Study of the Compton scattering of entangled annihilation photons

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TASKS:

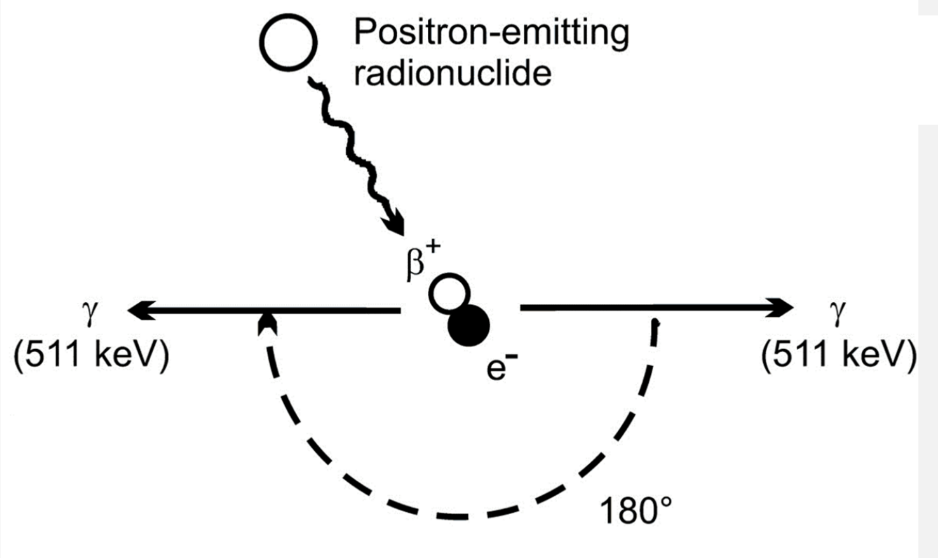
- Study of the Compton scattering kinematics for the pairs of decoherent annihilation gammas;
- Comparison of the scattering kinematics for the pairs of entangled and decoherent photons;

Importance:

- Compton scattering of entangled photons was not studied in details;
- Compton scattering of decoherent photons was not studied at all;
- Contradictions in the theoretical calculations of the cross-sections for entangled and decoherent photons;
- Possible applications for new generation of the positron emission tomography.

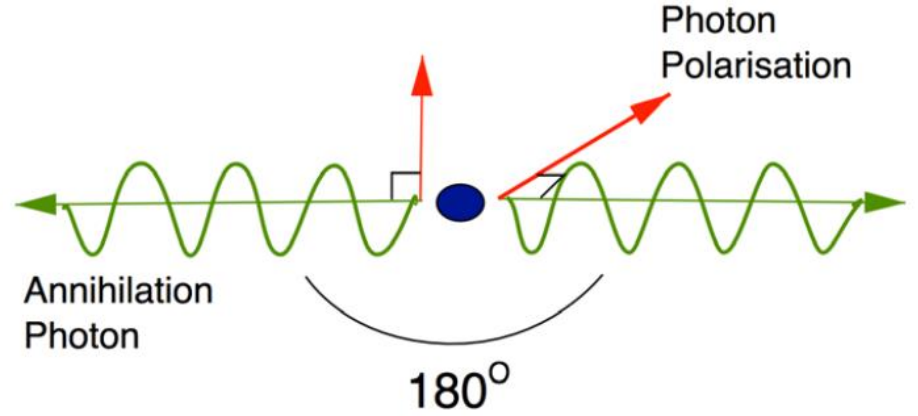
Entangled annihilation photons

(two-photon electron-positron annihilation at rest)



According to angular momentum conservation and parity symmetry the state vector of annihilation pair is:

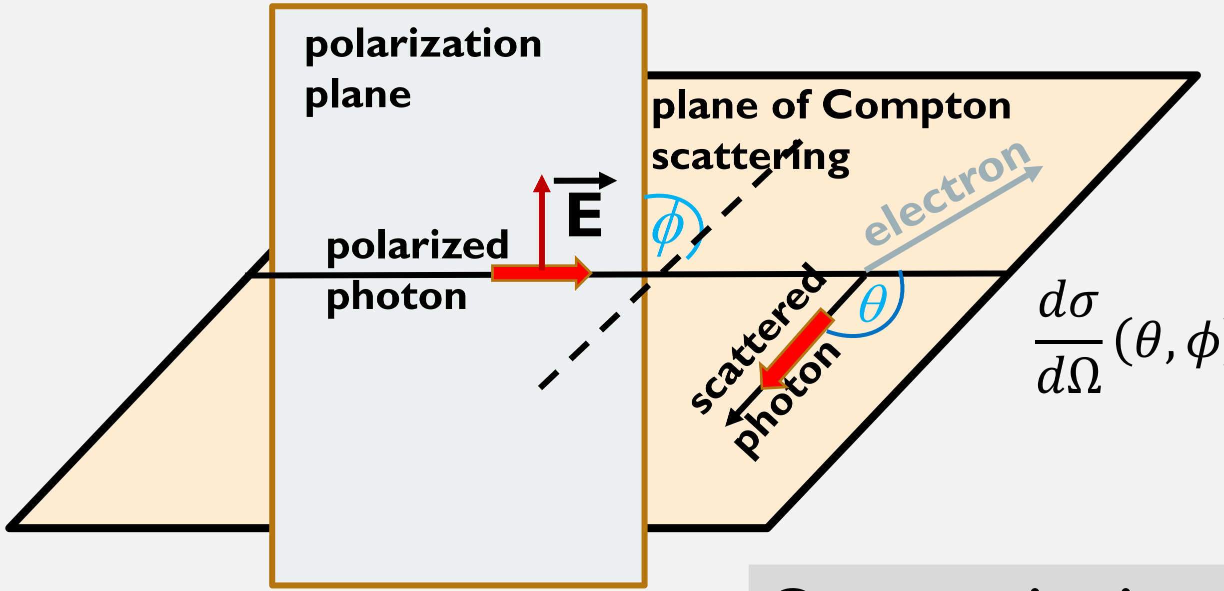
$$\Psi = |H\rangle_1 |V\rangle_2 + |V\rangle_1 |H\rangle_2$$



