Series summation and optical theorem

October 14, 2021

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Series and analyticity

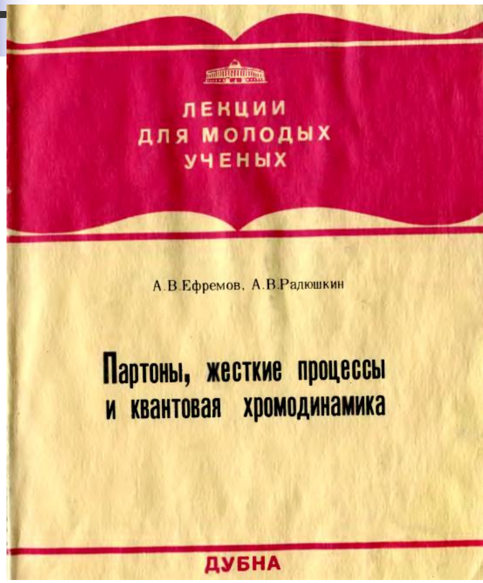
- Analytic properties in coupling complex plane: generic
- This talk: The particular case of the expansion over kinematic variables with the singularities fixed by unitarity and (generalized) optical theorem
- Relation to the work of and lessons from 7 heroes of the day (Happy Birthdays!!)

Main topics

- Parton distributions: from DIS to DVCS
- Analytic QCD coupling and (resummed) higher twists

- Analytic properties of graviton propagator
- Polarization and density matrix positivity

Back to 80's: QCD factorization



$$:\bar\psi(-\tfrac{x}{2})\Gamma\psi(\tfrac{x}{2}): = \sum_{n=0}^{\infty} :\bar\psi(0)\Gamma\overleftrightarrow{\partial}_{\mu_1}\ldots\overleftrightarrow{\partial}_{\mu_n}\psi(0): \frac{x^{\mu_1}\ldots x^{\mu_n}}{n!}, \quad (3.2)$$

проверяется по индукции

мы получаем прототип операторного разложения.

Затем, используя формулу

$$\langle P|:\bar\psi(0)\Gamma\partial_{\mu_1}\ldots\partial_{\mu_n}\psi(0):|P\rangle = a_n P_{\mu_1}\ldots P_{\mu_n} + (\text{свертки типа } p^2 g_{\mu_i\mu_j}\ldots) \quad (3.3)$$

и отождествляя коэффициенты a_n с моментами функций распределения партонов

$$a_n = \int_0^1 \frac{d\beta}{\beta}\beta^n f(\beta), \quad (3.4)$$

получаем обычную партонную формулу

$$T(P,q) = \int_0^1 \frac{d\beta}{\beta} t(q,\beta p) f(\beta). \quad (3.5)$$

RIVISTA DEL NUOVO CIMENTO VOL. 3, N. 2 1980

Hard Processes, Parton Model and QCD.

A. V. Efremov and A. V. Radyushkin

Laboratory of Theoretical Physics
Joint Institute for Nuclear Research - Dubna, USSR

(ricevuto il 6 Giugno 1979)

$$(3.2)\qquad :\bar\psi\left(-\frac{x}{2}\right)\Gamma\psi\left(\frac{x}{2}\right): = \sum_{n=0}^{\infty} :\bar\psi(0)\Gamma\overleftrightarrow{\partial}_{\mu_1}\ldots\overleftrightarrow{\partial}_{\mu_n}\psi(0): \frac{x^{\mu_1}\ldots x^{\mu_n}}{n!}$$

we obtain a prototype of the OPE.

Then, defining

$$(3.3)\qquad \langle P|:\bar\psi(0)\gamma_{\mu_1}\partial_{\mu_2}\ldots\partial_{\mu_n}\psi(0):|P\rangle = a_n P_{\mu_1}\ldots P_{\mu_n} + \text{terms containing } P^2 g_{\mu_i\mu_j}$$

and treating the coefficients a_n as the moments of parton distribution functions

$$(3.4)\qquad a_n = \int_0^1 \frac{d\xi}{\xi} f(\xi)\xi^n,$$

we obtain the standard parton formula

$$(3.5)\qquad T(P,q) = \int_0^1 \frac{d\xi}{\xi} t(q,\xi p) f(\xi).$$

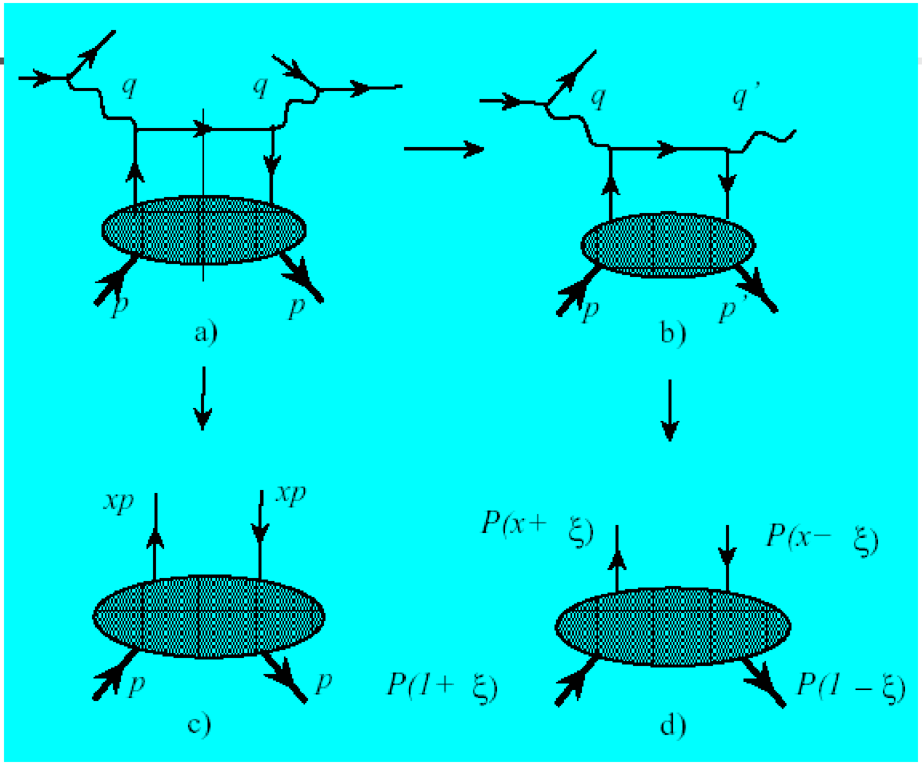
Why to "treat" series coefficients as a moments?

- Analyticity: pole in partonic Compton subprocess is transformed to cut in DIS amplitude
- Cut position is fixed by optical theorem stemming from unitarity $SS^{+}=1$, providing probability conservation
- Implies the probabilistic interpretation of parton model with momentum parton momentum fraction between 0 and 1

Factorization and analyticity

- Factorization provides analyticity
- Other proofs without series summation (EFP): separation of longitudinal and transverse momenta, analyticity is also preserved
- Generalized Parton Distributions (Mueller et al., Ji, Radyushkin; talk of S. Goloskokov): what about analyticity?

QCD Factorization for DIS and DVCS (AND VM production)



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}.$$

- Extra dependence on ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Unphysical regions

- DIS : Analytical function – polynomial in $1/x_B$ if $1 \le |x_B|$

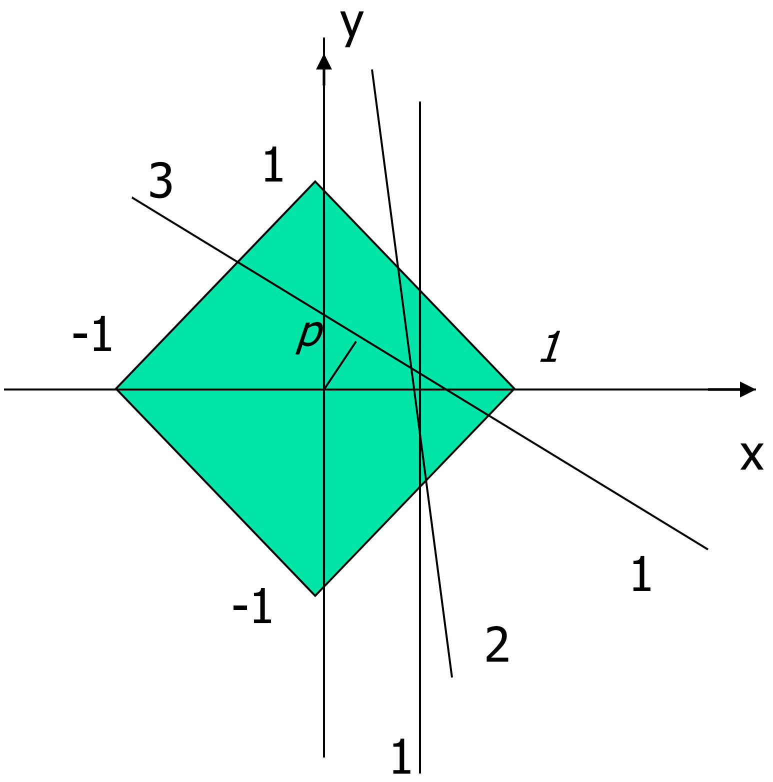
$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x,\xi)$
- Solved by using of Radyushkin's Double Distributions: Radon transform

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line-skewness
- Kinematics of DIS: $\xi = 0$
  ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$
  - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography



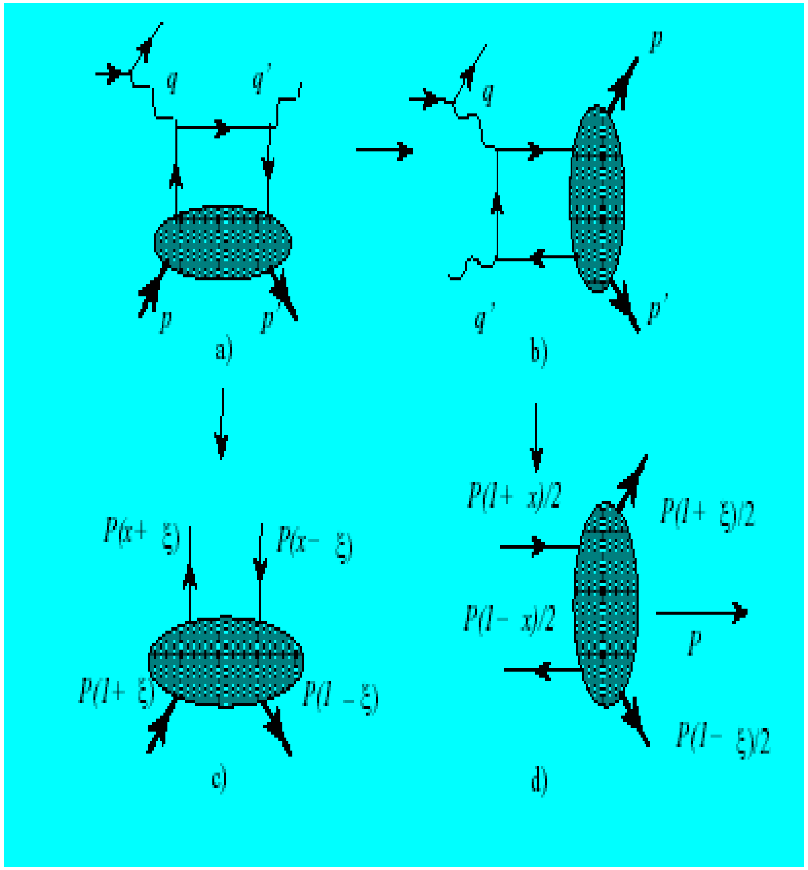
$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi|cos\phi|(H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi(H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons

- GPD -> Generalized Distribution Amplitudes



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in x weighted with x^n - are polynomials in $1/\xi$ of power *n+1*
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Holographic property (OT'05)

- Factorization Formula ->

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

- Analyticity -> Imaginary part -> Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

- "Holographic" equation (DVCS AND VM)

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y))\delta(z - x - \xi y)$$

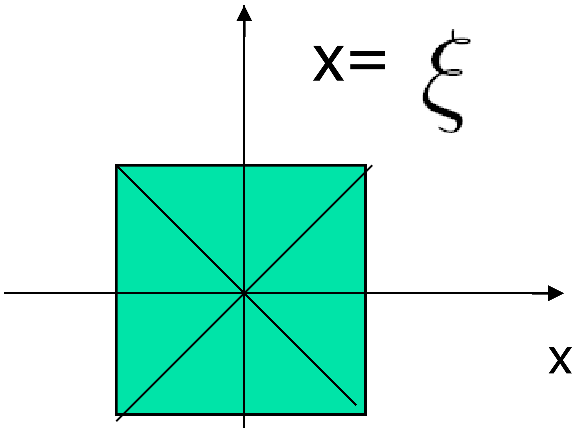
- Constant is the SUBTRATION one - due to the (generalized) M. Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

$$=-\left(\int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const \right)$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!



- Strategy (now adopted) of GPD's studies: start at diagonals (through Single Spin Asymmetry due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\mathcal{A}(\nu,Q^2)=\frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu',Q^2)}{(\nu'^2-\nu^2)}+\Delta \qquad \Delta=2\int_{-1}^{1}d\beta\,\frac{D(\beta)}{\beta-1}$$

$$\Delta^p_{\mathrm{CQM}}(2)=\Delta^n_{\mathrm{CQM}}(2)\approx 4.4, \qquad \Delta^p_{\mathrm{latt}}\approx\Delta^n_{\mathrm{latt}}\approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!

- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

Quadrupole formfactor

- D-term is related to Quadrupole gravitational FF (~ proton's "cosmological constant")

$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

- Vacuum – Cosmological Constant

$$\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$$

- Proton:

$$\Lambda = C(q^2)q^2$$

- Access: D-term in GPDs

From D-term to pressure

- *Inverse -> 1st moment (model)*
- *Kinematical factor: weighted pressure* $C \sim \langle p\, r^4 \rangle$ ($\langle p\, r^2 \rangle = 0$) M.Polyakov'03

$$T^Q_{\mu\nu}(\vec{r},\vec{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{\Delta}}\, \langle p', S'|\hat{T}^Q_{\mu\nu}(0)|p, S\rangle$$

$$T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\delta_{ij}$$

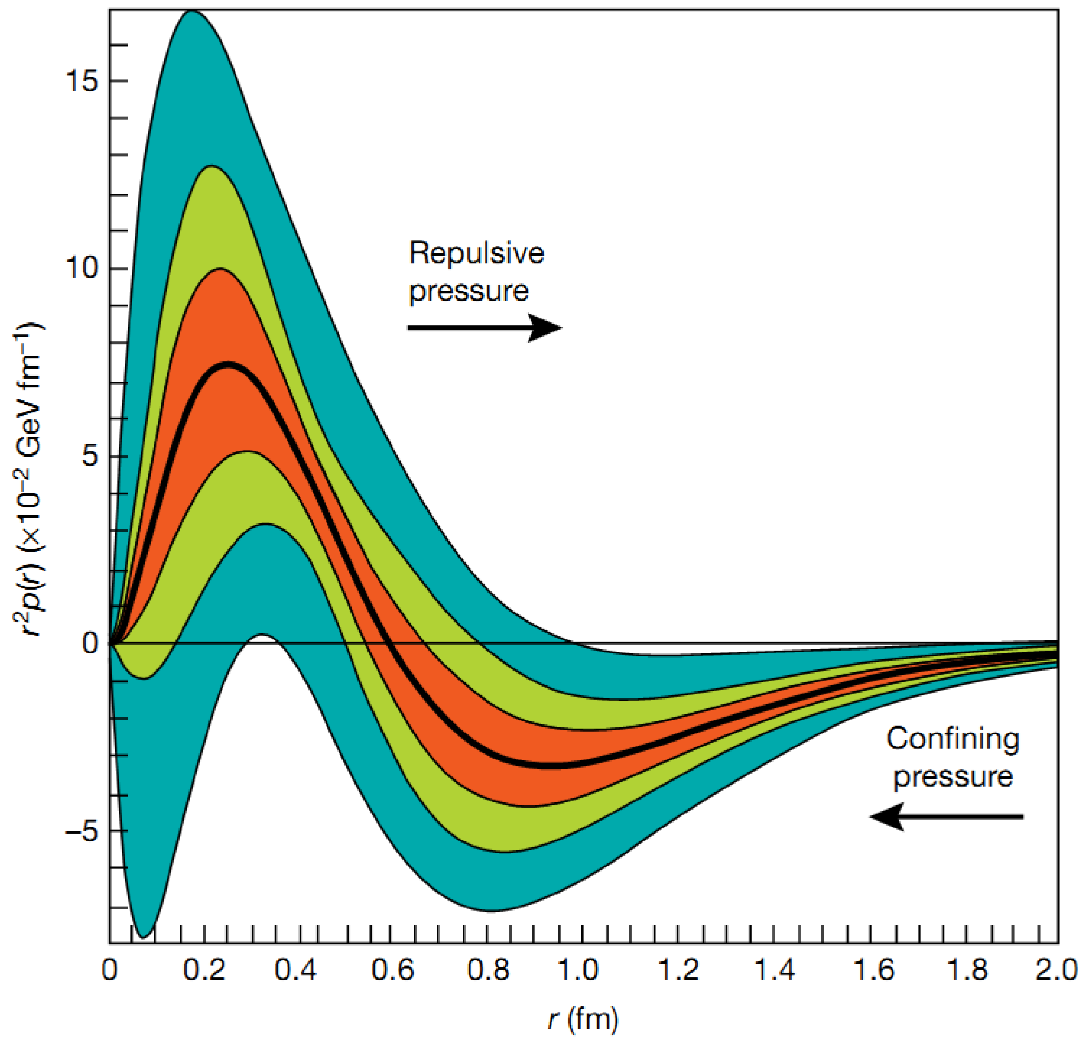
- *Justification: (Fourier inversed) consistency principle for Born gravitational scatterring? 2D<->3D?*

NATURE | www.nature.com/nature

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]



Analyticity and RG

- RG summation violates analyticity: “right” cuts lead to the “wrong” Landau pole
- QED – far UV
- QCD – IR
- Imposing of correct analytic properties: Analytic Perturbation Theory.
- **D.V. Shirkov**, I. Solovtsov, O. Solovtsova, A. Radyushkin, A. Bakulev, S. Mikhailov, N. Stefanis, N. Krasnikov, A. Pivovarov, A. Nesterenko,…

Higher Twist

Analytization is not a complete answer:
  Essentially non-perturbative
  $\sim\exp(-1/x^2)$

HT should be added, implied by LO already

$\Lambda^2 = \mu^2 \exp\left(- 4 \pi/\alpha\ (\mu^2)\ b_1\right)$

Interplay between PT and HT
  (cf Narison&Zakharov, Kataev&Parente)

Four-loop QCD analysis of the Bjorken sum rule

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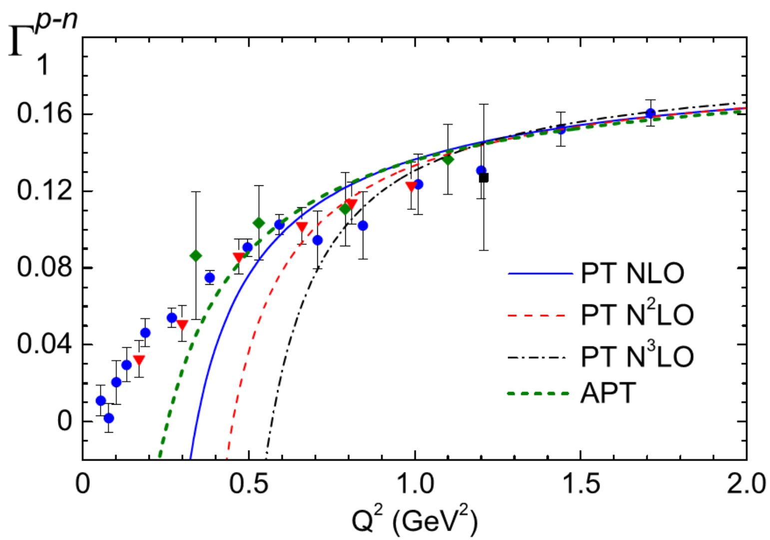
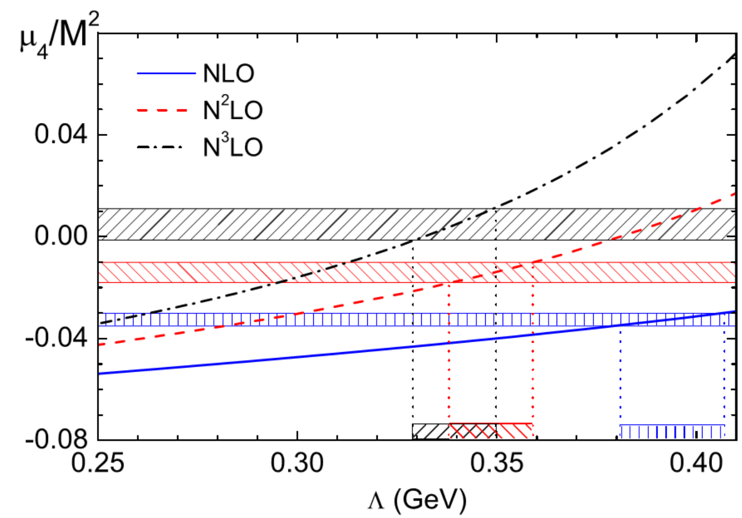
[c] Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russian Federation

Physics Letters B 706 (2012) 340–344

Bjorken SR

- PT/APT analysis based on record calculation of Baikov, Chetyrkin and Kuhn





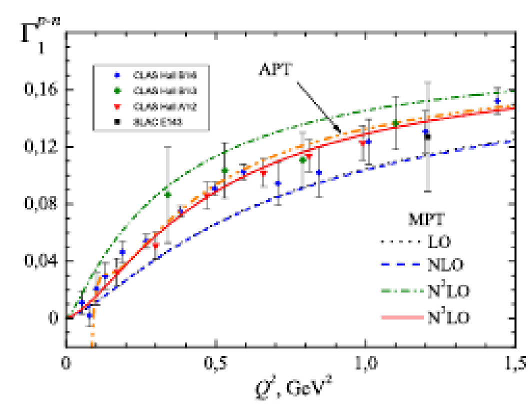
- HT decreases down to zero with PT order

Are HTs analytic?

