Gauge Field Theory Vacuum and Cosmological Inflation

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Congratulations

Happy 70th Anniversary !!!!!!!!

1. Effective Lagrangians in QED and YM theory

2. Chromomagnetic Condensation in YM theory

3. Quantum Energy Momentum Tensor

4. Solution of Friedmann Equations in Quantum Gauge Field Theory Vacuum

5. Inflation of the Type II and Type IV Universes

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Contribution of Vacuum Fluctuations to the Cosmological Constant

The calculation of the effective Lagrangian in QED by Heisenberg and Euler was the first example of a well-defined physically motivated prescription allowing to obtain a finite, gauge and renormalisation group-invariant result when investigating the vacuum fluctuations of quantised fields [29]. It appears that only the difference between vacuum energy in the presence and in the absence of external sources has a well-defined physical meaning [29, 30, 31, 32, 33, 34, 35, 36, 1, 2, 3, 4, 5]. Here we will follow this prescription and will derive the quantum equation of state for non-Abelian gauge fields by using the effective Lagrangian approach [37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,

Heisenberg-Euler, 1936; Schwinger 1951;Coleman-Weinberg 1973; Vanyashin-Terentev 1965; Skalozub:1975; Brown-Duff,1975; Duff — Ramon-Medrano,1975; Nielsen and Olesen 1978; Skalozub 1978; Nielsen 1978; Ambjorn-Nielsen-Olesen1979; Nielsen and Ninomiya,1979; Nielsen and Olesen 1979; Nielsen-Ninomiya 1980; Nielsen-Olesen 1979; Ambjorn-Olesen 1980; Ambjorn-Olesen 1980; Skalozub1980; Leutwyler 1980; Leutwyler 1981; Duff 1977 ; G.S 1976, 1977, 2018

Energy-Momentum Tensor of the Polarized Quantum Fields

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} < T^{GR}_{\mu\nu} >$$

Bunch, Davies 1977
Starobinski 1980
Mukhanov, Chibisov 1981
....

$$R^i_k - \frac{1}{2} \delta^i_k = \frac{1}{H^2}\left(R^i_l R^l_k - \frac{2}{3} R R^i_k - \frac{1}{2} \delta^i_k R^l_m R^m_l + \frac{1}{4} \delta^i_k R^2\right)$$

$$- \frac{1}{6M^2}\left(2R^{;i}_{;k} - 2\delta^i_k R^{;l}_{;l} - 2RR^i_k + \frac{1}{2}\delta^i_k R^2\right),$$

Energy-Momentum Tensor of the Polarized YM Vacuum

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} < T^{YM}_{\mu\nu} >$$

$$< T_{\mu\nu} >= T^{YM}_{\mu\nu}\Big[1 + \frac{b\ g^2}{96\pi^2} \ln \frac{2g^2\mathcal{F}}{\mu^4}\Big] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F},$$

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$\mathcal{F} = \frac{1}{4} g^{\alpha\beta} g^{\gamma\delta} G_{\alpha\gamma} G_{\beta\delta} \geq 0$ *is of Chromomagnetic type*

Heisenberg-Euler Effective Lagrangian in QED

$$\mathcal{L}_{eff} = \frac{\mathcal{E}^2 - \mathcal{H}^2}{2} - \pi mc^2 (\frac{mc}{h})^3 \int_0^\infty \frac{ds}{s^3} e^{-s} \{ \frac{as\cos(as)}{\sin(as)} \, \frac{bs\cosh(bs)}{\sinh(bs)} - 1 + \frac{a^2 - b^2}{3} s^2 \}$$

where dimensionless fields are

$$a = \frac{e\hbar\mathcal{E}}{m^2c^3}, \qquad b = \frac{e\hbar\mathcal{H}}{m^2c^3}$$

$$mc^2 = 8.2 \cdot 10^{-7} \; \frac{g \; cm^2}{s^2} \qquad \lambda_c = \frac{\hbar}{mc} = 3.86 \cdot 10^{-11} cm \qquad \frac{mc^2}{(\frac{\hbar}{mc})^3} = 1.43 \cdot 10^{25} \frac{g}{cm \; s^2}$$

$$\mathcal{E}_c = \frac{m^2c^3}{e\hbar} \sim 10^{16} \; Volt/cm \qquad \mathcal{H}_c = \frac{m^2c^3}{e\hbar} \sim 4.4 \cdot 10^{13} \; Gauss$$

Heisenberg-Euler Effective Lagrangian

In the limit of massless fermions one can get

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$$\mathcal{L}_e = -\mathcal{F} + \frac{e^2\mathcal{F}}{24\pi^2}\Big[\ln(\frac{2e^2\mathcal{F}}{\mu^4}) - 1\Big], \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}^2 - \vec{\mathcal{E}}^2}{2}, \quad \mathcal{G} = \vec{\mathcal{E}}\vec{\mathcal{H}} = 0,$$

Effective Lagrangian in Yang-Mills theory

The YM effective Lagrangian take the following form

$$\mathcal{L}^{(1)} = -\frac{1}{8\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s} \frac{(gF_1 s)\ (gF_2 s)}{\sinh(gF_1 s)\ \sinh(gF_2 s)} - $$
$$-\frac{1}{4\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s} (gF_1 s)\ (gF_2 s)[\frac{\sinh(gF_1 s)}{\sinh(gF_2 s)} + \frac{\sinh(gF_2 s)}{\sinh(gF_1 s)}]$$

$$F_1^2 = -\mathcal{F} - (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}, \qquad F_2^2 = -\mathcal{F} + (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}$$

Vanyashin and Terentev 1965
Duff and Ramon-Medrano 1975
Skalozub 1976

Bartalin, Matinyan and Savvidy 1976
Savvidy 1977
Matinyan and Savvidy 1978

N.Nielsen and Olesen 1978
Ambjorn, N.Nielsen and Olesen 1979
H.Nielsen and Ninomia 1979
H.Nielsen and Olesen 1979
Ambjorn and Olesen1980

Quantum Energy Momentum Tensor

$$\mathcal{L}_g = -\mathcal{F} - \frac{11N}{96\pi^2} g^2 \mathcal{F}\Big(\ln \frac{2g^2\mathcal{F}}{\mu^4} - 1\Big) \ , \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}_a^2 - \vec{\mathcal{E}}_a^2}{2} > 0, \quad \mathcal{G} = \vec{\mathcal{E}}_a \vec{\mathcal{H}}_a = 0 \ .$$

$$\mathcal{L}_q = -\mathcal{F} + \frac{N_f}{48\pi^2} g^2 \mathcal{F}\Big[\ln(\frac{2g^2\mathcal{F}}{\mu^4}) - 1\Big]$$

the energy momentum tensor by using the formula derived by Schwinger in [5]:

$$T_{\mu\nu} = (F_{\mu\lambda}F_{\nu\lambda} - g_{\mu\nu}\frac{1}{4}F^2_{\lambda\rho})\frac{\partial \mathcal{L}}{\partial \mathcal{F}} - g_{\mu\nu}(\mathcal{L} - \mathcal{F}\frac{\partial \mathcal{L}}{\partial \mathcal{F}} - \mathcal{G}\frac{\partial \mathcal{L}}{\partial \mathcal{G}}).$$

Dimensional Transmutation and Condensation

G.S. 1977

$$\mathcal{L}_g = -\mathcal{F} - \frac{11N}{96\pi^2} g^2 \mathcal{F}\Big(\ln \frac{2g^2\mathcal{F}}{\mu^4} - 1 \Big) ~, \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}_a^2 - \vec{\mathcal{E}}_a^2}{2} > 0, \quad \mathcal{G} = \vec{\mathcal{E}}_a \vec{\mathcal{H}}_a = 0 ~.$$

$$\mathcal{L}_q = -\mathcal{F} + \frac{N_f}{48\pi^2} g^2 \mathcal{F}\Big[\ln(\frac{2g^2\mathcal{F}}{\mu^4}) - 1 \Big]$$

The corresponding energy momentum tensor is

$$T_{\mu\nu} = T^{YM}_{\mu\nu}\Big[1 + \frac{b~g^2}{96\pi^2} \ln \frac{2g^2\mathcal{F}}{\mu^4}\Big] - g_{\mu\nu}\frac{b~g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0,$$

where $b = 11N - 2N_f$.

