Ultraperipheral proton-proton collisions at the Large Hadron Collider: survival factor

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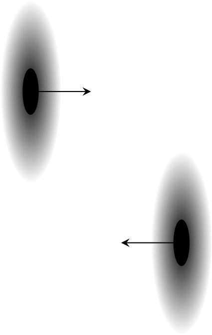
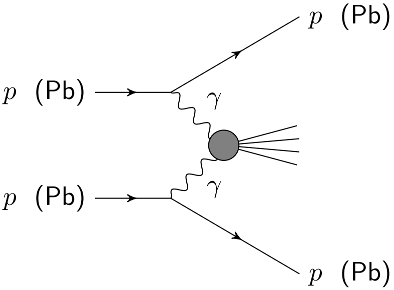
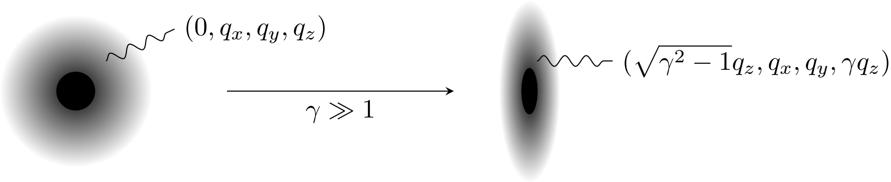
Summary

- Ultraperipheral collisions and equivalent photons approximation
  - assuming the colliding particles interact only electromagnetically
  - taking into account other interactions
- Survival factor
- Production of a pair of heavy charged particles ($pp \to pp\chi^+\chi^-$)
- Comparison to the ATLAS experiment ($pp \to pp\mu^+\mu^-$)

Ultraperipheral collisions at the LHC

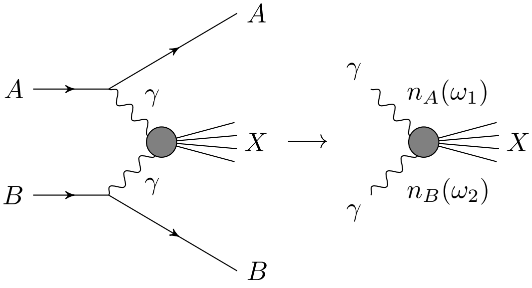






Photon virtuality: $Q^2 \equiv -q^2 = q_x^2 + q_y^2 + q_z^2 \ll (\gamma q_z)^2 \equiv \omega^2$

Equivalent photons approximation



$$\sigma(AB \to ABX) = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \, \sigma(\gamma\gamma \to X) \, n_A(\omega_1) \, n_B(\omega_2)$$

$$n(\omega) = \frac{2Z^2\alpha}{\pi\omega} \int_0^\infty \left[\frac{F(q_\perp^2 + (\omega/\gamma)^2)}{q_\perp^2 + (\omega/\gamma)^2} \right]^2 q_\perp^3 \mathrm{d}q_\perp$$

$q_\perp$ — photon transversal momentum, $F(Q^2)$ — Dirac form factor.

Proton form factor

$$F_p(Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau}$$

$$\tau = \frac{Q^2}{4m_p^2}. \; Q^2 \lesssim \Lambda^2_{\mathsf{QCD}} \Rightarrow \tau \lesssim 0.01.$$

Dipole approximation:

$$G_E(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2}$$

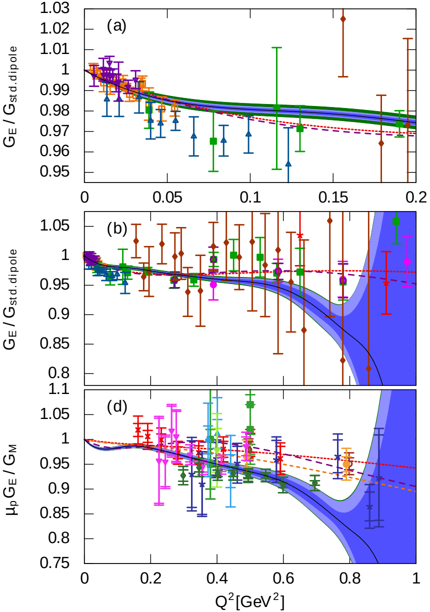
$$G_M(Q^2) = \frac{\mu_p}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2}$$

$$\mu_p = 2.79$$

$$\Lambda^2_{\mathsf{std.}} = 0.71 \text{ GeV}^2$$

$$r_p^2 = (0.8751 \text{ fm})^2 = 12/\Lambda^2$$

$$\Rightarrow \Lambda^2 = 0.61 \text{ GeV}^2$$



[1307.6227]

