Higher-Spin Theory at Mature Age

M.A.Vasiliev

Lebedev Institute, Moscow

Advances in Quantum Field Theory 2021

JINR, October 14, 2021

Space-Time and Particles

Maximally symmetric space-times are characterized by their symmetry group S.

Anti-de Sitter space: $AdS_d = O(d-1,2)/O(d-1,1)$, $S = O(d-1,2)$

Minkowski contraction $M^4 = ISO(1,3)/SO(1,3)$, $S = ISO(1,3)$

Particles are associated with UIRs S-modules Wigner 1939

realized by spaces of solutions to S-symmetric field equations.

In the special case of massless fields energy weight E_0 takes a spin-dependent minimal value compatible with unitarity

$$E_0 = E_0(\vec{s})\,, \qquad E_0(s) = s+1 \quad (d=4)$$

In this case $D(E_0,\vec{s})$ is indecomposable. The unitary module is its quotient $D'(E_0,\vec{s}) = D(E_0,\vec{s})/\tilde{D}(\tilde{E}_0,\tilde{\vec{s}})$

In physics factorization is realized by gauge symmetry becoming one of the fundamental physical principles.

Fronsdal Fields

Fronsdal fields 1978

All $m = 0$ HS fields are gauge fields

$\phi_{n_1 \dots n_s}$ is a rank-s symmetric tensor obeying $\phi^k{}_k{}^m{}_{m n_5 \dots n_s} = 0$

Gauge transformation:

$$\delta \phi_{n_1 \dots n_s} = \partial_{(n_1} \varepsilon_{n_2 \dots n_s)} \,, \qquad \varepsilon^m{}_{m n_3 \dots n_{s-1}} = 0$$

The challenge is to find a nonlinear deformation of Fronsdal field equations that respects a nonlinear deformation of the gauge symmetry

In 60-70th it was argued (Weinberg, Coleman-Mandula) that HS symmetries cannot be realized in a nontrivial local field theory in Minkowski space

Green light: AdS background with $\Lambda \neq 0$ Fradkin, MV, 1987

In agreement with no-go statements the limit $\Lambda \to 0$ is singular

Non-Locality of HS Gauge Theory

HS interactions contain higher derivatives:

A.Bengtsson, I.Bengtsson, Brink (1983); Berends, Burgers, van Dam (1984)

HS symmetries Fradkin, MV 1986 are infinite dimensional

Infinite towers of spins imply infinite towers of derivatives.

How (non)local is HS gauge theory?

HS symmetries do not commute with space-time symmetries

$$[T^n, T^{HS}] = T^{HS}, \qquad [T^{nm}, T^{HS}] = T^{HS}$$

Riemann geometry is not appropriate for HS theory

The mildest possibility: each vertex with fields of definite spins is local.

All vertices we have found so far up to the quintic order are spin-local:

local in the spinor space.

The worst option: HS theory is essentially nonlocal Sleight, Taronna 2017

Locality and Non-Locality

Equations of motion in perturbatively local field theory

$$E(\partial,\phi)=0\,,\qquad E(\partial,\phi)=\sum_{k=0,l=1}^{\infty} a^{n_1\dots n_k}_{a_1\dots a_l}\partial_{n_1}\dots\partial_{n_k}\phi^{a_1}\dots\phi^{a_l}$$

have a finite number of non-zero coefficients $a^{n_1\dots n_k}_{a_1\dots a_l}$ at any order l.

In non-local field theory this is not demanded.

Theories like HS theory involve infinite towers of fields: for instance Fronsdal fields of all spins. Hence a_i may take an infinite number of values. It makes sense to distinguish between the following cases

local: finite number of derivatives at any order

spin-local: finite number of derivatives at any order for any finite subset of fields

non-local: infinite number of derivatives at some order for some finite subset of fields.

Field Redefinitions

A local theory remains local under perturbatively local field redefinitions

$$\delta\phi^b = \sum_{k=0,l=1}^{\infty} a^{bn_1\ldots n_k}_{a_1\ldots a_l}\partial_{n_1}\ldots\partial_{n_k}\phi^{a_1}\ldots\phi^{a_l}$$

with a finite number of non-zero coefficients at any given order.

Application of a nonlocal field redefinition makes it seemingly non-local.

Given non-locally looking field theory, the essential question is whether or not it admits a choice of variables making it local or spin-local.

One of the central problems in the HS gauge theory is to find an appropriate setup making it (spin-)local.

In this talk it will be sketched how this problem is reformulated in terms of star-product functions leading to the proper setup.