Each photon in pair has no definite polarization but polarizations are orthogonal for photons in pair.
According to the theory the annihilation photons are maximally entangled.
But it was never experimentally proven!
The reason: difficulties in polarization measurements for high energy gammas.

Method of measurement of polarization for high energy photons

Only Compton scattering can be applied for the polarization measurements of gammas!



Differential cross-section of Compton is given by **Klein-Nishina formula**:

$$\frac{d\sigma}{d\Omega}(\theta,\phi) = \frac{1}{2}\cdot\frac{e^2}{m_e c^2}\cdot\frac{E_{\gamma_1}^2}{E_\gamma^2}\cdot\left(\frac{E_{\gamma_1}}{E_\gamma}+\frac{E_\gamma}{E_{\gamma_1}} - 2\sin^2\theta\cos^2\boldsymbol{\phi}\right)$$

Angle between the scattering plane and the polarization.

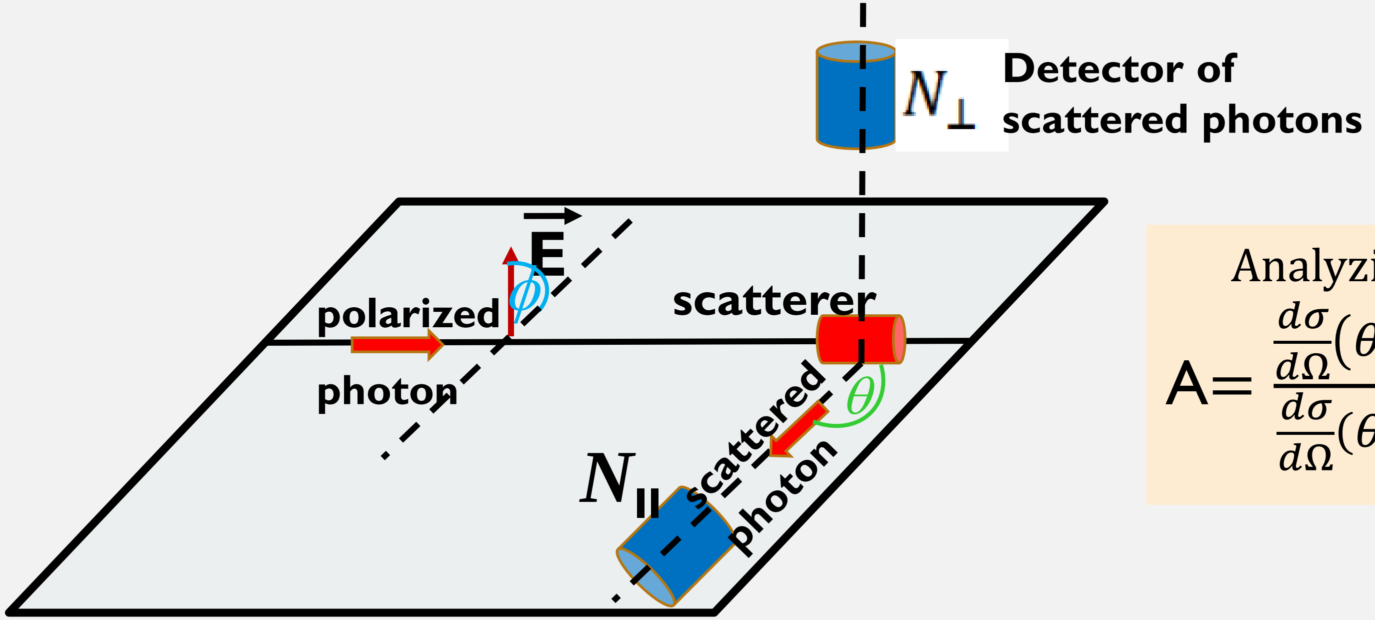
Cross-section is maximum for $\phi = \pi/2$ angle!

Preferentially the scattering plane is orthogonal to the polarization and the momentum of scattered photon is orthogonal to the initial polarization.

Compton polarimeters are used for the polarization measurements.

Compton polarimeters

The polarization of initial photons can be measured by registering the Compton scattered photons.