Proton EPA spectrum

Proton form factor with magnetic contribution taken into account:

$$F_p(Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau},$$

$$G_E(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2}, \qquad G_M(Q^2) = \frac{\mu_p}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2}$$

EPA spectrum:

$$n_p(\omega) = \frac{2Z^2\alpha}{\pi\omega} \int_0^\infty \left[\frac{F(q_\perp^2 + (\omega/\gamma)^2)}{q_\perp^2 + (\omega/\gamma)^2} \right]^2 q_\perp^3 \mathrm{d}q_\perp$$

$$= \frac{\alpha}{\pi\omega} \left\{ \left(1 + 4u - 2(\mu_p - 1)\frac{u}{v}\right) \ln\left(1 + \frac{1}{u}\right) - \frac{24u^2 + 42u + 17}{6(u+1)^2} \right.$$

$$+ \frac{\mu_p - 1}{(v-1)^4} \left[\frac{\mu_p - 1}{v - 1}(1 + 4u + 3v) - 2\left(1 + \frac{u}{v}\right) \right] \ln\frac{u+v}{u+1}$$

$$+ (\mu_p - 1)\frac{6u^2(v^2 - 3v + 3) + 3u(3v^2 - 9v + 10) + 2v^2 - 7v + 11}{3(u+1)^2(v-1)^3}$$

$$\left. -(\mu_p - 1)^2 \frac{24u^2 + 6u(v+7) - v^2 + 8v + 17}{6(u+1)^2(v-1)^4} \right\},$$

$$u = (\omega/\Lambda\gamma)^2, \; v = (2m_p/\Lambda)^2.$$

Photon-photon luminosity

$$\sigma(AB \to ABX) = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \sigma(\gamma\gamma \to X)\, n_A(\omega_1)\, n_B(\omega_2)$$

Let $s = 4\omega_1\omega_2$, $y = \frac{1}{2}\ln\frac{\omega_1}{\omega_2}$ ($\sqrt{s}$ — invariant mass of the system produced, y — its rapidity). Then

$$\sigma(AB \to ABX) = \int_0^\infty \mathrm{d}s\, \sigma(\gamma\gamma \to X) \frac{\mathrm{d}L_{AB}}{\mathrm{d}s},$$

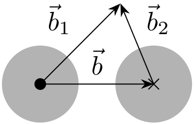
where

$$\frac{\mathrm{d}L_{AB}}{\mathrm{d}s} = \frac{1}{4} \int_{-\infty}^{\infty} \mathrm{d}y\, n_A\left(\tfrac{\sqrt{s}}{2}\mathrm{e}^{y}\right)\, n_B\left(\tfrac{\sqrt{s}}{2}\mathrm{e}^{-y}\right)$$

is the photon-photon luminosity.

EPA spectrum and cross section

With non-electromagnetic interactions of the colliding particles neglected:



$$n(\omega) = \frac{2Z^2\alpha}{\pi\omega} \int\limits_0^\infty \left[\frac{F(q_\perp^2 + (\omega/\gamma)^2)}{q_\perp^2 + (\omega/\gamma)^2}\right]^2 q_\perp^3 \,\mathrm{d}q_\perp,$$

$$\sigma(AB \to ABX) = \int\limits_0^\infty \mathrm{d}\omega_1 \int\limits_0^\infty \mathrm{d}\omega_2 \,\sigma(\gamma\gamma \to X)\, n_A(\omega_1)\, n_B(\omega_2).$$

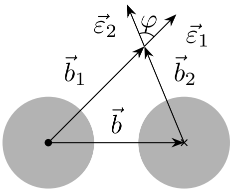
With non-electromagnetic interactions of the colliding particles taken into account:

$$n(\omega) = \int n(b,\omega)\,\mathrm{d}^2 b, \quad n(b,\omega) = \frac{Z^2\alpha}{\pi^2\omega}\left[\int\limits_0^\infty \frac{F(q_\perp^2 + (\omega/\gamma)^2)}{q_\perp + (\omega/\gamma)^2} J_1(bq_\perp)\, q_\perp^2 \,\mathrm{d}q_\perp\right]^2,$$

$$\sigma(AB \to ABX)$$
$$= \int\limits_0^\infty \mathrm{d}\omega_1 \int\limits_0^\infty \mathrm{d}\omega_2 \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \,\sigma(\gamma\gamma \to X)\, n_A(b_1,\omega_1)\, n_B(b_2,\omega_2)\, P_{AB}(|\vec{b}_1 - \vec{b}_2|).$$

$P_{AB}(b)$ is the probability for the colliding particles to survive after the collision with the impact parameter b.

