Chiral effects in external gravitational field

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Part I Motivation

QFT is to a great extent concentrated on anomalies (which distinguish QFT symmetries from classical FT symmetries)

In particular, in case of axial vector current J_5^α

$$\partial_\alpha J_5^\alpha = C_5 \vec{E} \cdot \vec{B} \qquad (\textit{gauge anomaly})$$

$$\nabla_\alpha \mathcal{J}_5^\alpha = C_{gr} \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} R^\alpha{}_{\beta\mu\nu} \tilde{R}^\beta{}_{\alpha\rho\sigma} \qquad (\textit{gravitational anomaly})$$

where $\vec{E}, \vec{B}$ electromagnetic fields, $R^\alpha{}_{\beta\gamma\delta}$ is Riemann tensor, $C_5 = 1/(4\pi^2)$, $C_{gr} = -1/(768\pi^2)$ for a Weyl spinor

Quantum hydrodynamics

Anomaly in terms of macroscopic quantities
(temperature T, chemical potential μ, 4-velocity of fluid u_α)
Son& Surowka (2009) For ideal fluid

$$J_5^\alpha = n_5 u^\alpha + C_\omega \omega^\alpha \qquad (\omega^\alpha = 1/2\epsilon^{\alpha\beta\gamma\delta} u_\beta \partial_\gamma u_\delta)$$

$$C_\omega = \mu^2 C_5 + T^2 C_T \qquad (C_T = 1/12)$$

where C_5 is the same as in front of the anomaly

The C_ω term is called chiral vortical effect

Charge density J_5^0 is a
mixture of microscopic and macroscopic helical motions,
suggesting the possibility of transitions between them.

Encouragement from Phenomenology

–The ratio η/s is smallest for the QGP (close to ideal fluid)

"Global Λ hyperon polarization in nuclear collisions"
STAR Collaboration, Nature 548, 62 (2017)

Quark-Gluon Plasma formed in nuclear collisions as a relativistic fluid at local thermodynamic equilibrium with acceleration and vorticity

Acceleration can be replaced by gravitational field
– QGP as a window to grav. interactions.

First attempted in "Thermal Hadronization and Hawking-Unruh Radiation in QCD",
P. Castorina, D. Kharzeev, H. Satz, Eur.Phys.J.C 52 (2007)

Physics of equilibrium vs physics of gravity

Probably, two most famous examples of similarity are:

∗ Transport induced by gradient of temperature $\vec{\nabla} T$ identical to that induced by acceleration $\vec{a}_{gr}$

$$\frac{\vec{\nabla} T}{T} \rightarrow -\vec{a}_{gr}$$

as a reflection of universality of the both (Luttinger (1964))

∗∗ hypothesis: gravity is not fundamental and could be replaced by macroscpic entropic force (E. Verlinde (2011)) :

$$\vec{F}(X_0)_{entropic} = T\vec{\nabla}_X S(X)|_{X_0}$$

(S is entropy, X is a characteristic of macrostate)

References to original papers

"Axial current in rotating and accelerating medium" , G. Prokhorov , O. Teryaev , V. I. Zakharov, 1805.12029 [hep-th];

"Thermodynamics of accelerated fermion gases and their instability at the Unruh temperature", G. Y. Prokhorov, O. V. Teryaev , V.I. Zakharov 1906.03529 [hep-th]

Also e-prints 2009.11402 [hep-th], 2109.06048 [hep-th]

"On chiral vortical effect in accelerated matter", P.G. Mitkin , V.I. 2103.01211 [hep-th] and in preparation

Part II Linear in acceleration effects

Interesting effect claimed first by

"Chiral and Gravitational Anomalies on Fermi Surfaces"
G. Basar, D. E. Kharzeev , I. Zahed, 1307.2234 [hep-th]

Considered motion of levels of the Fermi sphere at finite μ caused by external grav. field, a la Nielsen&Ninomiya

$$\partial_\alpha J_5^\alpha \;=\; \frac{\mu^2}{2\pi^2}(\vec{a}_{gr}\cdot\vec{\Omega})$$

where $\vec{a}_{gr}$ is the grav. acceleration, $\vec{\Omega}$ is the angular velocity

Upon substitution $\vec{a_{gr}} \;\to -\vec{\nabla}T/T$ looks as a novel chiral thermal effect