Unfolded Dynamics

First-order form of differential equations

$$\dot{q}^i(t) = \varphi^i(q(t)) \qquad \text{initial values:} \quad q^i(t_0)$$

Unfolded dynamics: multidimensional generalization

$$\frac{\partial}{\partial t} \to \mathsf{d}\,, \qquad q^i(t) \to W^\Omega(x) = dx^{n_1} \wedge \ldots \wedge dx^{n_p} W^\Omega_{n_1 \ldots n_p}(x)$$

$$\mathsf{d}\mathbf{W}^\Omega(\mathbf{x}) = \mathbf{G}^\Omega(\mathbf{W}(\mathbf{x}))\,, \qquad \mathsf{d} = \mathbf{dx^n}\partial_\mathbf{n} \qquad \mathbf{MV} \quad \mathbf{1988}$$

$G^\Omega(W)$: **function of "supercoordinates"** W^Ω

$$G^\Omega(W) = \sum_{n=1}^{\infty} f^\Omega{}_{\Phi_1 \ldots \Phi_n} W^{\Phi_1} \wedge \ldots \wedge W^{\Phi_n}$$

Covariant first-order differential equations

$d > 1$: **Compatibility conditions**

$$G^\Phi(W) \wedge \frac{\partial G^\Omega(W)}{\partial W^\Phi} \equiv 0$$

L_∞, A_∞, Q-**manifolds, etc** 1988, 2005

Space-Time as Vacuum Solution

Let ω^α be a set of one-forms:

$$G^\alpha(\omega) = -f^\alpha_{\beta\gamma}\omega^\beta \wedge \omega^\gamma \qquad \textbf{space-time}$$

Consistency: Jacobi identity for a Lie algebra s

Unfolded equations: flatness condition

$$\mathrm{d}\omega^\alpha + f^\alpha_{\beta\gamma}\omega^\beta \wedge \omega^\gamma = 0$$

$2 \times 2 = 4$: Spinor Language for $4d$ Models

$$x^n = \sigma^n_{\alpha\dot\beta} x^{\alpha\dot\beta} \quad n = 0,1,2,3, \quad \alpha = 1,2, \dot\alpha = 1,2$$

$$x^n x^m \eta_{nm} = \det|x^{\alpha\dot\alpha}| = x^{\alpha\dot\alpha} x^{\beta\dot\beta} \varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta} \qquad sl_2(\mathbb{C}) \sim o(3,1)$$

$Sp(4)$ **symmetric space-time** AdS_4 **as vacuum geometry**

$$\mathbf{R}_{\alpha\beta} := \mathbf{d}\omega_{\alpha\beta} + \omega_{\alpha\gamma}\omega_\beta{}^\gamma - \mathbf{H}_{\alpha\beta} = \mathbf{0}\,, \qquad \mathbf{R}_{\alpha\dot\beta} := \mathbf{d} + \omega_{\alpha\gamma}\mathbf{h}^\gamma{}_{\dot\beta} + \overline{\omega}_{\dot\beta\dot\delta}\mathbf{h}_\alpha{}^{\dot\delta} = \mathbf{0}$$

$$\mathbf{H}^{\alpha\beta} := \mathbf{h}^{\alpha\dot\alpha} \wedge \mathbf{h}^\beta{}_{\dot\alpha}\,, \qquad \overline{\mathbf{H}}^{\dot\alpha\dot\beta} := \mathbf{h}^{\alpha\dot\alpha} \wedge \mathbf{h}_\alpha{}^{\dot\beta}$$

$4d$ Massless Fields

Infinite set of integer spins 1988

$\omega(y,\bar y \mid x)\,, \quad C(y,\bar y \mid x) \quad f(y,\bar y) = \sum_{n,m=0}^{\infty} \frac{1}{n!m!} f_{\alpha_1\ldots\alpha_n,\dot\alpha_1\ldots\dot\alpha_m} y^{\alpha_1}\ldots y^{\alpha_n}\bar y^{\dot\alpha_1}\ldots\bar y^{\dot\alpha_m}$

$$\omega(\mu y,\mu\bar y|x) = \mu^{2(s-1)}\omega(y,\bar y|x)\,, \qquad C(\mu y,\mu^{-1}\bar y|x) = \mu^{\pm 2s}C(y,\bar y|x)$$