Polarization



$$\sigma(AB \to ABX) = \int_0^\infty \mathrm{d}s \left[\sigma_\parallel(\gamma\gamma \to X) \frac{\mathrm{d}L_{AB}^\parallel}{\mathrm{d}s} + \sigma_\perp(\gamma\gamma \to X) \frac{\mathrm{d}L_{AB}^\perp}{\mathrm{d}s} \right],$$

where

$$\frac{\mathrm{d}L_{AB}^\parallel}{\mathrm{d}s} = \frac{1}{4} \int_{-\infty}^{\infty} \mathrm{d}y \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \, n_A\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^y\right) n_B\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right) P_{AB}(b) \cos^2\varphi,$$

$$\frac{\mathrm{d}L_{AB}^\perp}{\mathrm{d}s} = \frac{1}{4} \int_{-\infty}^{\infty} \mathrm{d}y \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \, n_A\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^y\right) n_B\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right) P_{AB}(b) \sin^2\varphi$$

are photon-photon luminosities, $b = \sqrt{b_1^2 + b_2^2 - 2 b_1 b_2 \cos\varphi}$.

Survival factor

$$\sigma(AB \to ABX) \\ = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \, \sigma(\gamma\gamma \to X) \, n_A(b_1, \omega_1) \, n_B(b_2, \omega_2) \, P_{AB}(|\vec{b}_1 - \vec{b}_2|)$$

vs

$$\sigma(AB \to ABX)|_{P=1} = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \, \sigma(\gamma\gamma \to X) \, n_A(\omega_1) \, n_B(\omega_2)$$

$$\langle S_{AB}^{\gamma\gamma} \rangle = \sigma(AB \to ABX) / \, \sigma(AB \to ABX)|_{P=1}$$

$$S_{AB}^{\gamma\gamma} = \frac{\mathrm{d}L_{AB}/\mathrm{d}s \, \mathrm{d}y}{\mathrm{d}L_{AB}/\mathrm{d}s \, \mathrm{d}y|_{P=1}} = \frac{\mathrm{d}L_{AB}/\mathrm{d}\omega_1 \, \mathrm{d}\omega_2}{\mathrm{d}L_{AB}/\mathrm{d}\omega_1 \, \mathrm{d}\omega_2|_{P=1}}$$

$$S_{AB} = \frac{\mathrm{d}L_{AB}/\mathrm{d}s}{\mathrm{d}L_{AB}/\mathrm{d}s|_{P=1}}$$

Here L_{AB} is the luminosity neglecting photons polarizations: $L_{AB} = L_{AB}^{\parallel} + L_{AB}^{\perp}$.

Proton EPA spectrum

$$F_p(Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau},$$

$$G_E(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2}, \quad G_M(Q^2) = \frac{\mu_p}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2}, \quad \tau = \frac{Q^2}{4m_p^2},$$

$$n_p(b,\omega) = \frac{Z^2\alpha}{\pi^2\omega}\left[\int_0^\infty \frac{F(q_\perp^2 + (\omega/\gamma)^2)}{q_\perp + (\omega/\gamma)^2} J_1(bq_\perp)\, q_\perp^2 \mathrm{d}q_\perp\right]^2$$

$$= \frac{\alpha}{\pi^2\omega}\left[\frac{\omega}{\gamma}K_1\left(\frac{b\omega}{\gamma}\right) - \left(1 + \frac{(\mu_p - 1)\frac{\Lambda^4}{16m_p^4}}{\left(1-\frac{\Lambda^2}{4m_p^2}\right)^2}\right)\sqrt{\Lambda^2 + \frac{\omega^2}{\gamma^2}}\, K_1\left(b\sqrt{\Lambda^2+\frac{\omega^2}{\gamma^2}}\right)\right.$$

$$+ \frac{(\mu_p - 1)\frac{\Lambda^4}{16m_p^4}}{\left(1-\frac{\Lambda^2}{4m_p^2}\right)^2}\sqrt{4m_p^2 + \frac{\omega^2}{\gamma^2}}\, K_1\left(b\sqrt{4m_p^2+\frac{\omega^2}{\gamma^2}}\right)$$

$$\left. - \frac{1-\frac{\mu_p\Lambda^2}{4m_p^2}}{1-\frac{\Lambda^2}{4m_p^2}} \cdot \frac{b\Lambda^2}{2} K_0\left(b\sqrt{\Lambda^2+\frac{\omega^2}{\gamma^2}}\right)\right]^2,$$

