\, dp_2^E$$

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GEFICOM

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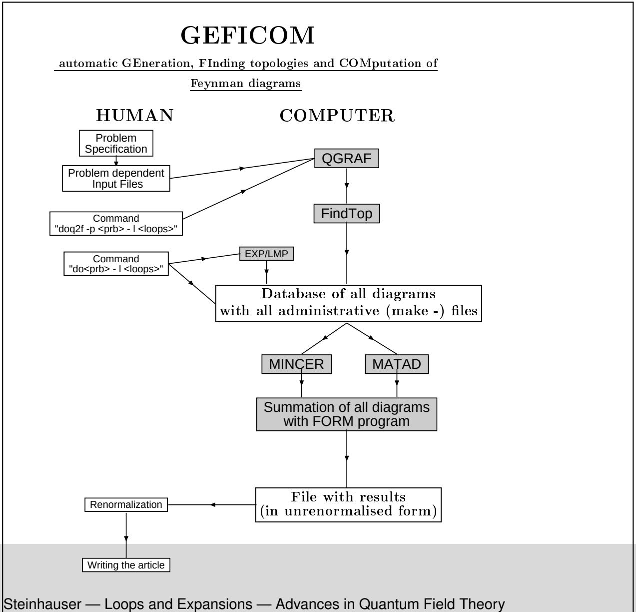

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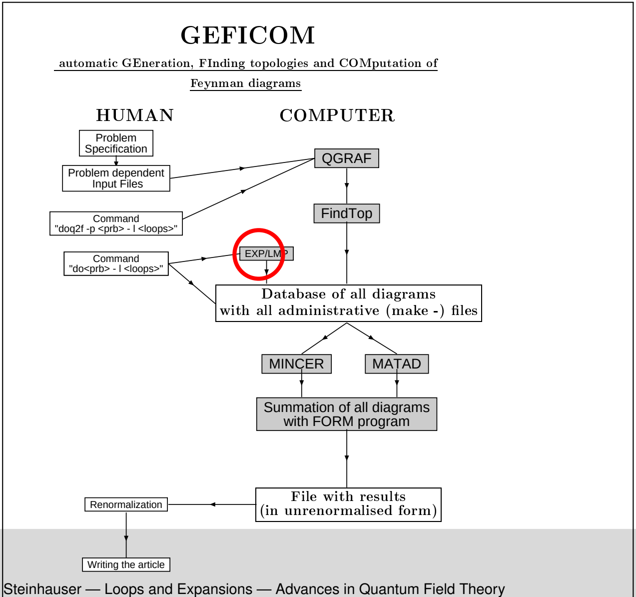
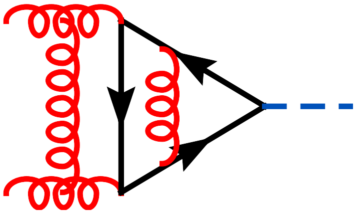


First application: gluon-Higgs vertex

[Chetyrkin,Kniehl,Steinhauser'98]



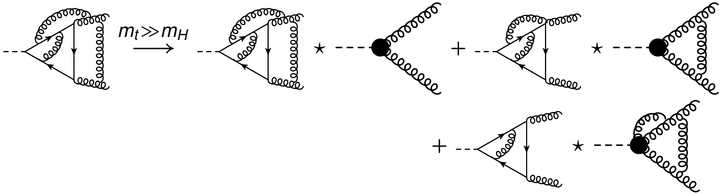
- $\mathcal{L}_{\text{eff}} = -\frac{H}{v} C_H (G_{\mu\nu})^2$
- 3-loop tadpoles
- "many" Feynman diagrams
- LET: $C_H = -\frac{1}{\zeta_{\alpha_s}} \frac{\partial}{\partial \log m_t} \zeta_{\alpha_s}$

➪ Gluon-Higgs coupling to 4 loops, N^3LO in 1998

Asymptotic expansion

Karlsruhe Institute of Technology

[Davies,Herren,Steinhauser'19]...

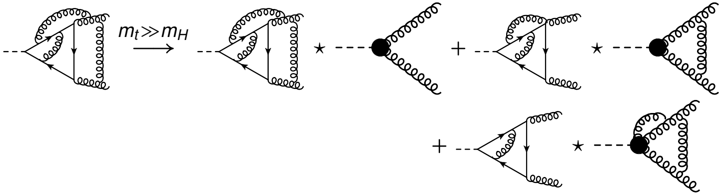


- automated nested Euclidian asymptotic expansion: `exp`

[Harlander,Seidensticker,Steinhauser'97;Seidensticker'97]

Asymptotic expansion

[Davies,Herren,Steinhauser'19]...