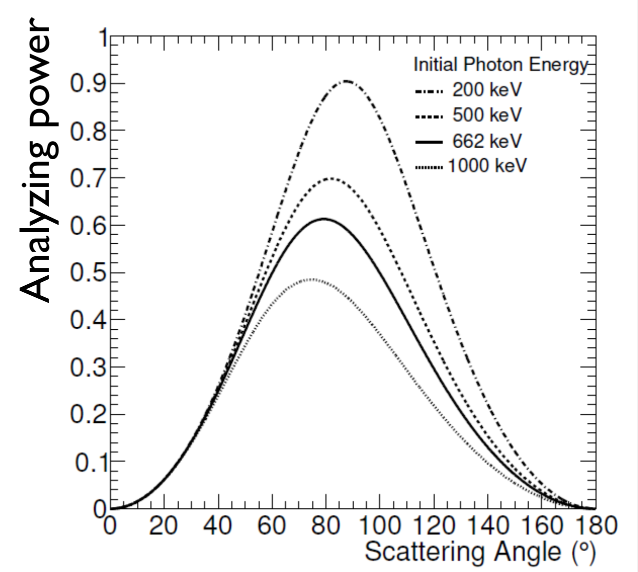


Analyzing power is a asymmetry in scattering of gammas:

$$A = \frac{N_\parallel - N_\perp}{N_\parallel + N_\perp}$$

Analyzing power for Compton polarimeter:

$$A = \frac{\frac{d\sigma}{d\Omega}(\theta, \phi=90^0) - \frac{d\sigma}{d\Omega}(\theta, \phi=0^0)}{\frac{d\sigma}{d\Omega}(\theta, \phi=90^0) + \frac{d\sigma}{d\Omega}(\theta, \phi=0^0)} = \frac{\sin^2\theta}{\frac{E\gamma_1}{E\gamma} + \frac{E\gamma}{E\gamma_1} - \sin^2\theta}$$



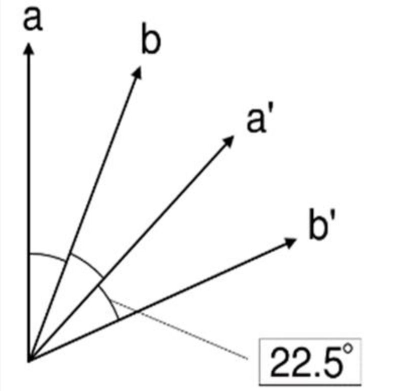
Maximum $A_{Max} = 0.7$ for 511 keV gamma's in ideal case (scattering angle=82^0).

A is significantly smaller than 1.
It provides the problems in polarization measurements for annihilation photons.

For comparison: optical polarimeters have A~1.

HOW TO PROVE THE ENTANGLEMENT OF PHOTONS?

Traditional method: CHSH inequality (not applicable) → ***Azimuthal correlations of scattered gammas***



N (***a***,***b***) - coincidences between counters with optical axes ***a*** and ***b***

$$E\left(\vec{a},\vec{b}\right)=\frac{N\left(\overrightarrow{a_{||}},\overrightarrow{b_{||}}\right)+N\left(\overrightarrow{a_{\perp}},\overrightarrow{b_{\perp}}\right)-N\left(\overrightarrow{a_{||}},\overrightarrow{b_{\perp}}\right)-N\left(\overrightarrow{a_{\perp}},\overrightarrow{b_{||}}\right)}{N\left(\overrightarrow{a_{||}},\overrightarrow{b_{||}}\right)+N\left(\overrightarrow{a_{\perp}},\overrightarrow{b_{\perp}}\right)+N\left(\overrightarrow{a_{||}},\overrightarrow{b_{\perp}}\right)+N\left(\overrightarrow{a_{\perp}},\overrightarrow{b_{||}}\right)}$$

Correlation function for ideal polarimeter (A=1):

$$S=E\left(\vec{a},\vec{b}\right)-E\left(\vec{a},\vec{b}`\right)+E\left(\vec{a},\vec{b}`\right)+E\left(\vec{a}`,\vec{b}`\right)$$

According to Bell's (CHSH) inequality:

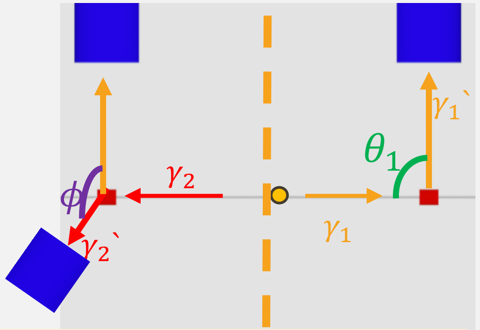
- **S<2 for non-entangled system**
- Maximum $S=2\sqrt{2}$ for entangled system.

For non-ideal polarimeter:

$$S \quad \Rightarrow \quad S` = S * A^2 \quad \Rightarrow$$

$S`<2$ for Compton polarimeters ($A^2<0.5$, ***always***)!