$\gamma\gamma$ luminosities in pp collisions

In the case of 13 TeV pp collisions [hep-ph/0608271, 1112.3243],

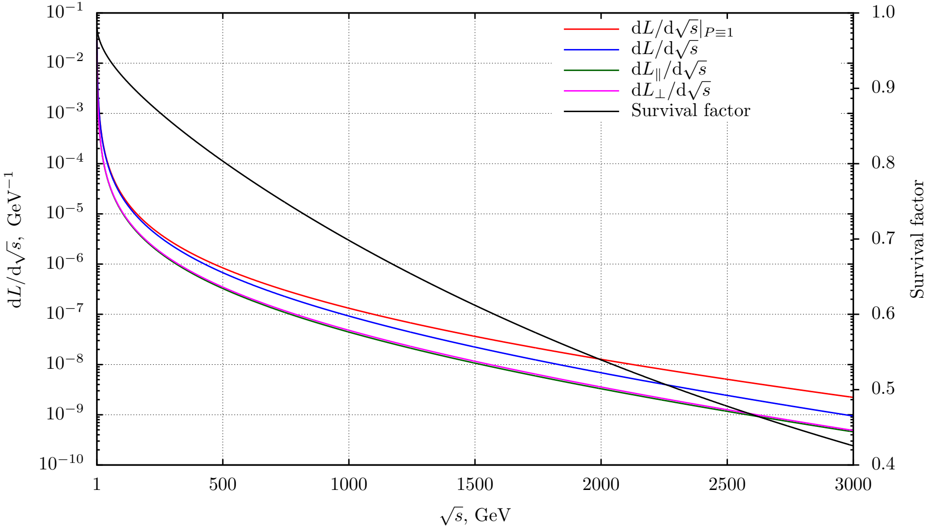
$$P_{pp}(b) = \left(1 - \mathrm{e}^{-\frac{b^2}{2B}}\right)^2, \; B = 21 \text{ GeV}^2.$$

Photon-photon luminosities:

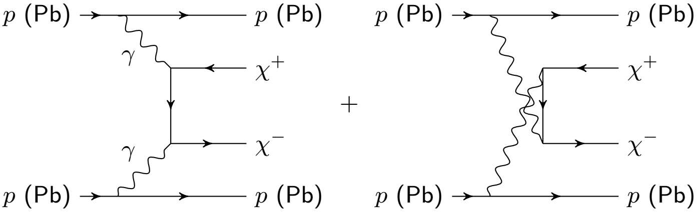
$$\frac{\mathrm{d}L_{pp}^{\parallel}}{\mathrm{d}s} = \frac{\pi^2}{2} \int_0^\infty b_1 \, \mathrm{d}b_1 \int_0^\infty b_2 \, \mathrm{d}b_2 \int_{-\infty}^{\infty} \mathrm{d}y \, n_p\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right) n_p\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right)$$
$$\times \left\{1 - 2\mathrm{e}^{-\frac{b_1^2+b_2^2}{2B}} \left[I_0\left(\frac{b_1 b_2}{B}\right) + I_2\left(\frac{b_1 b_2}{B}\right)\right] + \mathrm{e}^{-\frac{b_1^2+b_2^2}{B}} \left[I_0\left(\frac{2b_1 b_2}{B}\right) + I_2\left(\frac{2b_1 b_2}{B}\right)\right]\right\},$$

$$\frac{\mathrm{d}L_{pp}^{\perp}}{\mathrm{d}s} = \frac{\pi^2}{2} \int_0^\infty b_1 \, \mathrm{d}b_1 \int_0^\infty b_2 \, \mathrm{d}b_2 \int_{-\infty}^{\infty} \mathrm{d}y \, n_p\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right) n_p\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right)$$
$$\times \left\{1 - 2\mathrm{e}^{-\frac{b_1^2+b_2^2}{2B}} \left[I_0\left(\frac{b_1 b_2}{B}\right) - I_2\left(\frac{b_1 b_2}{B}\right)\right] + \mathrm{e}^{-\frac{b_1^2+b_2^2}{B}} \left[I_0\left(\frac{2b_1 b_2}{B}\right) - I_2\left(\frac{2b_1 b_2}{B}\right)\right]\right\}.$$

