Investigation of the spin-orbit strengths on the prediction of the closed shells for superheavy nuclei based on Two Center Shell Model

A.N.Bezbakh

Joint Institute for Nuclear Research

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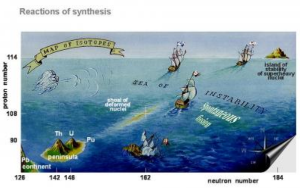
The motivation



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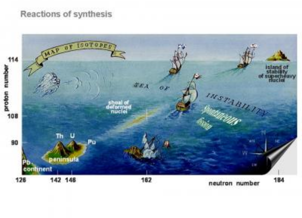

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- **The investigation of transfermium elements expands our knowledge of the single-particle structure, location of the shell closures, and decay modes of heaviest nuclei**

- **Increasing stability of nuclei approaching $N = 184$, and indication quite a large shell effects behind $\mathbf{Z = 114}$ $\Longrightarrow$ Are valid the predictions of relativistic and nonrelativistic mean-field models in our case ($Z = 120 - 126$, $N = 182 - 184$)?**
  Or the phenomenological model[1] **($Z = 126$)?**
  Note: the mic-mac models[2] predict $Z = 114$.

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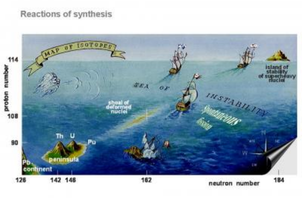

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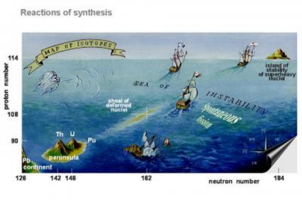

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Our Aim

To investigate the role of spin-orbital strengths on the position of the magic shell and how they affect the description of low-lying states

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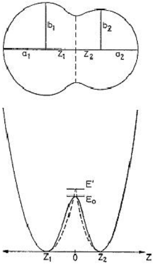
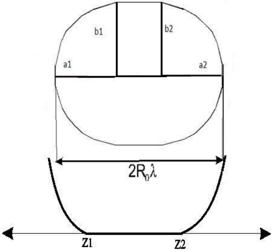
Two center shell model (TCSM)

$$H = (-\hbar/2m)\nabla^2 + V(\rho, z) + V_{l,s} + V_{l^2}$$ [3]



- $\lambda = L/2R_0$,
- $\beta = a/b = \beta_1 = \beta_2$ th
  case,
- $\varepsilon = E_0/E' = 0$,
- $\eta = (A_1 - A_2)/(A_1 + A_2) = 0$;

Other variables are fixed.



where the momentum-independent part is $V(\rho, z)$ and the momentum-dependent part consists of

$$V_{ls} = -\frac{2\hbar\kappa}{m\omega_0'}(\nabla V \times \mathbf{p})\,\mathbf{s}$$

$$V_{l^2} = -\kappa\mu\hbar\omega_0' l^2 + \kappa\mu\hbar\omega_0' \frac{N(N+3)}{2}\delta_{if}$$

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[3] J. Maruhn and W. Greiner, Z. Phys. A **251**, 431 (1972)

Modification of TCSM

In order to improve the description of spins and parities of the nuclear ground states, we introduce a weak dependence on $(N - Z)$ in the parameters $\kappa_{n,p}$ and $\mu_{n,p}$. For the actinide and transactinide region we suggest[4, 5]:

$$\kappa_n = -0.076 + 0.0058(N - Z) - 6.53 \times 10^{-5}(N - Z)^2 + 0.002A^{1/3}$$
$$\mu_n = 1.598 - 0.0295(N - Z) + 3.036 \times 10^{-4}(N - Z)^2 - 0.095A^{1/3}$$

and

$$\kappa_p = 0.0383 + 0.00137(N - Z) - 1.22 \times 10^{-5}(N - Z)^2 - 0.003A^{1/3}$$
$$\mu_p = 0.335 + 0.01(N - Z) - 9.367 \times 10^{-5}(N - Z)^2 + 0.003A^{1/3}$$

With those Eqs. we are able to describe correctly the ground-state spins of many heavy nuclei treated.

Note that the introduced additional dependence on $N - Z$ mainly supplies a better order of the single-particle levels near the Fermi surface.