Snyder H *et al* - 1948 Angular correlation of scattered annihilation radiation Phys. Rev. 73 440–8.



$$\boldsymbol{P_{12}(E_1,E_2,\phi)=\left(\frac{d\sigma}{d\Omega_1}\right)_{NP}\left(\frac{d\sigma}{d\Omega_2}\right)_{NP}\left[1-\alpha(\theta_1)\alpha(\theta_2)cos(2\phi)\right]}$$

Ratio of the numbers of scattered annihilation photons:

$$R(\theta)=\frac{N\left(\phi=\frac{\pi}{2}\right)}{N(\phi=0)}=1+\frac{2\sin^4\theta}{\gamma^2-2\gamma\sin^2\theta};$$

$$\gamma=2-cos\theta+(2-cos\theta)^{-1}$$

$$\boldsymbol{R=2.83\ for\ \theta=82^0;\ R=2.6\ for\ \theta=90^0}$$

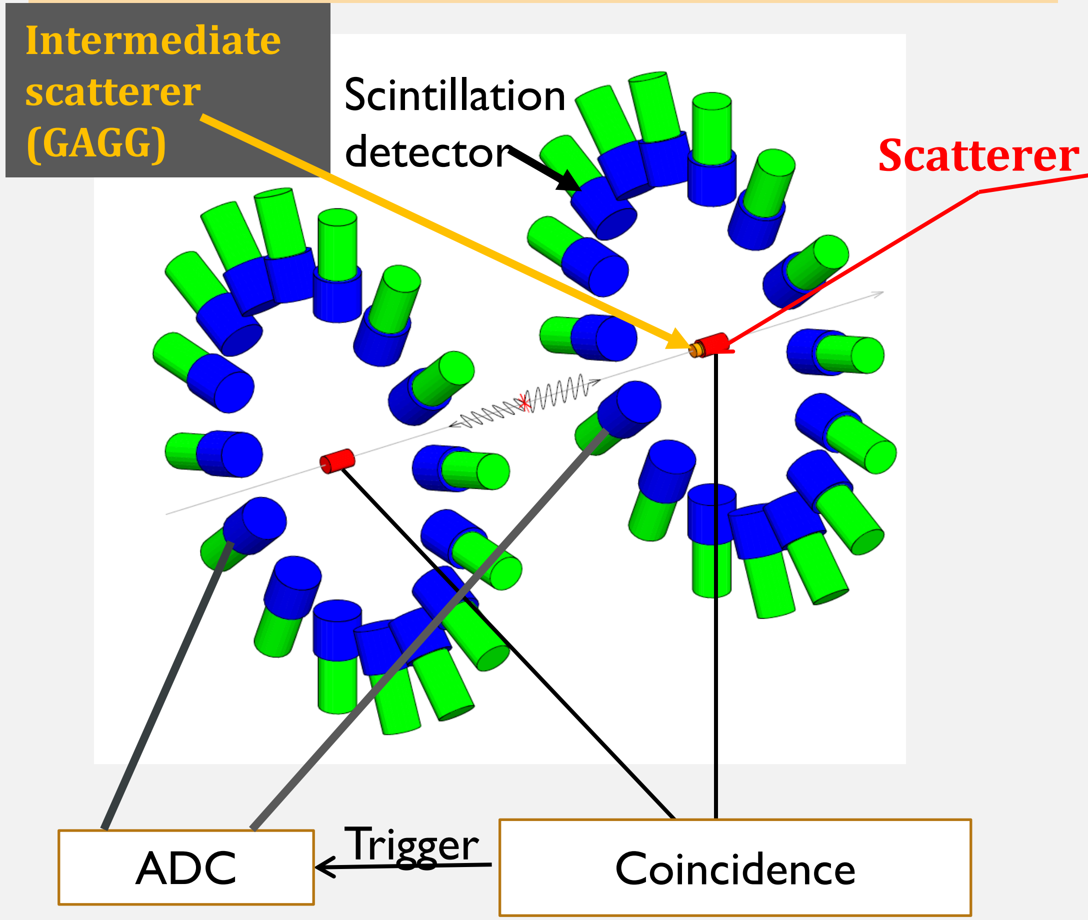
D. Bohm and Y. Aharonov (Phys. Rev. (1957) 108, 1070:
if **R > 2** ⇒ **gamma pair is entangled**.

For decoherent ***R=1;*** for non-entangled photons ***R<2***

H. Langhof, Zeitschrift fur Physik 160, 186-193 (1960) $\boldsymbol{R(82^0)=2.47\pm0.07}$

Experimental Setup

The experimental setup constructed in INR RAS (Moscow) will help to research scattering of both entangled and decoherent photons.