ω: **finite number of components (derivatives) for definite spin**

C: **infinite number of components (derivatives) for definite spin**

Fronsdal fields:

$$\omega(\mu y,\mu^{-1}\bar y|x) = \omega(y,\bar y|x)\,, \qquad C(0,0|x)$$

All other components of ω and C are derivatives of the Fronsdal fields

Free Field Unfolded Massless Equations

The full unfolded system for free massless bosonic fields is

$$\star \qquad R_1(y,\overline{y}\mid x) = \frac{i}{4}\left(\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}\ C(0,\overline{y}\mid x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\ C(y,0\mid x)\right)$$

$$\star\star \qquad \tilde{D}_0 C(y,\overline{y}\mid x) = 0$$

$$R_1(y,\bar{y}\mid x) := D_0^{ad}\omega(y,\bar{y}\mid x) \qquad D_0^{ad} = D^L - h^{\alpha\dot\beta}\left(y_\alpha\frac{\partial}{\partial\bar{y}^{\dot\beta}} + \frac{\partial}{\partial y^\alpha}\bar{y}_{\dot\beta}\right)$$

$$\tilde{D}_0 = D^L + h^{\alpha\dot\beta}\left(y_\alpha\bar{y}_{\dot\beta} + \frac{\partial^2}{\partial\mathbf{y}^\alpha\partial\bar{\mathbf{y}}^{\dot\beta}}\right) \qquad D^L = \mathsf{d}_x - \left(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^\beta} + \bar\omega^{\dot\alpha\dot\beta}\bar{y}_{\dot\alpha}\frac{\partial}{\partial\bar{y}^{\dot\beta}}\right)$$

$\star\star$ **implies that higher-order terms in y and $\bar{y}$ describe higher-derivative descendants of the primary HS fields Perturbative unfolded equations**

$$\mathsf{d}_x C = \sigma_- C + \textbf{lower-derivative and nonlinear terms}$$

$$\sigma_- := \mathbf{h}^{\alpha\dot\beta}\frac{\partial^2}{\partial\mathbf{y}^\alpha\partial\bar{\mathbf{y}}^{\dot\beta}}\,, \qquad \sigma_-^2 = 0\,.$$

σ_- **is the substitute of space-time differential in the unfolded dynamics formalism with respect to which spin-locality has to be defined in general unfolded system.**

HS Vertices

The problem: consistent nonlinear corrections 1988 in the local frame

$$\mathsf{d}_x\omega = -\omega * \omega + \Upsilon(\omega,\omega,C) + \Upsilon(\omega,\omega,C,C) + \dots\,,$$

$$\mathsf{d}_x C = -[\omega, C]_* + \Upsilon(\omega,C,C) + \dots$$

$$(f*g)(Y) = \int dSdT \exp iS_A T^A f(Y+S)g(Y+T)\,, \qquad Y^A = (y^\alpha, \bar{y}^{\dot\alpha})$$

$\Upsilon(\omega,\omega,C)$ **Central On-Mass-Shell Theorem** (1988)

$\Upsilon(\omega,C,C)$ **zero-form sector corrections** Didenko, Gelfond, Korybut, MV 2016-2018,

$\Upsilon(\omega,\omega,C,C)$, $\Upsilon(\omega,C,C,C)$ Didenko, Gelfond, Korybut, MV 2019, 2009.02811

$$\Upsilon(\omega,\omega,C,C) = \Upsilon^{\eta\eta} + \Upsilon^{\bar\eta\bar\eta} + \Upsilon^{\eta\bar\eta}\,,$$

where η is an arbitrary complex parameter of the $d=4$ HS theory.

$$\Upsilon^{\eta\eta}(\omega,\omega,C,C) = \Upsilon^{\eta\eta}_{\omega\omega CC} + \Upsilon^{\eta\eta}_{\omega C\omega C} + \Upsilon^{\eta\eta}_{C\omega\omega C} + \Upsilon^{\eta\eta}_{C\omega C\omega} + \Upsilon^{\eta\eta}_{CC\omega\omega} + \Upsilon^{\eta\eta}_{\omega CC\omega}\,,$$

$$\Upsilon^{\bar\eta\bar\eta}(\omega,\omega,C,C) = \Upsilon^{\bar\eta\bar\eta}_{\omega\omega CC} + \Upsilon^{\bar\eta\bar\eta}_{\omega C\omega C} + \Upsilon^{\bar\eta\bar\eta}_{C\omega\omega C} + \Upsilon^{\bar\eta\bar\eta}_{C\omega C\omega} + \Upsilon^{\bar\eta\bar\eta}_{CC\omega\omega} + \Upsilon^{\bar\eta\bar\eta}_{\omega CC\omega}\,,$$

$$\Upsilon^{\eta\bar\eta}(\omega,\omega,C,C) = \Upsilon^{\eta\bar\eta}_{\omega\omega CC} + \Upsilon^{\eta\bar\eta}_{\omega C\omega C} + \Upsilon^{\eta\bar\eta}_{C\omega\omega C} + \Upsilon^{\eta\bar\eta}_{C\omega C\omega} + \Upsilon^{\eta\bar\eta}_{CC\omega\omega} + \Upsilon^{\eta\bar\eta}_{\omega CC\omega}$$

