Flux Compactification for the Critical Non-Abelian Vortex and Quark masses

A. Yung

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1 Introduction

Seiberg and Witten 1994 : Confinement in the monopole vacuum of $\mathcal{N} = 2$ supersymmetric QCD

Cascade gauge symmetry breaking:

- SU(N)$\longrightarrow$ U(1)$^{N-1}$ condensate of adjoint scalars

  Example: SU(2)$\longrightarrow$ U(1)

- U(1)$^{N-1}$ $\longrightarrow$ 0 condensate of monopoles

At the last stage Abelian Abrikosov-Nielsen-Olesen flux tubes are formed.

Abelian confinement

Non-Abelian vortex strings

In the search for a non-Abelian confinement

Non-Abelian vortex strings were found in $\mathcal{N} = 2$ U(N) QCD

Hanany, Tong 2003

Auzzi, Bolognesi, Evslin, Konishi, Yung 2003

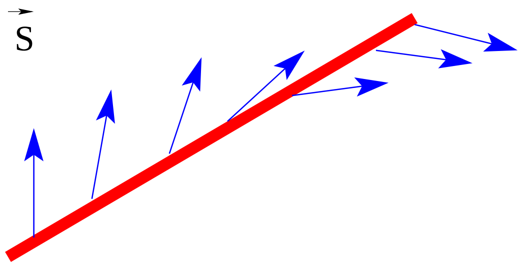
Shifman Yung 2004

Hanany Tong 2004

Non-Abelian string : Orientational zero modes

Rotation of color flux inside SU(N).

Non-Abelian vortex string is BPS and preserves $\mathcal{N} = (2, 2)$ supersymmetry on its world sheet.



Shifman and Yung, 2015: Non-Abelian vortex in $\mathcal{N}=2$ supersymmetric QCD can behave as a critical superstring

Idea:

Non-Abelian string has more moduli then Abrikosov-Nielsen-Olesen string.

It has translational + orientaional moduli

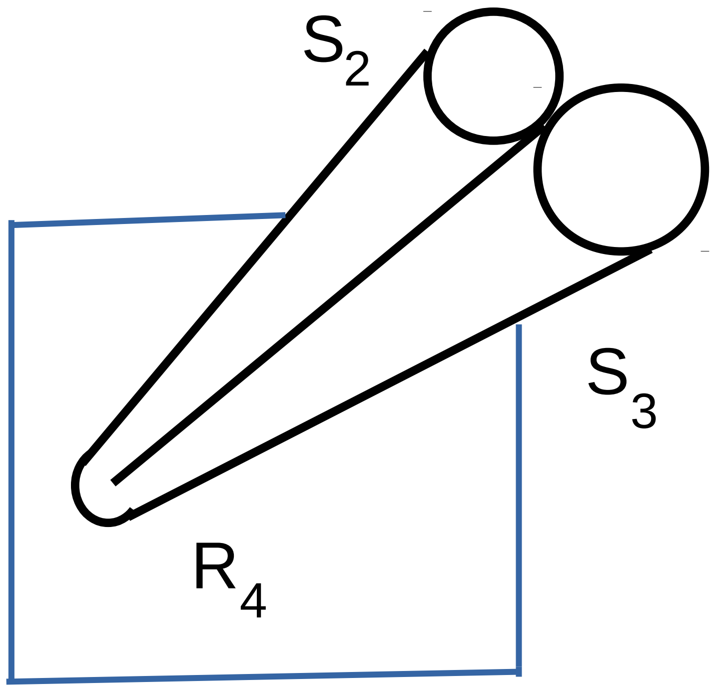
We can fulfill the criticality condition: In $\mathcal{N}=2$ QCD with U$(N=2)$ gauge group and $N_f=4$ quark flavors.

- The solitonic non-Abelian vortex have six orientational and size moduli, which, together with four translational moduli, form a ten-dimensional space.
- For $N_f=2N$ 2D world sheet theory on the string is conformal.

Target space of the world sheet 2D σ-model is

$$R^4 \times Y_6,$$

where Y_6 is a non-compact six dimensional Calabi-Yau manifold, the conifold.



Compactification on Y_6

Strings in the $U(N)$ theories are stable; they cannot be broken. Thus, we deal with the closed string.

We studied states of closed type IIA string propagating on $R^4 \times Y_6$ and interpreted them as hadrons in 4D $\mathcal{N} = 2$ QCD.

4D Massless states = Deformations of 10D metric preserving Ricci flatness

Deformations preserving Ricci-flat metric on a Calabi-Yau manifold are $h^{(1,1)}$ Kahler form deformations and $h^{(1,2)}$ deformations of the complex structure.

$h^{(p,q)}$ are Hodge numbers – numbers of harmonic (p, q) forms on Y_6.

Conifold == non-compact CY.

Looking for states with normalizable wave function over Y_6

String states localized near the conifold singularity.

The only 4D (logarithmically) normalizable state is associated with deformations of the complex structure

Singular conifold

$$w_1^2 + w_2^2 + w_3^2 + w_4^2 = 0$$

Deformed conifold

$$w_1^2 + w_2^2 + w_3^2 + w_4^2 = b$$

Complex modulus b – massless BPS baryon in 4D SQCD

Quark masses

Now let us try to introduce small quark masses in 4D SQCD.

On the field theory side this is the most natural deformation one can think of. The only deformation of 4D $\mathcal{N} = 2$ QCD without higher derivative terms allowed by $\mathcal{N} = 2$ supersymmetry.

However, on the string theory side this generates a mass term in the 2D world sheet σ-model breaking its conformal invariance on the classical level.

