Neutrino oscillations in quantum field theory and the solar neutrino problem

Vadim Egorov, Igor Volobuev (SINP MSU)

Introduction

The Standard Model, which is a gauge quantum field theory, allows one to describe, with high accuracy, a great amount of various elementary particle interaction processes in the framework of the S-matrix formalism and Feynman diagram technique. However, there is a belief that it cannot describe the phenomena of neutral kaon and neutrino oscillations, the latter being under intense theoretical and experimental investigation nowadays.

Unlike the scattering processes, neutrino oscillation occurs at macroscopic space and time intervals.

It is a well-known and experimentally confirmed phenomenon, which is usually understood as the transition from a neutrino flavor state to another neutrino flavor state depending on the distance traveled.

- S. Bilenky, “Introduction to the physics of massive and mixed neutrinos,”
  Lecture Notes in Physics **817** (2010) 1.

The standard way to describe the phenomenon is the quantum-mechanical approach in terms of plane waves. However, it is believed to be inconsistent, because the production of the neutrino flavor states is described within the SM, whereas their evolution is described within quantum mechanics. Such a description seems to be eclectic, since quantum field theory includes quantum mechanics as an indispensable part and must be able to describe all quantum phenomena. Moreover, the neutrino flavor states cannot be regarded as quantum states, i.e., states of a quantum system.

The production of such states without definite mass leads to violation of energy-momentum conservation.

The problem with violation of conservation of energy-momentum is allegedly solved in the wave packet treatment, although the corresponding calculations of amplitudes are very bulky.

Thus, a construction of a consistent and convenient description of neutrino oscillations within quantum field theory is of current interest.

For the first time, a description of neutrino oscillations within the framework of QFT and S-matrix approach was put forward in the paper

- I.Yu. Kobzarev, B.V. Martemyanov, L.B. Okun and M.G. Shchepkin,

"Sum rules for neutrino oscillations,"

Sov. J. Nucl. Phys. 35 (1982) 708.

In this approach the produced neutrinos are off-shell and described by the Feynman propagators in the coordinate representation.

Neutrino oscillation is now a usual interference process.

The incoming and outgoing leptons are described by plane waves, whereas the positions of nuclei are fixed by delta functions, which is a rather rough approximation.

As a result, in the momentum representation, there appear distance-dependent propagators of virtual neutrinos.

The idea was developed in the paper

- C. Giunti, C.W. Kim, J.A. Lee and U.W. Lee, "On the treatment of neutrino oscillations without resort to weak eigenstates," Phys. Rev. D **48** (1993) 4310.

The localization of the incoming and outgoing particles or nuclei is described with the help of wave packets, which makes the calculations of amplitudes very complicated.

The reason is:

The standard S-matrix formalism is not appropriate for describing the processes passing at finite distances and lasting finite time intervals.

Modified perturbative formalism

We put forward a modification of the perturbative formalism, which allows one to describe the processes passing at finite distances during finite time intervals.

The approach is based on the Feynman diagram technique in the coordinate representation supplemented by modified rules of passing to the momentum representation. The latter reflect the geometry of neutrino oscillation experiments and lead to a modification of the Feynman propagators of the neutrino mass eigenstates in the momentum representation.

The idea behind the approach comes from the paper

- R.P. Feynman, “Space-Time Approach to Quantum Electrodynamics,” Phys. Rev. **76** (1949), 769.

It was developed in the papers

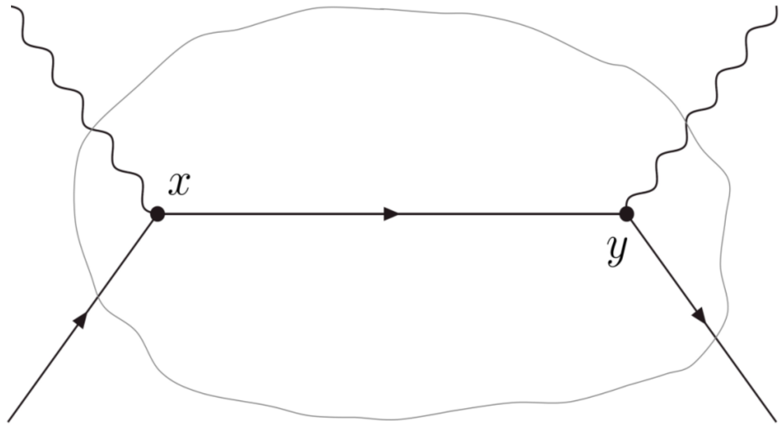
- V.O. Egorov and I.P. Volobuev, “Neutrino oscillation processes in a quantum field-theoretical approach,” Phys. Rev. D **97** (2018) no.9, 093002,
- V.O. Egorov and I.P. Volobuev, “Coherence length of neutrino oscillations in a quantum field-theoretical approach,” Phys. Rev. D **100** (2019) no.3, 033004.

