Monte Carlo simulation of the experimental setup for measurements of entangled annihilation photons

Sultan Musin

musin.sa@phystech.edu

MIPT, Moscow

INR RAS, Moscow

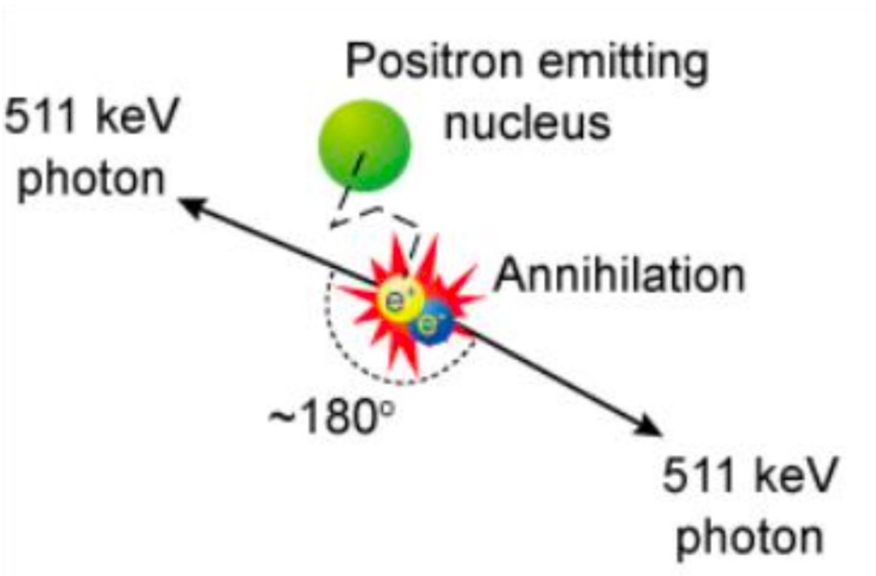
Goals

- Studying the kinematics of Compton scattering of entangled annihilation photons.
- Solving theoretical contradictions in calculation for Compton scattering kinematics of entangled and decoherent pairs of photons.

Objectives for this work

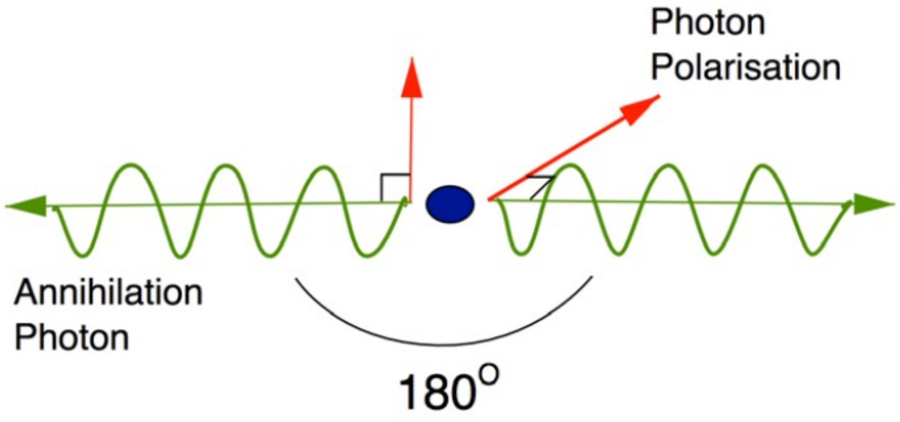
- Comparison of simulation results with experimental data.
- Estimation of the setup sensitivity to the measured effects.

Entangled annihilation photons



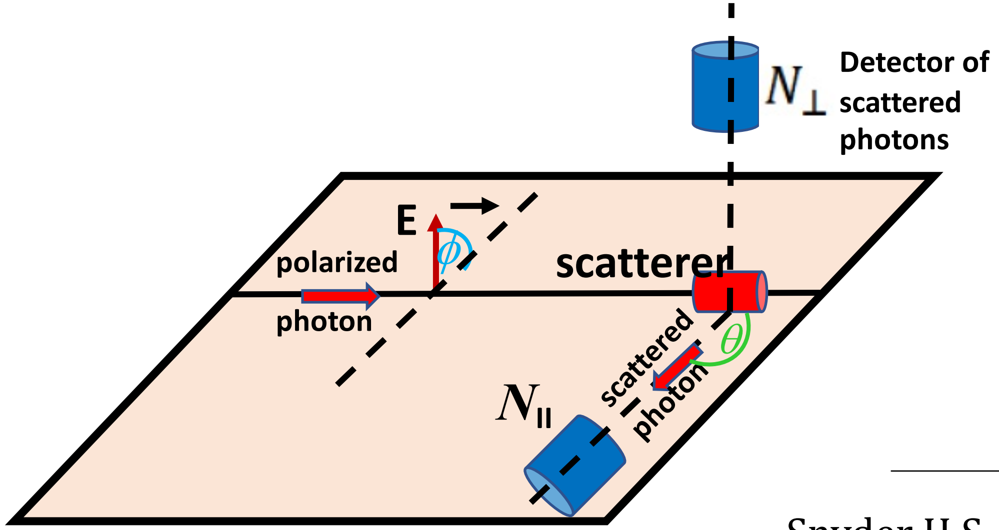
According to angular momentum conservation and parity symmetry the state vector of annihilation pair is:

$$\Psi = |H\rangle_1 |V\rangle_2 + |V\rangle_1 |H\rangle_2$$



Each photon in pair has no definite polarization but polarizations are orthogonal for photons in pair. According to the theory the annihilation photons are maximally entangled.

Methods for polarization measurements of high energy photons



Compton polarimeter

Klein-Nishina formula for polarized photons (not entangled)

$$\frac{d\sigma}{d\Omega}(\theta,\phi) = \frac{1}{2}\cdot\frac{e^2}{m_e c^2}\cdot\frac{E_{\gamma_1}^2}{E_\gamma^2}\cdot\left(\frac{E_{\gamma_1}}{E_\gamma}+\frac{E_\gamma}{E_{\gamma_1}} - 2\sin^2\theta\cos^2\phi\right)$$

Cross-section is maximum for $\phi = \pi/2$ angle!