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Quantum Energy Momentum Tensor

$$T_{\mu\nu} = T^{YM}_{\mu\nu}\Big[1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0,$$

$$T_{00} \equiv \epsilon(\mathcal{F}) = \ \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big) \qquad T_{ij} = \delta_{ij}\ \Big[\frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big)\Big] = \delta_{ij}\ p(\mathcal{F}).$$

The corresponding equation of state is

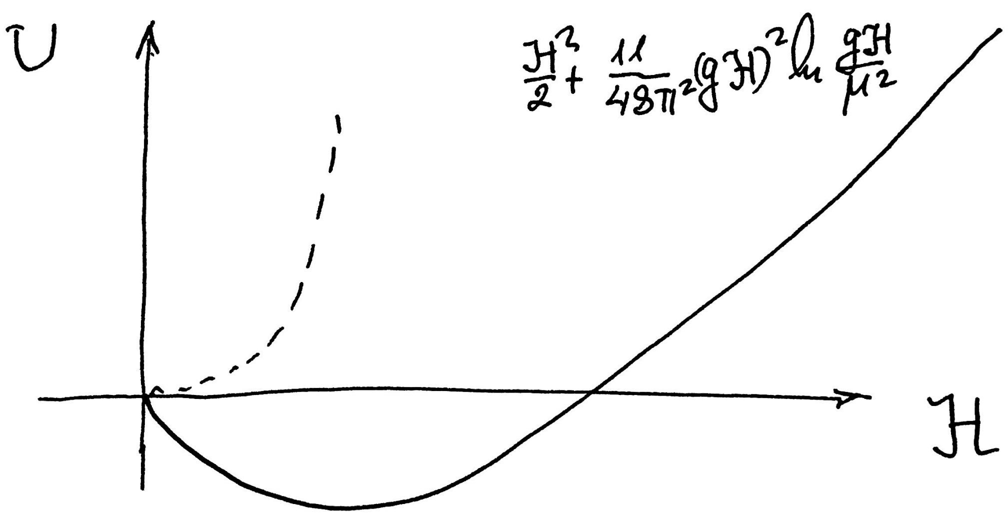
$$\epsilon(\mathcal{F}) = \ \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big), \qquad p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

$$\mathcal{F} \ = \ \tfrac{1}{4}g^{\alpha\beta}g^{\gamma\delta}G^a_{\alpha\gamma}G_{\beta\delta} \ \geq \ 0 \qquad \mathcal{G} = G^*_{\mu\nu}G^{\mu\nu} = 0$$

Dimensional Transmutation and Condensation

G.S. 1977

$$\epsilon(\mathcal{F}) = \ \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big)$$



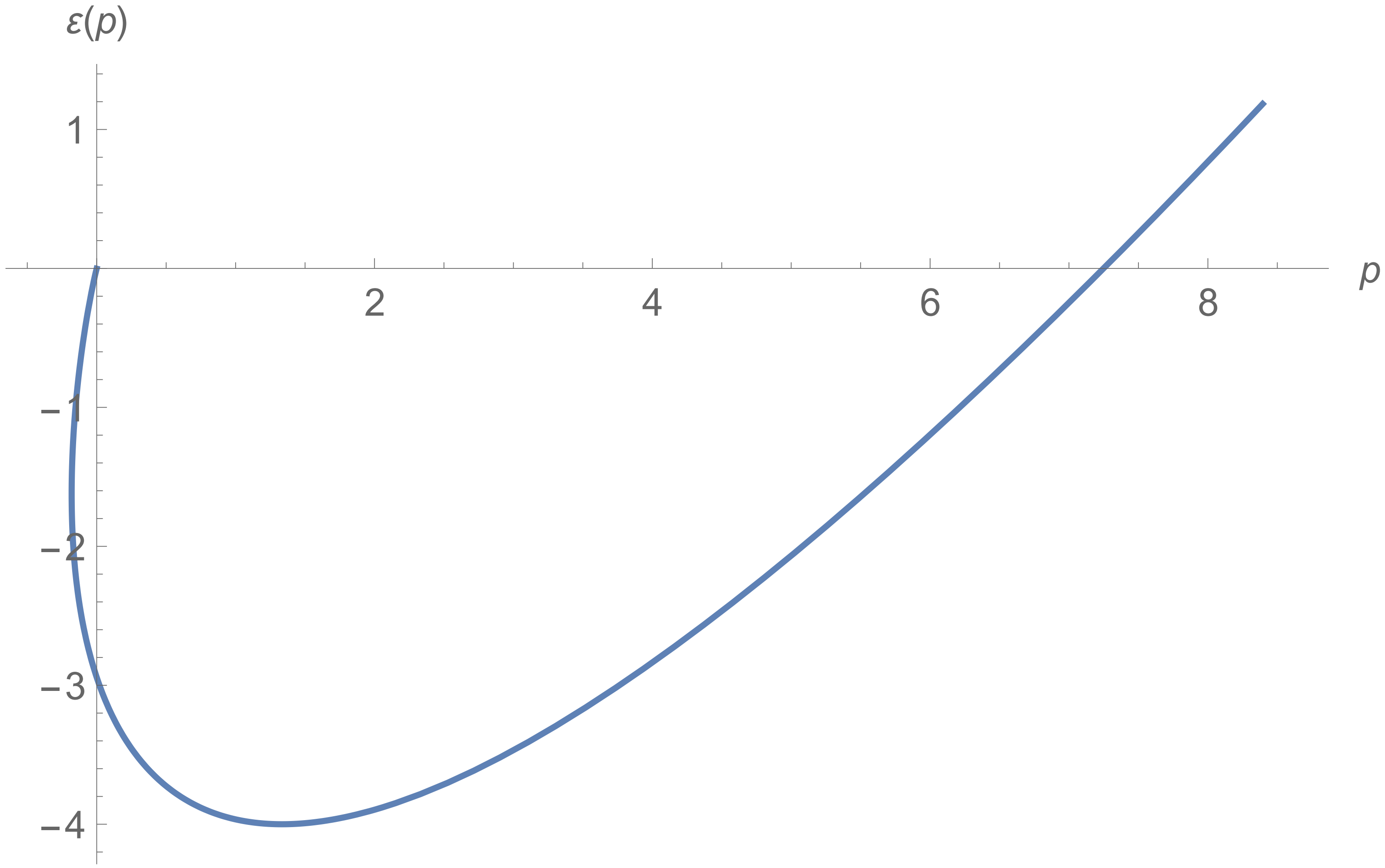
$$2g^2\mathcal{F}_{vac} = \mu^4 \exp\big(-\frac{96\pi^2}{b\ g^2(\mu)}\big) = \Lambda^4_{YM},$$

$$T^{\mu\nu}_{vac} = -g^{\mu\nu}\ \frac{b}{192\pi^2}2g^2\mathcal{F}_{vac},$$

$$p\ =\ -\epsilon\ >\ 0.$$

Ground State is Lorentz Invariant as it should

Yang-Mills Quantum Equation of State



$$\epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big), \qquad p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

Yang-Mills Quantum Equation of State

$$p = \frac{1}{3}\epsilon + \frac{4}{3}\frac{b\ g^2\mathcal{F}}{96\pi^2}\Lambda_{YM}^4 \qquad \text{and} \qquad w = \frac{p}{\epsilon} = \frac{\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} + 3}{3\Big(\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} - 1\Big)}$$

general parametrisation of the equation of state $p = w\epsilon$

Friedmann Evolution Equations

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0, \longrightarrow \quad \epsilon + p = \frac{4\mathcal{A}}{3}\ (2g^2\mathcal{F})\ \log\frac{2g^2\mathcal{F}}{\Lambda^4_{YM}},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p). \longrightarrow \quad \epsilon + 3p = 2\mathcal{A}\ (2g^2\mathcal{F})\ \Big(\log\frac{2g^2\mathcal{F}}{\Lambda^4_{YM}} + 1\Big).$$

the first equation can be solved for the field strength

$$2g^2\dot{\mathcal{F}} + 4(2g^2\mathcal{F})\frac{\dot{a}}{a} = 0 \qquad\qquad 2g^2\mathcal{F}\ a^4 = const \equiv \Lambda^4_{YM}\ a^4_0,$$