In terms of 10D supergravity we switch on a deformation and have to solve gravity equations of motion to find a new string vacuum.

Our task: To interpret quark masses in terms of a deformation of the 10D gravity.

Idea: Quark masses are described by a flux ”compactification” on the conifold.

Motivation

Since the massless b-baryon is a BPS state its mass is determined by its baryonic charge $Q_{U(1)_B} = 2$.

$$m_b = |m_1 + m_2 - m_3 - m_4|$$

where m_A, $A = 1, ..., 4$ are quark masses.

On the other hand, generically fluxes give rise to a potential for moduli on the Calabi-Yau manifold.

Therefore, we expect that

$\boxed{\text{Flux in 10D}}$ $\Longleftarrow\!\!\Longrightarrow$ $\boxed{\text{Quark masses in 4D}}$

For type IIA string theory.

even-form RR fluxes $\Longrightarrow$ potential for Kahler moduli

NS 3-form flux $\Longrightarrow$ potential for complex structure moduli

We focus on NS 3-form flux H_3

2 3-form flux

$$S_{10D} = \int d^{10}x\sqrt{-G}\,\Big\{R - \frac{1}{2}G^{MN}\,\partial_M\Phi\,\partial_N\Phi$$
$$- \frac{e^{-\Phi}}{12}\,H_{MNL}H^{MNL} - \frac{1}{2}\,e^{\frac{\Phi}{2}}\,F_4^2\Big\}, \qquad H_3 = dB_2$$

Use perturbation theory at small H_3. Solve equations for H_3 neglecting the back reaction on the metric and dilaton.

$$ds_{10}^2 = T\,\left[-(dt)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2\right] + ds_6^2$$

Conifold metric

$$ds_6^2 = dr^2 + \frac{r^2}{6}(e_{\theta_1}^2 + e_{\varphi_1}^2 + e_{\theta_2}^2 + e_{\varphi_2}^2) + \frac{r^2}{9}e_\psi^2$$

$$e_{\theta_1} = d\theta_1, \qquad e_{\varphi_1} = \sin\theta_1\, d\varphi_1\,,$$
$$e_{\theta_2} = d\theta_2, \qquad e_{\varphi_2} = \sin\theta_2\, d\varphi_2\,,$$
$$e_\psi = d\psi + \cos\theta_1 d\varphi_1 + \cos\theta_2 d\varphi_2$$

Equation for H_3

$$d(e^{-\Phi} * H_3) = e^{-\Phi} d(*H_3) = 0$$

Solution

$$H_3 = \mu \, \frac{dr}{r} \wedge \left(e_{\theta_1} \wedge e_{\varphi_1} - e_{\theta_2} \wedge e_{\varphi_2}\right)$$

3 Potential for the modulus b

Kinetic term for H_3

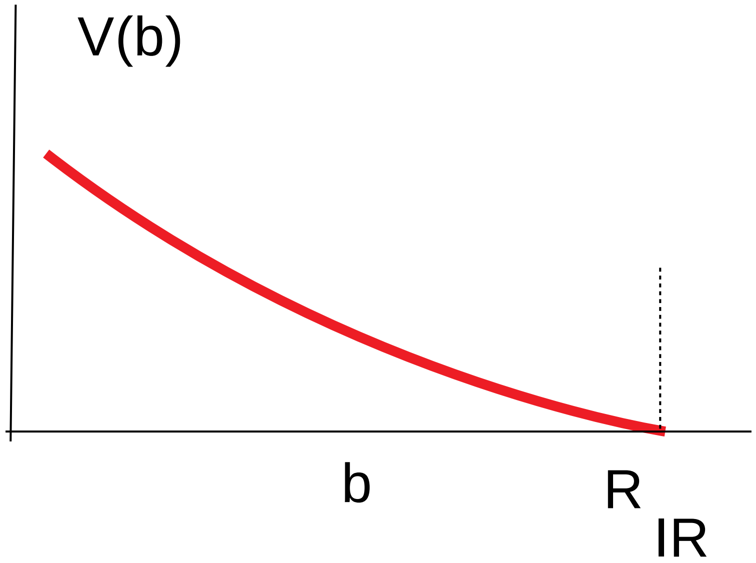
$$-\frac{e^{-\Phi}}{12}\int d^{10}x\sqrt{-G}\,H_{MNL}H^{MNL}$$

Produces the potential

$$V(b)=\mathrm{const}\,T^2\,e^{-\Phi}\,\mu^2\,\log\frac{R_{\mathrm{IR}}^3}{|b|}$$

where R_{IR} is the infrared cut-off.

Run-away vacuum



$$R_{IR} \longrightarrow \infty$$

No mass term for b.

4 Interpretation

Constraints

$$m_1 + m_2 - m_3 - m_4 = 0, \qquad m_1 m_2 - m_3 m_4 = 0$$

This leads us to

$$\mu^2 = |m_1 - m_2|^2, \qquad m_3 = m_1, \qquad m_4 = m_2$$

$$< |b| > \to \infty$$

Conifold degenerates

The sphere S_2 degenerates, radial coordinate shrinks, the size of the sphere S_3 goes to infinity.

Non-Abelian string becomes Abelian. No orientational modes. Only two size modes.

5 Conclusions

- The special choice of quark masses in 4D SQCD with $m_3 = m_1$ and $m_4 = m_2$ is interpreted as NS 3-form flux H_3.
- H_3 flux lifts the Higgs branch of the baryon b in 4D SQCD.
- The vacuum is of a run-away type, $< |b| > \to \infty$.
- At the run-away vacuum conifold degenerates to lower dimensions.
- This is interpreted as a flow from a non-Abelian vortex string to an Abelian one.