Interpretation as a puzzle

Gravimagnetic fields, (analogy between magnetic field and field of rotation),

$$\vec{B}_{gr} = 2\epsilon\vec{\Omega}, \quad \vec{E}_{gr} = -\epsilon\vec{\nabla}\phi_{gr}$$

where ϵ is energy of test particle

Analog (with all the coefficients) of the gauge anomaly
But: there is no place for such an anomaly in gravitational case since it is not "gauge invariant"

Need another explanation., or fresh appeoach

Effective theory

To describe equilibrium, effective interaction is introduced

$$\hat{H}_{eff} \; = \; \mu\hat{Q} \;\; or, \; in\ hydro \;\; \mathcal{L}_{eff} \; = \; \mu u_\alpha J^\alpha$$

where u_α is 4-velocity of element of a fluid, $\hat{Q}$ is conserved charge and μ is the associated chemical potential

Compare L_{eff} with corresponding gauge-potential interaction $\qquad \mathcal{L}_{fund} = eA_\alpha J^\alpha$

Clearly, effect of the thermodynamic interaction is calculable from known anomaly through the substitution:

$$eA_\alpha \;\to\; \mu u_\alpha$$

In this way we rederive directly the chiral vortical effect:
$(J_5^\alpha)_{vortical} \; = \; C_\omega \mu^2 \omega^\alpha$

Extra conservation law

Thermodynamic, effective interaction applies in infrared and is absent in the ultraviolet. Anomalous current is infrared sensitive while its divergence is ultraviolet sensitive and cannot be changed by thermodynamics

The way out: impose conservation of the vortical current in the media

$$\partial_\alpha(\mu^2\omega^\alpha) \;=\; 0$$

Conservation law of fluid helicity, non-Noether in nature known, since long, to hold in case of ideal fluid
(The helical motion is manifest in expression for $\mu^2\omega^\alpha$)

Fake phenomenology?

In presence of gravity the conservation law takes on form

$$\nabla_\alpha(\mu^2\omega^\alpha) \;=\; 0$$

where ∇_α is the covariant derivative.
This equation unifies ordinary derivative $\partial_\alpha J^\alpha$ and gravimagnetic "anomaly". The physical meaning is that equations of motion in accelerated frame in presence of gravity are the same as in rest frame without gravity.

However, phenomenology is made in flat-space terms, and in flat-space interpretation the current is not conserved

Thus equivalence principle imitates non-conservation of the current.

Unification of anomalies?

What we are getting (P.G. Mitkin+VIZ 2103.01211)

$$\partial_\alpha J_5^\alpha \ = C_5(\vec{\Omega}\cdot\vec{a_{gr}}) + C_{gr}R\tilde{R}$$

A kind of unification of anomalies since C_5 and C_{gr} enter same equation. Can rewrite

$$(\partial_\alpha J - a_\alpha)J_5^\alpha \ = \ C_{gr}R\tilde{R} \ \ where \ \ a_\alpha \equiv u^\beta\partial_\beta u_\alpha$$

Strange idea: $R\tilde{R}$ is to be constructed on the same a_α
The idea seems to be true:

$F\tilde{F} \ \sim \ R\tilde{R}$ if "potential" A_α is spin connection
(G. Volovik, 2104.01020)

P.G. Mitkin+VIZ, in preparation

III Duality between statistics and field theory

∗ Back to equilibrium of accelerated and rotated medium. Statistically, effective, or macroscopic interaction

$$\hat{H}_{eff} \;=\; \vec{\Omega}\cdot\hat{\vec{M}} + \vec{a}\cdot\hat{\vec{K}}$$

where $\vec{M}$ is angular momentum and $\vec{K}$ is the boost
∗∗On other hand, in FT effect of rotation and acceleration can be described in terms of an external grav. field

$$\hat{H}_{fund} \;=\; \frac{1}{2}\hat{\Theta}^{\alpha\beta}h_{\alpha\beta}$$

where $\Theta^{\alpha\beta}$ is the energy momentum tensor, $h_{\alpha\beta}$ is the grav. potentials accommodating the same $\vec{\Omega}$, $\vec{a}$
∗ ∗ ∗ Evaluate "external probes", $<\Theta^{\alpha\beta}>, <J_5^{\alpha}>$ for quantum particles. Results are expected to be the same.