Friedmann Evolution Equations

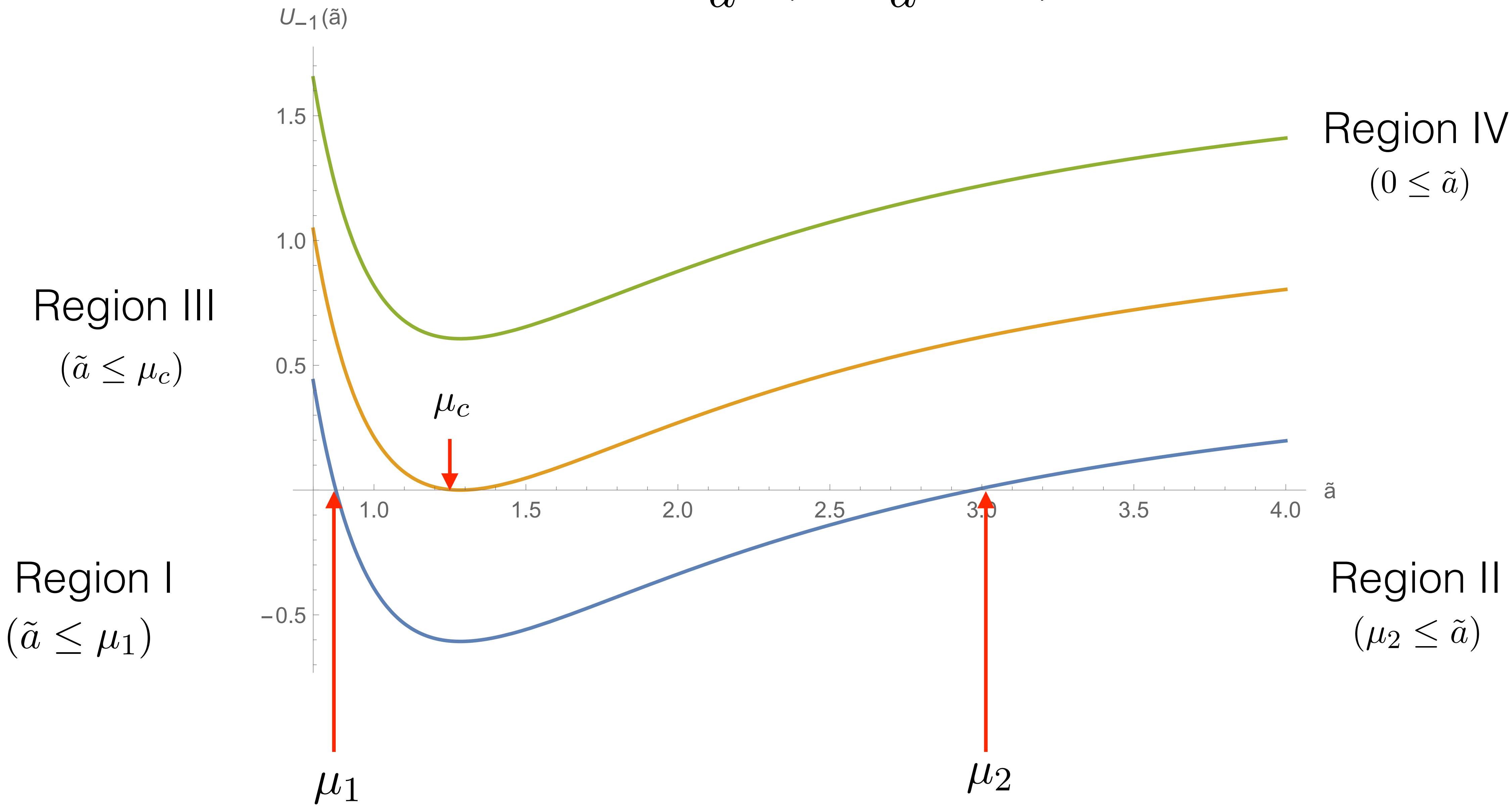
$$a(\tau) = a_0 \; \tilde{a}(\tau), \qquad ct = L \; \tau,$$

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

$$\frac{1}{L^2} = \frac{8\pi G}{3c^4}\; \mathcal{A}\; \Lambda_{YM}^4 \equiv \Lambda_{eff} \;\; ,$$

$$\mathcal{A} = \frac{b}{192\pi^2} = \frac{11N - 2N_f}{192\pi^2}.$$

$$U_{-1}(\tilde{a}) \equiv \frac{1}{\tilde{a}^2}\left(\log\frac{1}{\tilde{a}^4} - 1\right) + \gamma^2.$$



$$0 \leq \gamma^2 < \gamma_c^2$$

$$\gamma^2 = \gamma_c^2 = \frac{2}{\sqrt{e}}$$

$$\gamma_c^2 < \gamma^2$$

Type II Solution — Initial Acceleration of Finite Duration

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

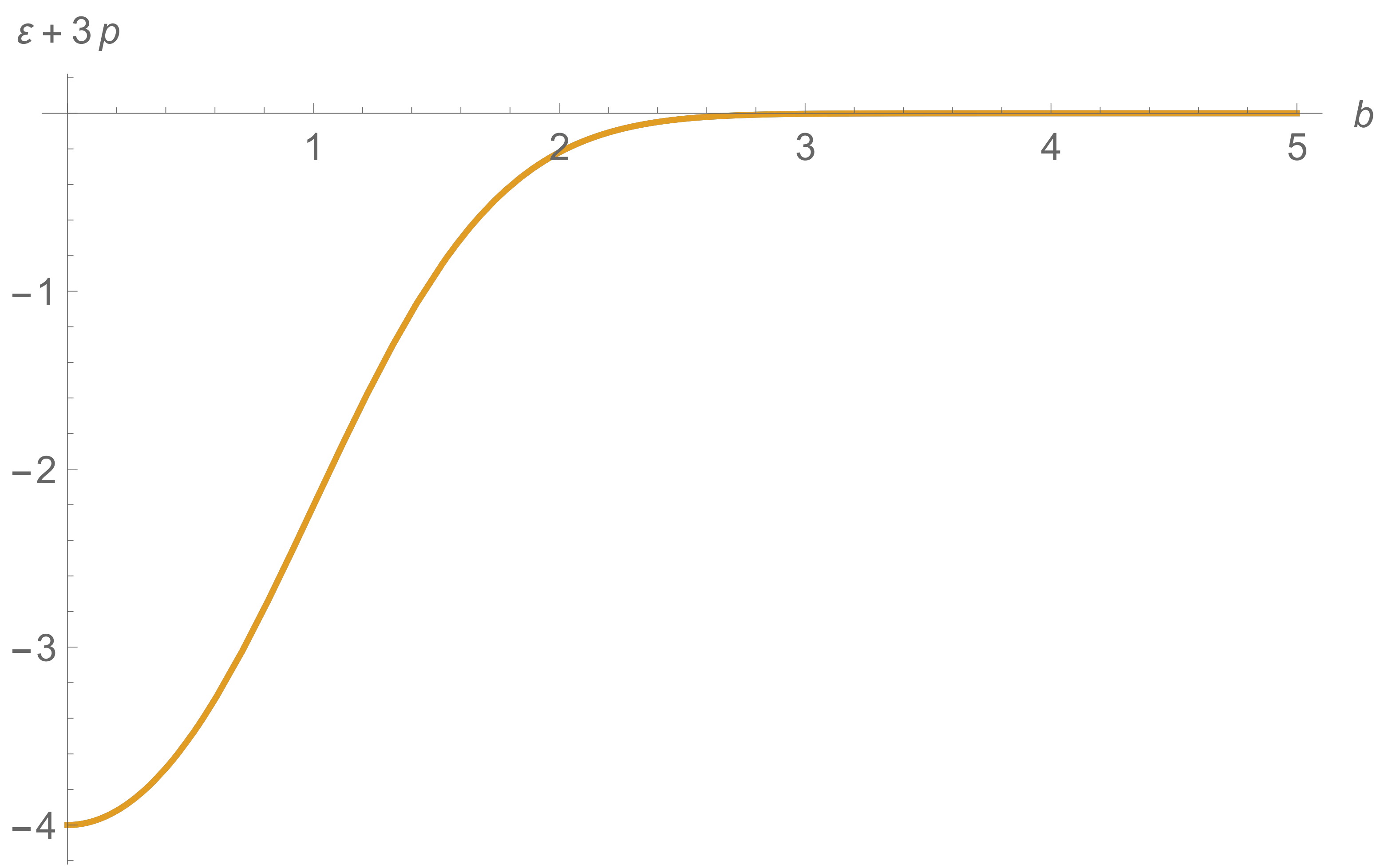
$$\tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0, \infty],$$

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\, e^{-\frac{b^2}{2}}\Big(\frac{\gamma^2\mu_2^2}{b^2}(e^{\frac{b^2}{2}} - 1) - 1\Big)^{1/2}.$$

$$\mu_2^2 = -\frac{2}{\gamma^2}\, W_-\Big(-\frac{\gamma^2}{2\sqrt{e}}\Big), \qquad 0 \le \gamma^2 < \frac{2}{\sqrt{e}} \text{ and } \tilde{a} \ge \mu_2.$$