Spin-Locality in $4d$ HS Theory

Nonlinear corrections have the form

$$F(P^{ij},\bar{P}^{kl})C(Y_1)\dots C(Y_n)\,,\qquad P^{ij}:=\frac{\partial}{\partial y_i^{\alpha}}\frac{\partial}{\partial y_{j\alpha}}\,,\qquad \bar{P}^{ij}:=\frac{\partial}{\partial \bar{y}_i{}^{\dot\alpha}}\frac{\partial}{\partial \bar{y}_{j\dot\alpha}}$$

with some non-polynomial functions $F(P^{ij},\bar{P}^{kl})$

Spin-locality: polynomiality of $F(P^{ij},\bar{P}^{kl})$ in either P or $\bar{P}$

Projector on fixed spins relates degree in P^{ij} and $\bar{P}^{kl}$ to each other!

Nonlinear System via Doubling of Spinors

Direct analysis of nonlinear deformation of the free unfolded equations is possible in the lower orders 1988 **but quickly gets complicated The efficient trick** MV 1992 **reduces the problem to De Rham cohomology with respect to additional spinor variables.**

$$\omega(Y;K|x) \longrightarrow W(Z;Y;K|x)\,, \qquad C(Y;K|x) \longrightarrow B(Z;Y;K|x)$$

$Y^A = (y^\alpha, \bar{y}^{\dot\alpha}),\; Z^A = (z^\alpha, \bar{z}^{\dot\alpha})$

Some of the nonlinear HS equations determine the dependence on Z_A **in terms of** **"initial data"** $\omega(Y;K|x)$ **and** $C(Y;K|x)$

$S(Z;Y;K|x) = \theta^A S_A(Z;Y;K|x)$ **is a connection along** Z^A $(\theta^A \equiv dZ^A)$

Klein operators $K = (k, \bar{k})$ **generate chirality automorphisms**

$$kf(A) = f(\tilde{A})k\,, \qquad A = (a_\alpha\,, \bar{a}_{\dot\alpha}) : \qquad \tilde{A} = (-a_\alpha\,, \bar{a}_{\dot\alpha})$$

Nonlinear HS Equations

$$\begin{cases} \mathrm{d}W + W \star W = 0 \\ \mathrm{d}B + W \star B - B \star W = 0 \\ \mathrm{d}S + W \star S + S \star W = 0 \\ \mathbf{S} \star \mathbf{B} - \mathbf{B} \star \mathbf{S} = \mathbf{0} \\ \mathbf{S} \star \mathbf{S} = \mathbf{i}(\theta^{\mathbf{A}}\theta_{\mathbf{A}} + \eta\theta^{\alpha}\theta_{\alpha}\mathbf{B} \star \mathbf{k} \star \kappa + \bar{\eta}\bar{\theta}^{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}}\mathbf{B} \star \mathbf{k} \star \bar{\kappa}) \end{cases} \qquad \mathbf{1992}$$

Inner Klein operators:

$$\kappa = \exp iz_\alpha y^\alpha\,, \qquad \bar{\kappa} = \exp i\bar{z}_{\dot{\alpha}}\bar{y}^{\dot{\alpha}}\,, \qquad \kappa \star f = \widetilde{f} \star \kappa\,, \qquad \kappa \star \kappa = 1$$

Dynamical content is located in the x-independent twistor sector

$\eta = \exp i\varphi$ **is a free phase parameter suggesting 3d bosonization.**

Perturbative Analysis

Vacuum solution

$$B_0 = 0\,, \qquad S_0 = \theta^A Z_A\,, \qquad W_0 = \frac{1}{2} w^{AB}(x) Y_A Y_B$$

$$\mathrm{d}W_0 + W_0 \star W_0 = 0\,, \qquad w^{AB} : AdS_4$$

$$[\mathbf{S_0}\,, \mathbf{f}]_\star = -\mathbf{2i}\mathrm{d}_\mathbf{Z}\mathbf{f}\,, \qquad \mathrm{d}_\mathbf{Z} = \theta^\mathbf{A} \frac{\partial}{\partial \mathbf{Z^A}}$$