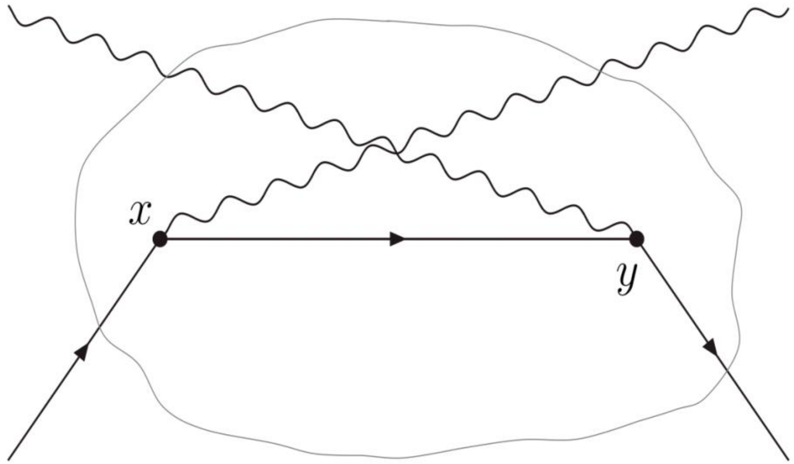
The modern perturbative S-matrix formalism and the diagram technique were formulated in the papers

- F.J. Dyson, “The S matrix in quantum electrodynamics,”
  Phys. Rev. **75** (1949) 1736,

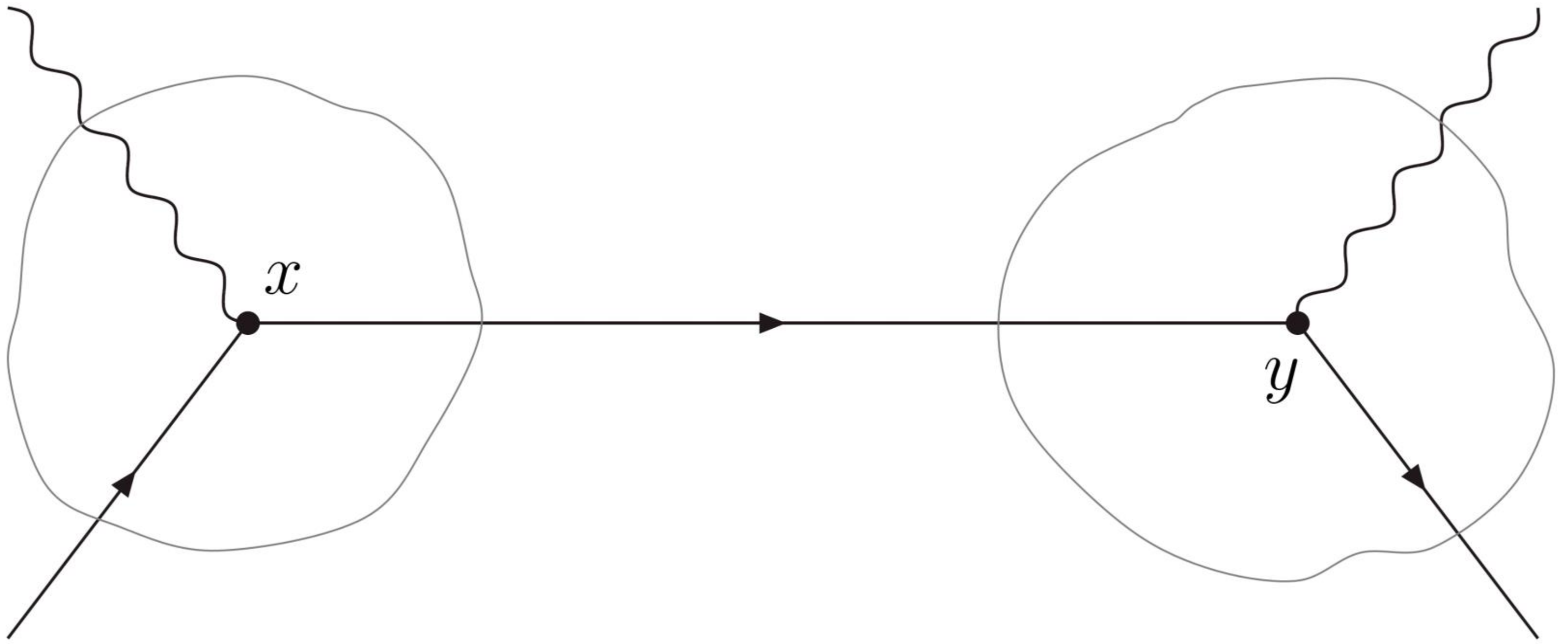
- R.P. Feynman, “Space-Time Approach to Quantum Electrodynamics,”
  Phys. Rev. **76** (1949), 769.