Type II Solution — Initial Acceleration of Finite Duration

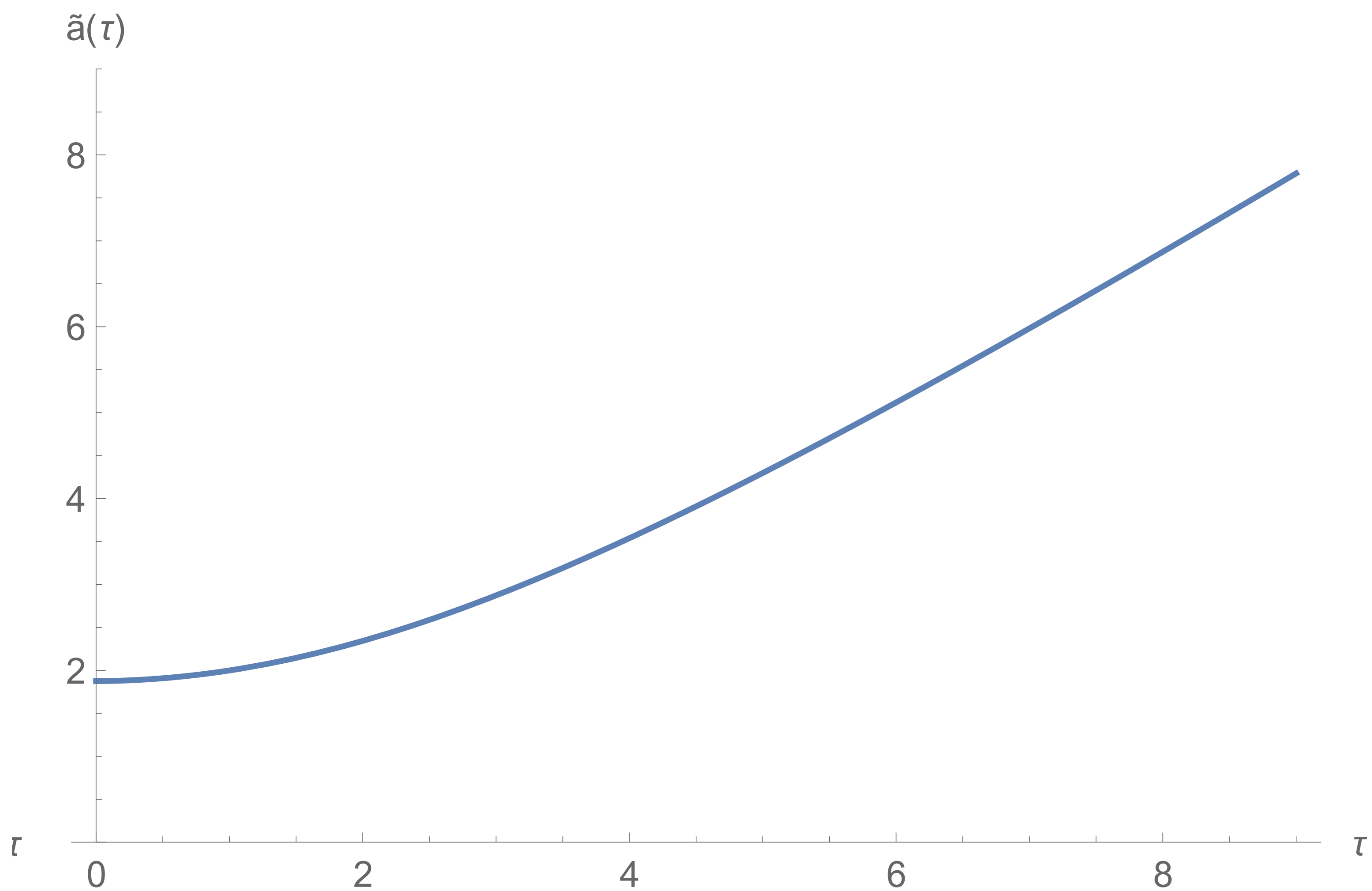
$$\epsilon + 3p = -\frac{2\mathcal{A}}{\mu_2^4}\ e^{-b^2(\tau)}(b^2(\tau) + \gamma^2\mu_2^2 - 2)\Lambda_{YM}^4, \qquad b \in [0, +\infty],$$



The r.h.s $\epsilon + 3p$ of the Friedmann acceleration equation (1.4) always negative

Type II Solution *Initial Acceleration of Finite Duration*

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\ e^{-\frac{b^2}{2}}\Big(\frac{\gamma^2\mu_2^2}{b^2}(e^{\frac{b^2}{2}}-1)-1\Big)^{1/2}. \qquad \tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0,\infty],$$

