Energy dependent ratios of level-density parameters in superheavy nuclei

Azam Rahmatinejad
BLTP, JINR, Dubna, Russia

In collaboration with
T. Shneidman, G. Adamian, N. V. Antonenko, P. Jachimowicz, and M. Kowal

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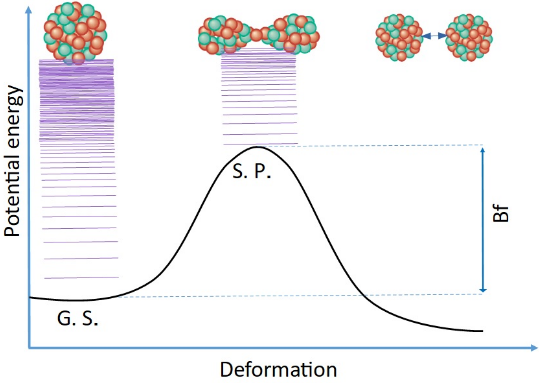
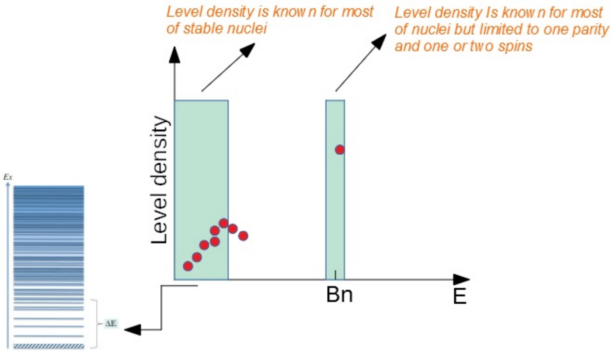
Outline

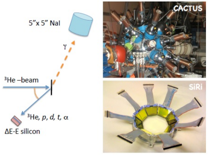
Why level density is important in superheavy region?!

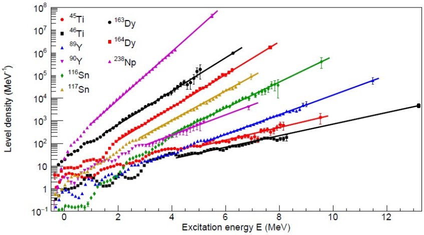
Level density is the number of levels per energy unit (MeV).



$$\frac{\Gamma_f}{\Gamma_n} \propto \frac{\rho_f}{\rho_n}$$


Level density is know n for most of stable nuclei
Level density Is know n for most of nuclei but limited to one parity and one or two spins
Level density
Bn
E
Ex
ΔE


5"x 5" NaI
γ
³He –beam
³He, p, d, t, α
ΔE-E silicon
CACTUS
SiRi


Level density (MeV⁻¹)
Excitation energy E (MeV)
45Ti
46Ti
89Y
90Y
116Sn
117Sn
163Dy
164Dy
238Np

Fermi Gas Model (FGM)

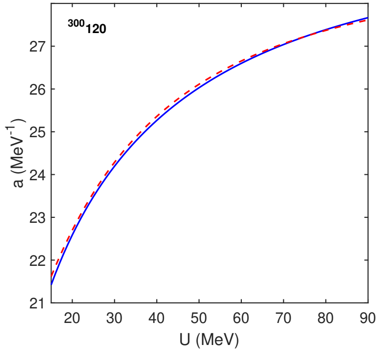
$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12a^{\frac{1}{4}}U^{\frac{5}{4}}}\exp(2\sqrt{aU})$$

$$\frac{\Gamma_n}{\Gamma_f} = \frac{4A^{2/3}a_f\,(U-B_n)}{K_0 a_n\left[2a_f^{1/2}(U-B_f)^{1/2}-1\right]}\times\exp\left[2a_n^{1/2}(U-B_n)^{1/2}-2a_f^{1/2}(U-B_f)^{1/2}\right]$$

$$U = aT^2$$

$$a = \frac{A}{12}$$

$$a(A,U) = \tilde{a}(A)\left[1+\frac{1-\exp(-\frac{U}{E_D})}{U}\delta E_{sh}\right]$$