Standard scattering process





Scattering process in neutrino oscillation experiment setting



Distance-dependent propagators

To take into account the geometry of the experiment we have to integrate with respect to the coordinates x and y in such a way that the distance between the points x and y along the unit vector directed from the source to the detector is fixed and equal to L. This can be achieved by introducing the delta function $\delta((\vec{\mathbf{y}}-\vec{\mathbf{x}})\vec{\mathbf{n}}-\mathbf{L})$ into the integrand. Formally, this is equivalent to replacing the standard Feynman fermion propagator $\mathbf{S^c}(\mathbf{y}-\mathbf{x})$ by $\mathbf{S^c}(\mathbf{y}-\mathbf{x})\delta((\vec{\mathbf{y}}-\vec{\mathbf{x}})\vec{\mathbf{n}}-\mathbf{L})$.

The Fourier transform of this product is called *the distance-dependent fermion propagator in the momentum representation*:

$$S^{c}\left(p,\vec{L}\right)\equiv\int \mathrm{d}^4 z\ \mathrm{e}^{\mathrm{i}pz}\ S^{c}\left(z\right)\delta\left(\vec{n}\vec{z}-L\right).$$

The integral can be evaluated exactly:

$$S^{c}\left(p,\vec{L}\right)=\mathrm{i}\,\frac{\hat{p}+m+\vec{\gamma}\vec{n}\left(\vec{p}\vec{n}-\sqrt{\left(\vec{p}\vec{n}\right)^2+p^2-m^2}\right)}{2\sqrt{\left(\vec{p}\vec{n}\right)^2+p^2-m^2+\mathrm{i}\varepsilon}}\,\mathrm{e}^{-\mathrm{i}\left(\vec{p}\vec{n}-\sqrt{\left(\vec{p}\vec{n}\right)^2+p^2-m^2}\right)L}.$$

$\vec{L}=L\vec{n},\quad \hat{p}=p_\mu\gamma^\mu$. In what follows, we assume $\vec{p}\vec{n}\sim\left|\vec{p}\right|$.

In the paper

- W. Grimus and P. Stockinger, “Real oscillations of virtual neutrinos,” Phys. Rev. D **54** (1996) 3414

it was rigorously proved that the particles propagating at macroscopic distances are almost on the mass shell, i.e. for such particles

$$\left|p^2 - m^2\right| / \left(\vec{p}\vec{n}\right)^2 << 1.$$

In this approximation we get the distance-dependent propagator in the simple form:

$$S^{\mathrm{c}}\left(p,\vec{L}\right) = \mathrm{i}\,\frac{\hat{p}+m}{2\,\vec{p}\vec{n}}\,\mathrm{e}^{-\mathrm{i}\frac{m^2-p^2}{2\,\vec{p}\vec{n}}L}.$$

While constructing amplitudes, we use this propagator instead of the usual Feynman propagator.