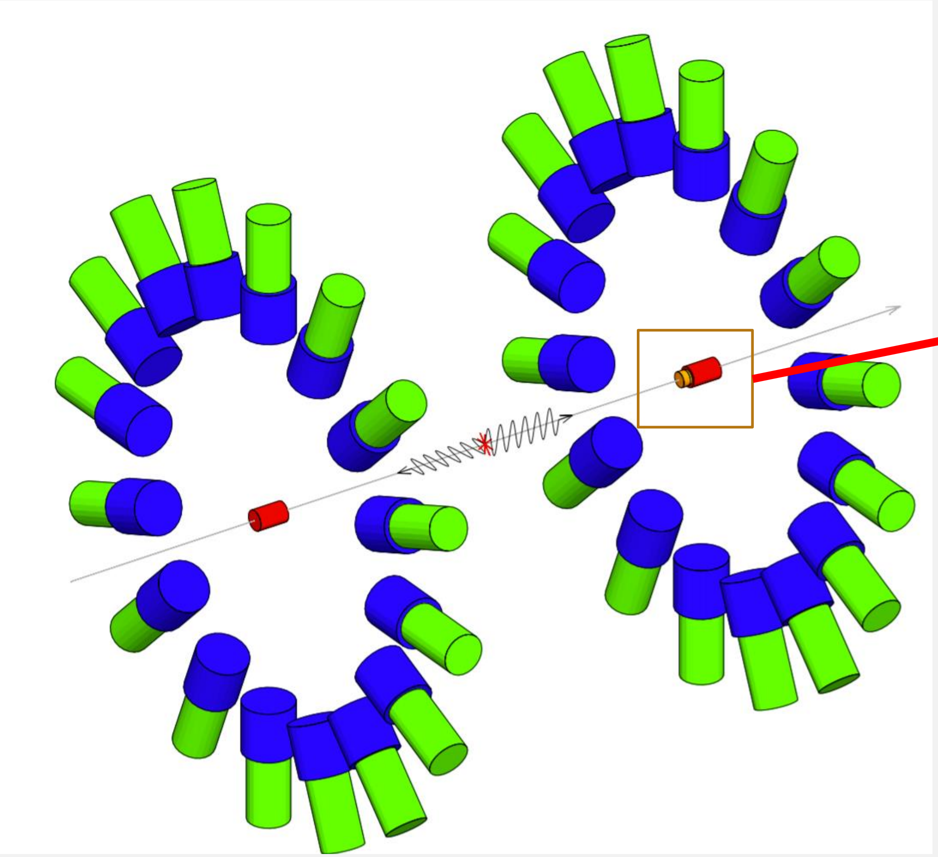


The experimental setup

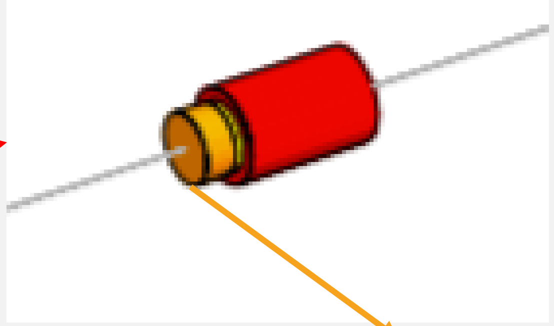
- The setup consists of 2 arms
- Each arms comprises 16 scintillation detector (NaI) positioned at $\frac{\pi}{8}$ angles to each other
- In each arm 16 independent Compton polarimeters can be distinguished (pair of perpendicularly positioned scintillation detectors (NaI))

Principle of production of decoherent pairs

Decoherence is the transition from entangled to mixed quantum state as a result of interaction with environment (intermediate scatterer).



Intermediate scatterer: scintillator GAGG



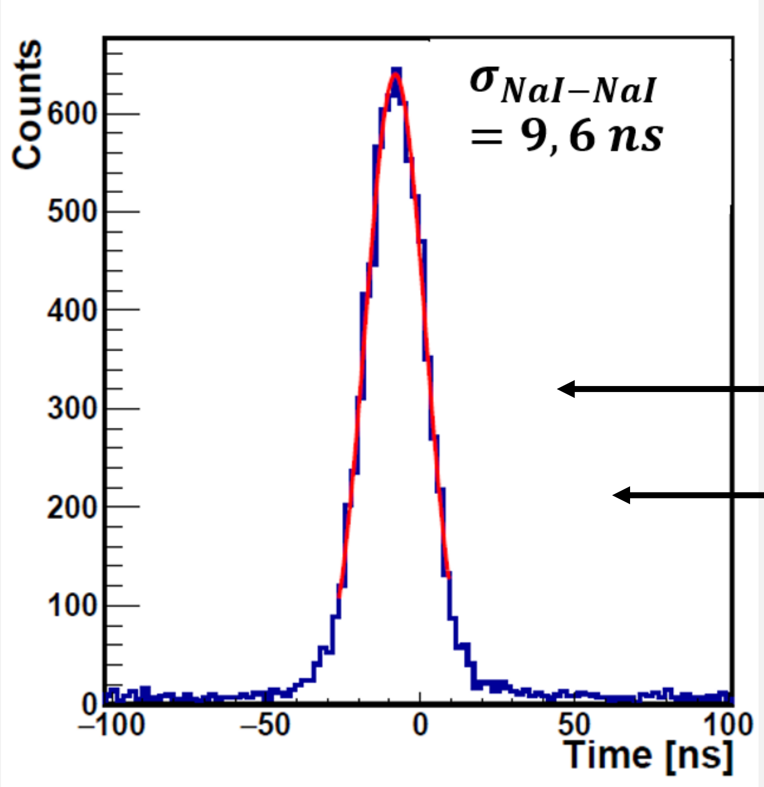
To produce decoherent photons ***intermediate GAGG scatterer*** is placed before one of the ***main scatterers***.