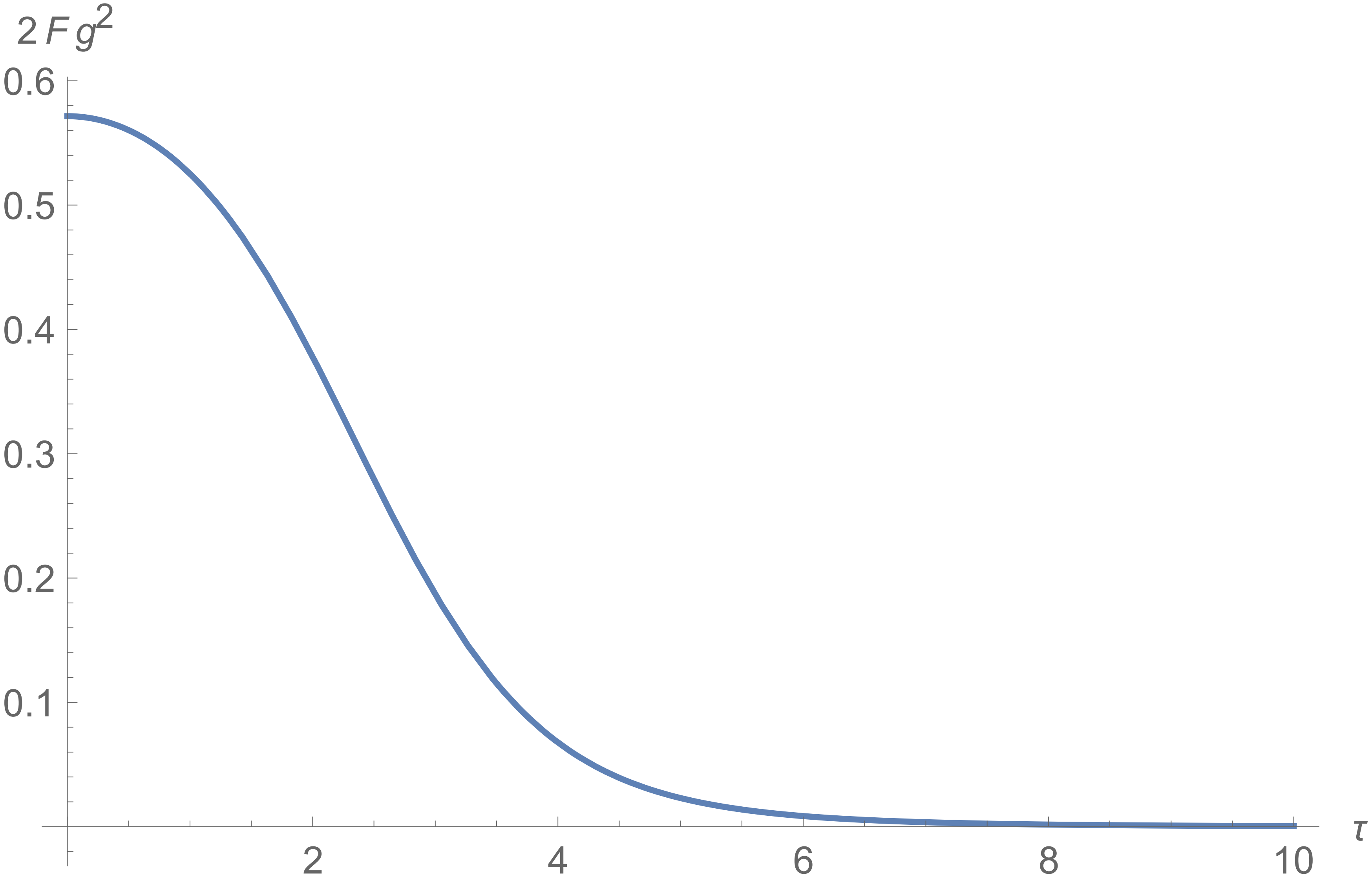


The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor‡

$$a(t) \simeq\ ct, \qquad a(\eta) \simeq a_0 e^{\eta}. \tag{5.87}$$

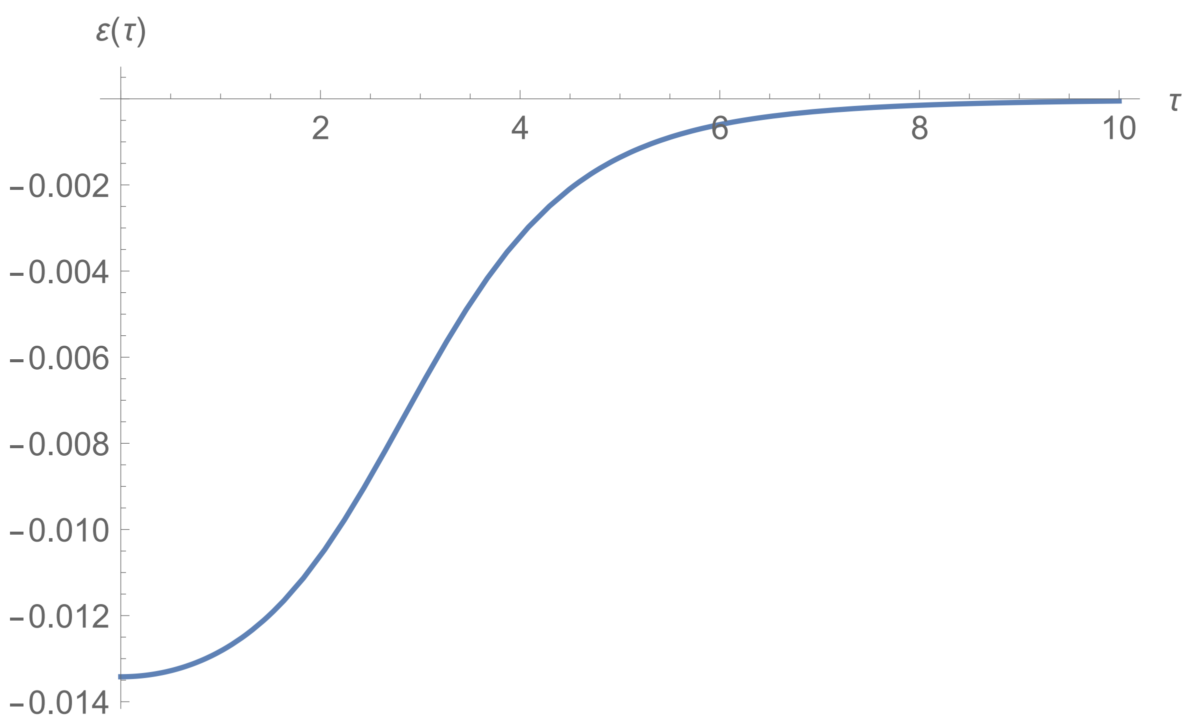
Evolution of the Field Strength

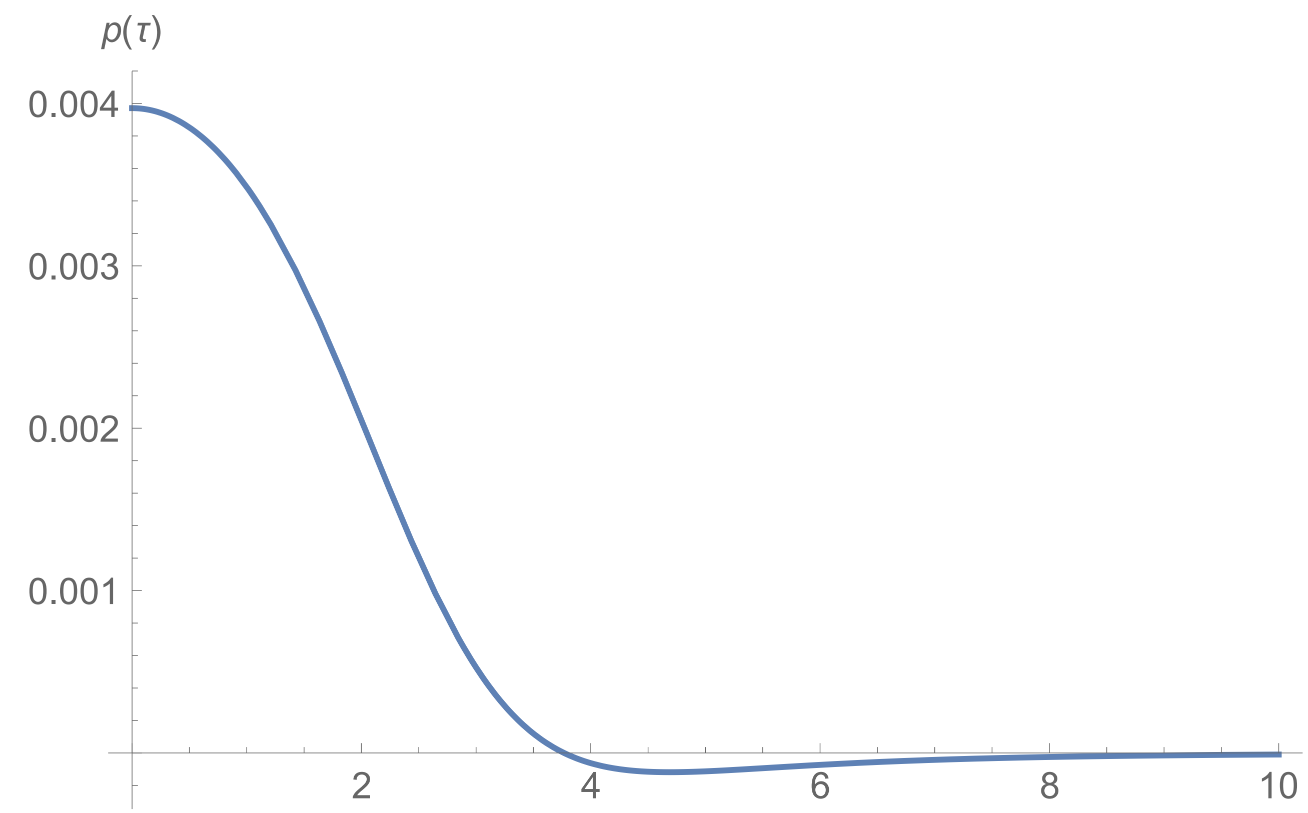
$$2g^2\mathcal{F} \;= \frac{\Lambda_{YM}^4}{\tilde{a}^4(\tau)}$$



Evolution of Energy Density and Pressure

$$\epsilon = \frac{\mathcal{A}}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big)\Lambda_{YM}^4, \qquad p = \frac{\mathcal{A}}{3\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} + 3\Big)\Lambda_{YM}^4.$$