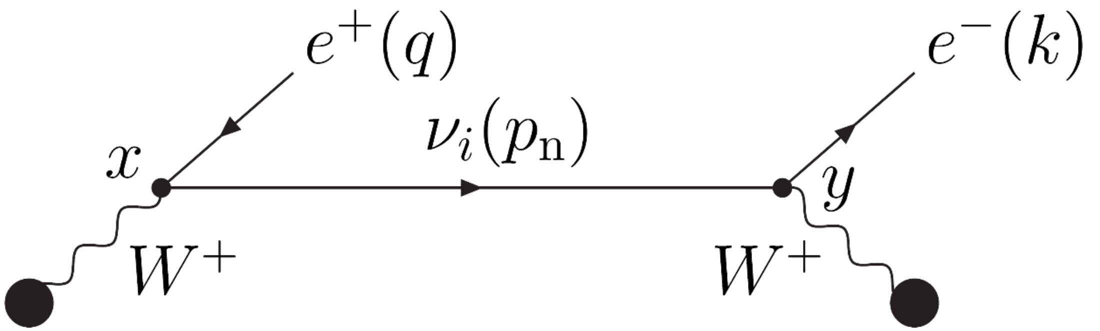
Neutrino oscillations

We work in the minimal extension of the SM by the right neutrino singlets. The weak interaction Lagrangian of leptons looks like

$$L_{\text{int}}^{\text{lep}} = -\frac{g}{2\sqrt{2}}\left(\sum_{i,k=1}^{3}\bar{l}_i\,\gamma^{\mu}\left(1-\gamma^{5}\right)U_{ik}\,\nu_k\,W_{\mu}^{-} + \text{h.c.}\right) + \frac{g\sin^2\theta_{\text{w}}}{\cos\theta_{\text{w}}}\sum_{i=1}^{3}\bar{l}_i\,\gamma^{\mu}\,l_i\,Z_{\mu} -$$
$$-\frac{g}{4\cos\theta_{\text{w}}}\sum_{i=1}^{3}\bar{l}_i\,\gamma^{\mu}\left(1-\gamma^{5}\right)l_i\,Z_{\mu} + \frac{g}{4\cos\theta_{\text{w}}}\sum_{k=1}^{3}\bar{\nu}_k\,\gamma^{\mu}\left(1-\gamma^{5}\right)\nu_k\,Z_{\mu}\,.$$

We also use the one-dimensional approximation, where the neutrino momenta are assumed to be directed from the centre of a source to the centre of a detector.

First, we consider the processes, where neutrinos are produced and detected in the charged-current interaction with nuclei.



The squared amplitude summed over the particle polarizations factorizes in the approximation of massless neutrinos:

$$\left\langle |M|^2 \right\rangle = \left\langle |M_{\mathrm{P}}|^2 \right\rangle \left\langle |M_{\mathrm{D}}|^2 \right\rangle \frac{1}{4(\vec{p}_{\mathrm{n}}\vec{n})^2} \left[1 - 4 \sum_{\substack{i,k=1 \\ i>k}}^{3} |U_{1i}|^2 |U_{1k}|^2 \sin^2\left(\frac{\Delta m_{ik}^2}{4\vec{p}_{\mathrm{n}}\vec{n}} L \right) \right].$$

Production process

Detection process

$$\Delta m_{ik}^2 = m_i^2 - m_k^2$$

Integrating over the particle momenta, we get:

$$\frac{\mathrm{d}^3 W}{\mathrm{d}^3 p} = \underbrace{\frac{\mathrm{d}^3 W_{\mathrm{P}}}{\mathrm{d}^3 p}}_{\text{Probability of } \nu(\vec{p}) \text{ production}} W_{\mathrm{D}} \underbrace{\left[1 - 4\sum_{\substack{i,k=1 \\ i>k}}^{3} \left|U_{1i}\right|^2 \left|U_{1k}\right|^2 \sin^2\left(\frac{\Delta m_{ik}^2}{4|\vec{p}|} L\right)\right]}_{P_{ee}(|\vec{p}|, L) \text{ is the standard oscillating factor}} . \quad \left(\vec{p}\vec{n} = |\vec{p}| \right)$$