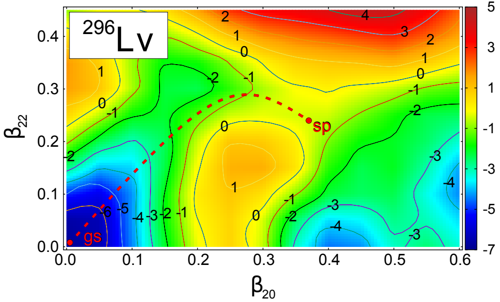


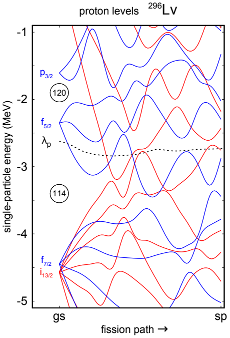
To keep simplicity of FGM for more realistic studies on survival probabilities

- To use a proper theoretical model which includes shell and pairing effects for the calculation of level densities.
- To analyze energy and shell correction dependencies of level-density parameter through fitting the FGM with microscopic calculations.
- To analyze energy dependent level-densities parameter ratios which are important for the estimation of the probabilities of de-excitation cascades via light particles emission in competition with splitting and thus for the determination of the survival probabilities.

(1): Single particle energies

MM method: M. Kowal, P. Jachimowicz, A. Sobiczewski, Phys. Rev. C 82, 014303 (2010); P. Jachimowicz, M. Kowal, J. Skalski, Phys. Rev. C 89, 024304 (2014); Phys. Rev. C 95, 034329 (2017)





(2): Data

Mass, potential energy, shell correction and deformations of nuclei.

Ground state								
Z	N	A	Mass (MeV)	E_{tot} (MeV)	ELD (MeV)	δE_{sh} (MeV)	β_2	β_4
118	172	290	196.44	-5.58	0.1	-5.68	-0.12	0.01
118	173	291	197.38	-5.41	1.12	-6.53	0.08	-0.04
118	174	292	197.75	-5.92	0.29	-6.22	0.08	-0.04
118	175	293	198.83	-5.84	1.52	-7.37	0.08	-0.05
118	176	294	199.61	-6.18	0.04	-6.22	-0.09	-0.01
118	177	295	200.89	-6.12	0.93	-7.06	-0.09	-0.01
118	178	296	201.85	-6.5	0.14	-6.64	-0.09	-0.01
118	179	297	203.81	-6.00	0.88	-6.88	0.03	-0.02
118	180	298	204.91	-6.46	0.04	-6.50	-0.04	-0.01

(2): Data

Saddle point								
Z	N	A	Mass (MeV)	E_{tot} (MeV)	ELD (MeV)	δE_{sh} (MeV)	β_2	β_4
118	172	290	202.19	0.17	2.11	-1.94	0.27	-0.04
118	173	291	203.78	0.99	3.74	-2.75	0.27	-0.06
118	174	292	203.84	0.17	0.45	-0.28	0.36	0.02
118	175	293	205.45	0.78	1.52	-0.74	0.35	0.03
118	176	294	205.71	-0.08	0.38	-0.46	0.36	0.03
118	177	295	207.53	0.51	1.48	-0.97	0.35	0.03
118	178	296	207.97	-0.39	0.44	-0.83	0.36	0.03
118	179	297	210.02	0.21	1.28	-1.07	0.36	0.03
118	180	298	210.69	-0.68	0.32	-1.00	0.37	0.03

(3) Model: Superfluid formalism

Nucleus is considered as a system of independent quasiparticles.

The thermal equilibrium is assumed between neutron and proton subsystems.

$$\Omega = -\beta \sum_{\tau=p,n} \sum_k (\varepsilon_{\tau k} - \lambda_\tau - E_{\tau k}) + 2\sum_k \log[1+\exp(-\beta E_{\tau k})] - \beta \frac{\Delta_\tau^2}{G_\tau}$$

The BCS equations, which determine the temperature dependence of Δ_τ and λ_τ , are derived from Ω.

$$N_\tau = \sum_k \left(1 - \frac{\varepsilon_{\tau k} - \lambda_\tau}{E_{\tau k}} \tanh \frac{\beta E_{\tau k}}{2}\right), \frac{2}{G_\tau} = \sum_k \frac{\tanh(\beta E_{\tau k})/2}{E_{\tau k}}$$

λ_τ, Δ_τ: chemical potential, pairing gap.