First-order fluctuations

$$B_1 = C(Y)\,, \qquad S = S_0 + S_1\,, \qquad W = W_0(Y) + W_1(Y) + W_0(Y)C(Y)$$

Order-n equations containing S have the form

$$\mathrm{d}_Z U_n(Z;Y|dZ) = V[U_{<n}](Z;Y|\theta) \qquad \mathrm{d}_Z V[U_{<n}](Z;Y|\theta) = 0$$

can be solved as

$$U_n(Z;Y|\theta) = \mathrm{d}^*_Z V[U_{<n}](Z;Y|\theta) + \mathbf{h(Y)} + \mathrm{d}_Z \epsilon(Z;Y|\theta)$$

$$\mathrm{d}^*_Z V(Z;Y|\theta) = (Z^A - Q^A)\frac{\partial}{\partial \theta^A} \int_0^1 \frac{dt}{t} V(tZ + (1-t)Q; Y|t\theta)$$

Interpretation

The contracting homotopy freedom encodes:

All possible gauge choices in d_z-exact forms $\mathsf{d}_z\epsilon(Z;Y|dZ)$

All possible choices of field variables in d_z cohomology $\mathbf{h(Y)}$

Any unfolded HS system is associated with one or another solution to the nonlinear HS system.

Unfolded equations that appear in the sector of d_Z cohomology automatically reproduce consistent HS vertices solving the Hochschild cohomology problem.

How to single out the proper (e.g., minimally nonlocal) frames?

Shifted Homotopy

Contracting homotopy $\Delta_{q,\beta}$

$$\Delta_{q,\beta} f(z,y,\theta) := \int \frac{d^2u d^2v}{(2\pi)^2} e^{(iv_\alpha u^\alpha)} \int_0^1 \frac{dt}{t} (z-u+q)^\alpha \frac{\partial}{\partial \theta^\alpha} f(tz+(1-t)(u-q), \beta v+y, t\theta)$$

Obeys resolution of identity

$$\{ \mathsf{d}_Z \,, \Delta_a \} + h_a = Id\,.$$

with the cohomology projector

$$h_{q,\beta}(f(z,y,\theta)) = \int \frac{d^2u d^2v}{(2\pi)^2} \exp i v_\alpha u^\alpha f(u-q, \beta v + y, 0)$$

Spin-local limit: $\beta \to -\infty$ **with** $Q_A = \beta \frac{\partial}{\partial Y^A}$

Didenko, Gelfond, Korybut, MV 1909.04876

Local vertices up to the quintic order!

Conclusion

The shifted homotopy scheme is proposed leading to spin-local HS vertices derived from the nonlinear equations.

A class of new local vertices is found up to the quintic order.

Didenko, Gelfond, Korybut, MV 1909.04876, 2009.02811

Indications that HS gauge theory is spin-local in higher orders

Main problem on the agenda:

spontaneous breaking of HS symmetries

in the Coxeter HS models 1804.06520

Congratulations to all heroes of the conference!

Vladimir Belokurov

Konstantin Chetyrkin

Dmitry Kazakov

Nikolay Krasnikov

Anatoly Radyushkin

Vladimir Smirnov

Alexey Vladimirov

Properties

- General applicability
- Manifest (HS) gauge invariance
- Invariance under diffeomorphisms
- Clear group-theoretical interpretation of fields and equations in terms of modules and Chevalley-Eilenberg (Hochschild in HS theory) cohomology of a symmetry algebra s

  Background fields: flat connection of s

  Fields: s-modules

  Equations: covariant constancy conditions
- Local degrees of freedom are in 0-forms $C^i(x_0)$ at any $x = x_0$

  (as $q(t_0)$) infinite-dimensional module dual to the space of single-particle states: $C^i(x_0)$ moduli of solutions
- Independence of ambient space-time

  Geometry is encoded by $G^\Omega(W)$

Coxeter HS Equations

Unfolded equations for 1804.06520 $\mathcal{C}$-HS theories remain the same except

$$iS\star S = dZ^{An}dZ_{An} + \sum_i \sum_{v\in\mathcal{R}_i} \eta_i B \frac{dZ_n^A v^n dZ_{Am} v^m}{(v,v)} \star \kappa_v$$

κ_v are generators of $\mathcal{C}$ acting trivially on all elements except for dZ_n^A

$$\kappa_v \star dZ_A^n = R_v{}^n{}_m dZ_A^m \star \kappa_v\,.$$

η_i is a coupling constant on the conjugacy class $\mathcal{R}_i$ of $\mathcal{C}$.