Finally, since the experimental setting determines the direction of the neutrino momentum, but not its magnitude, to find the probability of detecting an electron we have to integrate this expression with respect to $|\vec{p}|$:

$$\frac{\mathrm{d}W}{\mathrm{d}\Omega} = \int_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{\mathrm{d}^3 W}{\mathrm{d}^3 p} |\vec{p}|^2 \,\mathrm{d}|\vec{p}| = \int_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{\mathrm{d}^3 W_{\mathrm{P}}}{\mathrm{d}^3 p} W_{\mathrm{D}}\, P_{ee}\left(|\vec{p}|, L\right) |\vec{p}|^2 \,\mathrm{d}|\vec{p}| \,.$$

Similar formulas can be obtained for the processes, where neutrinos are produced and detected in both charged and neutral-current interactions with electrons.

For processes in an external magnetic field the distance-dependent propagator should be replaced by

$$S_i^{\mathrm{c}}\left(p,\vec{L}\right)=\mathrm{i}\,\frac{\hat{p}\left(1-\mathrm{i}\,\vec{j}\,\vec{\gamma}\right)}{4p^0}\,\mathrm{e}^{\mathrm{i}\left(\frac{p^2-m_i^2}{2|\vec{p}|}+\mu_0 m_i H_\perp\right)L}+\mathrm{i}\,\frac{\hat{p}\left(1+\mathrm{i}\,\vec{j}\,\vec{\gamma}\right)}{4p^0}\,\mathrm{e}^{\mathrm{i}\left(\frac{p^2-m_i^2}{2|\vec{p}|}-\mu_0 m_i H_\perp\right)L}$$

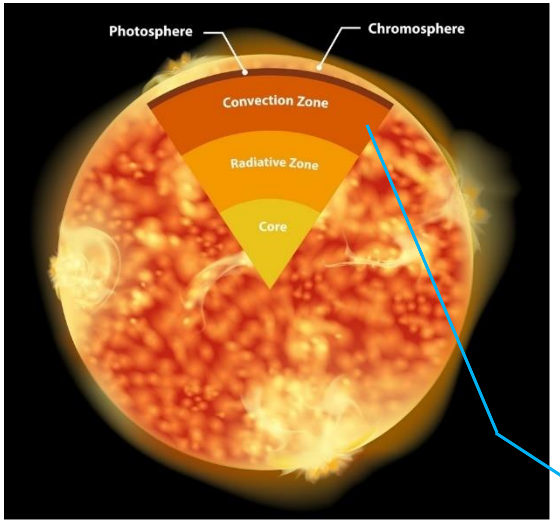
Solar neutrino problem

The solar neutrino problem consists in a large discrepancy between the flux of solar neutrinos predicted by the Standard Solar Model and the flux measured directly in experiments.

Experiment	BP2000	Measured	Measured/BP2000
Chlorine	$7.6^{+1.3}_{-1.1}$	2.56 ± 0.23	0.34 ± 0.06
GALLEX + GNO	128^{+9}_{-7}	$74.1^{+6.7}_{-7.8}$	0.58 ± 0.07
SAGE	128^{+9}_{-7}	$75.4^{+7.8}_{-7.4}$	0.59 ± 0.07
^{8}B-Kamiokande	$5.05\ [1.00^{+0.20}_{-0.16}]$	$2.80\ [1.00 \pm 0.14]$	0.55 ± 0.13
^{8}B-Super-Kamiokande	$5.05\ [1.00^{+0.20}_{-0.16}]$	$2.40\ [1.00^{+0.04}_{-0.03}]$	0.48 ± 0.09
hep-Super-Kamiokande	9.3	$11.3(1 \pm 0.8)$	~ 1