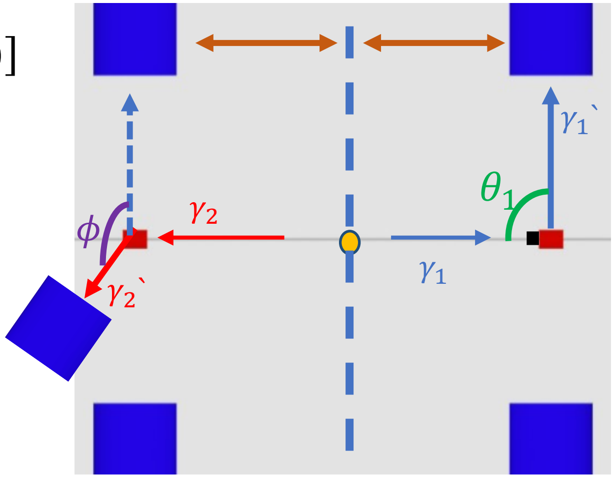
Snyder H S, Pasternack S and Hornbostel J, - 1948 Angular correlation of scattered annihilation radiation Phys. Rev. 73 440–8

$$P_{12}(E_1, E_2, \phi) = \left(\frac{d\sigma}{d\Omega_1}\right)_{NP}\left(\frac{d\sigma}{d\Omega_2}\right)_{NP}[1 - \alpha(\theta_1)\alpha(\theta_2)\cos(2\phi)]$$

Ratio of the numbers of scattered annihilation photons:

$$R_{theory}(\theta) = \frac{N\left(\phi = \frac{\pi}{2}\right)}{N(\phi = 0)} = 1 + \frac{2\sin^4\theta}{\gamma^2 - 2\gamma\sin^2\theta}; \qquad \gamma = 2 - cos\theta + (2 - cos\theta)^{-1}$$

$$R = 2.6 \ \ for\ \theta = 90^0$$

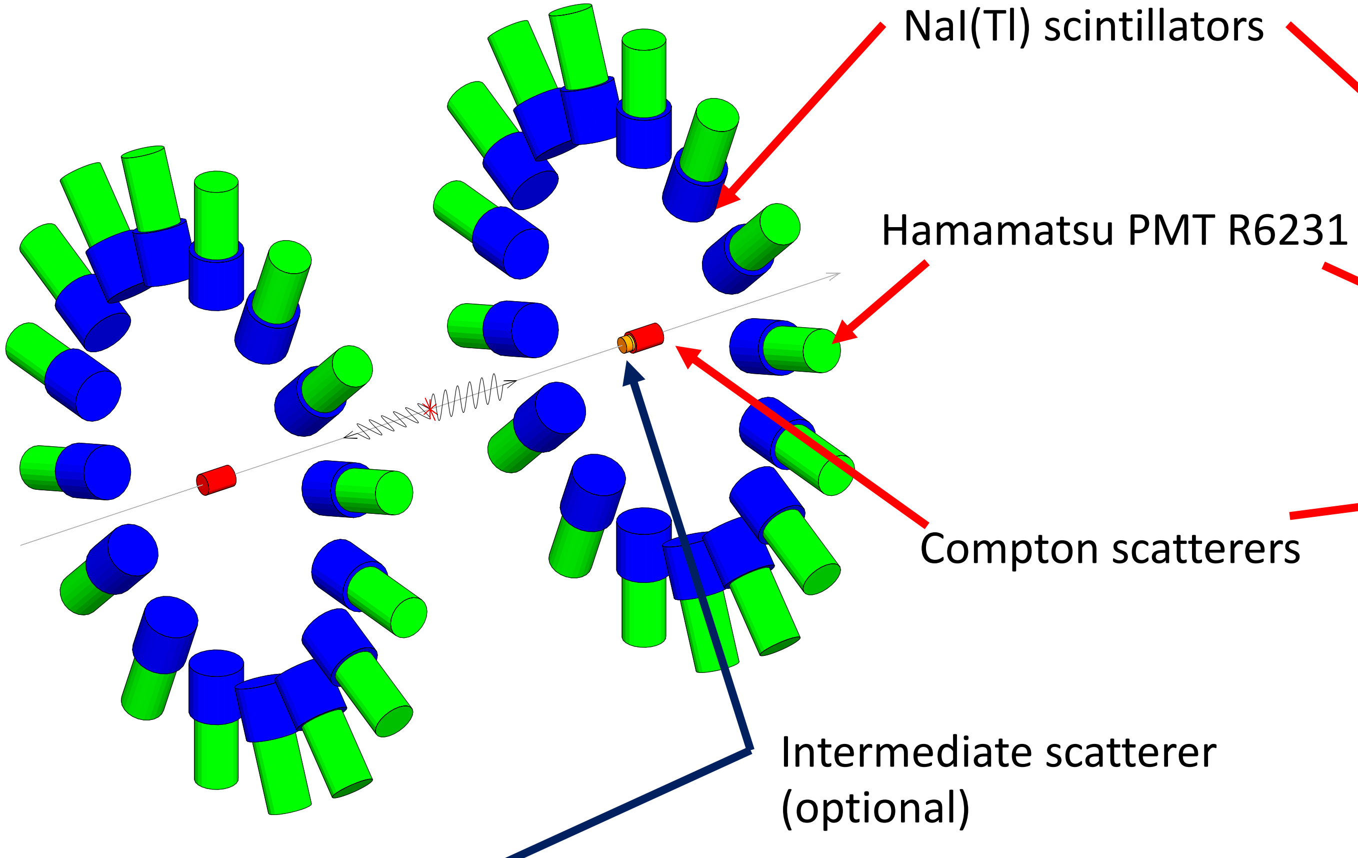


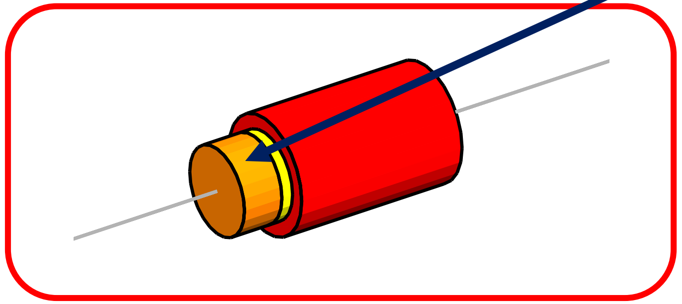
Two Compton Polarimeters
for angular correlations measurements