Survival factor in pp collisions with the energy 13 TeV



$pp \to pp\chi^+\chi^-$

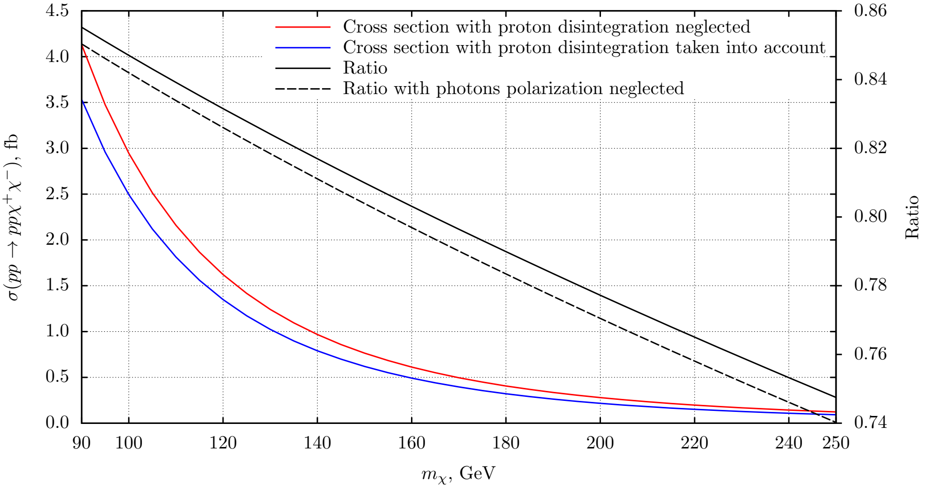


Breit-Wheeler cross sections [Phys.Rev. 46, 1087 (1934)]:

$$\sigma_{\parallel}(\gamma\gamma \to \chi^+\chi^-) = \frac{4\pi\alpha^2}{s}\left[\left(1+\frac{4m_\chi^2}{s}-\frac{12m_\chi^4}{s^2}\right)\ln\frac{1+\sqrt{1-4m_\chi^2/s}}{1-\sqrt{1-4m_\chi^2/s}} - \left(1+\frac{6m_\chi^2}{s}\right)\sqrt{1-\frac{4m_\chi^2}{s}}\right]$$

$$\sigma_{\perp}(\gamma\gamma \to \chi^+\chi^-) = \frac{4\pi\alpha^2}{s}\left[\left(1+\frac{4m_\chi^2}{s}-\frac{4m_\chi^4}{s^2}\right)\ln\frac{1+\sqrt{1-4m_\chi^2/s}}{1-\sqrt{1-4m_\chi^2/s}} - \left(1+\frac{2m_\chi^2}{s}\right)\sqrt{1-\frac{4m_\chi^2}{s}}\right]$$

$pp \to pp\chi^+\chi^-$



Fiducial cross section

Cuts: $p_T > \hat{p}_T$, $|\eta| < \hat{\eta}$.
p_T — transversal momentum, η — pseudorapidity.

$$\frac{\mathrm{d}\sigma_{\text{fid.}}(pp \to pp\chi^+\chi^-)}{\mathrm{d}s} = \int\limits_{\max\left(\hat{p}_T,\, \frac{\sqrt{s}}{2\cosh\hat{\eta}}\sqrt{1-\frac{4m_\chi^2}{s}}\right)}^{\frac{\sqrt{s}}{2}\sqrt{1-\frac{4m_\chi^2}{s}}} \mathrm{d}p_T \left[\frac{\mathrm{d}\sigma_\parallel(\gamma\gamma \to \chi^+\chi^-)}{\mathrm{d}p_T}\frac{\mathrm{d}\hat{L}^\parallel}{\mathrm{d}s} + \frac{\mathrm{d}\sigma_\perp(\gamma\gamma \to \chi^+\chi^-)}{\mathrm{d}p_T}\frac{\mathrm{d}\hat{L}^\perp}{\mathrm{d}s}\right]$$