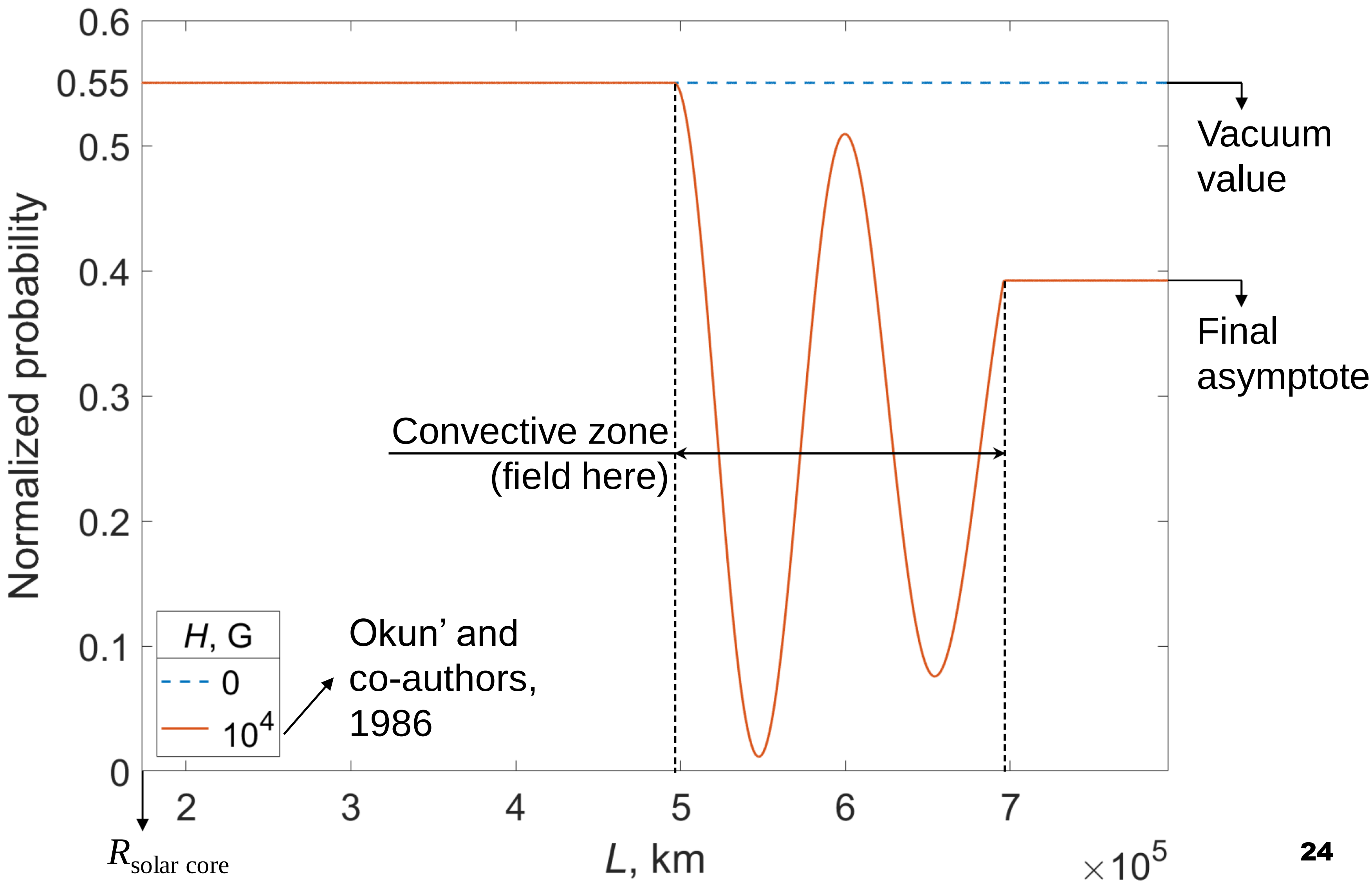
For the neutrinos produced in the solar core, one must average the process probability over the ball of radius 173 000 km. In so doing, the energy-dependent oscillations vanish, no matter what the production reaction is. We arrive at:

$$\overline{W} = \sum_{i=1}^{3} \left|U_{1i}\right|^4 \cos^2\big(\underbrace{\mu_0 m_i \overline{H} L}\big)$$