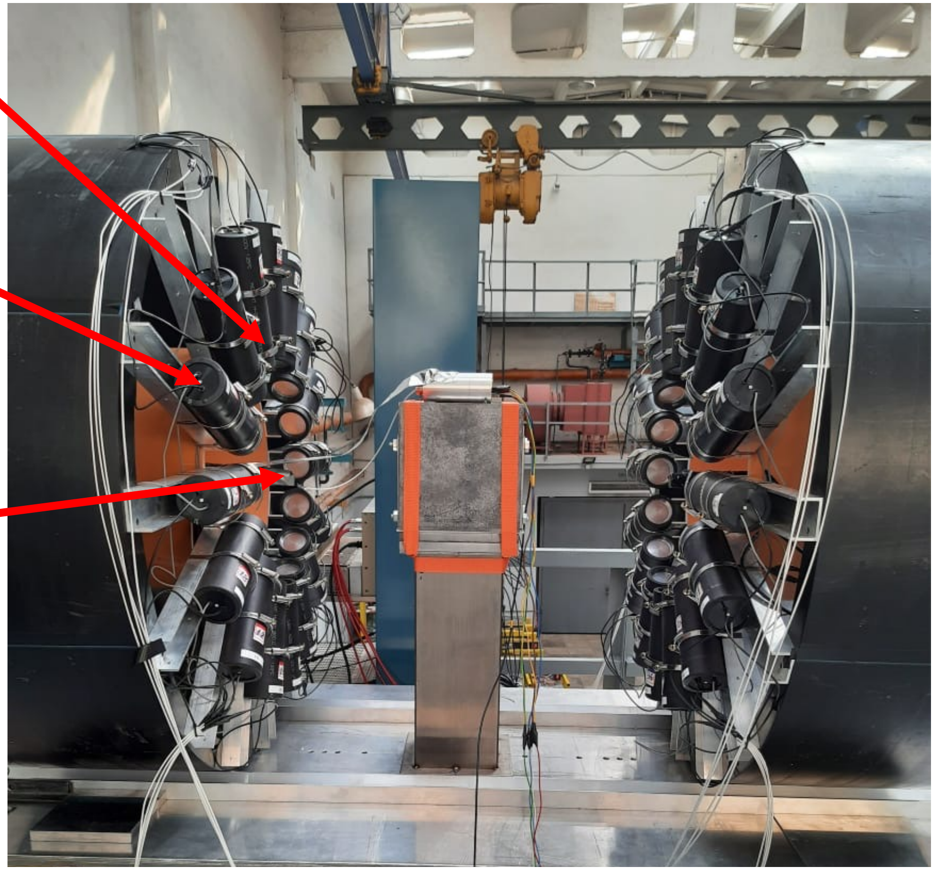
According to D. Bohm and Y. Aharonov (Phys. Rev. (1957) 108, 1070) the measurements of angular correlations would provide the experimental test of the entanglement if R>2.

For decoherent photons ***R=1*** for non-entangled photons ***R<2***

Experimental setup



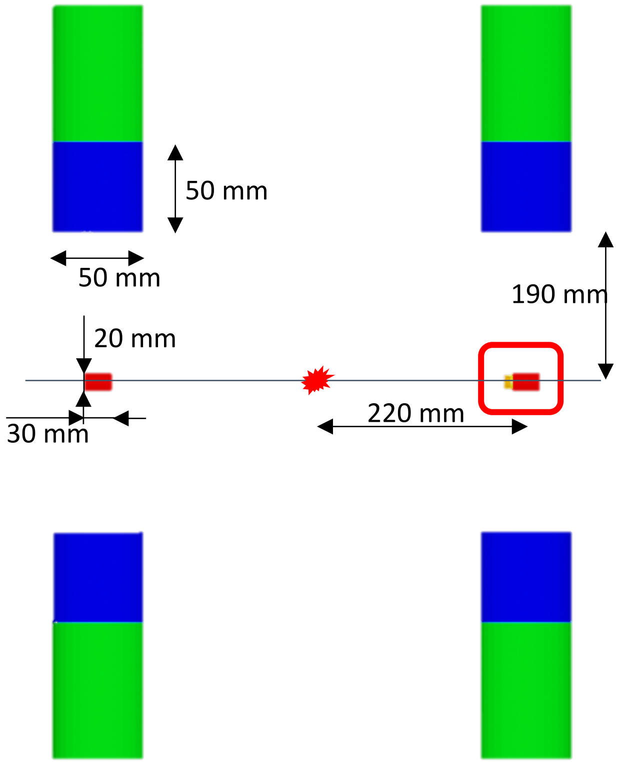


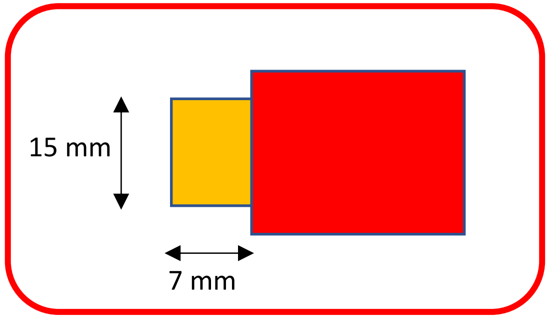


Experimental setup in INR RAS, Moscow

- 2 arms
- 16 NaI(Tl) scintillation detectors in each arm = 16 independent Compton polarimeters
- 22.5 degrees between every NaI(Tl) detector

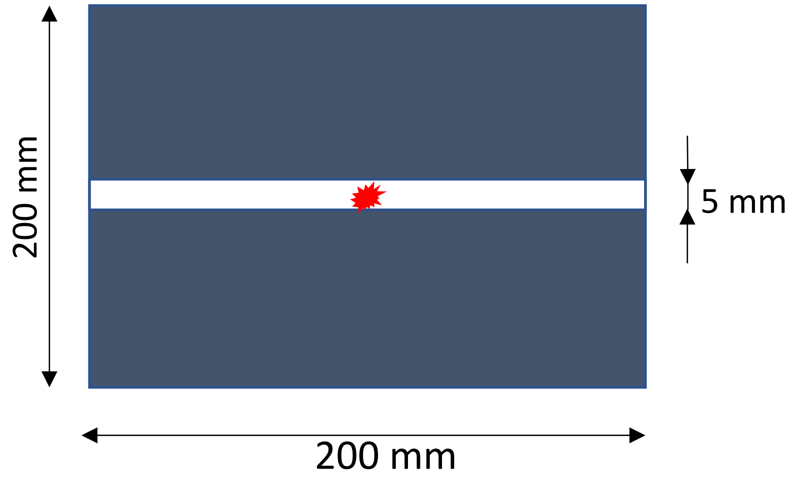
Geometry of the setup





Intermediate GAGG scatterer is placed before one of the main scatterers.

^{22}Na – positron source. Positrons annihilate with environmental electrons and produce a pair of entangled photons with energy 511 keV each

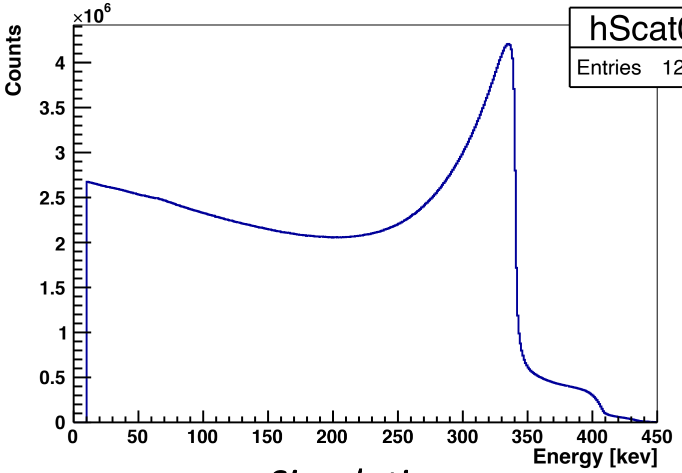


Collimator

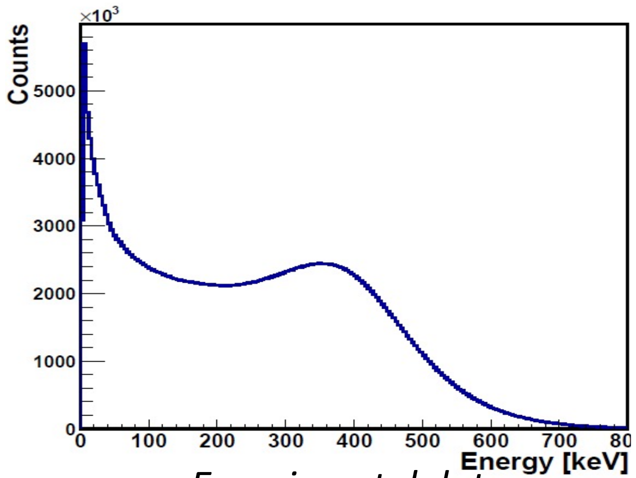
Polarizations of photons are not defined separately, but polarization directions are transverse to each other.