If first interaction in intermediate scatterer occurs then pair of gammas becomes decoherent. The decoherent pairs are easily distinguished from the entangled ones by analyzing time and energy spectra in GAGG scintillator.

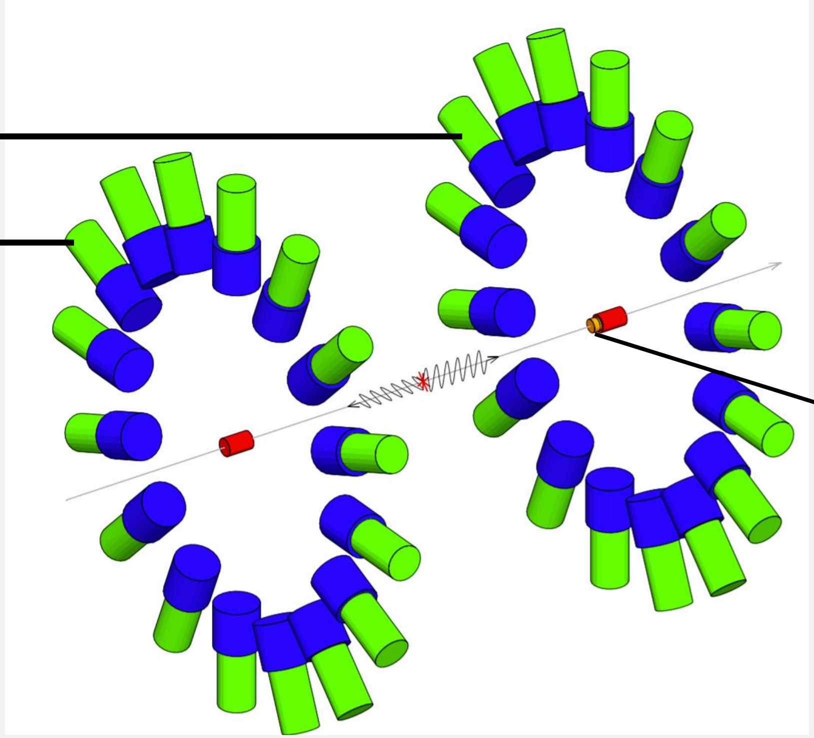
Event selection using time spectra

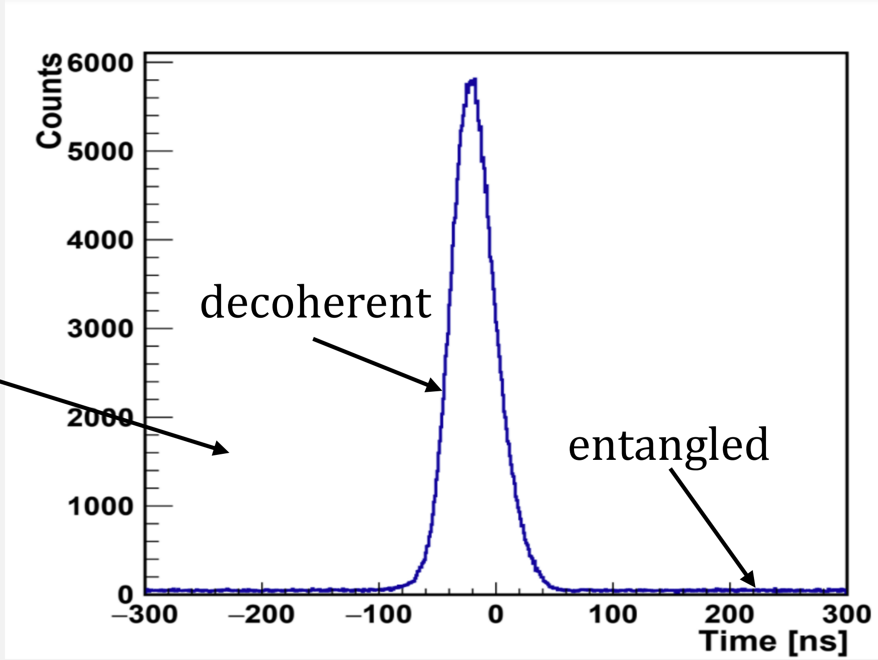
Both, time and amplitude spectra were used for the event selection

Peak in spectrum of time difference between intermediate scatterer and main scatterer accounts = interaction in intermediate scatterer ⇒ production of decoherent pair