- Infinite sum of zero-momentum poles in Q^2 may be converted to cut (OT'13)

$$S(Q^2) = \sum_{1}^{\infty} a_n \left(\frac{M^2}{Q^2}\right)^n. \qquad a_n = \int_{-\infty}^{+\infty} f(x) x^{n-1} \qquad S(Q^2) = \int_{-\infty}^{\infty} dx \frac{f(x) M^2}{Q^2 - x M^2}.$$

- Was applied to BjSR Gabdrakhmanov, Khandramai, OT'15
- APT and HT are both analytic: cancellation of Landau pole instead?



Are ChPT series analytic?

- Positive powers of Q^2 -inverse moments of distributions?
- Cf – Recursive relations for "quasirenormalizible" theories for partial waves implying t-channel uniratity and analyticity by M. Polyakov, Semenov-Tian-Shansky, Smirnov, Vladimirov
- Have similarity with relations for non-renormalizible theories directly found recently by Kazakov

Graviton propagator, renormalization scale and black-hole like states

X. Calmet[a], R. Casadio[b], A.Yu. Kamenshchik[b,c,*], O.V. Teryaev[d,e]

Physics Letters B 774 (2017) 332–337

Hawking radiation and the Bloom–Gilman duality

R Casadio[1], Alexander Yu Kamenshchik[1,2,6] and Oleg V Teryaev[3,4,5]

Class. Quantum Grav. 35 (2018) 155001

APT for graviton propagator

- Graviton propagator with resummed matter insertions (Donoghue et al): no Landau pole but conjugated poles in complex plane described by Lambert eq.

$$G^{-1}(p^2) = 2p^2\left[1 - \frac{N p^2}{120\pi m_p^2}\ln\left(-\frac{p^2}{\mu^2}\right)\right] \qquad N = N_s + 3N_f + 12N_V, \qquad z\ln z = -A, \qquad z = -\frac{p^2}{\mu^2} \qquad A = \frac{120\pi m_p^2}{N\mu^2}.$$

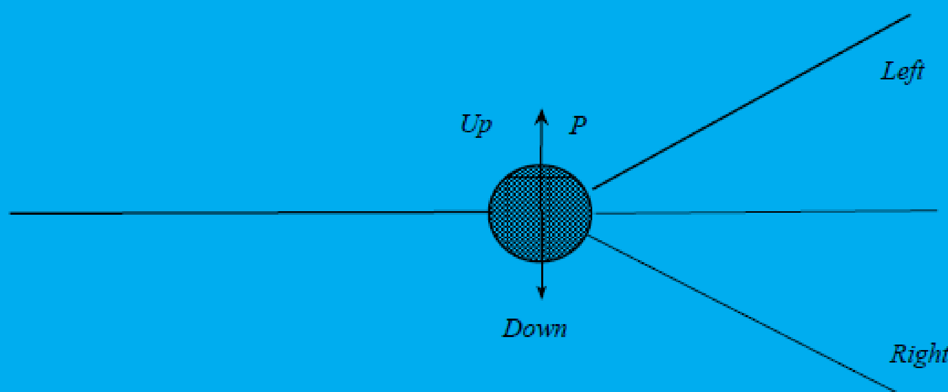
- May be interpreted as BH precursors
- Strong dependence on μ^2
- Implementation of APT provides the imaginary part to propagator (SSA)
- BG-like duality: widths related to Hawking: lower bound for lifetime

$$\tau \simeq \alpha\, \ell_P \left(\frac{m_0}{m_p}\right)^3 \gtrsim 0.7\, \tau_P, \qquad \alpha_{SB} = 5120\pi$$

- Various choices of full set in optical theorem: Quark <-> hadron similar to matter <-> radiation

Single Spin Asymmetries and imaginary phases

Simplest example - (non-relativistic) elastic pion-nucleon scattering $\pi\vec{N} \to \pi N$



$M = a + ib(\vec{\sigma}\vec{n})$ $\vec{n}$ is the normal to the scattering plane.

Density matrix: $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P})$,

Differential cross-section: $d\sigma \sim 1 + A(\vec{P}\vec{n})$, $A = \frac{2Im(ab^*)}{|a|^2+|b|^2}$

Single Spin Asymmetries

Main properties:

– Parity: transverse polarization

– Imaginary phase – can be seen from T-invariance or technically - from the imaginary i in the (quark) density matrix

Various mechanisms – various sources of phases

Phases in QCD

- QCD factorization – soft and hard parts-
- Phases form soft, hard and overlap
- Assume (generalized) optical theorem – phase due to on-shell intermediate states – positive kinematic variable (= their invariant mass)
- Hard: Perturbative (a la QED: Barut, Fronsdal

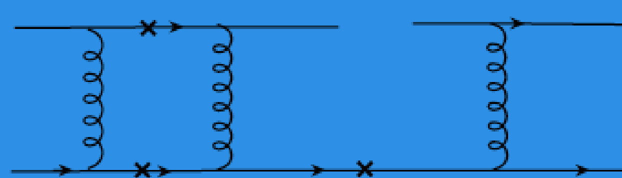
(1960):

Kane, Pumplin, Repko (78) Efremov (78)

Perturbative PHASES IN QCD

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like $q - e$ scattering in DIS):

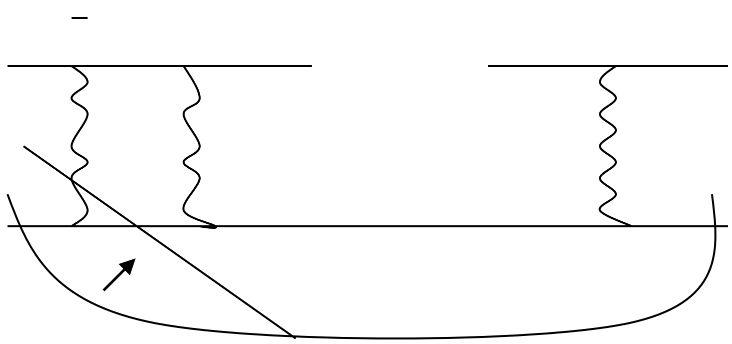


$$A \sim \frac{\alpha_S m p_T}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

Short+ large overlap– twist 3

- Quarks – only from hadrons
- Various options for factorization – shift of SH separation



- New option for SSA: Instead of 1-loop twist 2 – Born twist 3: Efremov, OT (85, Ferminonc poles); Qiu, Sterman (91, GLUONIC poles)

SSA and quest for twist resummation

- Twist 3: $A \sim M/P_T$
- $A < 1$ -> higher twists needed
- Moment representation:
  $A \sim < MP_T/(M^2 + P_T^2) >$
- TMDs - Infinite tower of twists

Approach to polarization in HIC (talk by VI Zakharov) : vortices in pionic superfluid (V.I. Zakharov,OT: 1705.01650;PRD96,09623)

- Pions may carry the axial current due to quantized vortices in pionic superfluid (Kirilin,Sadofyev,Zakharov'12)

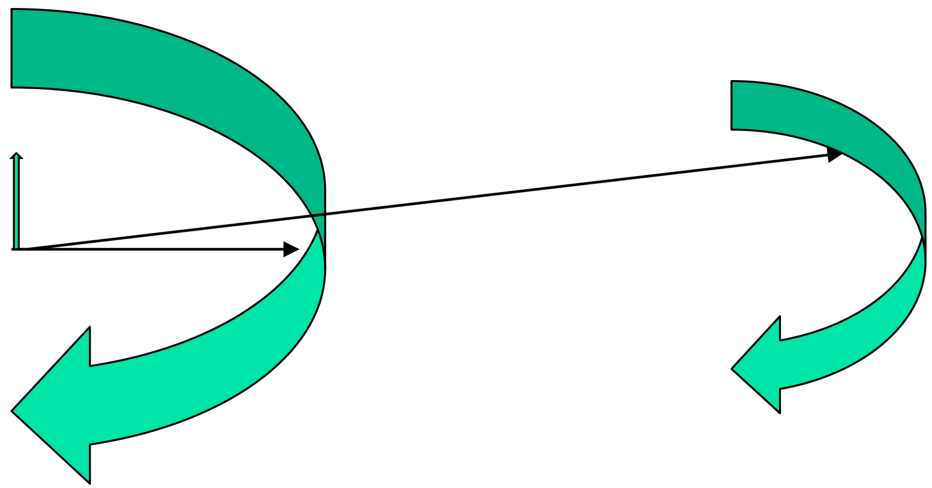
$$j_5^\mu = \frac{1}{4\pi^2 f_\pi^2}\epsilon^{\mu\nu\rho\sigma}(\partial_\nu\pi^0)(\partial_\rho\partial_\sigma\pi^0) \qquad \frac{\pi_0}{f_\pi} = \mu\cdot t + \varphi(x_i) \qquad \oint \partial_i\varphi dx_i = 2\pi n$$

$$\partial_i\varphi = \mu v_i$$

- Core of the vortex- baryonic degrees of freedom- polarization
- Transition to heavy d.o.f.: Dissipation (counterpart of absorptive phases)

Core of quantized vortex

- Constant circulation – velocity increases when core is approached
- S



- Helium ($v < v_{sound}$) bounded by intermolecular distances
- Pions ($v<c$) –> (baryon) spin in the center

Polarization in HIC and density matrix positivity

- Current values of polarization ~ 5 %
- What guarantees that $P < 1$?
- Universal properties of QCD matter? Hydrodynamical resummations?

Conclusions

- Optical theorem strongly constrains the series summation
- Sometimes tends to violate the unitarity, correct analytic properties can be reinforced
- The representation in terms of moments can be helpful for pdf and HT (where it is process-dependent)
- ChPT? Gravity?
- Optical theorem and density matrix positivity: need for resummations

Thanks!

- Happy Birthdays!
- Happy Anniversaries!
- Many happy returns!