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[4] G.G. Adamian, N.V. Antonenko, and W. Scheid, Phys. Rev. C **81**, 024320 (2010)

[5] A.N. Kuzmina, G.G. Adamian, N.V. Antonenko, and W. Scheid, Phys. Rev. C **85**, 014319 (2012)

Potential energy and E_{sh}

$$E = E_{LDM} + \delta E_{mic}$$

- The Coulomb and surface energies
- The shell E_{sh} and pairing corrections

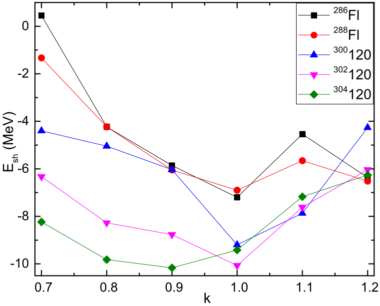
Note

The stability of SHN correlates with the shell correction energy E_{sh} in the ground state. The larger $|E_{sh}|$, the greater the stability of SHN with respect to spontaneous fission and $\alpha-$decay.

Calculated results: Position of shell closure

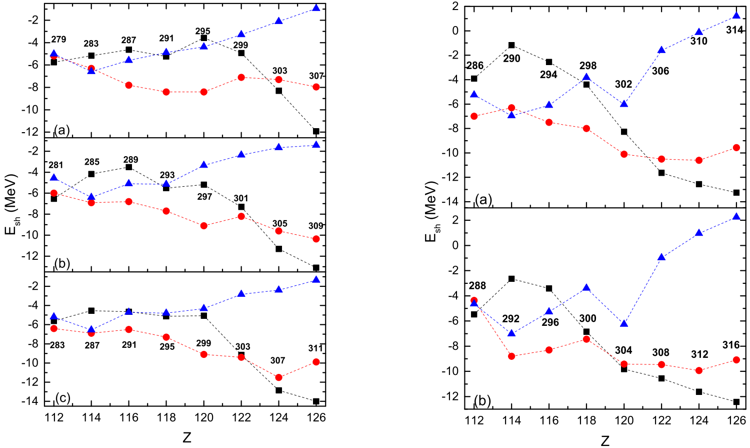
$$H = (-\hbar/2m)\nabla^2 + V(\rho, z) + k V_{l,s} + V_{l^2}$$

In order to study the influence of spin-orbit (sl) strengths in the region of SHN with the modified TCSM, we take the sl-term as $k V_{ls}$ and study how the results depend on the coefficient k varying from 0.8 to 1.2. The value $k = 1$ corresponds to the parameters defined in Eqs. for κ and μ .



Calculated results: Position of shell closure

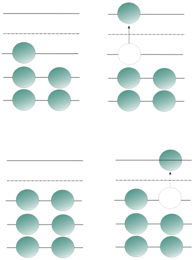
$k = 0.8$ (squares), 1.0 (circles), and 1.2 (triangles)



The stability of the nuclei with $Z > 120$ decreases with increasing k

The strength of spin-orbit interaction is crucial to define the position of the shell closures in nuclei beyond Pb. The 20% variation of the spin-orbit strength can strongly shift the position of the minimum of E_{sh}.

One-quasiparticle energies E_μ

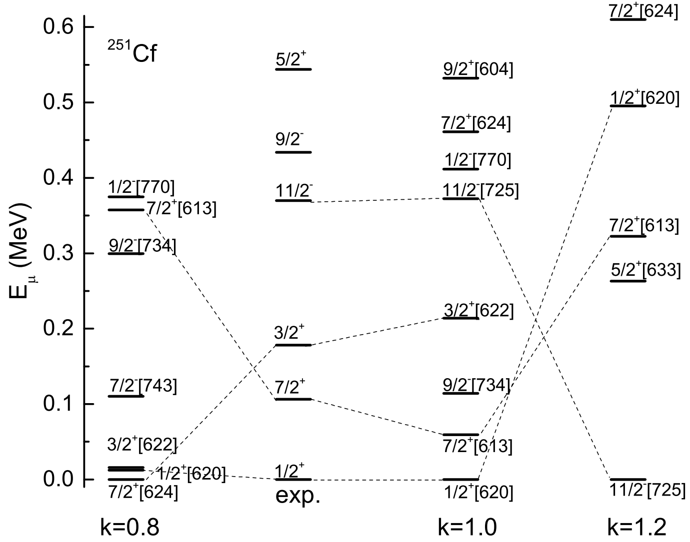


Calculating the potential energy surface as a function of collective coordinates with the TCSM, we find the ground-state potential minimum in which the energies of the low-lying one-quasiparticle states are obtained.

$$E_\mu = \sqrt{(e_\mu - e_F)^2 + \Delta^2} - \sqrt{(e'_\mu - e_F)^2 + \Delta^2}$$

Single-particle states μ with energies e_μ,
the pairing-energy gap parameter Δ,
the Fermi energy e_F.