$E_{\tau k} = \sqrt{(\varepsilon_k - \lambda)^2 + \Delta^2}$: quasiparticle energies.

G_τ: The constant of the pairing interaction.

(4) Thermodynamic relations

$$E_\tau(T) = \sum_k \varepsilon_{k,\tau}\left(1 - \frac{\varepsilon_{k,\tau} - \lambda_\tau}{E_{k,\tau}} tanh \frac{\beta E_{k,\tau}}{2}\right) - \frac{\Delta_\tau^2}{G_\tau},$$

$$U(T) = \sum_\tau E_\tau(T) - E_\tau(0).$$

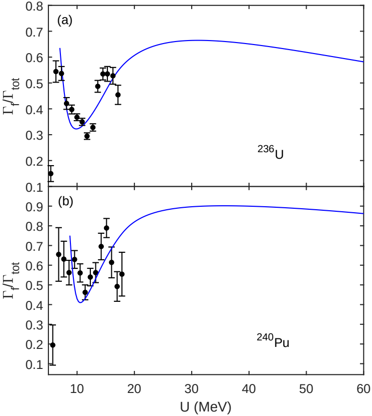
$$S(T) = \sum_\tau \sum_k \{ln[1 + exp(-\beta E_{k,\tau})] + \frac{\beta E_{k,\tau}}{1 + exp(\beta E_{k,\tau})}\}.$$

$$\rho_i(U) = \frac{\exp(S)}{(2\pi)^{\frac{3}{2}}\sqrt{D}}$$

Fission and neutron emission probabilities

$$\frac{\Gamma_n}{\Gamma_f} = \frac{gA^{2/3}}{K_0} \frac{\int_0^{U-B_n} \varepsilon \rho_{GS}\left(U - B_n - \varepsilon\right) d\varepsilon}{\int_0^{U-B_f} \rho_{SP}\left(U - B_f - \varepsilon\right) d\varepsilon}$$

$$\frac{\Gamma_f}{\Gamma_{tot}} = \frac{1}{1 + \Gamma_n/\Gamma_f}$$



A. Rahmatinejad, A. N. Bezbakh, T. M. Shneidman, G. Adamian, N. V. Antonenko, P. Jachimowicz, and M. Kowal, Phys. Rev. C **103**, 034309, (2021).

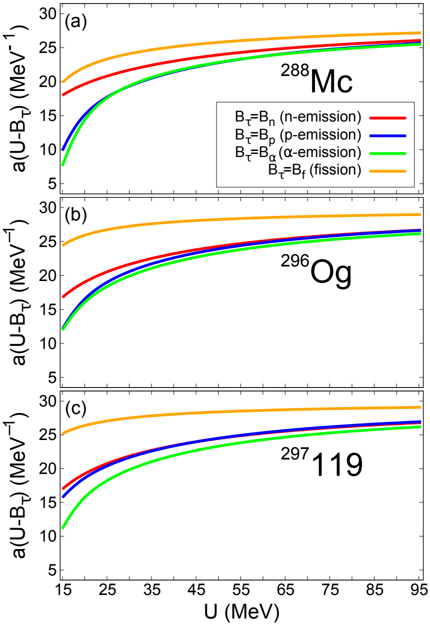
Experimental data:

E. Cheifetz, H. C. Britt, and J. B.Wilhelmy, Phys. Rev. C**24**, 519 (1981).

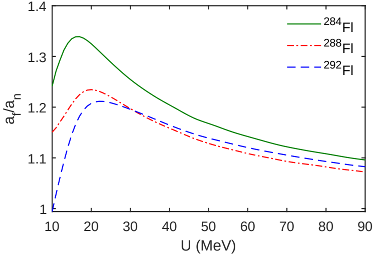
$$\frac{a_f}{a_n} = \frac{a_{sp}(A, U - B_f)}{a_{gs}(A-1, U - B_n)}$$

$$\frac{a_p}{a_n} = \frac{a_{gs}(A-1, U - B_p)}{a_{gs}(A-1, U - B_n)}$$

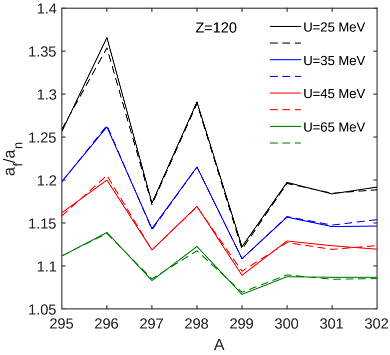
$$\frac{a_\alpha}{a_n} = \frac{a_{gs}(A-4, U - B_\alpha)}{a_{gs}(A-1, U - B_n)}$$



a_f / a_n

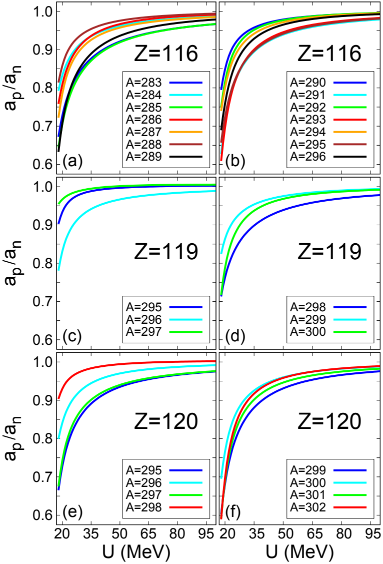


284Fl
288Fl
292Fl
a_f/a_n
U (MeV)

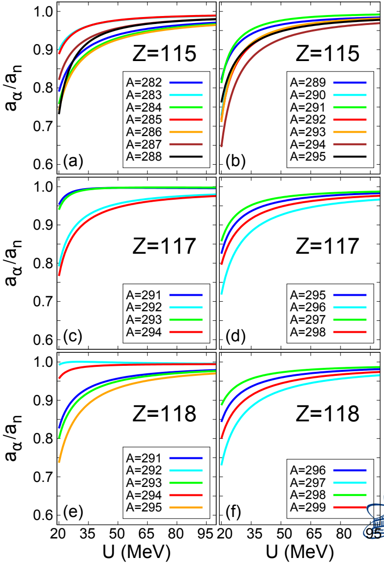


Z=120
U=25 MeV
U=35 MeV
U=45 MeV
U=65 MeV
a_f/a_n
A

$a_{p(\alpha)}/a_n$



1.0
0.9
0.8
0.7
0.6
a_p/a_n
Z=116
(a)
A=283
A=284
A=285
A=286
A=287
A=288
A=289
Z=116
(b)
A=290
A=291
A=292
A=293
A=294
A=295
A=296
Z=119
(c)
A=295
A=296
A=297
Z=119
(d)
A=298
A=299
A=300
Z=120
(e)
A=295
A=296
A=297
A=298
Z=120
(f)
A=299
A=300
A=301
A=302
20 35 50 65 80 95
U (MeV)



1.0
0.9
0.8
0.7
0.6
a_α/a_n
Z=115
(a)
A=282
A=283
A=284
A=285
A=286
A=287
A=288
Z=115
(b)
A=289
A=290
A=291
A=292
A=293
A=294
A=295
Z=117
(c)
A=291
A=292
A=293
A=294
Z=117
(d)
A=295
A=296
A=297
A=298
Z=118
(e)
A=291
A=292
A=293
A=294
A=295
Z=118
(f)
A=296
A=297
A=298
A=299
20 35 50 65 80 95
U (MeV)

Collective enhancements

$$\rho(U) = \sum_c \rho_i(U - U_c)\tau_c \simeq \rho_i(U)K_{coll}$$

$$K_{coll} = \sum_c \exp\left(-\beta U_c\right)\tau_c$$

$$\tau_c = 2I_c + 1$$

$$U_c = \hbar\omega_\beta(n_\beta + 1/2) + \hbar\omega_\gamma(2n_\gamma + |K|/2 + 1) + \frac{\hbar^2}{2\Im}\left[I_c(I_c + 1) - K^2\right]$$