Some (artistic) analogies

Main Topics

- Equivalence Principle: way to merge strongest and weakest interactions
- Gravitational Formfactors: EP for spin and its Extension
  - D-term, pressure and inflation
  - Spin-1 and average shear
- Heavy ion collisions : highest vorticity and acceleration
  - Anomalous current and polarization (EFT/TD/Gravity)
  - Unruh radiation

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Strong interactions and gravity

- $E_{EM}/E_G \sim e^2/(m/M_{Pl})^2$ $\qquad M_{Pl} \sim 10^{18}$ GeV
- For 2 particles with M_{Pl} mass at Compton wavelength distance $(1/M_{Pl})$: $E_G \sim (G = 1/M_{Pl}^2)\, M_{Pl}^2 / (1/M_{Pl}) = M_{Pl}$
  $g \sim (G = 1/M_{Pl}^2)\, M_{Pl} / (1/M_{Pl})^2 = M_{Pl}$
- Gravitational interaction is strongly suppressed ~ $(\Lambda/M_{Pl})^2$
- Equivalence Principle
- I: Acceleration <-> Gravity
- HIC: $a \sim \Lambda$, $a/g \sim \frac{c^2}{v_\oplus^2} \cdot \frac{R_\oplus}{R_A} \sim 10^{30}$
- $M_{Pl} \to \Lambda$ ("GeV Gravity")
- II: Coupling to Energy-Momentum Tensor

Electromagnetism vs Gravity (OT'99)

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q)$$

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2 e_q P^\mu$$

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2 P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2 e_q M \phi(q)$$

$$M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle

EP and hadron structure

- "Microscopic" EP (coupling of gravity to EMT)
- +
- Conservation law
  (Momentum SR to get local from LC pdf's):
  $\int dx\, x\, (\Sigma\, q(x) + G(x))=1)$
- =
- "Macroscopic" EP (universal falling) :
- Tested VERY precisely

Gravitational Formfactors (Pagels'66, Ji'97)

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- No M_{Pl}! May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons Ji's SRs
- Describe interaction with both classical and TeV gravity

Ji's and 1st moment "mass" SRs: Generalized Parton Distributions imply models for both EM and Gravitational Formfactors (Selyugin,OT '09)

- Smaller mass square radius (attraction vs repulsion ☺): follows from Regge behaviour of GPDs ~ $x^{\alpha(t)}$ (cf AdS QCD)

$$\rho(b) = \sum_q e_q \int dx q(x,b) \qquad = \int d^2q F_1(Q^2 = q^2) e^{i\vec{q}\,\vec{b}}$$

$$= \int_0^\infty \frac{qdq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\mathrm{Gr}}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

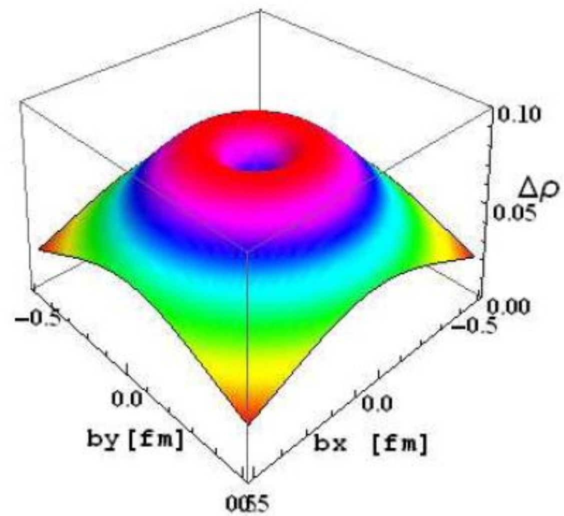


FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$

spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle

Equivalence principle

- Newtonian – "Falling elevator" – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on (quantum!) SPIN – known since 1962 (Kobzarev and Okun'; ZhETF paper contains acknowledgment to Landau: probably his last contribution to theoretical physics before car accident); rederived from conservarion laws - Kobzarev and V.I. Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW
LETTERS

VOLUME 68 | 13 JANUARY 1992 | NUMBER 2

Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

Physics Department, FM-15, University of Washington, Seattle, Washington 98195

(Received 25 September 1991)

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background
- New high precision EDM experiments: gravity is essential (NN Nikolaev,Vergeles,Silenko,…)

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

EP and quantum measurement

- If spin is just a geometric vector, EP for Earth’s rotation is “trivial”: spin rotates with Earth’s angular velocity like Foucault pendulum
- Non-trivial if quantum measurement (quite practical here) is performed in the rotating frame

Equivalence principle for moving particles EPII vs EPI

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics
- Matrix elements DIFFER $h_{zz} = h_{xx} = h_{yy} = h_{00}$

$$\mathcal{M}_g = (\epsilon^2 + p^2) h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)
- Arbitrary fields – Obukhov, Silenko, OT '09,'11,'13,16,17: also the same dynamocs for classical and quantum rotators ("EP for strong fields")

Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction ($\sim\mathbf{P}^2$) is compensated by $1/\mathbf{P}^2$ in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99) and
- Flip by "gravitoelectric" field: relic neutrino (Anisotropic Universe: Kamenshchik,OT'15)? Black hole?

$$\frac{d\sigma_{+-}}{d\sigma_{++}} = \frac{tg^2(\frac{\phi}{2})}{(2\gamma - \gamma^{-1})^2}$$

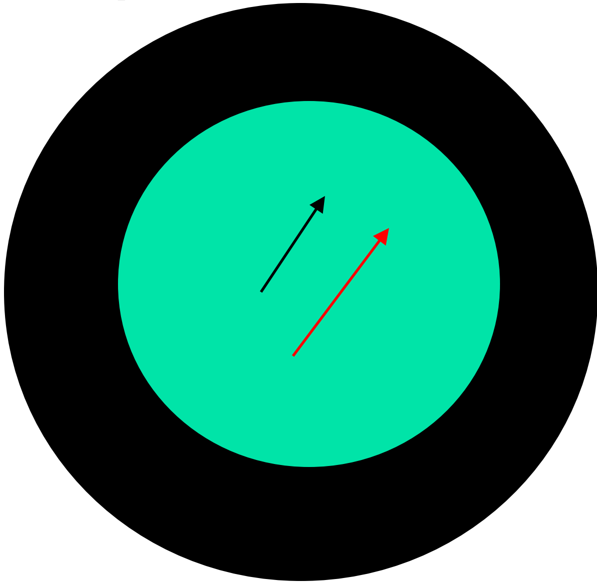
Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $<P+q|T^{mn}|P-q> = P^{\{m}<P+q|J^{n\}}|P-q>/e$ up to the terms linear in q
- Gravitomagnetic g=2 for any spin
- Special role of g=2 for ANY spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also g=2 for Black Holes. Indication of "quantum" nature?!

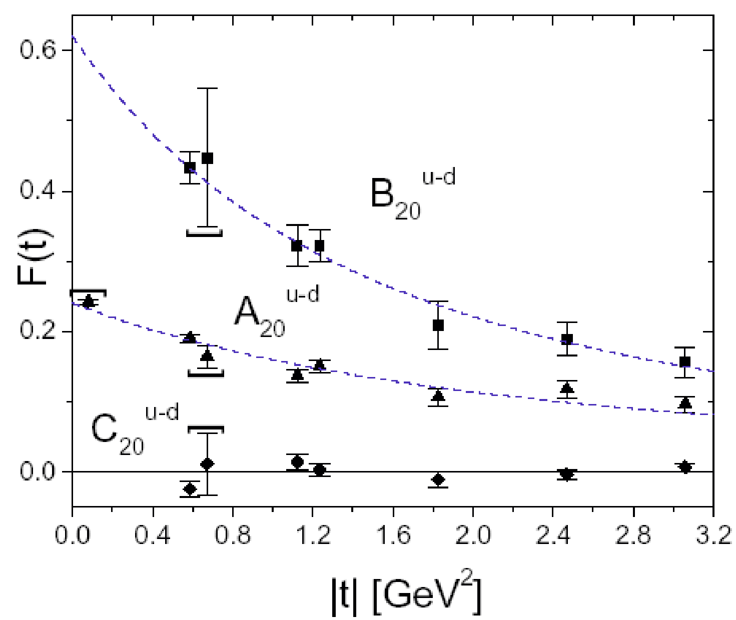
Cosmological implications of PNEP

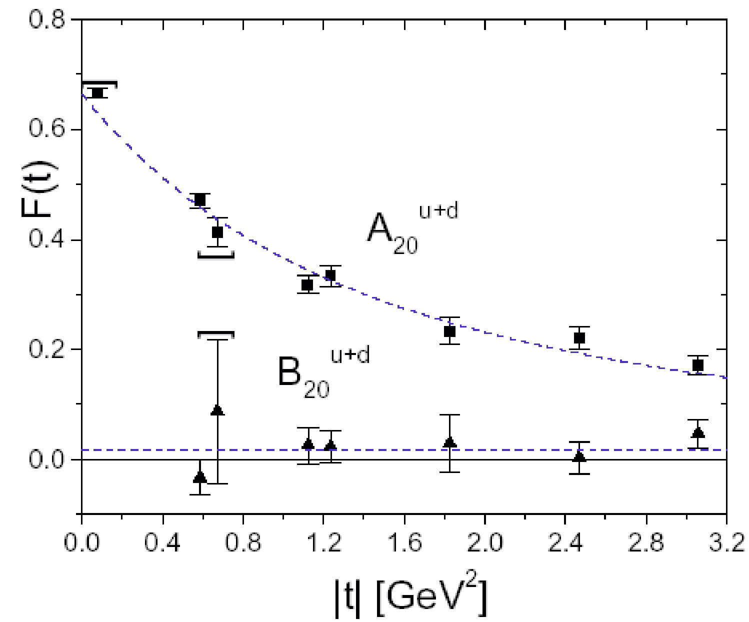
- Necessary condition for Mach's Principle (in the spirit of **S.Weinberg's** textbook-Section 9.7)
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



Generalization of Equivalence principle

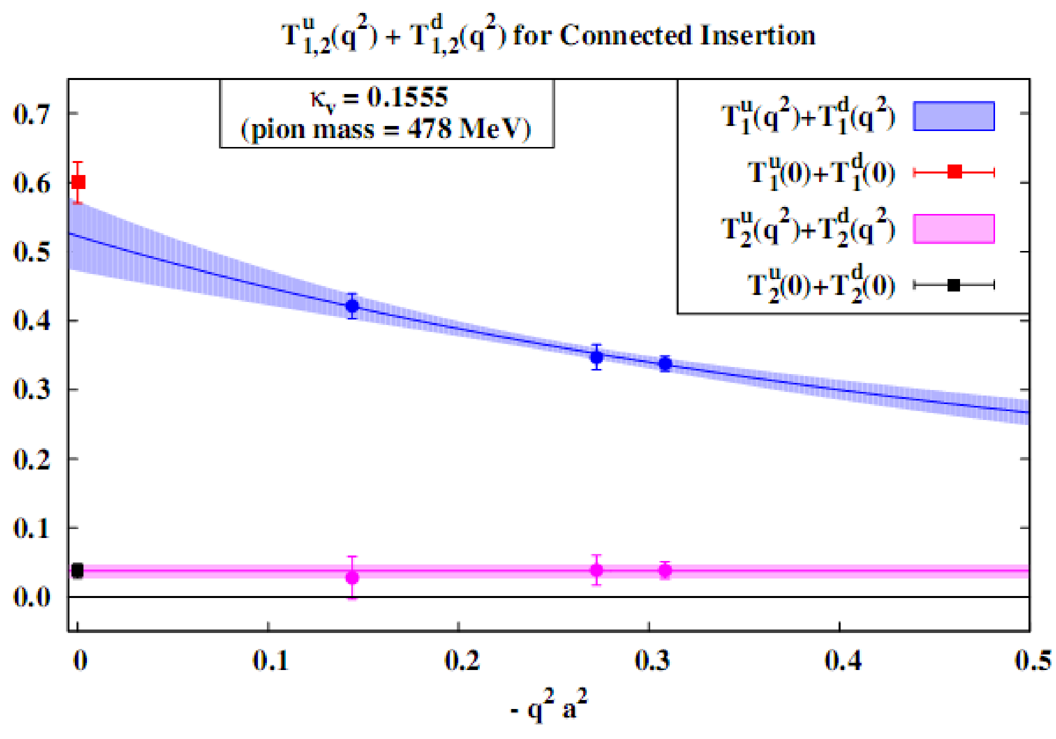
- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





More recent lattice study (M. Deka,…K.-F. Liu et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In NLO pQCD – violated (LF:S.Brodsky et al.)
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 71)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement?! Nucleons do not break even by black holes?! Match BH complementarity?! "GeV Gravity"?
- Support by recent observation of smallness of EP-forbidden "Cosmological Constant"

Exact Equipartition and Pivot

- Important notion introduced by C. Lorce to relate transverse spin SR's of Ji&Yuan and Leader et al.
- Naïve interpretation of ExEP: common (approximately, averagely) pivot for quarks and gluons:
- $\langle J_{T(q,G)} \rangle = \langle x_0 \rangle \langle P_{L(q,G)} \rangle$
- Can this be satisfied for some of pivot choices?