Differential photon-photon cross sections:

$$\frac{\mathrm{d}\sigma_\parallel(\gamma\gamma \to \chi^+\chi^-)}{\mathrm{d}p_T} = \frac{8\pi\alpha^2 p_T}{s(p_T^2 + m_\chi^2)} \cdot \frac{1 - \frac{2(p_T^4 + 2m_\chi^4)}{s(p_T^2 + m_\chi^2)}}{\sqrt{1 - \frac{4(p_T^2 + m_\chi^2)}{s}}}$$

$$\frac{\mathrm{d}\sigma_\perp(\gamma\gamma \to \chi^+\chi^-)}{\mathrm{d}p_T} = \frac{8\pi\alpha^2 p_T}{s(p_T^2 + m_\chi^2)} \cdot \frac{1 - \frac{2p_T^4}{s(p_T^2 + m_\chi^2)}}{\sqrt{1 - \frac{4(p_T^2 + m_\chi^2)}{s}}}$$

Fiducial cross section

Fiducial luminosity:

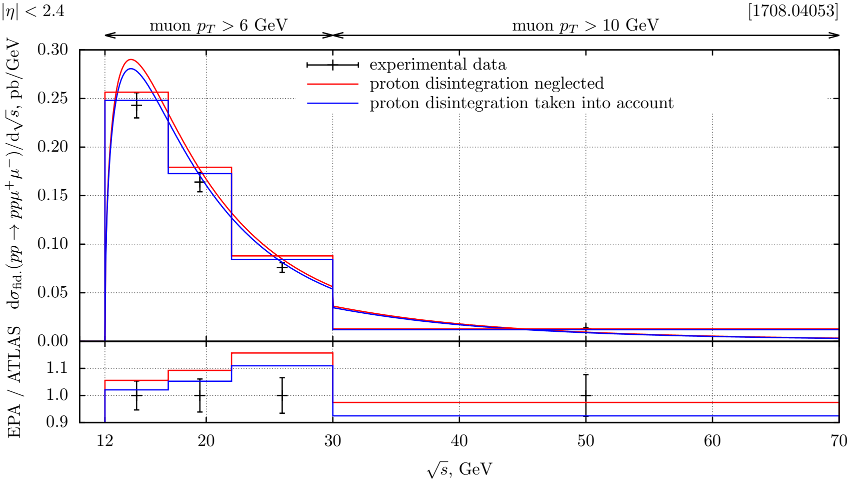
$$\frac{\mathrm{d}\hat{L}^{\parallel}}{\mathrm{d}s} = \frac{1}{4} \iint \mathrm{d}^2 b_1 \, \mathrm{d}^2 b_2 \, P(b) \int_{-\hat{y}}^{\hat{y}} \mathrm{d}y \, n\left(b_1, \tfrac{\sqrt{s}}{2}\, \mathrm{e}^{y}\right) \, n\left(b_2, \tfrac{\sqrt{s}}{2}\, \mathrm{e}^{-y}\right) \, \cos^2 \varphi$$

$$\frac{\mathrm{d}\hat{L}^{\perp}}{\mathrm{d}s} = \frac{1}{4} \iint \mathrm{d}^2 b_1 \, \mathrm{d}^2 b_2 \, P(b) \int_{-\hat{y}}^{\hat{y}} \mathrm{d}y \, n\left(b_1, \tfrac{\sqrt{s}}{2}\, \mathrm{e}^{y}\right) \, n\left(b_2, \tfrac{\sqrt{s}}{2}\, \mathrm{e}^{-y}\right) \, \sin^2 \varphi$$

Rapidity cut:

$$\hat{y} = \operatorname{arcsinh}\left[\frac{\sqrt{s}\, p_T}{2(p_T^2 + m_\chi^2)} \left(\sinh\hat{\eta} - \sqrt{\cosh^2\hat{\eta} + \frac{m_\chi^2}{p_T^2}} \cdot \sqrt{1 - \frac{4(p_T^2 + m_\chi^2)}{s}}\right)\right]$$

ATLAS experiment: $pp \to pp\mu^+\mu^-$



Integrated cross section:

- Experiment: 3.12 ± 0.07 (stat.) $\pm\, 0.10$ (syst.) pb.
- With proton disintegration neglected: 3.39 pb.
- With proton disintegration taken into account: 3.26 pb.

Conclusions

- Ultraperipheral collisions is a way to study photon-photon collisions at the LHC.
- With EPA, many calculations can be performed analytically. Numerical integration is required, Monte Carlo simulation is not.
- Survival factor and photon-photon luminosities in pp collisions were calculated for invariant mass up to 3 TeV.
- Good agreement with the ATLAS experiment.
- We are going to publish our code as a library at Github before the end of the year.