Photons lose their quantum entanglement state after passing through an intermediate scatterer in one arm and become decoherent.

Geant4 simulation without intermediate scatterer (entangled photons)



Simulation



Experimental data

Energy deposition In Compton scatterer

Energy deposition for Compton scattering

$$E_{dep} = \frac{E_0}{1 + \frac{m_e c^2}{2E_0 \sin^2 \frac{\theta}{2}}}$$

For θ = 90 *in Compton scatterer*:

$$E_{dep} = \frac{1}{2} E_0 = \frac{1}{2} * 511\ keV = 255.5\ keV$$

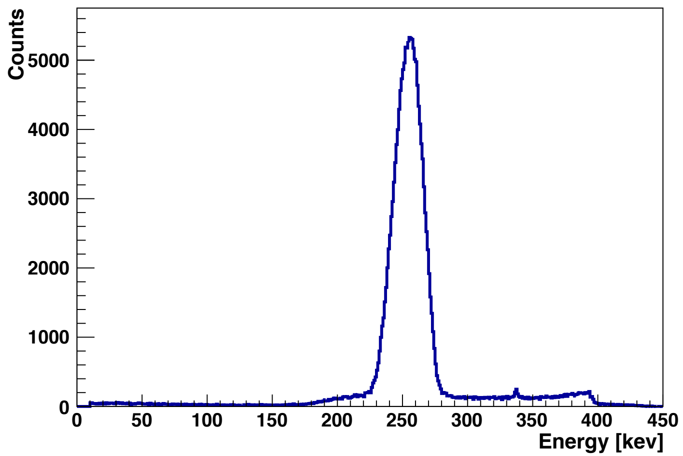
Simulation results

$$E_{dep}^{compt} = (254.3 \pm 10.7)\ keV$$

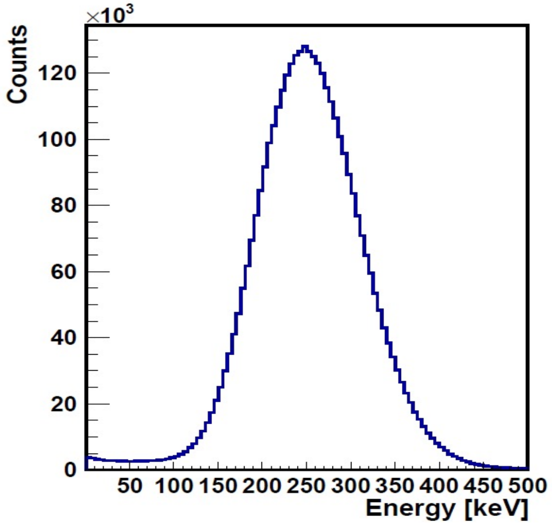
Experimental results

$$E_{dep}^{compt} = (254.3 \pm 10.7)\ keV$$

Energy resolution is defined by solid angle



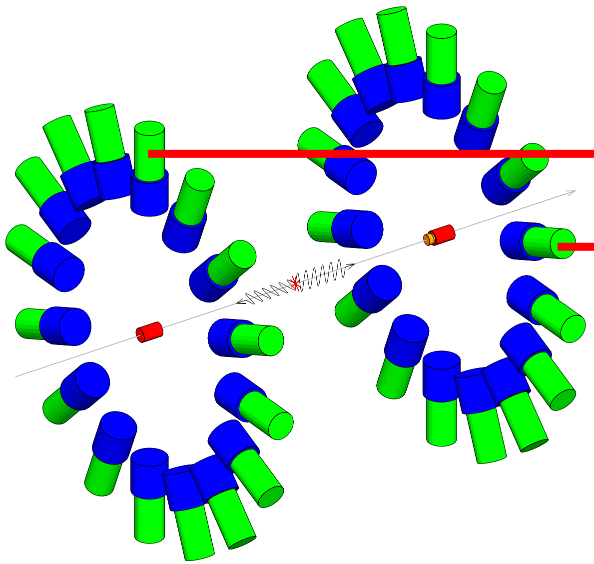
Simulation

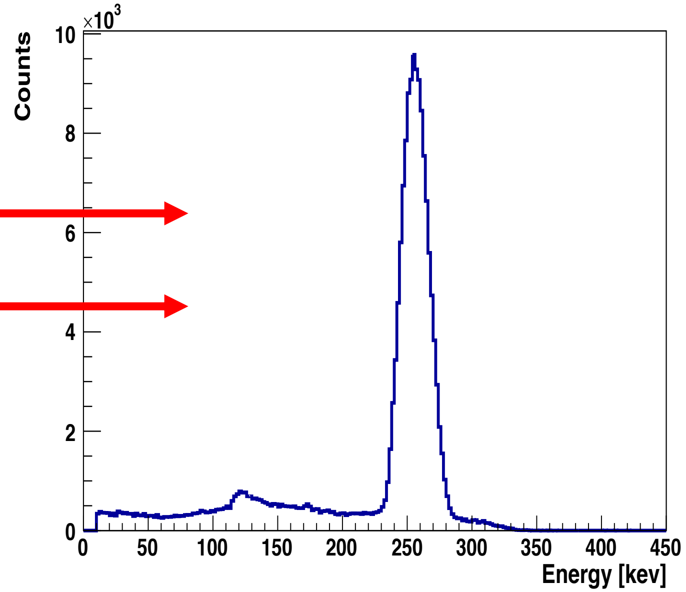


Experimental data

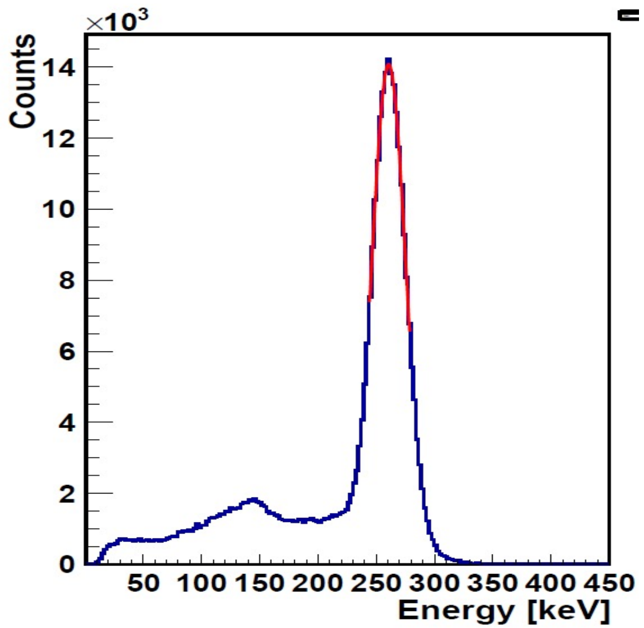
Energy deposition In Compton scatterer with further hit to NaI(Tl)