Gravitational formfactors and pressure in hadron pairs production

- Back to GDA region
- -> moments of H(x,x) - define the coefficients of powers of cosine!– $1/\xi$
- Higher powers of cosine in t-channel – threshold in s -channel
- Larger for pion than for nucleon pairs because of less fast decrease at x ->1
- Stability defines the sign of GDA

$$\mathcal{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x) \frac{x^n}{\xi^{n+1}} + \Delta\mathcal{H}.$$

Quantum roots of classical stability

- GPDs

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x) - H(x,\xi)}{x - \xi + i\epsilon}$$

- =-($\int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x-\xi+i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$)
- Sufficient condition: positive (because of forward limit!) H is a decreasing function of ξ at any x

- GDA's $H(z,\xi) = sign(\xi)\Phi(\frac{z}{\xi}, \frac{1}{\xi})$

$$\mathscr{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x) \frac{x^n}{\xi^{n+1}} + \Delta\mathscr{H}.$$

- Positivity of GDA balance between unitarity and stability
- Soft PION theorem – positivity of DA!?

Gravitational FFs from Belle data on GDAs

S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

- Gravitational FFs are related to twist-2 GDAs

$$A_{\lambda_1\lambda_2} = T_{\mu\nu}\varepsilon^{\mu}(\lambda_1)\varepsilon^{\nu}(\lambda_2)/e^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz \frac{2z-1}{z(1-z)}\Phi^q(z,\xi,W^2)$$

$$\int dz(2z-1)\Phi_q^+(z,\xi,W^2) = \frac{2}{(P^+)^2}\langle \pi^+(p_1)\pi^-(p_2)|T_q^{++}(0)|0\rangle$$

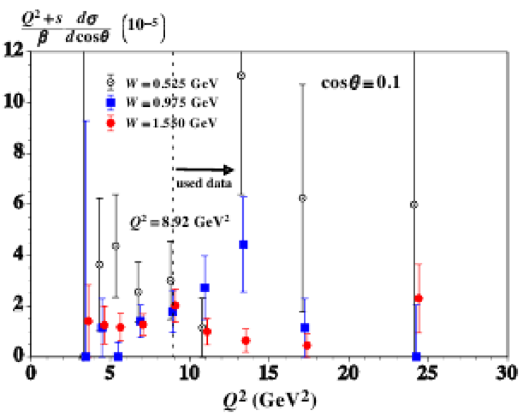
$$\langle \pi^0(p_1)\pi^0(p_2)|T^{\mu\nu}(0)|0\rangle = \frac{1}{2}\left[(sg^{\mu\nu} - P^{\mu}P^{\nu})\Theta_1 + \Delta^{\mu}\Delta^{\nu}\Theta_2\right]$$

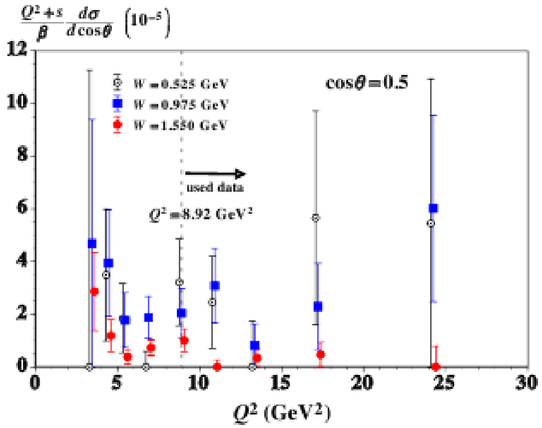
$$P = p_1 + p_2 \,,\ \Delta = p_1 - p_2$$

M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003

- Belle data and scaling : W=0.525,0.975, 1.55 GeV

$$\frac{(Q^2+s)d\sigma}{\beta d|\cos\theta|} \propto \left|\Phi^{\pi^0\pi^0}(z,\cos\theta,W,Q)\right|^2$$





Phase shifts and resonances

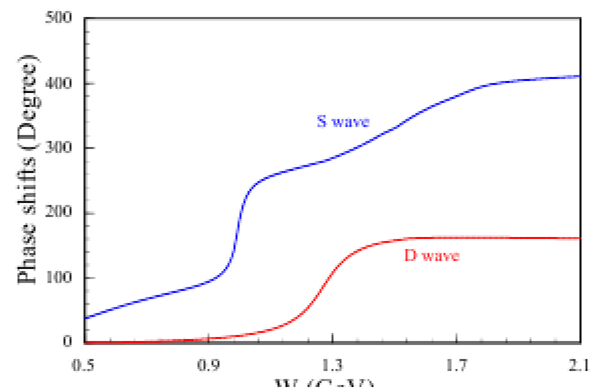
- Leading harmonics

$$\sum_q \Phi_q^+\left(z,\xi,W^2\right) = 18n_f z(1-z)(2z-1)[B_{10}(W)+B_{12}(W)P_2(2\xi-1)]$$

$$= 18n_f z(1-z)(2z-1)[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)]$$

$$\tilde{B}_{10}(W)=\bar{B}_{10}(W)e^{i\delta_0}, \tilde{B}_{12}(W)=\bar{B}_{12}(W)e^{i\delta_2}$$

- S/D shifts



- $f_0(500)$, $f_2(1270)$ contributions

$$\bar{B}_{12}(W) = \beta^2 \frac{10 g_{f_2\pi\pi} f_{f_2} M_{f_2}^2}{9\sqrt{2}\sqrt{(M_{f_2}^2 - W^2)^2 - \Gamma_{f_2}^2 M_{f_2}^2}}$$

$$\bar{B}_{10}(W) = \frac{5 g_{f_0\pi\pi} f_{f_0}}{3\sqrt{2}\sqrt{(M_{f_0}^2 - W^2)^2 - \Gamma_{f_0}^2 M_{f_0}^2}}$$

Fits and results

- Collection

$$\Phi_q^+\left(z,\xi,W^2\right)=N_h z^{\alpha}(1-z)^{\alpha}(2z-1)[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)]$$

$$\tilde{B}_{10}(W)=[\frac{-3+\beta^2}{2}\frac{5R_\pi}{9}F_h(W^2)+\frac{5g_{f_0\pi\pi}f_{f_0}}{3\sqrt{2}\sqrt{(M_{f_0}^2-W^2)^2-\Gamma_{f_0}^2M_{f_0}^2}}]e^{i\delta_0}$$

$$\tilde{B}_{12}(W)=[\beta^2\frac{5R_\pi}{9}F_h(W^2)+\beta^2\frac{10g_{f_2\pi\pi}f_{f_2}M_{f_2}^2}{9\sqrt{2}\sqrt{(M_{f_2}^2-W^2)^2-\Gamma_{f_2}^2M_{f_2}^2}}]e^{i\delta_2}$$

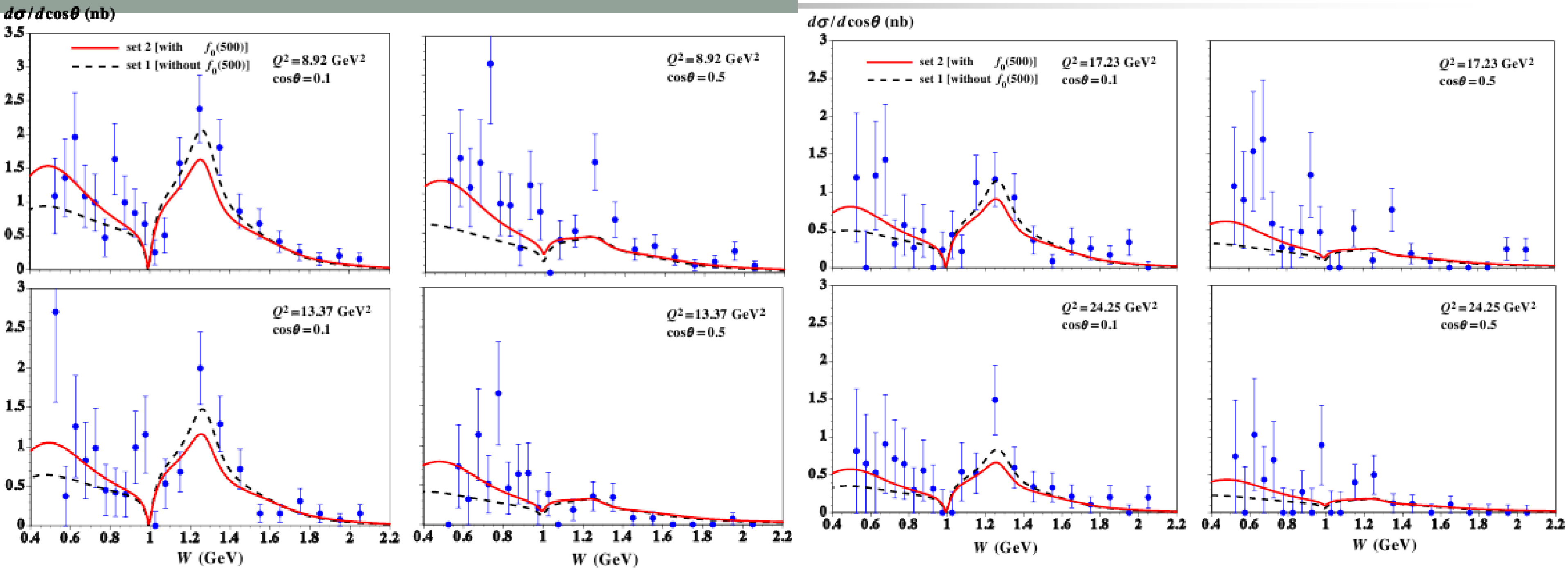
$$F_h(W^2)=\frac{1}{\left[1+\frac{W^2-4m_\pi^2}{\Lambda^2}\right]^{n-1}}$$