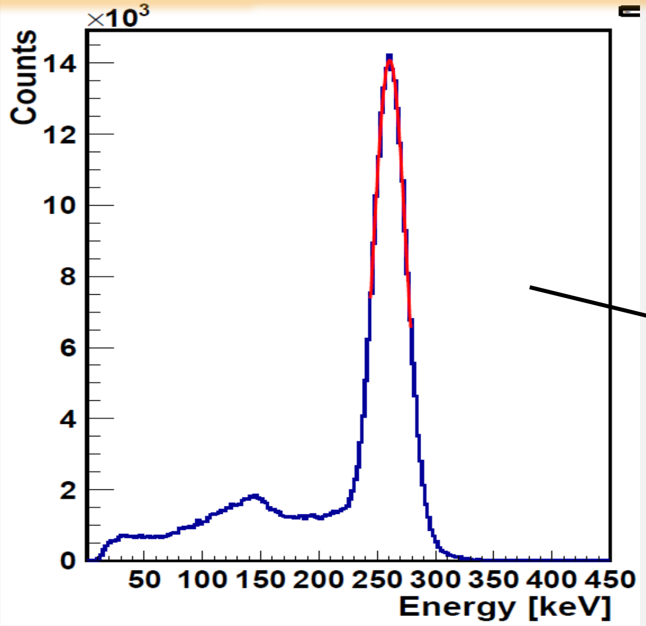
Time difference between two scintillation detectors (NaI) of opposite arms

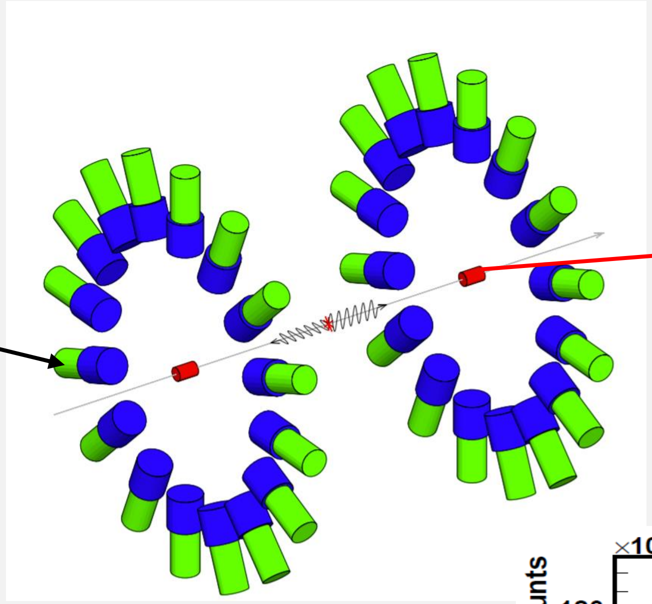


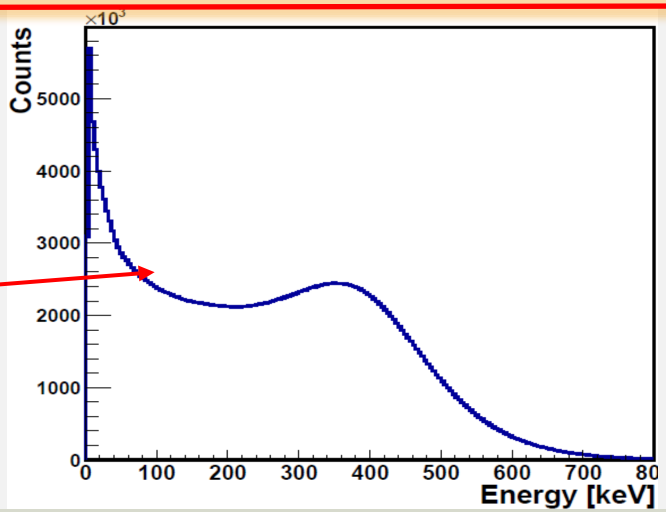


Spectrum of time difference between main scatterer and intermediate scatterer

Amplitude spectra in scintillation detectors and scatterers







Energy deposition in main scatterer

Energy deposition in scintillation detector(NaI)

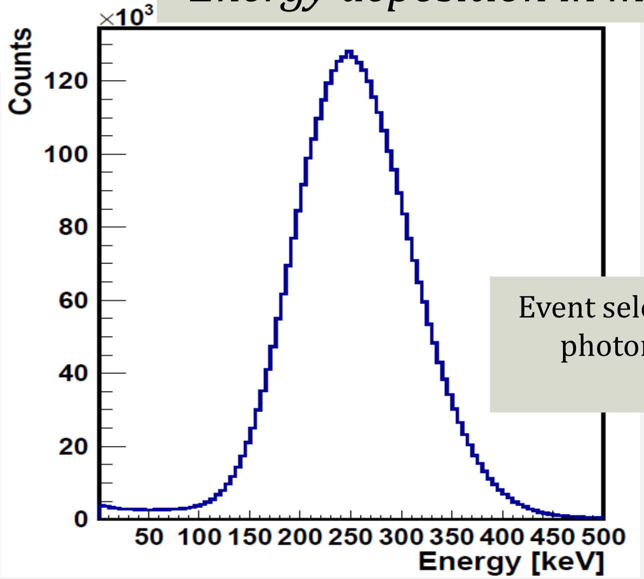
For energy of scattered photon after Compton scattering at an angle θ: $E_{\gamma`}(\theta) = \frac{E_\gamma}{1+\frac{E_\gamma}{m_e c^2}(1-cos\theta)}$

$E_{\gamma(\gamma`)}$ - energies of incident and scattered photons.