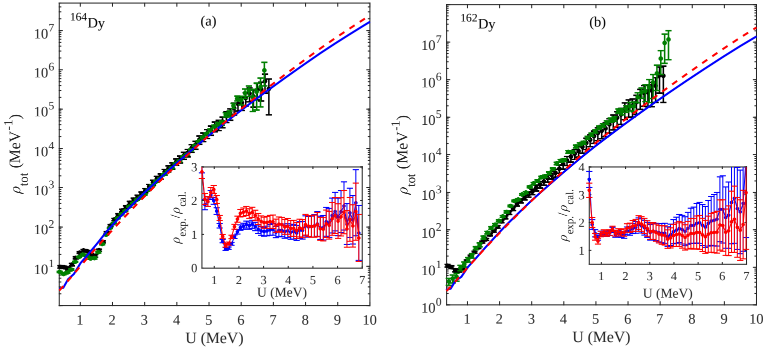
$$K_{coll} = K_{rot}K_{vib}$$

$$K_{vib} = \exp\left(0.0555A^{2/3}T^{4/3}\right)$$

$$K_{rot} = \begin{cases} 1, & \text{for spherical nuclei} \\ \Im_\perp T, & \text{for deformed nuclei,} \end{cases}$$

$$\Im_\perp = \Im_{r.b}f(\beta_2, \beta_4)$$

$$f(\beta_2, \beta_4) = 1 + \sqrt{5/16\pi}\beta_2 + (45/28\pi)\beta_2^2 + (15/7\pi\sqrt{5}\beta_2\beta_4)$$



A. Rahmatinejad, T. M. Shneidman, N. V. Antonenko, A. N. Bezbakh, G. G. Adamian, and L. A. Malov, *Phys. Rev. C* **101**, 054315 (2020).

Experimental level densities:

Green symbols: M. Guttormsen *et al.*, Phys. Rev. C **68**, 064306 (2003), and

H.T. Nyhus *et al.*, Phys. Rev. C **85**, 014323 (2012).

Black symbols: T. Renstrøm *et al.*, Phys. Rev. C **98**, 054310 (2018).

Collective states in α-emission channel

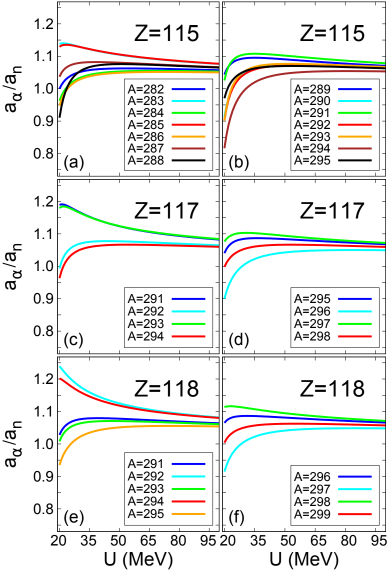
Excitations in mass asymmetry motion and relative vibrations of α-particle and daughter nucleus

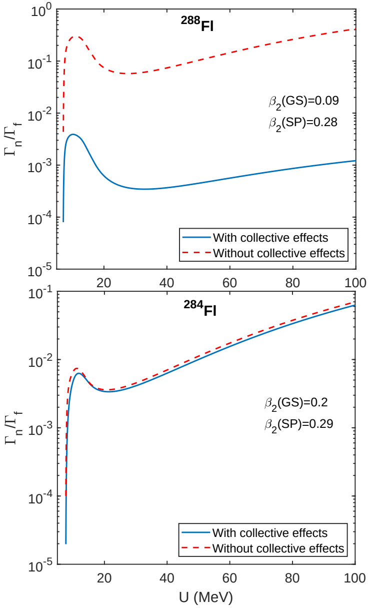
$$K_\alpha(\beta) = \sum_c \exp(-\beta U_c)\tau_c$$