- Best fit with (2) and without (1) f_0

	Set 1	Set 2
α	0.801±0.042	1.157± 0.132
Λ	1.602±0.109	1.928±0.213
a	3.878± 0.165	3.800± 0.170
b	0.382± 0.040	0.407± 0.041
f_{f0}	------	0.0184± 0.034

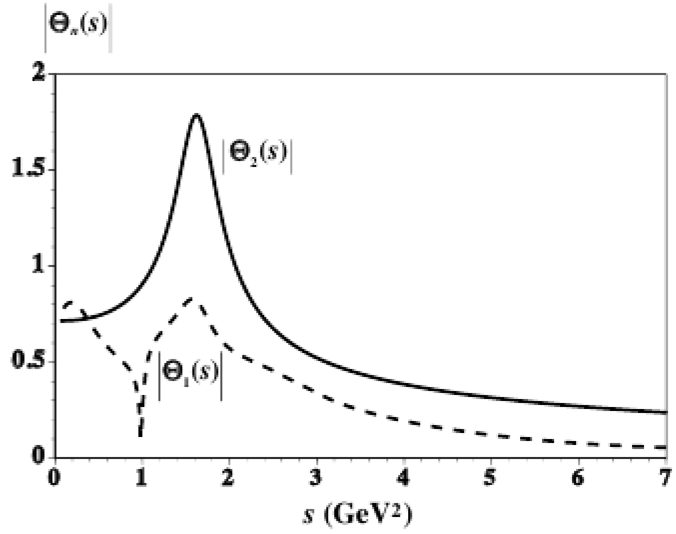
$\frac{\chi^2}{NOF}=1.22$ $\frac{\chi^2}{NOF}=1.09$

Description of data



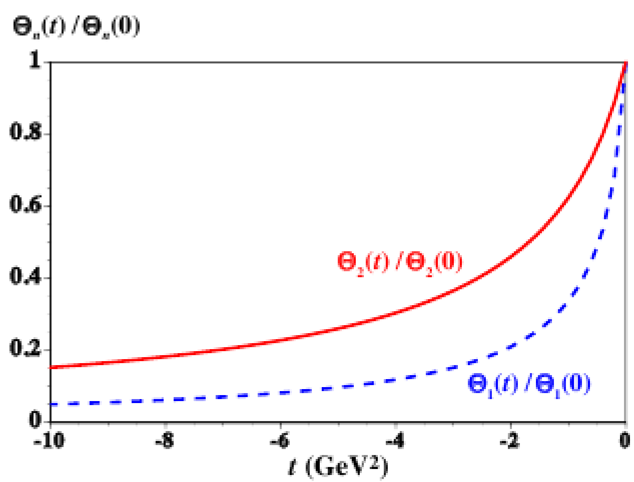
Formfactors

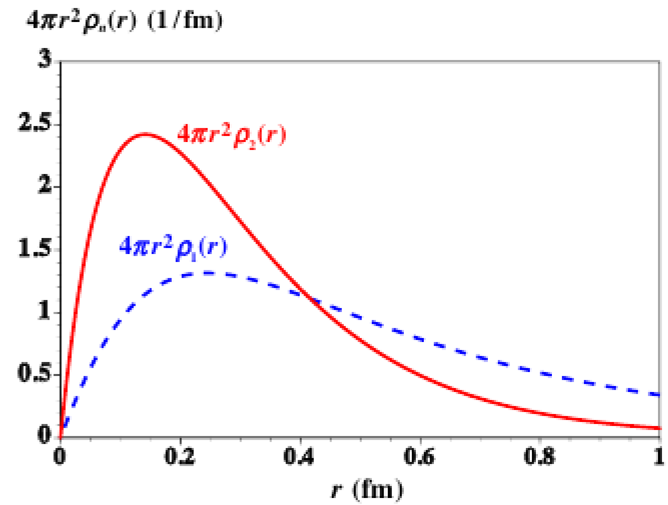
- Resonance structure in pressure – related Θ_1



Time-like -> space-like

- Dispersion relation and Fourier transform





$$<r^2>=6\int_{4m^2}^{\infty}\frac{\mathrm{Im}(F(s))}{s^2}$$

- Mass radius

$$\sqrt{\langle r^2\rangle} = 0.69 \text{ fm for } \Theta_2$$

Spin 1 EMT and inclusive processes

- Forward matrix element -> density matrix
- Contains P-even term: tensor polarization $S^{\alpha\beta}$
- Symmetric and traceless: correspond to (average) shear forces
- For spin ½: P-odd vector polarization requires another vector (q) to form vector product

SUM RULEs

- Efremov,OT'81 : zero sum rules:
- Current conservation: 1st moment: also in parton model by Close and Kumano (90)
- EMT conservation: 2nd moment (forward analog of Ji's SR: AGM $=\langle A_T \rangle = 0$)
- Average shear force (compensated between quarks and gluons)
- Gravity and (Ex)EP (zero average shear separately for quarks and gluons) – OT'09

Manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

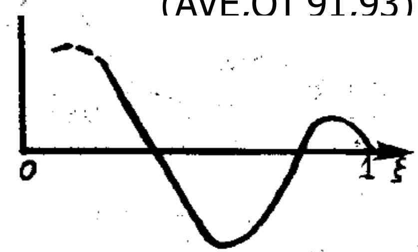
- Tensor polarization - coupling of EMT to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}}.$$

$$\int_0^1 C_i^T(x)dx = 0$$

$$\langle P,S|\bar{\psi}(0)\gamma^\nu D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2S^{\nu\nu_1}P^{\nu_2}...P\nu_n\int_0^1 C_q^T(x)x^n dx$$

(AVE,OT'91,93)



$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu(1-\delta(\mu^2)) + 2M^2S^{\mu\nu}\delta_1(\mu^2)$$

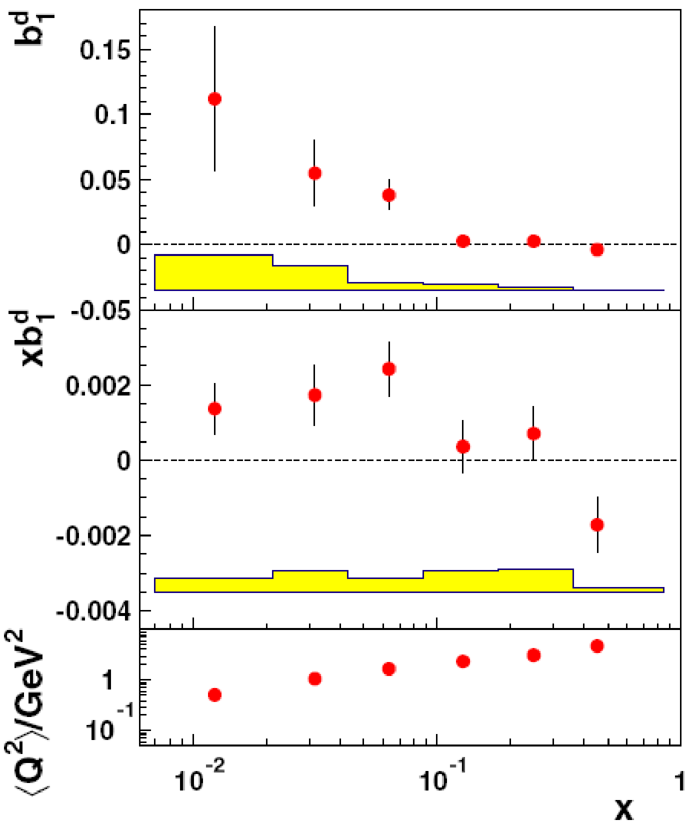
$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu\delta(\mu^2) - 2M^2S^{\mu\nu}\delta_1(\mu^2)$$

$\sum_q \int_0^1 C_i^T(x)xdx = \delta_1(\mu^2)$ =0 for ExEP

HERMES – data on tensor spin structure function

PRL **95**, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (collective tensor polarized glue << sea)



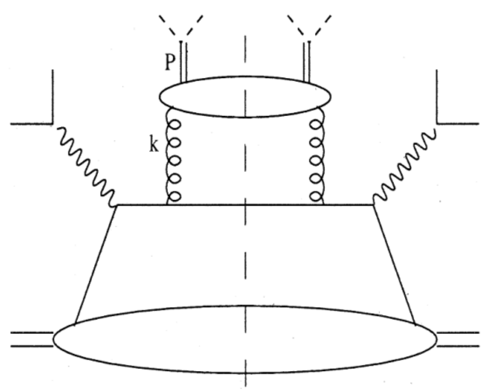
Where else to test?

- EIC
- DY@J-PARC
- ET'81-any hard process ("multi-messenger")
- Possibility: hadronic tensor SSA@NICA

Fragmentation functions

- Tensor polarized fragmentation functions: (Szymanowski, Schaefer, OT'99)

A. Schäfer et al. / Physics Letters B 464 (1999) 94–100



- Suggestion'21: zero SRs (analogous to momentum SR) may probe the (Ex)EP for hadrons inside partons (EIC: gluons)

More on vector mesons and ExEP

- J=1/2 -> J=1. QCD SR/model/lattice calculation of Rho's AMM gives g close to 2 (g=2 exactly in AdS QCD).
- Why?
- Maybe because of similarity of moments and ExEP
- $g-2=\langle E_u(x)\rangle$; $B=\langle xE_u(x)\rangle$
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP: Gluons momentum fraction sizable

EP: Where is the fastest possible rotation and acceleration?

- Non-central heavy ion collisions (Angular velocity ~ c/Compton wavelength) $\Omega \sim \frac{c}{\lambda_p} = \frac{m_p c^2}{\hbar}$
- ~25 orders of magnitude faster than Earth's rotation $\eta_{\rm rot} = \frac{\Omega}{\omega_\oplus} = \frac{c}{R_A} \cdot \frac{T_\oplus}{2\pi} = \frac{1}{2\pi} \cdot \frac{cT_\oplus}{R_A} \approx 10^{27}$
- Differential rotation – vorticity
- P-odd :May lead to various P-odd effects (Chiral magnetic/vortical effects)
- Acceleration: even larger ratio with the gravity of Earth

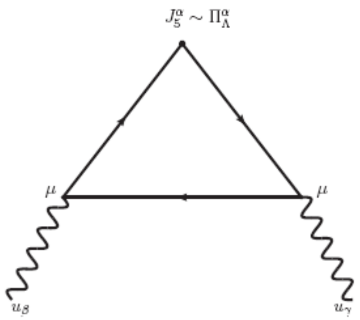
$$\eta_{\rm acc} = \frac{c}{R_A} \cdot \frac{c}{g_\oplus} = \eta_{\rm rot} \frac{2\pi c}{T_\oplus g_\oplus} \approx 10^{30} \qquad \eta_{\rm acc} = \frac{c}{R_A} \cdot \frac{c}{g_\oplus} = \frac{c^2}{v_\oplus^2} \cdot \frac{R_\oplus}{R_A}$$

Effective field theory: Anomalies

- 4-Velocity is also a GAUGE FIELD (V.I. Zakharov et al)

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$

- Triangle anomaly leads to polarization of quarks and hyperons
(Rogachevsky, Sorin, OT '10)
- Analogous to anomalous gluon contribution to nucleon spin
(Efremov,OT'88)
- 4-velocity instead of gluon field potential and vorticity ------//-------
(chromo)magnetic field strength!