Geant4 simulation without intermediate scatterer (entangled photons)





Simulation



Experimental data

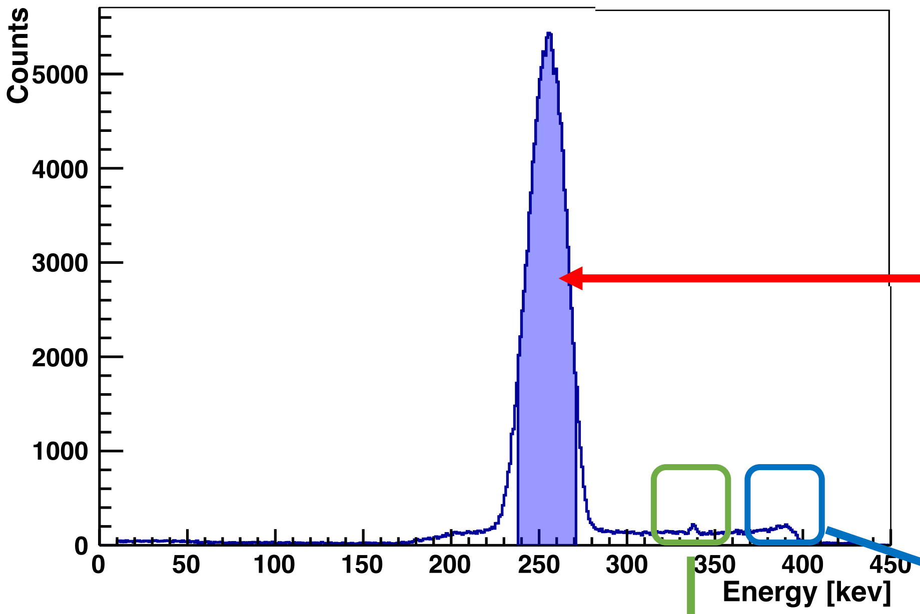
Energy deposition In NaI(Tl) detector

$$E_{dep}^{NaI} = (256.2 \pm 10.7)\ keV$$

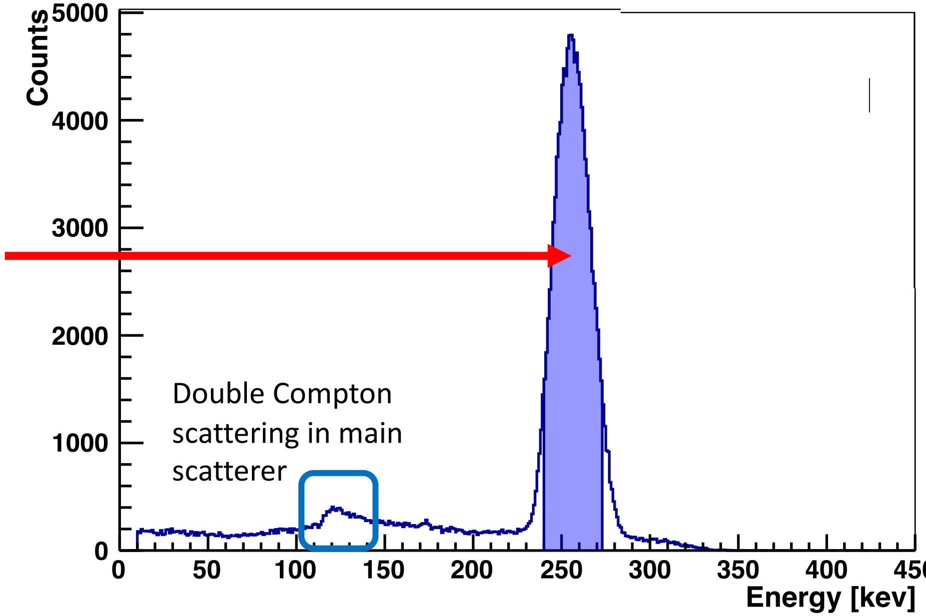
$$E_{dep}^{NaI} = (256.2 \pm 10.7)\ keV$$

Events selection

Energy deposition In Compton scatterer with further hit to NaI(Tl)

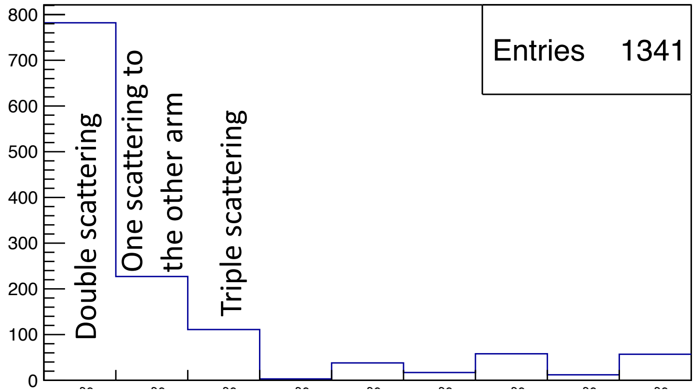


Energy deposition In NaI(Tl) detector

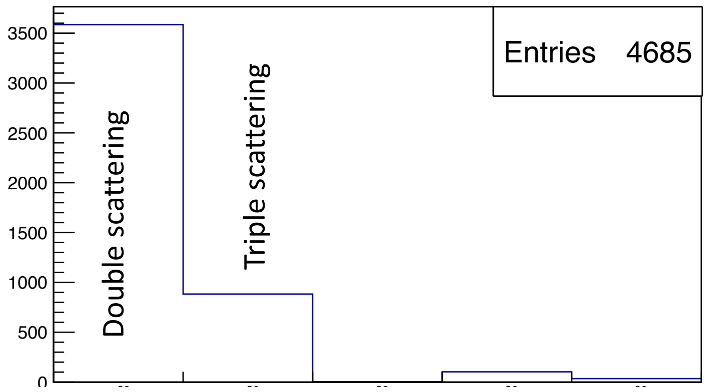


One Compton scattering
1.5σ width

Processes in scatterer, E_{dep} ∈[334 ; 342] keV



Processes in scatterer, E_{dep} ∈[370 ; 400] keV



Angular correlations for entangled photons

To simulate the quantum entanglement the Geant4 11.0(beta) class was used, written by the authors of *Watts, D.P., Bordes, J., Brown, J.R. et al. Photon quantum entanglement in the MeV regime and its application in PET imaging. Nat Commun 12, 2646 (2021).*

$$dP = \frac{{k_1}^2{k_2}^2(\varepsilon_1\varepsilon_2 - \varepsilon_1 sin^2\theta_2 - \varepsilon_2 sin^2\theta_1 + 2sin^2\theta_1 sin^2\theta_2 sin^2\phi)}{4\pi^2 k_0^2(\frac{40}{9} - 3\ln 3)^2} d\Omega_1 d\Omega_2$$