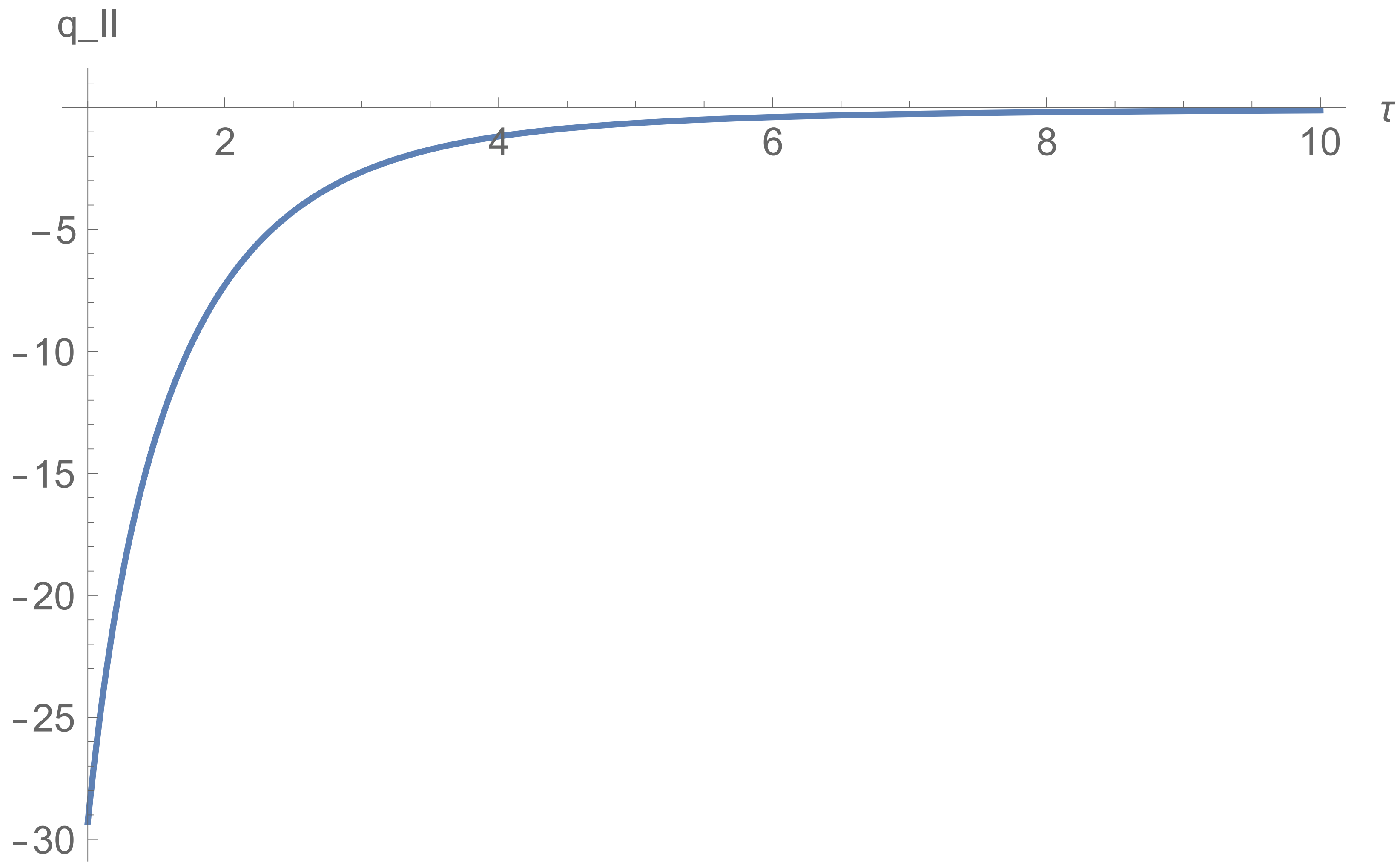
Evolution of the Hubble parameter

deceleration parameter

$$q = -\frac{\ddot{a}}{a}\frac{1}{H^2}.$$

$$q = \frac{\frac{1}{\tilde{a}^4}\Big(\log\frac{1}{\tilde{a}^4} + 1\Big)}{\frac{1}{\tilde{a}^4}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - \frac{k\gamma^2}{\tilde{a}^2}}$$

Type II Solution *Deceleration of finite duration*



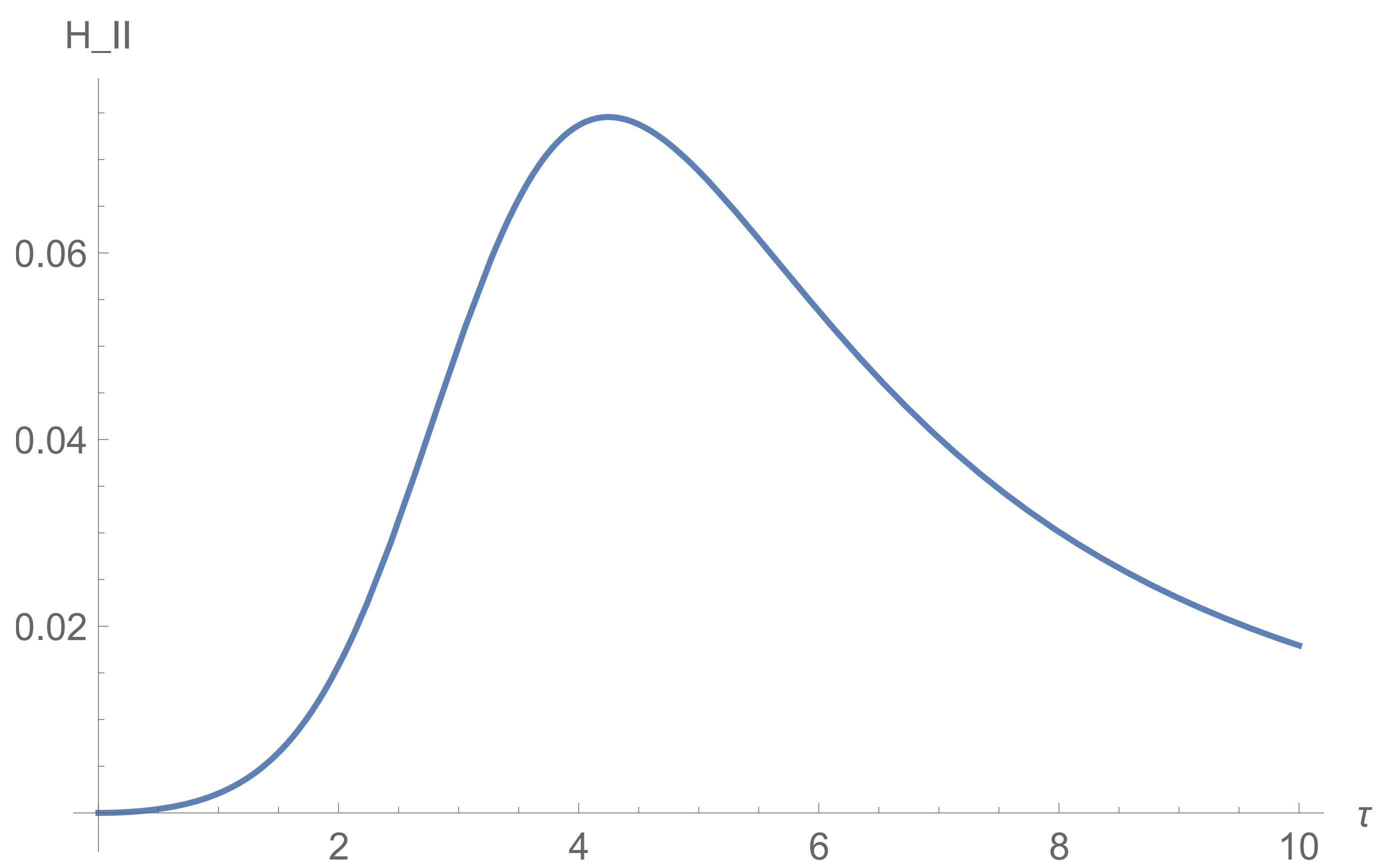
The deceleration parameter of the Type II solution is always negative:

$$q_{II} = \frac{b^2 + \gamma^2\mu_2^2 - 2}{b^2 + \gamma^2\mu_2^2(1 - e^{b^2/2})} < 0 \qquad q_{II} \propto -\frac{2}{b^2} \qquad q_{II} \propto -\frac{b^2}{\gamma^2\mu_2^2}e^{-b^2/2} \to 0.$$

Hubble Parameter

$$L^2H^2 = L^2\Big(\frac{\dot{a}}{a}\Big)^2 = \frac{1}{\tilde{a}^2}\Big(\frac{d\tilde{a}}{d\tau}\Big)^2 = \frac{1}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big) - \frac{k\gamma^2}{\tilde{a}^2(\tau)}$$

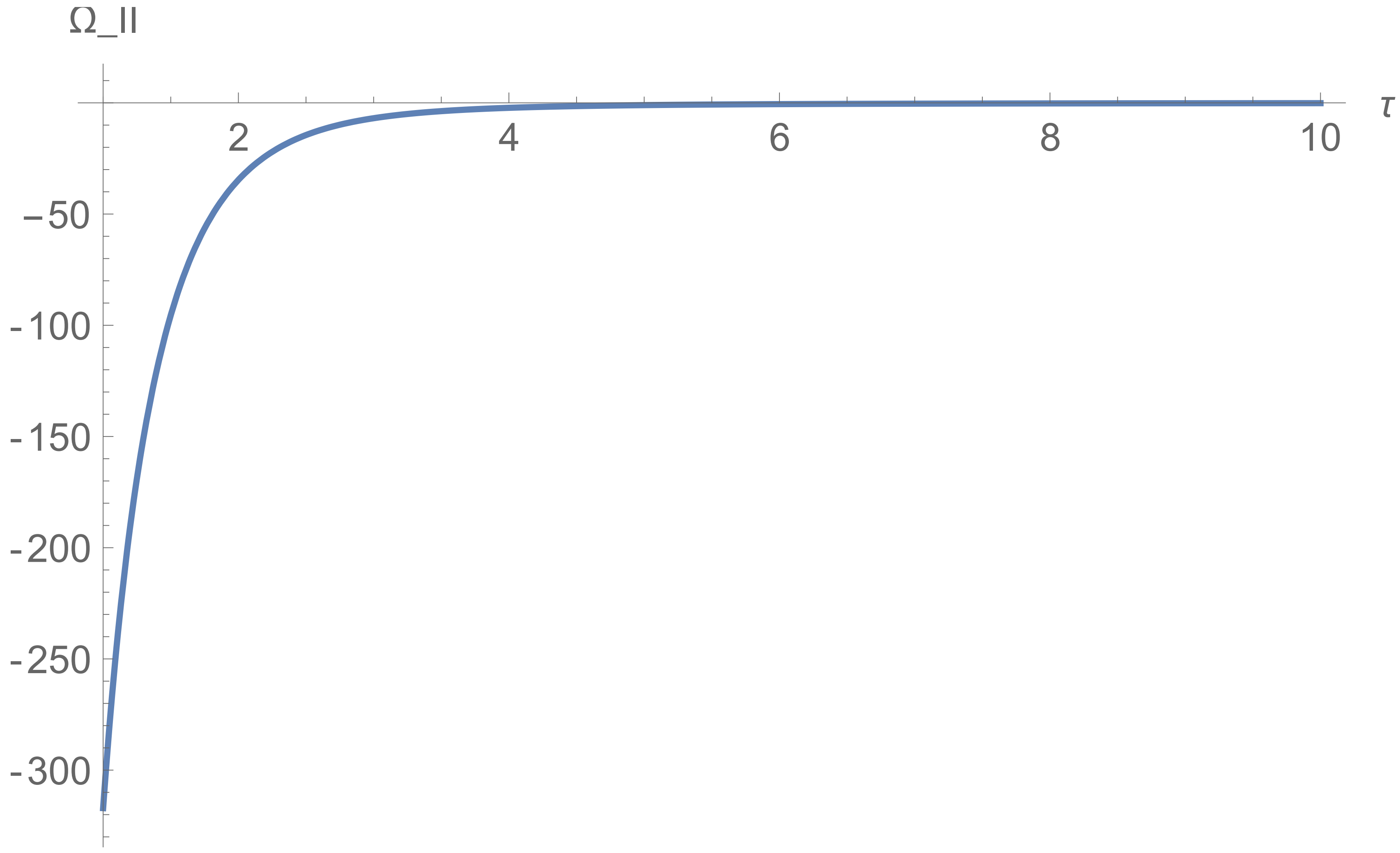
$$L^2H^2 = \frac{e^{-b^2}}{\mu_2^4}\Big(\gamma^2\mu_2^2(e^{b^2/2} - 1) - b^2\Big).$$