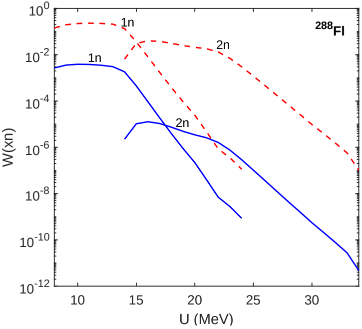
$$U_c = \hbar\omega_{ma} n_{ma} + \hbar\omega_b(2n_b + |K|)$$

$$\tau_c = 2|K| + 1$$

T. M. Shneidman, G. G. Adamian, N. V. Antonenko, R. V. Jolos, and Shan-Gui Zhou, Phys. Rev. C 92, 034302 (2015).




288Fl
β2(GS)=0.09
β2(SP)=0.28
With collective effects
Without collective effects
284Fl
β2(GS)=0.2
β2(SP)=0.29
With collective effects
Without collective effects
Γn/Γf
U (MeV)


288Fl
1n
2n
1n
2n
W(xn)
U (MeV)

Summary

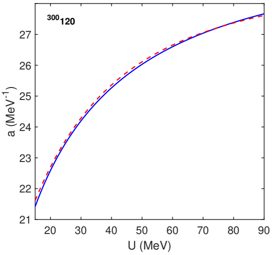
- With nuclear level densities at SP and GS one can evaluate competition between neutron emission and fission.
- Generally, the level-density parameter ratios increase with excitation energy and reach an asymptotic value less than 1.1 for a_f/a_n, and less than unity for $a_{p(\alpha)}/a_n$.
- Because of large difference in the shell corrections at the saddle point and at the ground state as well as different rates of their damping with excitation energy, the ratios a_f/a_n have a peak at energy lower than 30 MeV.
- The account of collective effects due to cluster degrees of freedom in the level densities of α-emission residue enhances the ratio a_α/a_n to the values larger than unity.
- The values of a_α/a_n and a_f/a_n larger than those obtained microscopically effectively account the collective enhancement of level density.
- More details in: arXiv:2108.12484v1 [nucl-th]

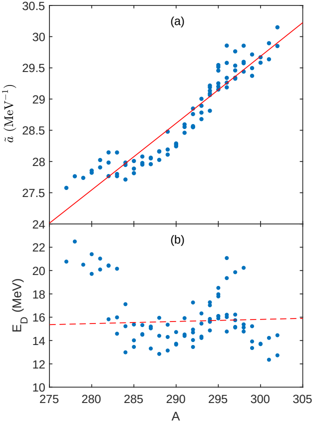
Thank you for your attention!

Fermi Gas Model (FGM)

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12a[U]^{\frac{1}{4}}U^{\frac{5}{4}}}\exp(2\sqrt{a[U]U})$$

$$a(A,U) = \tilde{a}(A)\left[1 + \frac{1-\exp(-\frac{U}{E_D})}{U}\delta E_{sh}\right]$$





$$\tilde{a} = a_1 A + a_2 A^2$$

$a_1 = 0.09$ MeV^{-1},
$a_2 = 2.89 \times 10^{-4}$ MeV^{-1},
$E_D \approx 15$ MeV.

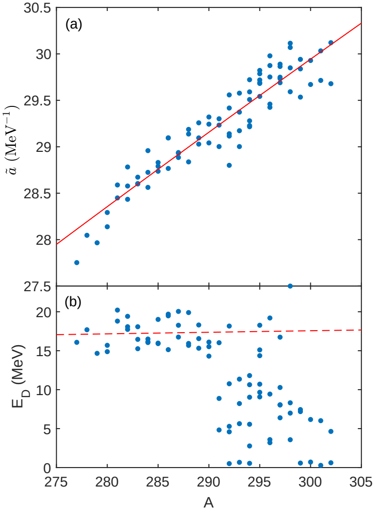
Saddle point

$$|\delta E_{sh}| \leq 1.7 \text{ MeV}$$

$$\delta E_{sh} \rightarrow (\delta E_{sh} - \Delta)$$

$$\tilde{a} = a_1 A + a_2 A^2$$

$a_1 = 0.1217 \text{ MeV}^{-1}$,
$a_2 = -7.3 \times 10^{-5} \text{ MeV}^{-1}$,
$E_D \approx 17$ MeV.