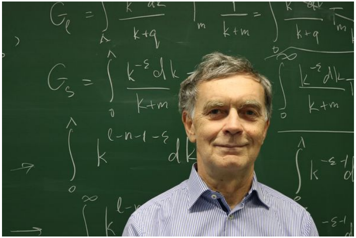


- automated nested Euclidian asymptotic expansion: `exp`

[Harlander,Seidensticker,Steinhauser'97;Seidensticker'97]

Without Kostja — no `exp`

Volodya



Loops – expansions – more loops

- $>$ 2000 Hamburg
- N^3LO Hamiltonian for Potential NRQCD [Kniehl,Penin,Smirnov,Steinhauser'02]
  ...and many applications
- Mercator professorship

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- > 2000 Hamburg
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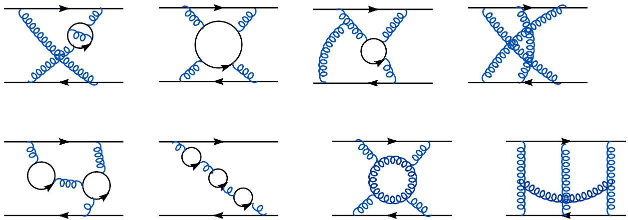
- Mercator professorship
- > 2004 Karlsruhe
- annual visits
- Static potential: 2008,. . . ,2009,. . . ,2016 [Lee,Smirnov,Smirnov,Steinhauser]



Loops – expansions – more loops

- 2013: Forschungspreis der Alexander von Humboldt-Stiftung (KIT + Humboldt University Berlin)

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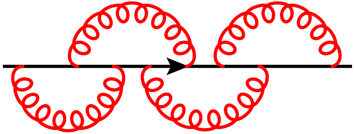

- 4-loop $\overline{\mathrm{MS}}$-on-shell relation

[Marquard,Smirnov,Smirnov,Steinhauser,Wellmann'15'16]

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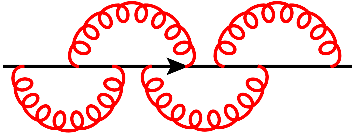
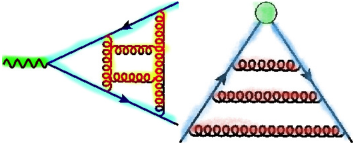


- 4-loop $\overline{\mathrm{MS}}$-on-shell relation

[Marquard,Smirnov,Smirnov,Steinhauser,Wellmann'15'16]

- massive 3-loop form factor
- massless 4-loop form factor



[Henn,Lee,Smirnov,Smirnov,Steinhauser,'16,. . . , + von Manteuffel,Schabinger'21]

4-loop form factor

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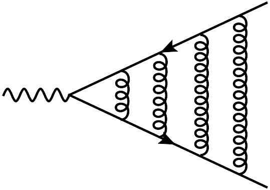
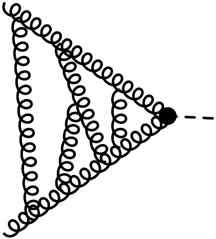


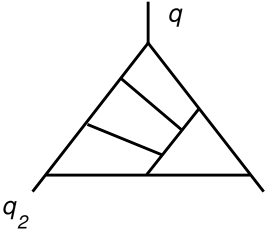


[. . . ; Henn,Smirnov,Smirnov,Steinhauser'16; von Manteuffel,Schabinger'16'19; Henn,Lee,Smirnov,Smirnov,Steinhauser'16;

Lee,Smirnov,Smirnov,Steinhauser'17'19; von Manteuffel,Panzer,Schabinger'20;

Lee,von Manteuffel,Schabinger,Smirnov,Smirnov,Steinhauser'21; . . .]