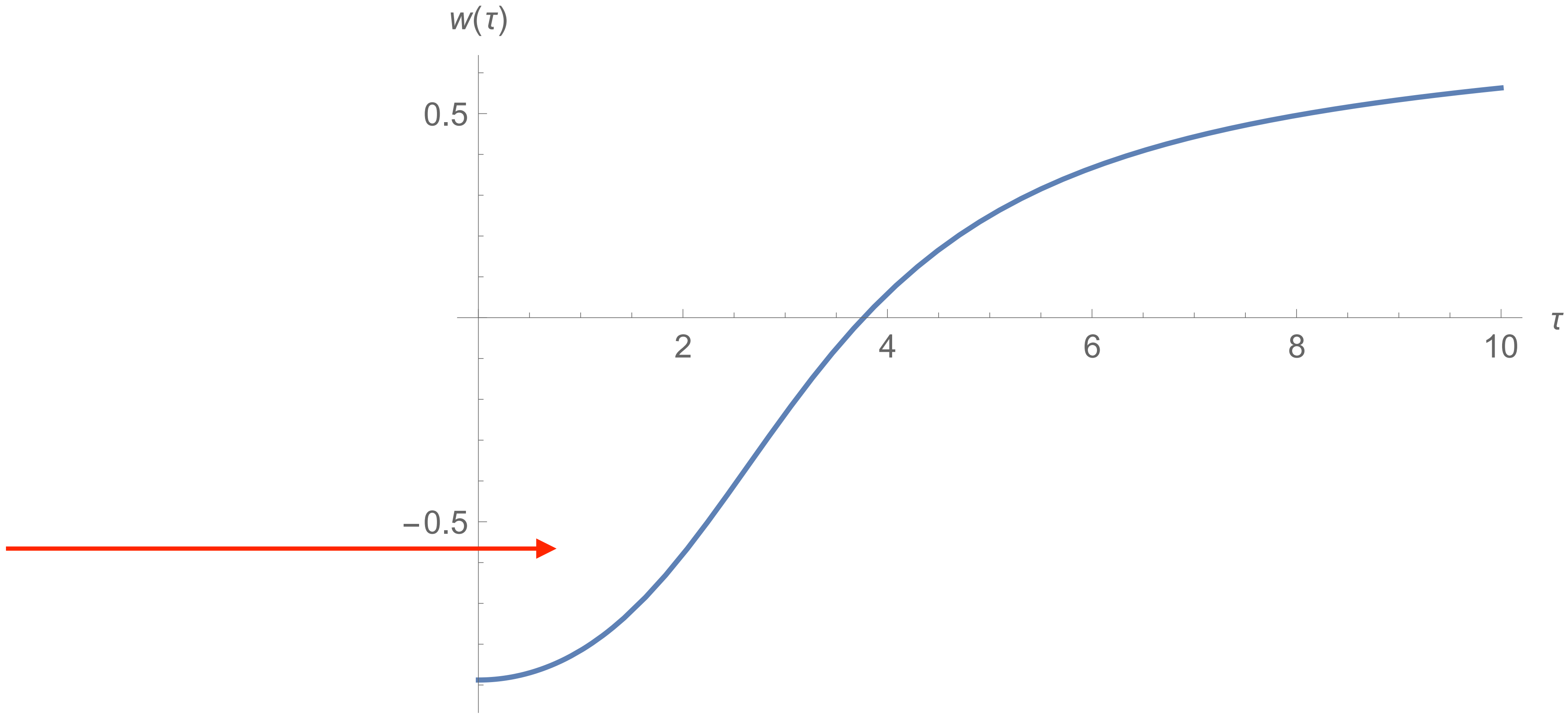
Type II Solution   *Density Parameter*

$$\Omega_{vac} \equiv \frac{8\pi G}{3c^4}\frac{\epsilon}{H^2}$$

$$\Omega_{vac} - 1 = -\frac{\gamma^2}{(\frac{d\tilde{a}}{d\tau})^2} = -\frac{\gamma^2 \mu_2^2 e^{b^2/2}}{\gamma^2 \mu_2^2 (e^{b^2/2} - 1) - b^2}$$



Type II Solution — Effective Parameter w

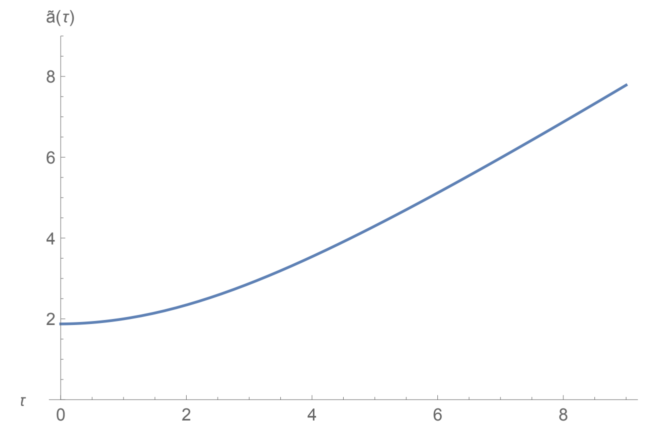


For the equation of state $p = w\epsilon$ one can find the behaviour of the effective parameter w

$$w_{II} = \frac{b^2(\tau) + \gamma^2\mu_2^2 - 4}{3\Big(b^2(\tau) + \gamma^2\mu_2^2\Big)}, \qquad -1 \leq w_{II},$$

$$w = \frac{p}{\epsilon} = \frac{\log\frac{1}{\tilde{a}^4(\tau)} + 3}{3\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big)}.$$

Type II Solution   Initial Acceleration of Finite Duration



The number of e-foldings

typical parameters around $\gamma^2 = 1.211$, $\mu_2^2 \simeq 1.75$ we get $\tau_s = 10^{23}$ and $\mathcal{N} \simeq 53$. $\mathcal{N} = \ln \frac{a(\tau_s)}{a(0)}$.

$$t_s^{GUM} = \frac{L_{GUM}}{c}\tau_s \simeq 4.2 \times 10^{-13}\ sec, \qquad \text{where } L_{GUM} \simeq 1.25 \times 10^{-25} cm$$

$$a(0) = L_{GUM}\frac{\mu_2}{\gamma} \simeq 1.5 \times 10^{-25} cm, \qquad a(t_s) = L_{GUM}\frac{\mu_2}{\gamma}e^{\mathcal{N}} \simeq 1.25 \times 10^{-2} cm,$$