,where

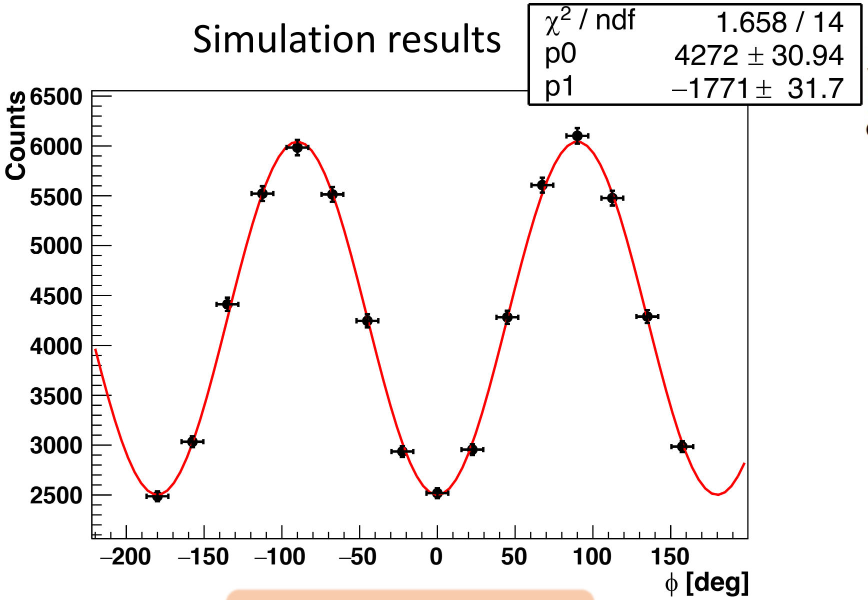
$$\varepsilon_1 = \frac{k_1}{k_0} + \frac{k_0}{k_1}$$

$$\varepsilon_2 = \frac{k_2}{k_0} + \frac{k_0}{k_2}$$

$$N(\varphi) = \alpha - \beta\cos 2\varphi$$

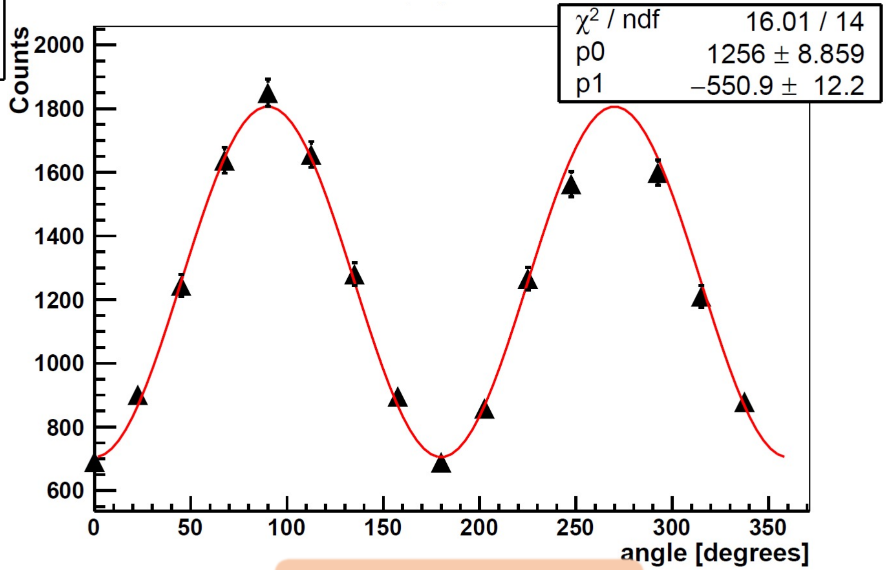
$$\rightarrow \quad R = \frac{N(\varphi = 90°)}{N(\varphi = 0°)} = \frac{\alpha + \beta}{\alpha - \beta}$$