4-loop form factor



- reduction: FIRE [A. Smirnov], LiteRed [Lee]
- MIs: $q^2 \neq 0$ and $q_2^2 = (q_2 + q)^2 = 0$
- integrals simple if $q_2^2 = q^2$

- idea: introduce arbitray q_2^2 ➪ differential equations [Henn,Smirnov,Smirnov'14]
- boundary conditions: $q_2^2 = q^2$
- use canonical basis [Henn'13'14] [Lee'14] [Gituliar,Magerya'16; Meyer'16; Prausa'17]
  solution: iterated integrals ➪ harmonic polylogarithms (HPLs) [Remiddi,Vermaseren'99][Maitre'05]
- $q_2^2 \to 0$: $\left(q_2^2/q^2\right)^{a\epsilon}$ extract term with $a = 0$

4-loop form factor ... more details

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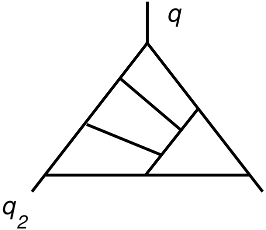
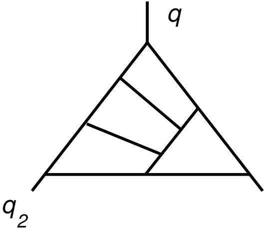


➪ 76 MIs

$q_2^2 \neq 0$ ➪ 332 MIs $\qquad x = q_2^2/q^2$

4-loop form factor ... more details



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primary basis

canonical basis

$$f(x,\epsilon) \xrightarrow{f = T\cdot g \text{ [Lee'14]}} \boxed{\begin{array}{l} g'(x,\epsilon) = \epsilon\, A(x)\cdot g(x,\epsilon) \\ A(x) = \frac{a}{x} + \frac{b}{x-1} \end{array}}$$

4-loop form factor ... more details

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$\downarrow$

solve in terms of HPLs

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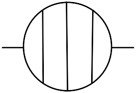
boundary conditions for $x = 1$:



4-loop

2-point functions

[Baikov,Chetyrkin,Kühn'05'08;. . . ; Lee,Smirnov,Smirnov'11]

4-loop form factor ... more details

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↓

solve in terms of HPLs
boundary conditions for $x = 1$:

↓

get $x = 0$ result ("naive"):

1. $g_{x\to 0} = x^{\epsilon a} h(\epsilon)$ a: 332×332 matrix

➪ $x^{\epsilon a}$ is 332×332 matrix; each element is linear combination of $x^{k\epsilon}$ terms with $k \leq 0$

4-loop form factor ... more details

primary basis

canonical basis

$f(x,\epsilon)$ $\xrightarrow{f=T\cdot g \text{ [Lee'14]}}$ $g(x,\epsilon)$

$\downarrow$

solve in terms of HPLs
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get $x = 0$ result ("naive"):

1. $g_{x\to 0} = x^{\epsilon a} h(\epsilon)$
2. expand HPLs for $x \to 0$

match 1. and 2.

➪ $h(\epsilon)$

➪ get $x^{0\epsilon}$ terms $\widehat{=}$ "naive"

4-loop form factor ... more details

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$f(x, \epsilon)$ $\xrightarrow{f=T\cdot g \text{ [Lee'14]}}$ $g(x, \epsilon)$

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$f(0, \epsilon)$ $\xleftarrow{f=T\cdot g}$