Observable for AVE: polarization

STAR, Nature 548 (2017) 62-65

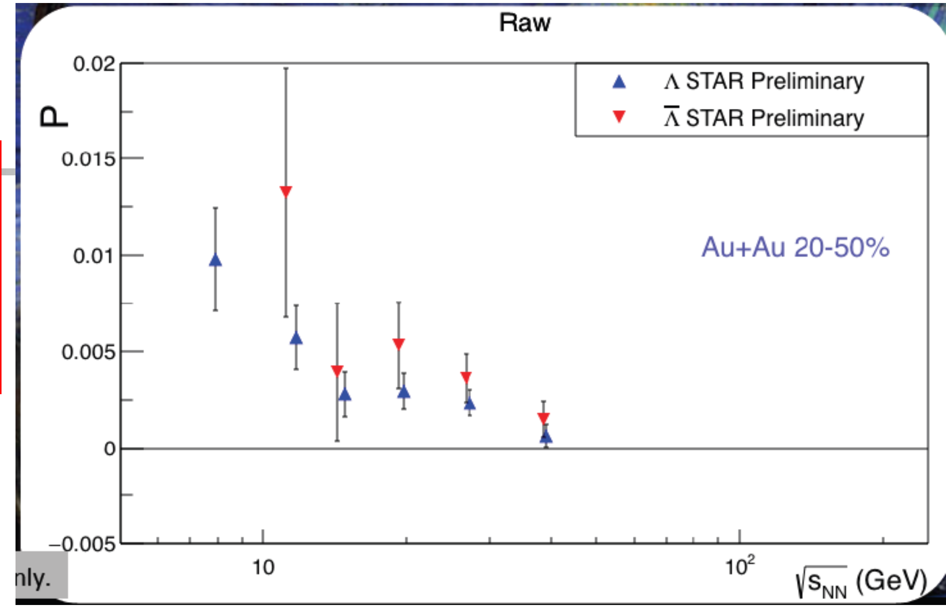
O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortaic effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)

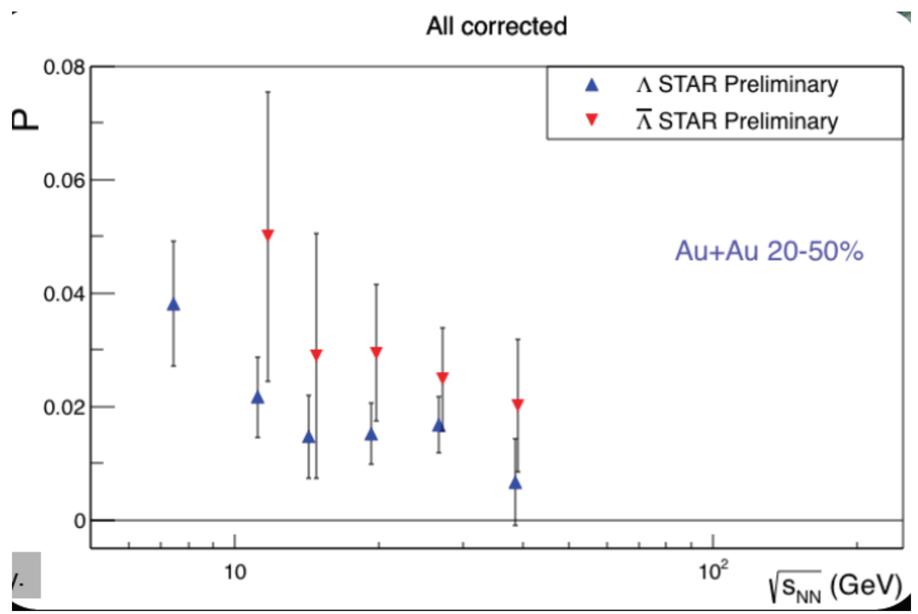
One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \qquad (6)$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.



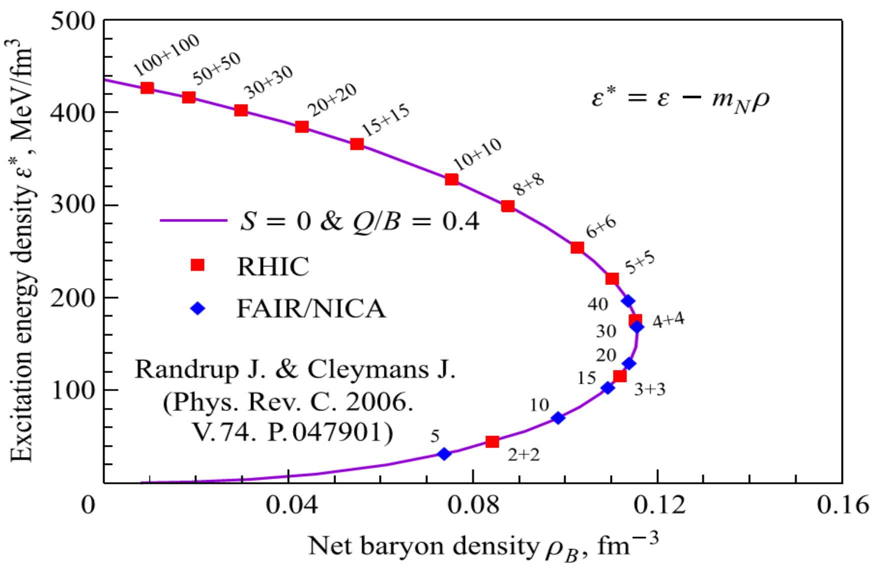


NICA (Nuclotron based Ion Colider fAcility)

– the flagship project in HEP of Joint Institute for Nuclear Research (JINR)

Main targets of "NICA Complex":

- ***study of hot and dense baryonic matter***
- *investigation of hadronic spin structure through various polarization phenomena*
- *development of accelerator facility for HEP @ JINR providing intensive beams of relativistic ions from p to Au*

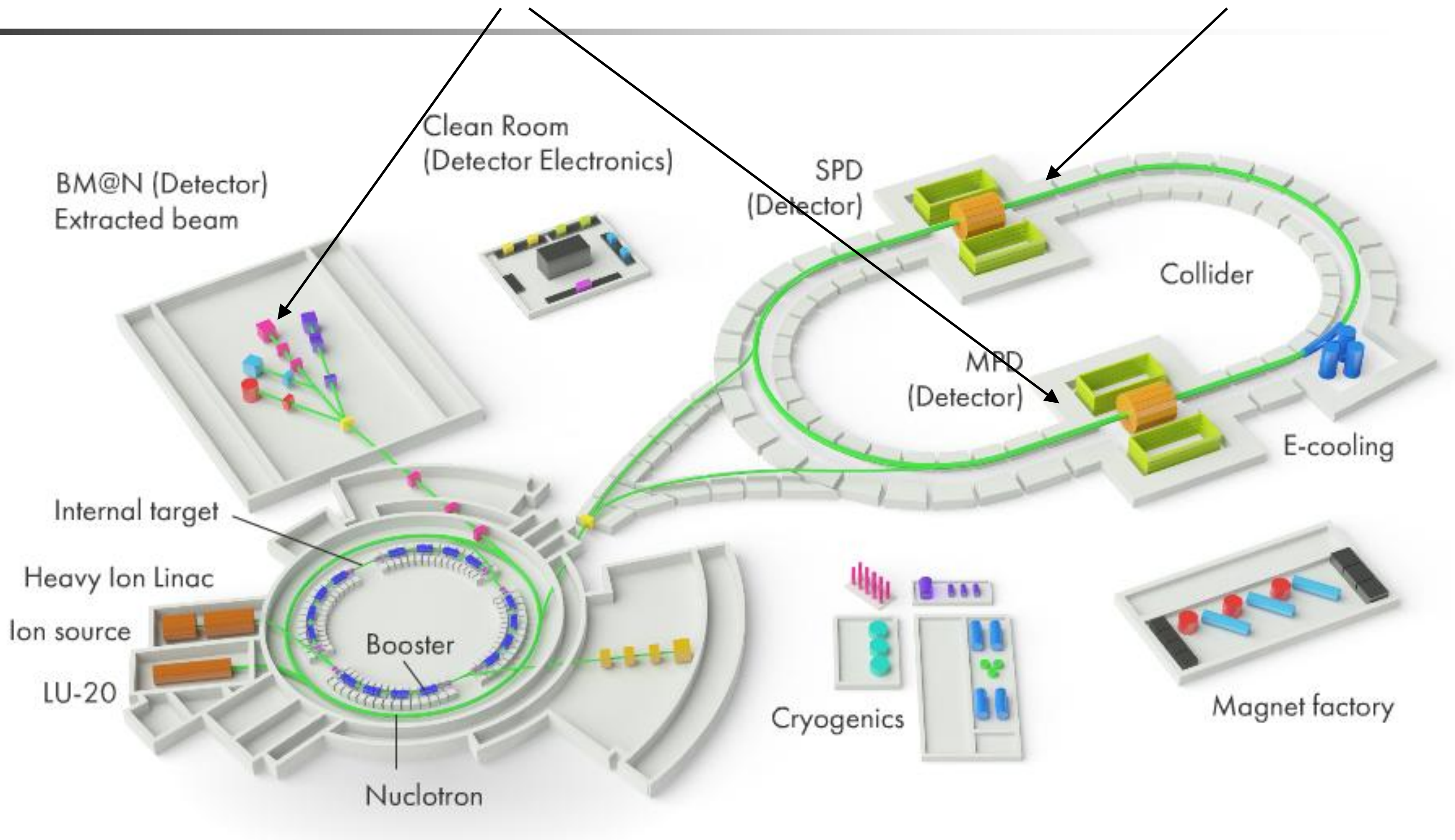


polarized protons and deuterons with energy up to

$\sqrt{S_{NN}}$ = 11 GeV (Au^{79+}, **L ~ 10^{32}** $cm^{-2}\ c^{-1}$)

$\sqrt{S}$ = 27 GeV (p, L ~ **10^{32}** $cm^{-2}\ c^{-1}$)