Simulation results

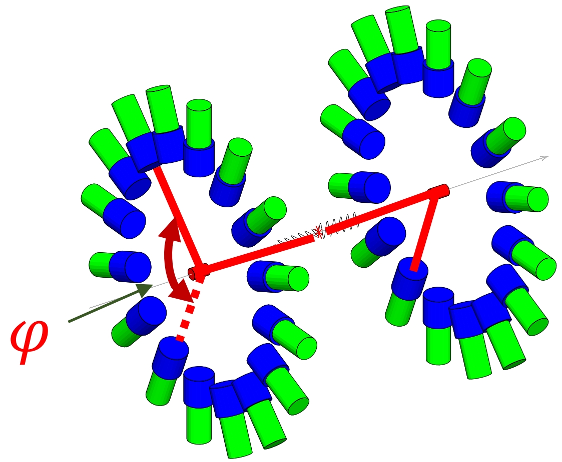


$R = 2.42 \pm 0.05$

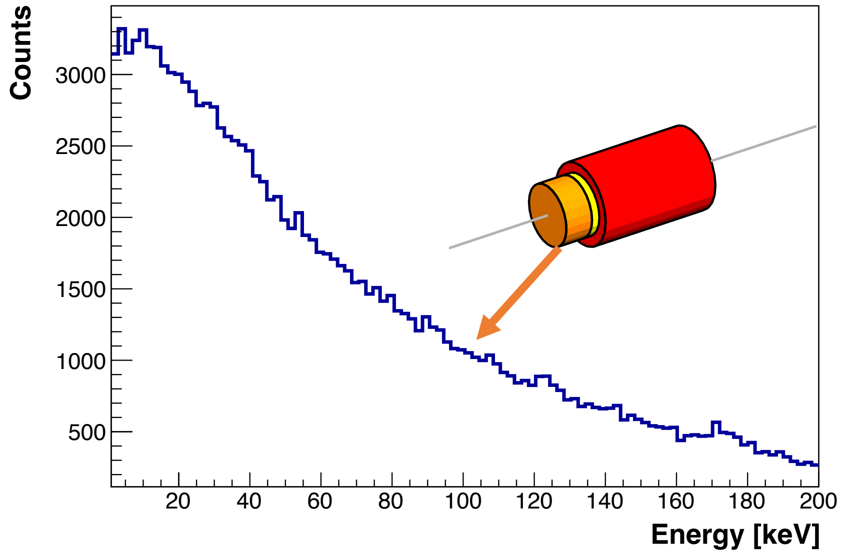
Experimental data



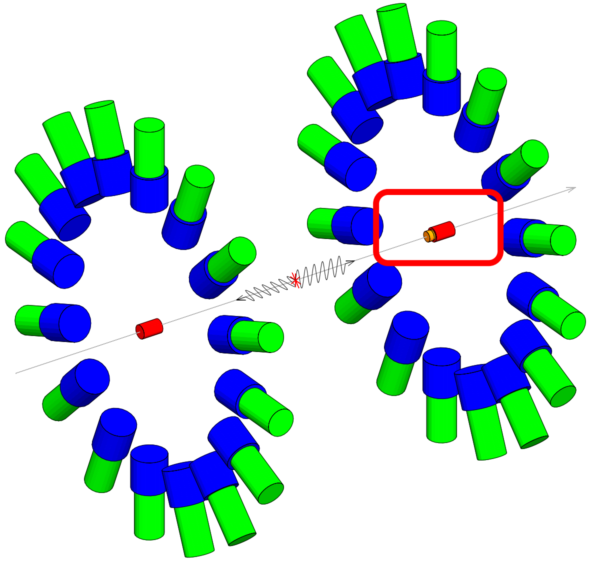
$R = 2.56 \pm 0.07$



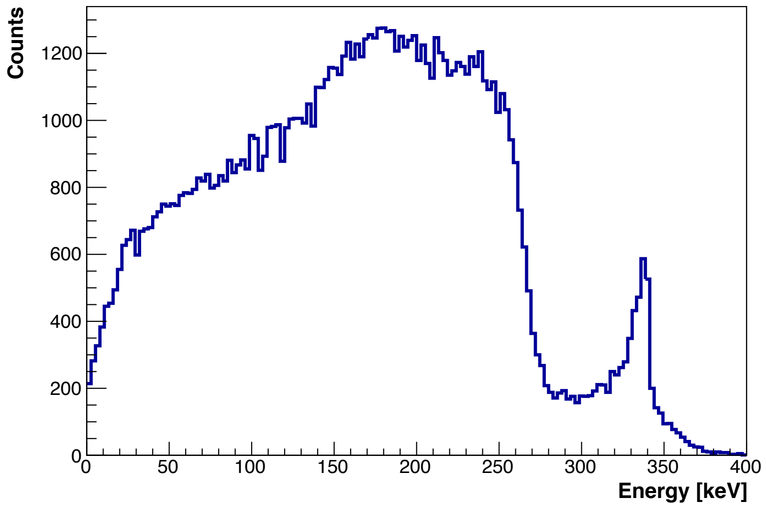
Case of decoherent photons



Energy deposition in intermediate scatterer



Simulation

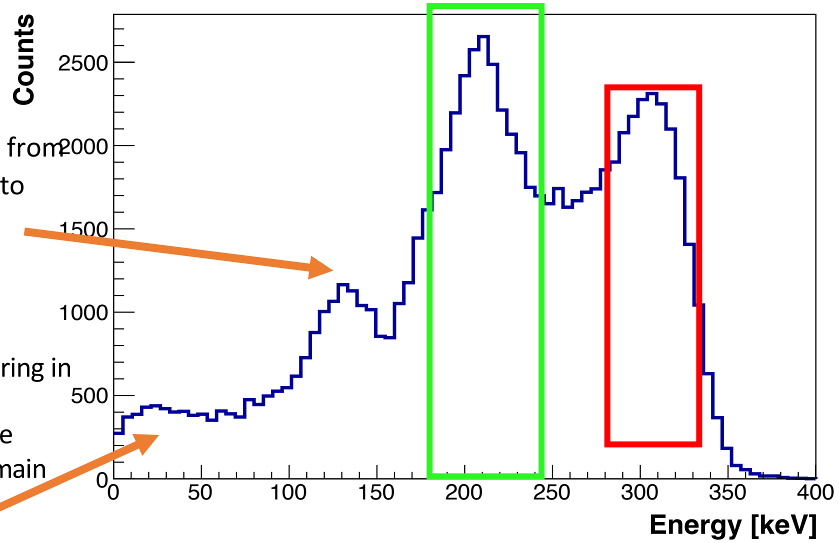


Energy deposition in main Compton scatterer

Events selection in case of decoherent photons

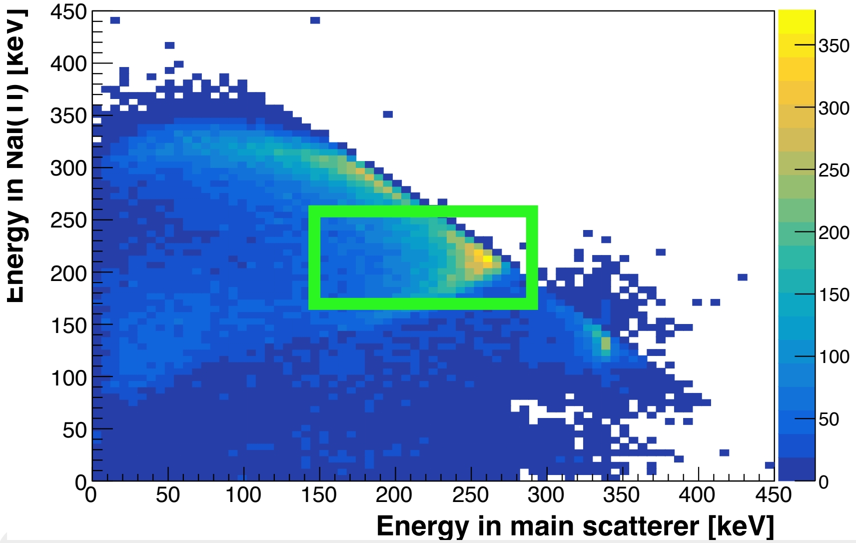
Back scattering from main scatterer to intermediate

Double scattering in intermediate scattering, one scattering in main scatterer

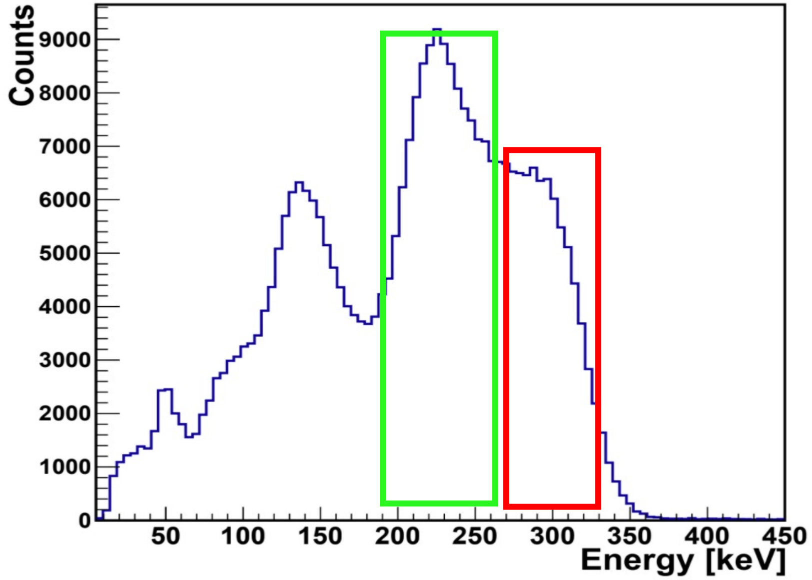


Deposited energy correlation in scintillation detector (NaI) for decoherent photons