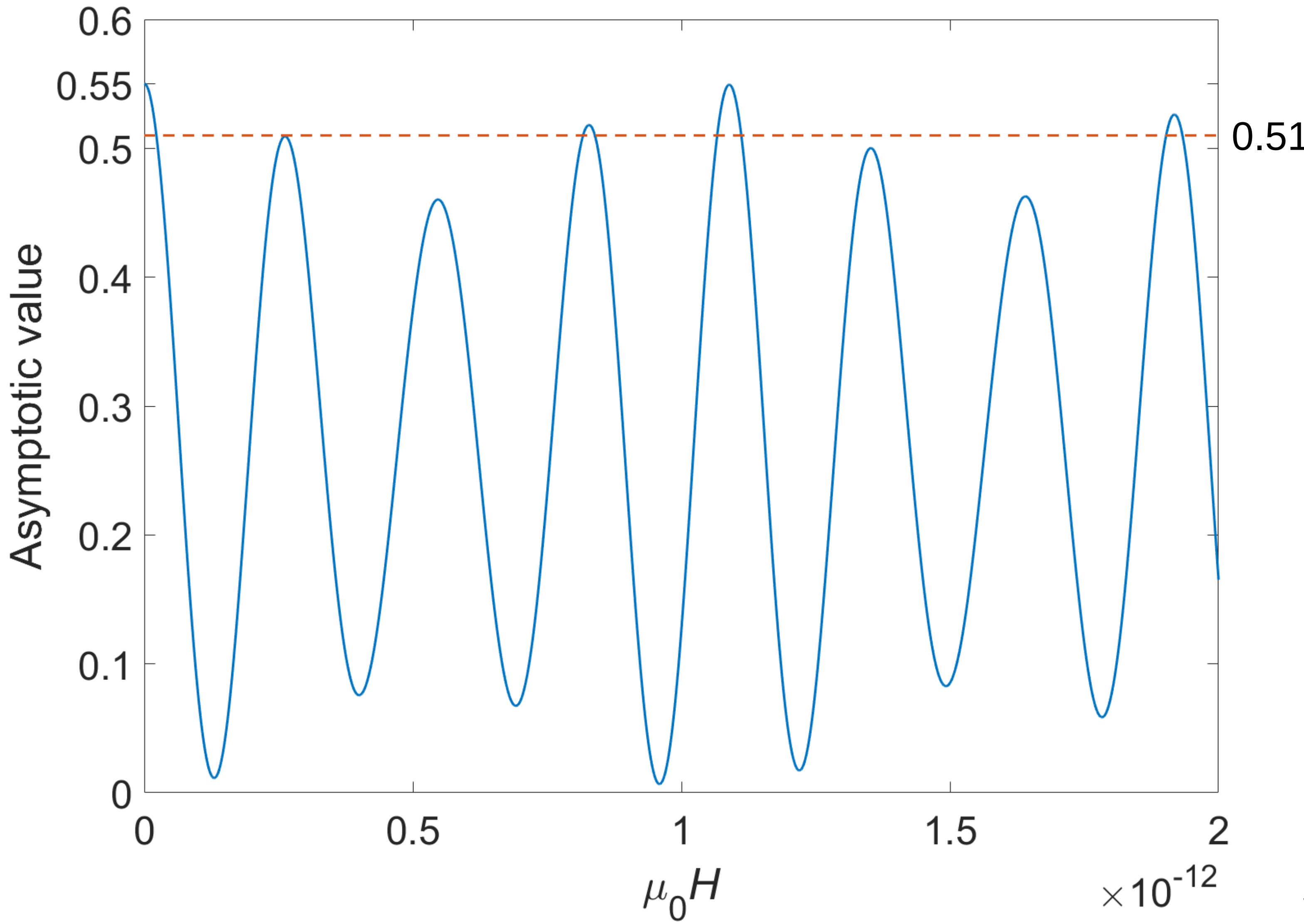


The phase, accumulated by the neutrino on its path through the area of the field in the Sun. Here we assume that the convective zone of the Sun has thickness L_{conv} = 200 000 km and its magnetic field is constant.

Borexino and GEMMA experiments set a limit on the neutrino magnetic moment: $\mu_{\nu_e} \le 2.8 \cdot 10^{-11} \mu_B$.



Since L_{conv} is almost fixed and the product $\mu_0 \overline{H}$ is under variation, it is useful to plot $\overline{W}\left(\mu_0 \overline{H}\right)$:



Conclusion

- A new perturbative formalism is developed, which allows one to consistently describe neutrino oscillations within the Standard Model without use of the neutrino flavor states.

- It is also shown that, in a magnetic field, the energy-dependent oscillations of solar neutrinos cannot be visible here on Earth. Only the energy-independent oscillation average is observable, which can be a part of the solution to the solar neutrino problem.

Thank you!