NICA: heavy ions and hadrons

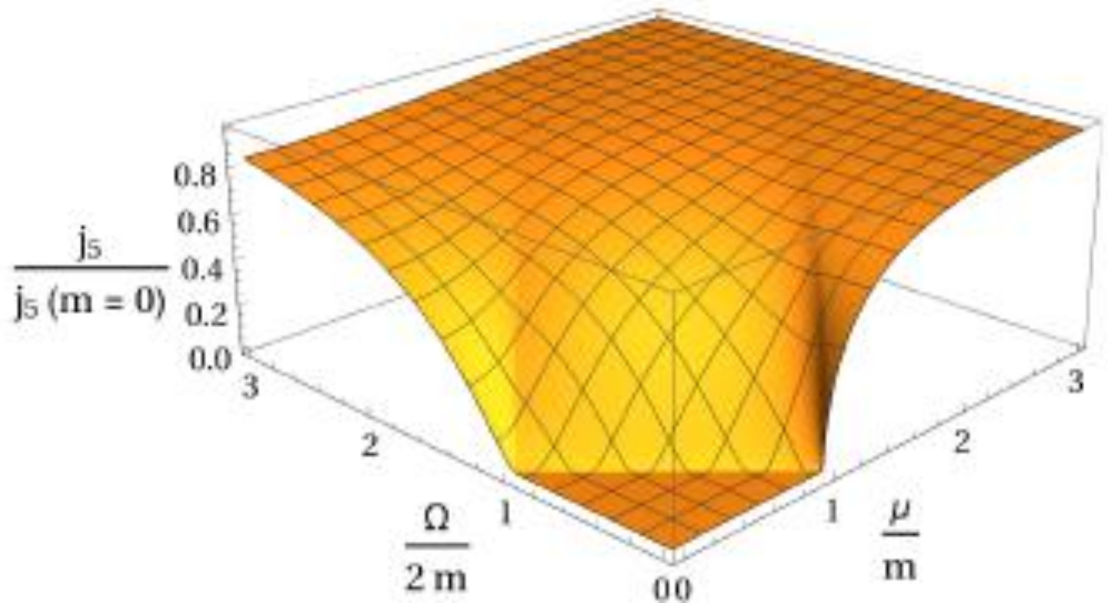


Why EP and rotating frame?

- Statistical approach (F. Becattini et al.; “Standard Model” for polarization): spin equilibrium in rotating frame
- Decrease with energy: explained by decrease of (relevant) hydrodynamic vorticity (Betz, Torrieri, Csernai, Becattini, Karpenko, Lisa,…)
- Interesting to compare with quantum measurement essential for EP: Landau&Lifshitz v. 5, Section 8 (“Law of entropy increasing”): possible relation of inequivalence of time directions due to quantum measurements
- EP/quantum measurement/statistics interplay (recall history of Kobzarev&Okun publication)?!
- Cf.: EP violation (modification?) due to thermal effects (non-zero AGM: Buzzegoli, Kharzeev’21)

Comparison of approaches: Axial ("anomalous" without anomaly) current in TD approach: Vilenkin'82,..

- Prokhorov, Zakharov, OT'18:Threshold effects in chemical potential and angular velocity
- From equilibrated spin of massive hadrons to EFT for spin of massless quarks



Rotated and accelerated frame: Wigner function and Zubarev density operator

- G. Prokhorov, V. Zakharov,OT ’19:
- Imaginary chemical potential due to acceleration appears!

$$
\begin{aligned}
\langle j_\mu^5 \rangle = \frac{\omega_\mu + i\,\mathrm{sgn}(\omega a) a_\mu}{2(g_\omega - i g_a)} \int \frac{d^3 p}{(2\pi)^3} \\
\times \{ n_F(E_p - \mu - g_\omega/2 + i g_a/2) \\
- n_F(E_p - \mu + g_\omega/2 i g_a/2) \\
+ n_F(E_p + \mu - g_\omega/2 + i g_a/2) \\
- n_F(E_p + \mu + g_\omega/2 - i g_a/2) \} + \text{c.c.},
\end{aligned}
$$

$$
\rho = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} = 2 \int \frac{d^3 p}{(2\pi)^3} \left(\frac{|\mathbf{p}| + ia}{1 + e^{\frac{|\mathbf{p}|}{T} + \frac{ia}{2T}}} + \frac{|\mathbf{p}| - ia}{1 + e^{\frac{|\mathbf{p}|}{T} - \frac{ia}{2T}}} \right)
$$

$$
+4 \int \frac{d^3 p}{(2\pi)^3} \frac{|\mathbf{p}|}{e^{\frac{2\pi |\mathbf{p}|}{a}} - 1} \qquad (T > T_U)
$$

in red: *modifications* compared to the *Wigner function*

- In the first integral, the **acceleration** enters as an **imaginary chemical potential** $\pm \frac{ia}{2}$ [G.P., O. Teryaev, V. Zakharov, Phys. Rev. D 98, no. 7, 071901 (2018)].

Statistics vs geometry: Unruh effect (Becattini'18; Prokhorov, OT, Zakharov'19)

- Results for energy density of thermal system in Minkowski space coincide with the early known for the space with conical singularity (e.g. cosmic strings)

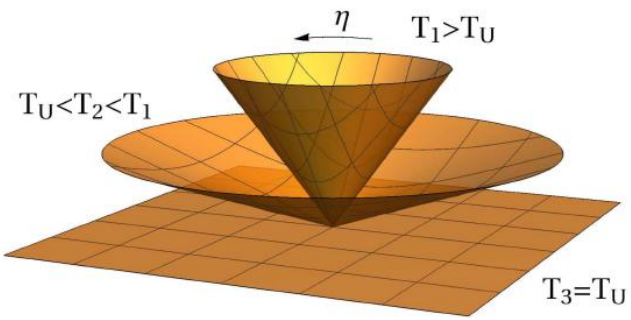
$$\rho_{s=0} = \frac{\pi^2 T^4}{30} + \frac{T^2|a|^2}{12} - \frac{11|a|^4}{480\pi^2},$$

$$\rho_{s=1/2} = \frac{7\pi^2 T^4}{60} + \frac{T^2|a|^2}{24} - \frac{17|a|^4}{960\pi^2}$$

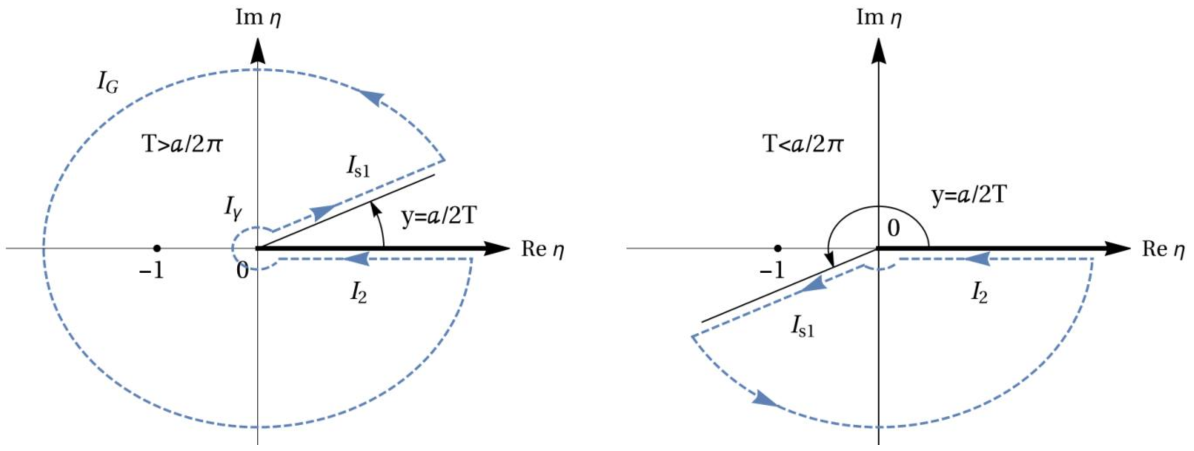
$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x\, c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Energy density turns to zero for $T=T_U=a/(2\pi)$ (~"physical conditions of renormalization". also simple explanation of coefficient)

Instability for high accelerations



- Normally $T>T_U$
- Fast accelration without thermalization: instability



- EP ~ fall to BH?
- Censorship: Origin for fast thermalization?

CONCLUSIONS

- Equivalence principle: allows to study interplay of QCD and gravity
- Modern accelerators (LHC, RHIC, CEBAF EIC,FAIR,NICA… also “effective gravity” labs

Conclusions/Outlook

- EMT coupling
- Separate couplings of quarks and gluons to gravity: ExEP, pressure, shear, cosmological constant
- EP in QCD matter ("GeV gravity")
- Anomalous transport and polarization
- Unruh radiation

OUTLOOK

- EP-I/EPII intersections
- Pressure, shear, EoS for hadrons
- Helicity flip (Relic neutrino, PTOLEMY,Dark matter)

BACKUP

Is D-term independent?

- Fast enough decrease at large energy ->

$$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2 - \nu^2} + \mathrm{C}_0$$

$$\mathrm{C}_0 = \Delta - \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2}$$

$$= \Delta + \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,x)}{x}.$$

- FORWARD limit of Holographic equation

$$\Delta = \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,0) - H^{(+)}(x,x)}{x}$$

$$= 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0) - H(x,x)}{x},$$

$$\mathrm{C}_0(t) = 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0,t)}{x}$$

“D – term” 30 years before…

- Cf Brodsky, Close, Gunion’72 (seagull ~ pressure) – but NOT DVMP
- D-term – a sort of renormalization constant
- May be calculated in effective theory if we know fundamental one
- OR
- Recover through special regularization procedure (D. Mueller)?

ExEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible

ExEP and Sivers function

- Sivers function – process dependent (effective) one
- T-odd effect in T-conserving theory- phase
- FSI – Brodsky-Hwang-Schmidt model
- Unsuppressed by M/Q twist 3
- Process dependence- colour factors
- After Extraction of phase – relation to universal (T-even) matrix elements

ExEP and Sivers function -II

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (OT'07, **hep-ph/0612205**): $x\, f_{T}(x) :\; xE(x)$
- Burkardt SR for Sivers functions is then related to Ji's SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x\, f_{T}(x) = \sum_{q,G} \int dx x E(x) = 0$$

ExEP and Sivers function for deuteron

- EEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones –Brodsky, Gardner)

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T^{\mu\rho}_B - x^\rho T^{\mu\nu}_B$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

- Delicate effect of NP QCD

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu}) \partial^\rho (J^\alpha_{5S} x^\nu) = 0.$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha} \langle P|J^\alpha_{5S}|P+q\rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P|J^\gamma_{5S}|P+q\rangle$$

$$\langle P, S|J^5_\mu(0)|P+q, S\rangle = 2MS_\mu G_1 + q_\mu (Sq) G_2,$$

$$q^2 G_2|_0 = 0$$

Holography vs NLO

- Depends on factorization scheme
- Special role of scheme preserving the coefficient function
- Nucleon as (scheme dependent) black hole – 3D information encoded in 2D

C vs Cbar (=Λ)

- Cancellations of Cbars – negative pressure
- Cf Chaplygin gas: ($p=-A/\varrho$) – analog of cosmological constant
- Cancellation in vacuum; Pauli (divergent), Zel'dovich (finite)
- Flavour structure of pressure: DVMP!