In the important case of the Coxeter group B_p

$$iS\star S = dZ^{An}dZ_{An} + \sum_{v\in\mathcal{R}_1} \eta_1 B \frac{dZ_n^A v^n dZ_{Am} v^m}{(v,v)} \star \kappa_v + \sum_{v\in\mathcal{R}_2} \eta_2 B \frac{dZ_n^A v^n dZ_{Am} v^m}{(v,v)} \star \kappa_v$$

with arbitrary η_1 and η_2 responsible for the

HS and stringy/tensorial features, respectively

$\eta_2 \neq 0$ for $p \geq 2$

The framed construction leads to a proper massless spectrum.

Jacobi for Cherednik imply consistency of field equations.

Klein Operators and Single-Trace Operators

Enlargement of the field spectra of the rank- $p>1$ Coxeter HS models: $C(Y^n_A;k_v)$ depend on p copies of oscillators Y^n_A and Klein operators k_v.

Qualitative agreement with enlargement of the boundary operators in tensorial boundary models

Klein operators of Coxeter reflections permute master field arguments

At $p=2$ the star product of two master fields $(C(Y_1;Y_2|x)k_{12})$ gives

$$(C(Y_1;Y_2|x)k_{12})\star(C(Y_1;Y_2|x)k_{12})=C(Y_1;Y_2|x)C(Y_2;Y_1|x)$$

$p=2$ system: strings of fields with repeatedly permuted arguments

$$C_{n\,string}:=\underbrace{C(Y_1;Y_2|x)\star C(Y_2|;Y_1|x)\star C(Y_1;Y_2|x)\ldots}_{n}$$

are analogous of the single-trace operators in AdS/CFT.

$C(Y_1;Y_2|x)$ and $C(Y_1;Y_2|x)C(Y_2;Y_1|x)$: single-trace-like

$C(Y_1;Y_2|x)C(Y_1;Y_2|x)$: double-trace-like.

For $p>2$ fields carry p arguments permuted by S_p generated by k_{ij}

Relation with space-time locality

Conceptual problem with the space-time definition of locality in AdS.

Lorentz-covariant derivatives D_n commute to a constant: background AdS curvature

$$[D_n\,, D_m] = R_{nm}$$

giving meaning to non-polynomial functions of D_n demands a particular ordering prescription

Y variables provide an appropriate ordering for D_n derivatives

HS Theories and String Theory

HS theories: $\Lambda \neq 0$, $m = 0$, **symmetric fields** $s = 0, 1, 2, \ldots \infty$

First Regge trajectory

String Theory: $\Lambda = 0$, $m \neq 0$ **except for a few zero modes**

Infinite set of Regge trajectories

What is a HS symmetry of a string-like extension of HS theory?

MV 2012, 2018, Gaberdiel and Gopakumar 2014-2018

String Theory as spontaneously broken HS theory?! $(s > 2; m > 0)$.

Most Urgent Problems

Appropriate scheme leading to (spin-local) choice of variables.

Didenko, Gelfond, Korybut, MV 2016 - 2019

String-like and Tensor-like HS theory!?

MV 1804.06520; Degtev, MV 1905.11267

Quantum Gravity Challenge

QG effects should matter at ultra-high energies of Planck scale

$$M_P = 10^{19} GeV$$

A distinguished theoretic possibility is to conjecture that the regime of ultra high (transPlanckian) energies exhibits some high symmetries that are spontaneously broken at low energies

Idea: to understand what kind of higher symmetries can be introduced in relativistic theory and to see consequences

HS gauge theory:

theory of higher symmetries consistent with unitary QFT

Must be beautiful

and can affect fundamental concepts of gravity and quantum mechanics

It is and it does!