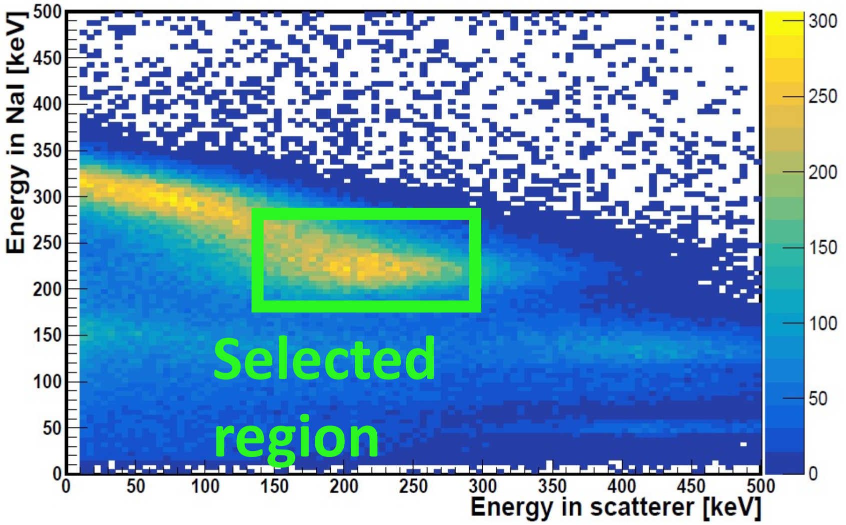
Simulation



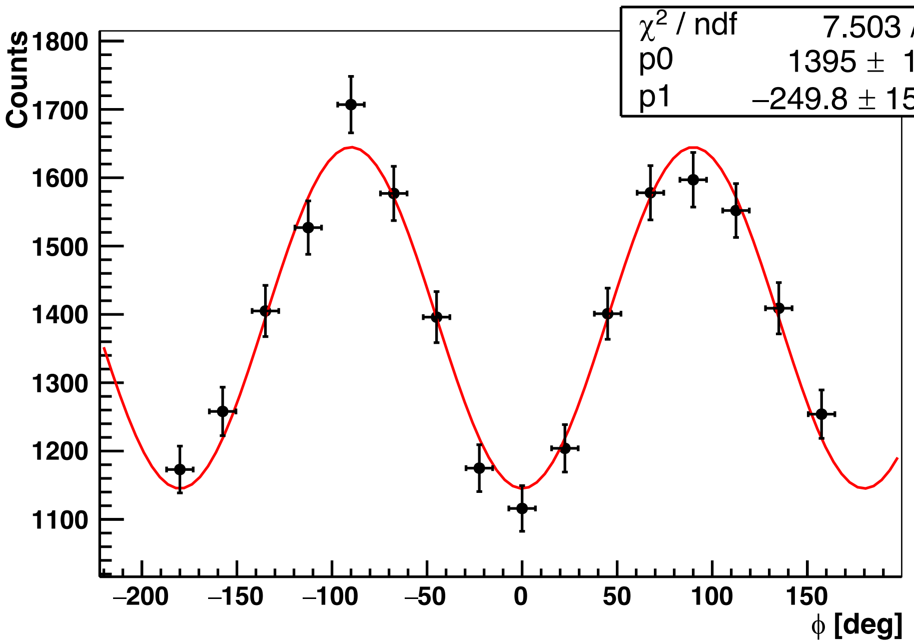
Deposited energy correlation between scatterer and scintillation detector (NaI) for decoherent photons



Experiment

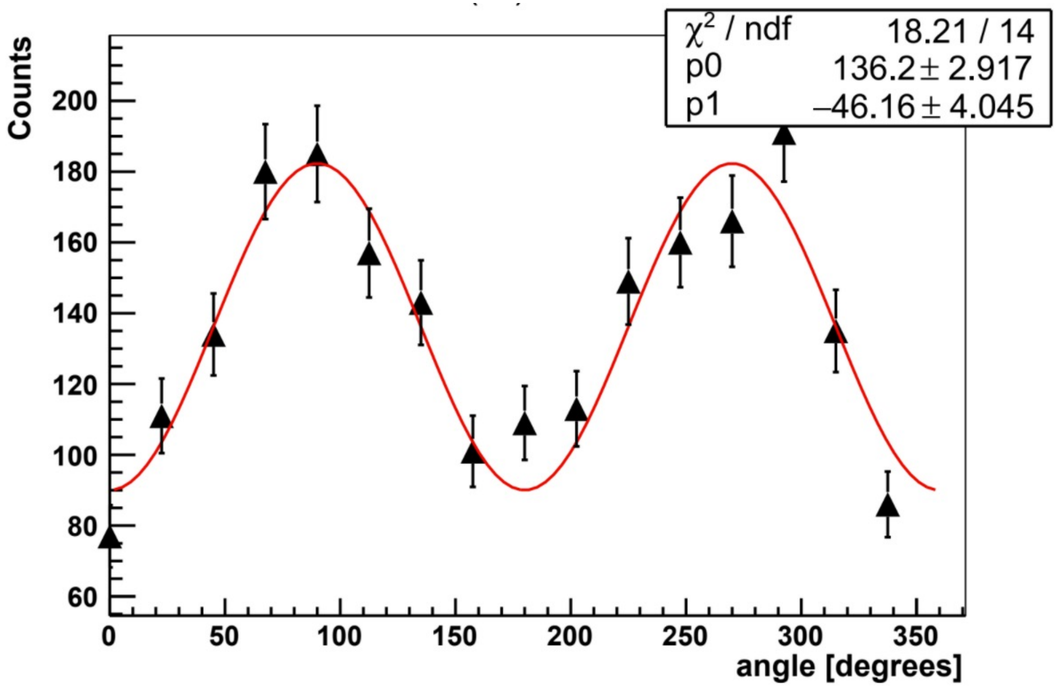


Angular correlations for decoherent photons



Simulation results

$\mathrm{R} = 1.44 \pm 0.03$



Experimental data

$\mathrm{R} = 2.04 \pm 0.15$

Geant4 incorrectly treats the entanglement after the first interaction.

This Monte Carlo model doesn't fit the experimental data. It is to be fixed in future Geant4 development.

Conclusion

- Monte Carlo simulation of the experimental setup was carried out.
- Energy spectra in detectors of the setup were obtained and compared to the experimental data.
- Angular correlation functions of two event groups, entangled photons and decoherent photons, were examined.
- There is an agreement between the experiment and Monte Carlo simulation for entangled photons.
- The contradiction between current kinematics model for decoherent photons and experimental data was found.
- Monte Carlo model doesn’t fit the experimental data in case of decoherent photons. It is to be fixed in future Geant4 development.

Thank you for your attention