$\downarrow$

332 MIs

$\downarrow$

76 MIs

$$
\begin{aligned}
F_q^{(4)}\Big|_{\epsilon^0} = {}& n_f C_F^3 \Big(\frac{1153615}{126}\zeta_7 + \frac{6316}{9}\zeta_5\zeta_2 + \frac{229468}{135}\zeta_3\zeta_2^2 + \frac{547270}{81}\zeta_3^2 + \frac{1341628}{2835}\zeta_2^3 - \frac{3467995}{162}\zeta_5 - \frac{192737}{81}\zeta_3\zeta_2 - \frac{1420133}{1215}\zeta_2^2 \\
& - \frac{38482147}{972}\zeta_3 + \frac{12734681}{648}\zeta_2 + \frac{7837713013}{419904} \Big) + n_f C_A C_F^2 \Big(\frac{5669}{4}\zeta_7 - \frac{249194}{135}\zeta_5\zeta_2 - \frac{417244}{405}\zeta_3\zeta_2^2 - \frac{11438080}{729}\zeta_3^2 \\
& - \frac{38510}{63}\zeta_2^3 + \frac{4959127}{243}\zeta_5 + \frac{408107}{81}\zeta_3\zeta_2 + \frac{8388679}{2916}\zeta_2^2 + \frac{2642551543}{26244}\zeta_3 - \frac{111491363}{2187}\zeta_2 - \frac{326984889779}{3779136} \Big) \\
& + n_f C_A^2 C_F \Big(\frac{6943}{24}\zeta_7 + \frac{2755}{9}\zeta_5\zeta_2 + \frac{1912}{45}\zeta_3\zeta_2^2 + \frac{202210}{27}\zeta_3^2 + \frac{128953}{1620}\zeta_2^3 - \frac{34844257}{3240}\zeta_5 - \frac{119489}{108}\zeta_3\zeta_2 - \frac{1251893}{540}\zeta_2^2 \\
& - \frac{111467677}{1944}\zeta_3 + \frac{123861583}{3888}\zeta_2 + \frac{21075909203}{279936} \Big) + n_f \frac{d_F^{abcd} d_F^{abcd}}{N_F} \Big(-1240\zeta_7 + \frac{992}{3}\zeta_5\zeta_2 - \frac{3952}{15}\zeta_3\zeta_2^2 + \frac{680}{9}\zeta_3^2 + \frac{41620}{189}\zeta_2^3 \\
& + \frac{95098}{27}\zeta_5 + \frac{92}{3}\zeta_3\zeta_2 + \frac{7552}{45}\zeta_2^2 - \frac{21566}{9}\zeta_2 - \frac{13414}{27}\zeta_3 + \frac{3190}{3} \Big) + n_f^2 C_F^2 \Big(\frac{689582}{729}\zeta_3^2 + \frac{191252}{945}\zeta_2^3 + \frac{187364}{1215}\zeta_5 \\
& + \frac{177800}{729}\zeta_3\zeta_2 + \frac{4777}{135}\zeta_2^2 - \frac{90719803}{13122}\zeta_3 + \frac{44208841}{13122}\zeta_2 + \frac{5325319081}{944784} \Big) + n_f^2 C_A C_F \Big(-\frac{1714}{3}\zeta_3^2 - \frac{2836}{315}\zeta_2^3 + \frac{150886}{135}\zeta_5 \\
& - \frac{436}{3}\zeta_3\zeta_2 - \frac{3722}{135}\zeta_2^2 + \frac{2897315}{486}\zeta_3 - \frac{5825827}{1296}\zeta_2 - \frac{3325501813}{279936} \Big) + n_f^3 C_F \Big(-\frac{2194}{135}\zeta_5 - \frac{820}{81}\zeta_3\zeta_2 + \frac{3322}{135}\zeta_2^2 - \frac{20828}{243}\zeta_3 \\
& + \frac{145115}{729}\zeta_2 + \frac{10739263}{17496} \Big) + n_{q\gamma} \frac{d_F^{abc} d_F^{abc} C_F}{N_F} \Big(26624\zeta_7 + 1792\zeta_5\zeta_2 + \frac{35584}{15}\zeta_3\zeta_2^2 + \frac{30688}{9}\zeta_3^2 - \frac{179152}{189}\zeta_2^3 - \frac{170224}{27}\zeta_5 \\
& - \frac{8656}{3}\zeta_3\zeta_2 + \frac{60416}{45}\zeta_2^2 - \frac{100624}{9}\zeta_2 - \frac{71552}{27}\zeta_3 - \frac{89360}{9} \Big) + n_{q\gamma} \frac{d_F^{abc} d_F^{abc} C_A}{N_F} \Big(-\frac{13972}{3}\zeta_7 - 1840\zeta_5\zeta_2 - \frac{784}{5}\zeta_3\zeta_2^2 \\
& - 13008\zeta_3^2 - \frac{618328}{189}\zeta_2^3 + \frac{54436}{9}\zeta_5 + \frac{15112}{3}\zeta_3\zeta_2 - \frac{95692}{45}\zeta_2^2 + \frac{45976}{9}\zeta_3 + \frac{107984}{9}\zeta_2 + \frac{65264}{9} \Big) + n_{q\gamma} n_f \frac{d_F^{abc} d_F^{abc}}{N_F} \Big(2304\zeta_3^2 \\
& + \frac{545792}{945}\zeta_2^3 - \frac{8512}{9}\zeta_5 - \frac{1888}{3}\zeta_3\zeta_2 + \frac{12448}{45}\zeta_2^2 - \frac{3520}{9}\zeta_3 - \frac{16928}{9}\zeta_2 - \frac{11296}{9} \Big)
\end{aligned}
$$

+ contributions without closed fermion loop.

[Lee,von Manteuffel,Schabinger,Smirnov,Smirnov,Steinhauser'21]

Congratulations