Contracting homotopy

Contracting homotopy

$$\Delta_{(1-\beta)q\,,\beta}(f) = \int \frac{d^2ud^2v}{(2\pi)^2}\int d^3_+\tau\delta(1-\sum_{i=1}^{3}\tau_i)\Big[\frac{(1-\beta)\tau_1}{1-\beta(1-\tau_2)}\Big]^{p-1}$$

$$\exp i[v_\alpha u^\alpha + \tau_1 z_\alpha y^\alpha - \tau_2 q_\alpha y^\alpha]\frac{(1-\beta\tau_1)(z+q)^\beta - \beta\tau_3(u+q)^\beta}{1-\beta(1-\tau_2)}\frac{\partial}{\partial\theta^\beta}$$

$$\phi\Big(\tau_1 z + \frac{\tau_2\tau_3\beta}{1-\beta(1-\tau_2)}u - \tau_2 q, v+\tau_3 y, \theta, \frac{1-\tau_3-\beta\tau_1}{1-\beta(1-\tau_2)}\Big)$$

p **is the degree of** f **in** θ

and cohomology projector

$$h_{(1-\beta)q\,,\beta}(f) = \int_0^1 d\tau\zeta^{-2}\int\frac{d^2ud^2v}{(2\pi)^2}\exp i[v_\alpha u^\alpha + \tau(1-\beta)\zeta^{-1}y_\alpha q^\alpha]$$

$$\phi(\tau(\beta u-(1-\beta)q)\zeta^{-1},(1-\tau)(v+y\zeta^{-1}),\tau)$$

$$\zeta := (1-\beta\tau)$$

are well defined for any $-\infty < \beta < 1$

Pre-Ultra-Locality Theorem

Though β-shifted homotopy is well defined for any $-\infty < \beta < 1$ it is not guaranteed that the limit $\beta \to -\infty$ is well defined.

It is shown that the sufficient condition for it to be well defined is that the r.h.s. of the equation for the order-m correction S_m to S

$$-2i\mathrm{d}_z S_m = -\sum_{k=1}^{m-1} S_k * S_{m-k} + B_m * \gamma$$

belongs to $\mathcal{H}_2^{+0}$

$$-\sum_{k=1}^{m-1} S_k * S_{m-k} + B_m * \gamma \in \mathcal{H}_2^{+0}$$

Moreover, in this case the correction to dynamical field equations turns out to be Pre-ultra-local which means that arguments of the zero-forms C turn out to be y-independent. By Pfaffian locality theorem contractions between the arguments of C must be zero at least in the order C^2.

Example

The simplest vertex is

$$\Upsilon^{\eta\eta}_{\omega\omega CC} = \frac{\eta^2}{4}\int_{[0,1]^2} \mathrm{d}\sigma \mathrm{d}\sigma' \, \sigma\sigma' \int d^3_+\tau \delta(1-\sum_{i=1}^{3}\tau_i)\tau_1 \, (\partial_{1\alpha}\partial_2^\alpha)^2$$

$$\exp\Big[i(\tau_2\sigma+\tau_3\sigma'+\tau_1\sigma\sigma')\partial_{1\alpha}\partial_2^\alpha\Big]\omega(y-(1-\tau_3)\sigma y,\bar{y};K)\,\bar{*}\,\omega(\tau_3\sigma' y,\bar{y};K)\bar{*}$$

$$\bar{*}\, C(\tau_2\sigma\partial_1+(1-\sigma'(1-\tau_2))\partial_2,\bar{y};K)\,\bar{*}\, C(-\tau_1\sigma\partial_1-\sigma'\tau_1\partial_2,\bar{y};K)\,,$$

where ∂_1 and ∂_2 are derivatives over the arguments of the first and second facors of $\omega(y)$.

$\bar{*}$ is the star product with respect to the barred variables $\bar{y}$

Ultralocality: no y-dependence and contractions between the arguments of zero-forms C.

Substituting into the r.h.s. of unfolded equations

$$\mathrm{d}_x\omega = -\omega * \omega + \Upsilon(\omega,\omega,C) + \Upsilon(\omega,\omega,C,C) + \dots$$

singling out $y^s\bar{y}^{s-2}$ components and integrating over the homotopy parameters σ_i and τ_i we obtain a local deformation of Fronsdal equations.

Spin-Locality

Space-time is described by a chosen flat (vacuum) connection of the global space-time symmetry s.

σ_-, which is the most negative grade part of the flat connection is directly related to the space-time geometry.

HS currents $J^s_{s_1 s_2}$ form s-modules different from the HS fields C_s.

$$C_{s_1} \otimes C_{s_2} = \sum_{t=0}^{\infty} J^t_{s_1 s_2}$$

Very much like it makes sense to distinguish between HS fields C_s associated with different g-modules characterized by different spins s HS currents $J^s_{s_1 s_2}(C_{s_1}, C_{s_2})$ should be considered as different s-modules to be distinguished from the HS massless modules carried by C_s.