Calculated results: Dependence of one-quasiparticle spectra on spin-orbit strength



exp.:https://www.nndc.bnl.gov

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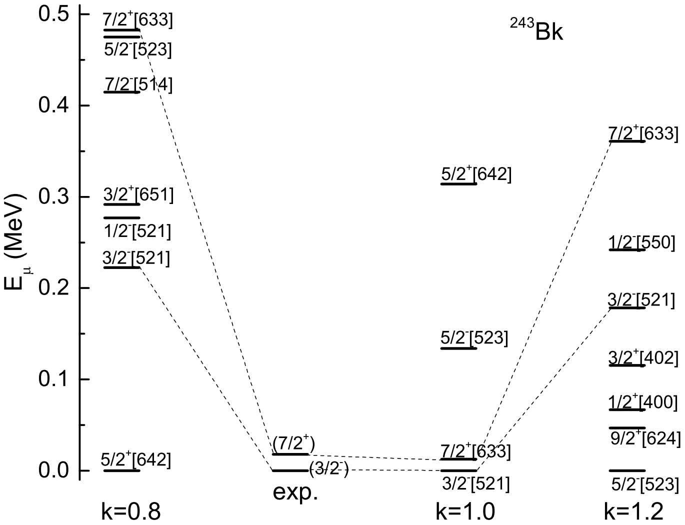

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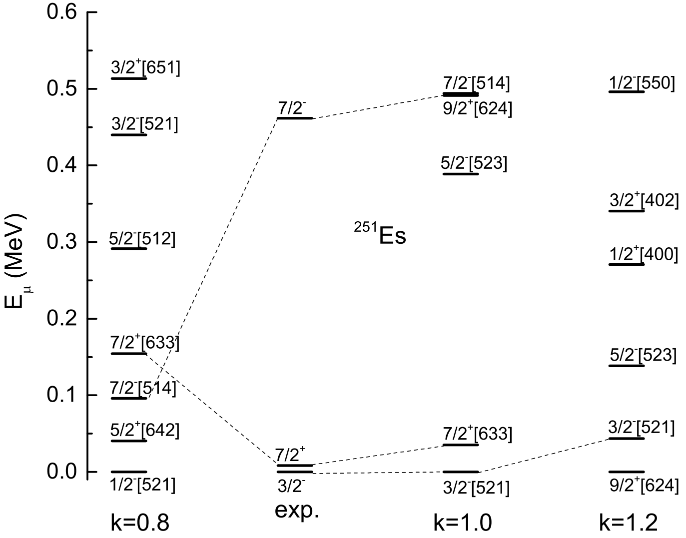

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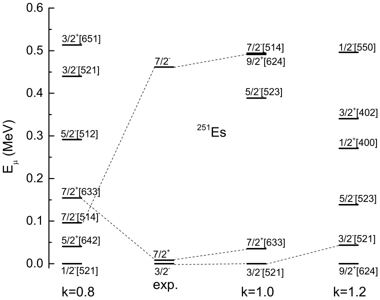

Briefly

- The experimental energies, spins, and parities are well described (within 250 keV) with $k = 1.0$.
- The calculated results obtained at $k = 0.8$ and 1.2 are less consistent with the experimental data – the ground-state spins and parities can not be reproduced.
- In most cases, the one-quasiparticle spectra become denser with $k = 0.8$ or 1.2.
- At $k = 1$ we have the best description of low-lying one-quasiparticle states.

SUMMARY

- As shown, the quality of the description of low-lying one-quasiparticle states crucially depends on the spin-orbit strength. The spin-orbit strength taken in the modified TCSM at $k = 1$ allows us to describe well the low-lying one-quasiparticle spectra in heavy nuclei.

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[6] A.N. Kuzmina, G.G. Adamian, N.V. Antonenko, and W. Scheid, Phys. Rev. C **85**, 014319 (2012)

[7] G.G. Adamian, L.A. Malov, N.V. Antonenko, H. Lenske, K. Wang, and S.-G. Zhou, Eur. Phys. J. A **54**, 170

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- The shape of the island of stability is also defined by the spin-orbit strength. At $k > 1$, it is located between $Z = 112$ and 120, while at $k \leq 1$ it is extended to $Z = 126$. The experiments on production of the $Z = 120$ nucleus could help us to answer the question of whether there is a shelf of stability beyond $Z = 120$.

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