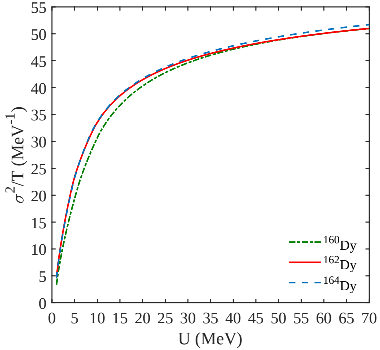


$$\rho_{tot}(U) = \frac{\rho(U)}{\sqrt{2\pi\sigma^2}}$$

$$\sigma^2 = \frac{1}{2}\sum_{\tau=p,n}\sum_{k} m_{\tau k}^2 \cosh^{-2}\left(1/2\beta E_{\tau k}\right),$$

$m_{\tau k}$: The single-particle spin projections.

$$\Im = \frac{\hbar^2\sigma^2}{T}$$



Nucleus	$\Im_{r.b.}$ ($\hbar^2$/MeV)
^{160}Dy	65.46
^{162}Dy	66.83
^{164}Dy	68.21

$$\Im_{r.b.} = 0.4MR^2$$

Assumption of a decoupling between intrinsic and collective degrees of freedom

$$U = U_i + U_c$$

$$\rho(U) = \int \rho_i(U_i)\rho_{coll}(U - U_i)dU_i$$

$$\rho_{coll}(U - U_i) = \sum_c \delta(U - U_i - U_c)\tau_c(U_c).$$

$$\tau_c(U_c) = 2I_c + 1$$

$$\rho(U) = \sum_c \rho_i(U - U_c)\tau_c(U_c)$$

$$\rho(U) \simeq \sum_c \left[\rho_i(U) - U_c \frac{\partial \rho_i(U)}{\partial U}\right] \tau_c(U_c)$$

$$= \sum_c \left[\rho_i(U) - \frac{U_c}{T}\rho_i(U)\right] \tau_c(U_c).$$

$$\rho(U) \simeq \rho_i(U) \sum_c \exp(-\frac{U_c}{T})\tau_c(U_c)$$

$$K_{coll} = \sum_c \exp(-\frac{U_c}{T})\tau_c(U_c)$$

$$U_c = \hbar\omega_\beta(n_\beta + 1/2) + \hbar\omega_\gamma(2n_\gamma + |K|/2 + 1) + \frac{\hbar^2}{2\Im}\left[I_c(I_c+1) - K^2\right]$$

n_β, n_γ: the quantum numbers of harmonic oscillator energies.

K: the projection of I_c on the symmetry axis.

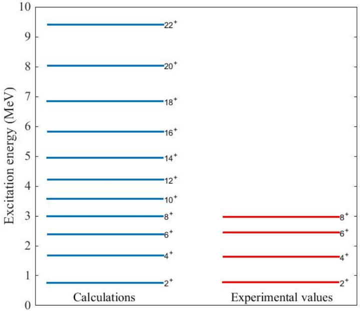
Yrast band

Quantum numbers:
$K = 0,\ I = 0, 2, 4, \ldots$
$n_\beta = n_\gamma = 0$
$U_c = \frac{\hbar^2}{2\Im}\left[I(I+1)\right].$