More on statistical approach

- The scheme known to work in case of pure rotation. Inclusion of acceleration is recent, see Becattini (2017)
- Statistical averaging involves density operator $\hat{\rho}$ where $\hat{\rho} \ = \ \frac{1}{Z}\exp\left(- b_\alpha \hat{P}^\alpha + \bar{\omega}_{\alpha\beta}\hat{J}^{\alpha\beta}\right)$ where $\hat{J}^{\alpha\beta}$ are generators of the Lorentz transformations $\bar{\omega}_{\alpha\beta} \ = \ \partial_\alpha(u_\beta/T) - \partial_\beta(u_\alpha/T)$,
- The boost operators $\hat{K}^\alpha$ are conserved but do not commute with $\hat{H}$. A novel feature!
- Duality with gravity looks very questionable since in statistics dipole moments and spins are treated on equal while in case of gravity dipole moments are essentially forbidden by CP invariance

Statistics-gravity duality at work

Evaluate energy density Θ_{00} of quantum massless spinors as function of independent a, T exploiting 'novel' density operator (G. Prokhorov, O. Teryaev, VZ+references)

$$\rho_{vac} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2}$$

First ever evaluation of vac. energy without subtractions.

$$get \qquad \rho_{vac}(T_{Unruh}) = 0$$

as is expected from general covariance

One-loop exact evaluation of the Unruh temperature

On the other side of duality

Energy density of same quantum particles in geometrical terms (metrics determined by external gravitational field)

metric is Euclidean Rindler space with boundary and conical singularity on the boundary

$$\epsilon_{vac} = \Big(\frac{7\pi^2 T^4}{60} + \frac{T^2}{24 r^2} - \frac{17}{960\pi^2 r^4}\Big)$$

where r is the distance along the cone, related to acceleration (the result known since long)

Statistical calculation in flat space fits exactly field theory on a manifold with a boundary

Axial current on gravitational background

Consider slowly rotating, accelerated gas of massless fermions. Axial current $\vec{J}_5$, by dimension and polynomiality

$$\vec{J}_5 = c_T T^2 \vec{\Omega} + c_a a^2 \vec{\Omega}$$

c_T term is thermal contribution, calculable in terms of Fermi distribution

c_a term is the vacuum contribution, exists in absence of any medium

Statistical approach misses the anomaly,

or vacuum component

Forced to switch to a less ambitious form of duality.

Matching anomaly and Hawking radiation

Constant c_a is determined by field theory at $T = 0$.
Namely, in a simple enough geometry with intrinsic rotation Ω and acceleration a, chiral gravitational anomaly:

$$\partial_\alpha J_5^\alpha = c_a \partial_\alpha \left(a^2 \Omega^\alpha \right)$$

At spatial infinity $a_\infty = 0$, The difference between the currents at infinity and at finite a is uniquely fixed by the anomaly. Thus, c_a is related to C_{gr}

c_T term is like subtraction constant,
to be yet determined at this point.

Axial current, cnt'd

Coefficient c_T determined by pure thermal field theory (a=0). The result of a standard calculation $\vec{J}_5 = \vec{\Omega}(T^2/6)$

Combining the two terms (M. Stone (2018))

$$\vec{J}_5 \ = \ \vec{\Omega}\Big(\frac{T^2}{6} - \frac{a^2}{6(2\pi)^2}\Big)$$

The current is vanishing at $T \ = \ T_{Unruh}$ as it should vanish provided that Minkowski vacuum is stable under rotation

Crossing into the black hole

At black-hole horizon $T = T_{Unruh}$
Going to $T < T_{Unruh}$ corresponds crossing into the BH
Statistical approach reveals instability due to quantum particles (G. Prokhorov et al. (2019)) which fits nicely field theory (A. Polyakov et al. 1803.09168)

Higher spins

Original calcukations now address mostly the issue of higher-spin quantum particles.
Vacuum component $c_a \sim (2S^3 - S)$, difficult to recoincile with thermal component $c_T \sim S$
Also, for higher spins negative modes

Conclusions

We had three parts:

- "Introduction"–phenomenology of heavy-ion collisions indicates, that gravitational effects might be relevant
- "Linear in acceleration terms"-governed by equivalence principle, conservation of fluid helicity, role of "gravimagnetic anomaly" is clarified, hints on "unification of anomalies"
- "Higher orders in acceleration"–possible extension of the equivalence principle, instabilities, higher spins at the horizon