Spin-locality implies that nonlinear corrections to field equations has to be local in terms of original fields C_s and all currents $J^s_{s_1 s_2}$.

Spin-Locality Versus Space-Time Locality

For theories with a finite number of fields C_s and currents $J^s_{s_1 s_2}(C,C)\,,\ldots$ spin-locality implies usual space-time locality with a finite number of derivatives of the original fields C_s

$$D^L C^s(Y;K|x) = i\lambda h^{\alpha\dot\beta}\Big(y_\alpha \bar y_{\dot\beta} - \frac{\partial^2}{\partial y^\alpha \partial \bar y^{\dot\beta}}\Big) C^s(Y;K|x) + \sum_{s_1,s_2=0}^{\infty} J^s_{s_1 s_2}(Y;K|x) + \ldots .$$

As a result, interpretation of C in terms of space-time derivatives acquires J-dependent corrections affecting space-time equations

$$L^{FR} C_s = \sum_{s_1,s_2=0}^{\infty} J^s_{s_1 s_2}[C_{s_1}, \mathcal{C}_{s_2}] + \sum_{t=0}^{\infty} \sum_{s_1,s_2,s_3} J^s_{s_1 t}\Big[C_{s_1}, J^t_{s_2 s_3}\Big] + \ldots,$$

where (abusing notation) C_s is a spin-s Fronsdal field,

$L^{FR} C_s$ is the l.h.s. of free Fronsdal equations.

Once currents in corrections to Fronsdal equations are treated as independent fields (corresponding to independent operators of the boundary operator algebra) these terms are still local containing a finite number of derivatives of each current.

Historical Comments

Cubic vertices of A.Bengtsson, I.Bengtsson, Brink (1983); Berends, Burgers, van Dam (1984) are local as well as their AdS extension Fradkin, MV 1987

Importance of the locality issue in HS theory was stressed in Prokushkin, MV 1998

Proposal for interpretation of locality in terms of star-product functions MV 2015

Analysis of HS corrections in nonlinear HS equations via conventional homotopy led the authors to misleading conclusions instead of the proper interpretation that conventional homotopy is not appropriate Skvortsov, Taronna et al 2015-2017

Analysis of locality of HS corrections in nonlinear HS equations via separation of variables MV 2016 and shifted homotopy formalism Didenko, Gelfond, Korybut, MV 2016-2019 led to proper answer first at the lowest level and then far beyond.

Free Fields as s-Modules

Let W^α contain p-forms $\mathcal{C}^i$ (e.g. 0-forms) and G^i be linear in ω and C

$$G^i(\omega,\mathcal{C}) = -\omega^\alpha (T_\alpha)^i{}_j \wedge \mathcal{C}^j \,.$$

Compatibility condition implies that $(T_\alpha)^i{}_j$ form some representation T of s, acting in a carrier space V of $\mathcal{C}^i$. The unfolded equation is

$$D_\omega \mathcal{C} = 0$$

$D_\omega \equiv \mathrm{d} + \omega$: covariant derivative in the s-module V.

The covariant constancy equation : linear equations in a chosen s-symmetric background described by the flat connection ω : $(D_\omega)^2 = 0$.

s: global symmetry

$$\delta\mathcal{C}^i(x) = \varepsilon^\alpha(x)(T_\alpha)^i{}_j \mathcal{C}^j(x)\,, \qquad D_\omega \varepsilon = 0$$

Unfolding and holographic duality

Unfolding unifies various dualities including holographic duality

Extension of space-time without changing dynamics by letting the exterior derivative d and differential forms W live in a larger space

$$\mathsf{d} = dX^n \frac{\partial}{\partial X^n} \to \tilde{\mathsf{d}} = dX^n \frac{\partial}{\partial X^n} + d\widehat{X}^{\hat{n}} \frac{\partial}{\partial \widehat{X}^{\hat{n}}}\,, \qquad dX^n W_n \to dX^n W_n + d\widehat{X}^{\hat{n}} \widehat{W}_{\hat{n}}\,,$$

$\widehat{X}^{\hat{n}}$ are additional coordinates

$$\tilde{\mathsf{d}} W^\Omega(X, \widehat{X}) = G^\Omega(W(X, \widehat{X}))$$

Two unfolded systems in different space-times are equivalent (dual) if they have the same unfolded form. Given unfolded system generates a class of holographically dual theories in different dimensions.

Useful applications:

$sp(8)$-invariant formulation of $4d$ massless equations 2001

derivation of superfield formulations of SUSY models (Misuna, MV (2013))

HS holography 2012,2015