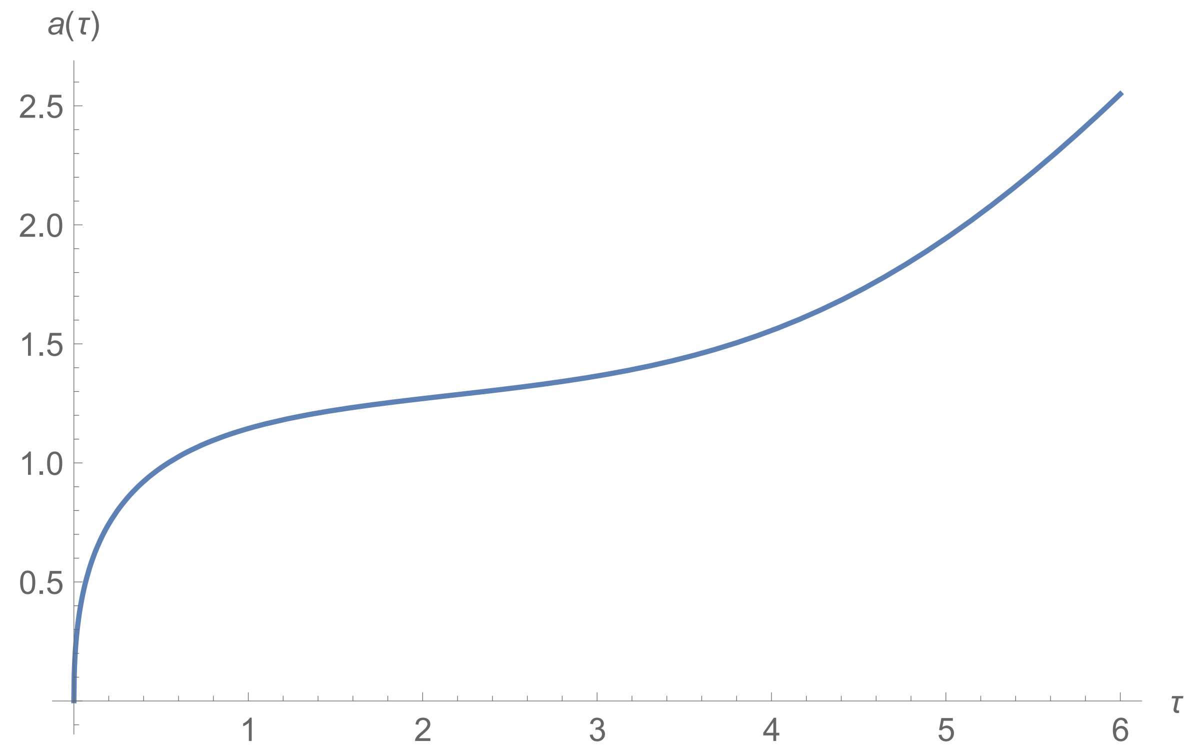
The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor[‡]

$$a(t) \simeq\ ct, \qquad a(\eta) \simeq a_0 e^{\eta}. \tag{5.87}$$

Type IV Solution - Late time Acceleration

The Type IV solution is defined in the region $\gamma^2 > \gamma_c^2$

$$\gamma_c^2 = \frac{2}{\sqrt{e}},$$



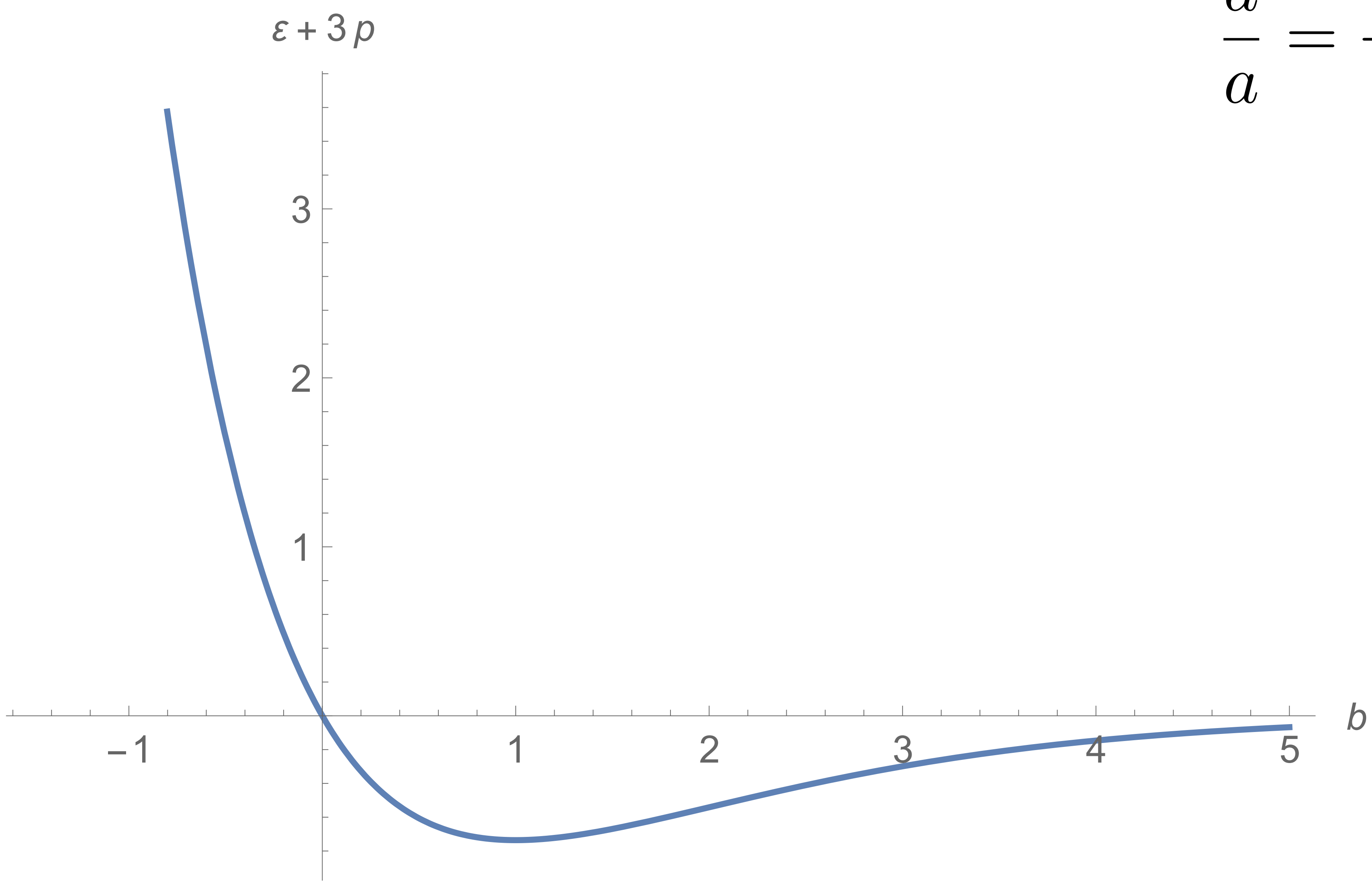
$$q_{IV} = \frac{b}{b + \frac{1}{2}(1 - \frac{\gamma^2}{\gamma_c^2} e^{2b})},$$

$$H = \sqrt{\frac{2}{e}} \frac{e^{-2b}}{L} \Big(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b \Big)^{1/2} \simeq \frac{1}{ct}.$$

$$\Omega_{vac} = 1 - \frac{\gamma^2}{(\frac{d\tilde{a}}{d\tau})^2} = 1 - \frac{\gamma^2 e^{2b}}{\gamma_c^2 \Big(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b \Big)} \to 0.$$

Strong Energy Dominance Condition is Violated

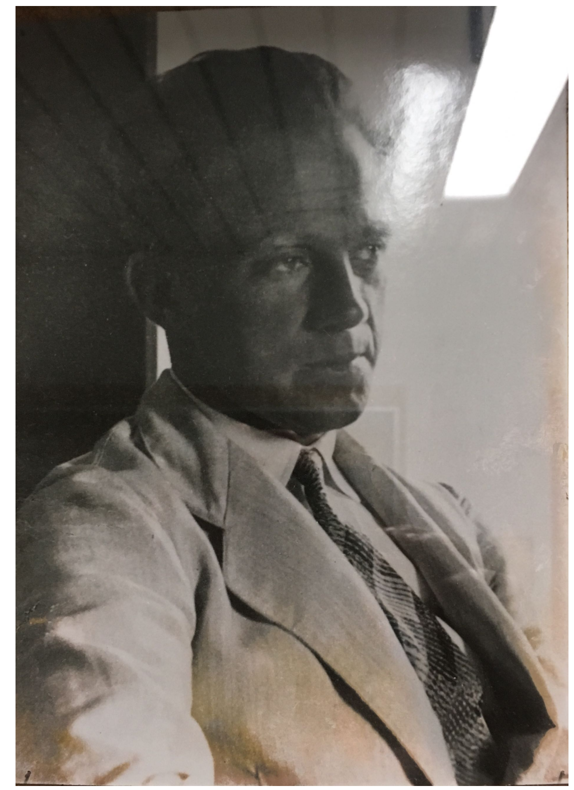
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p).$$



$\epsilon + 3p$ of the Friedmann acceleration equation is positive when $b < 0$ and is negative when $b > 0$.

Heisenberg and Euler 1936

Hans Euler



Werner Heisenberg

Happy Birthdays !