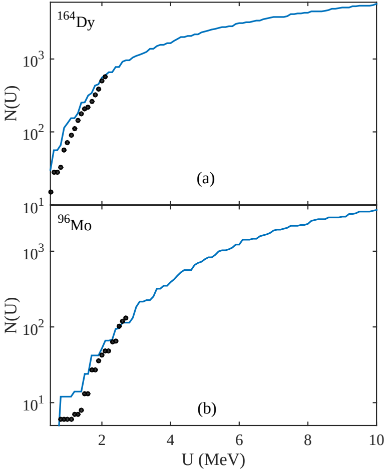
$E(2^+) = \frac{2\times 3}{2\Im}$

$\Im_{exp} = \frac{3}{E(2^+)}.$

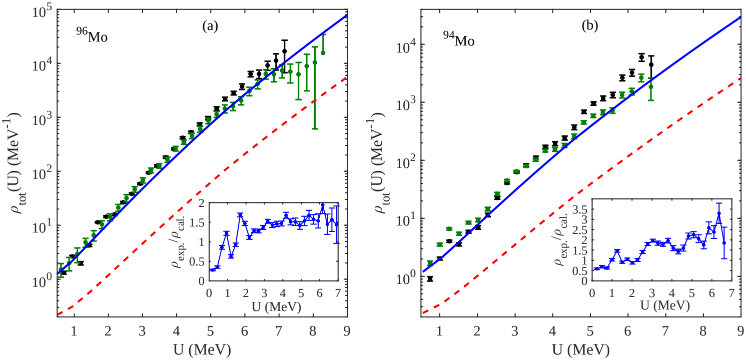
$\Im = \Im_{r.b}\left(1 - a_1 e^{-a_2 I(I+1)}\right)$
$a_1 = 0.89,\ a_2 = 0.006$



D. Abriola, and A. A. Sonzogni, Nuclear Data Sheets **107**, 2423 (2006).



B. Singh and J. Chen, Nuclear Data Sheets **147**, 1 (2018).
D. Abriola, and A. A. Sonzogni Nuclear Data Sheets **109**, 2501 (2008).



Green symbols: R. Chankova *et al.*, Phys. Rev. C **73**, 034311 (2006).

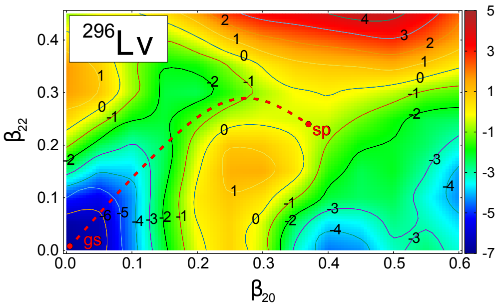
Black symbols: H. Utsunomiya *et al.*, Phys. Rev. C **88**, 015805 (2013).

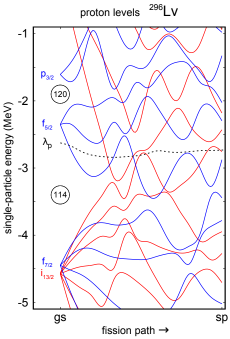
PES: Ground states and saddle points

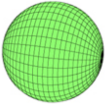
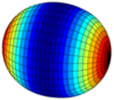
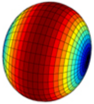
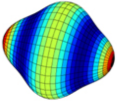
MM method: M. Kowal, P. Jachimowicz, A. Sobiczewski, Phys. Rev. C 82, 014303 (2010); P. Jachimowicz, M. Kowal, J. Skalski, Phys. Rev. C 89, 024304 (2014); Phys. Rev. C 95, 034329 (2017)

$$
\begin{aligned}
GS : R(\vartheta, \varphi) = cR_0\{1 \quad & + \quad \beta_{20}\mathrm{Y}_{20} + \beta_{30}\mathrm{Y}_{30} + \beta_{40}\mathrm{Y}_{40} \\
& + \quad \beta_{50}\mathrm{Y}_{50} + \beta_{60}\mathrm{Y}_{60} + \beta_{70}\mathrm{Y}_{70} \\
& + \quad \beta_{80}\mathrm{Y}_{80}\}
\end{aligned}
$$

$$
\begin{aligned}
SP : R(\vartheta, \varphi) = cR_0\{1 \quad & + \quad \beta_{20}\mathrm{Y}_{20} + \frac{\beta_{22}}{\sqrt{2}}[\mathrm{Y}_{22} + \mathrm{Y}_{2-2}] \\
& + \quad \beta_{40}\mathrm{Y}_{40} + \beta_{60}\mathrm{Y}_{60} + \beta_{80}\mathrm{Y}_{80}\}
\end{aligned}
$$





$\beta_{\lambda\mu} = 0$	$\beta_{20} > 0$	$\beta_{20} < 0$	$\beta_{40} > 0$
			
$\beta_{22} \neq 0$	$\beta_{30} \neq 0$	$\beta_{32} \neq 0$	$\beta_{20} \gg 0$