In our experiment energy $E_\gamma = m_e c^2$.

$$\Rightarrow E_{\gamma`}(\theta = 90^0) = \frac{m_e c^2}{2} \cong 255{,}5\ keV$$

Taking into account the, modeling yields for *scintillation detector(NaI)* $E_{scint} = 257\ keV$ and *scatterer* $E_{scat} = 253\ keV$



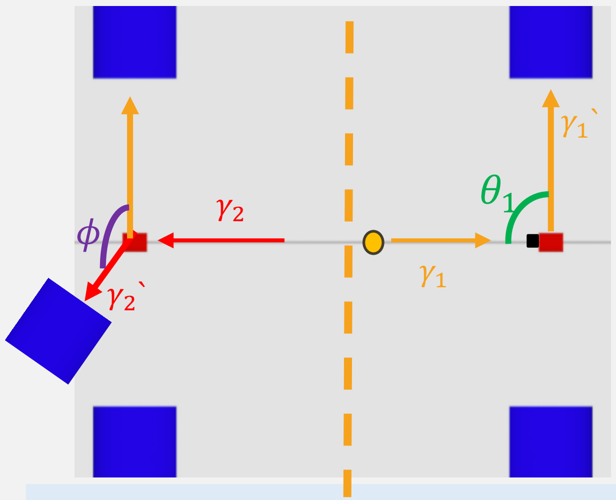
Event selection: only scattered photons detected by NaI scintillator

Energy deposition in main scatterer if scattered photons are registered by scintillation detectors (NaI)

Asymmetry in angular distribution of entangled scattered photons

Every shoulder contains 16 detectors ⇒ for each angle there are 16 different pairs of scintillation detectors, which add to the total ***number of coincidences for chosen angle(N)***

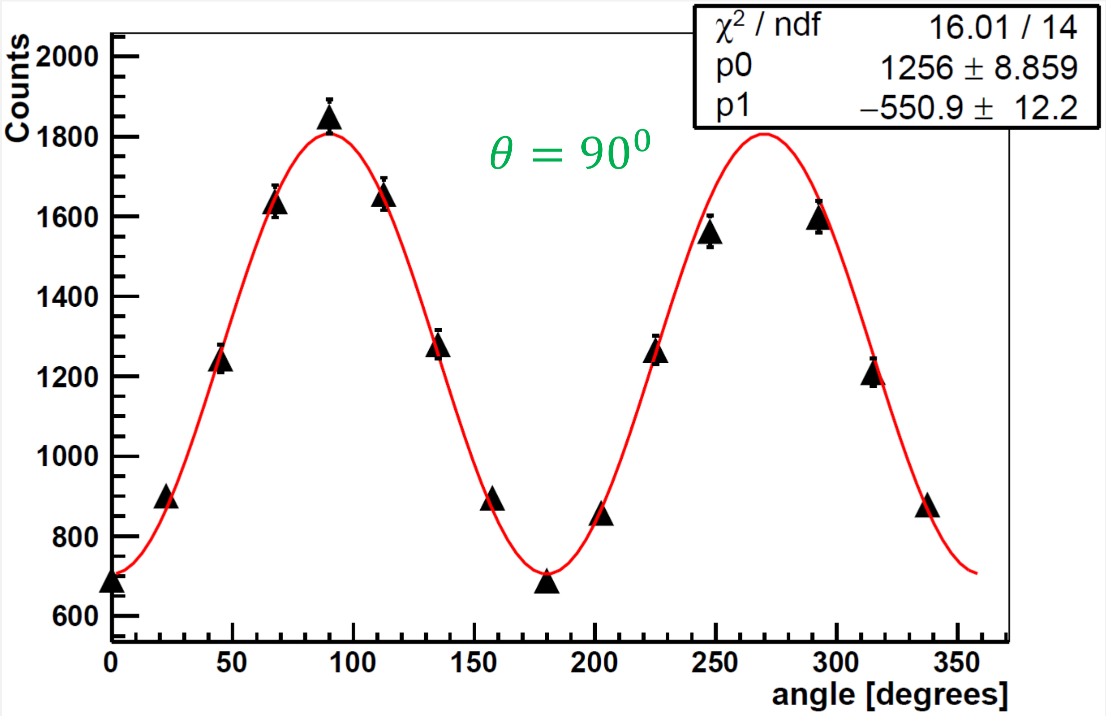
$$\boldsymbol{P_{12}(E_1, E_2, \phi) = \left(\frac{d\sigma}{d\Omega_1}\right)_{NP}\left(\frac{d\sigma}{d\Omega_2}\right)_{NP}[1 - \alpha(\theta_1)\alpha(\theta_2)cos(2\phi)]} \Rightarrow N(\phi) = A + B \cdot \cos(2\phi)$$



$$\alpha(\theta_i) = P_{\gamma_i} = \frac{\sin^2\theta_i}{\epsilon_i + \frac{1}{\epsilon_i} - \sin^2\theta_i}$$

$\left(\frac{d\sigma}{d\Omega_i}\right)_{NP}$ - for non-polarized photons.

$$\epsilon = \frac{E_{\gamma}`}{E_\gamma}.$$



Ratio of the numbers of scattered annihilation photons:

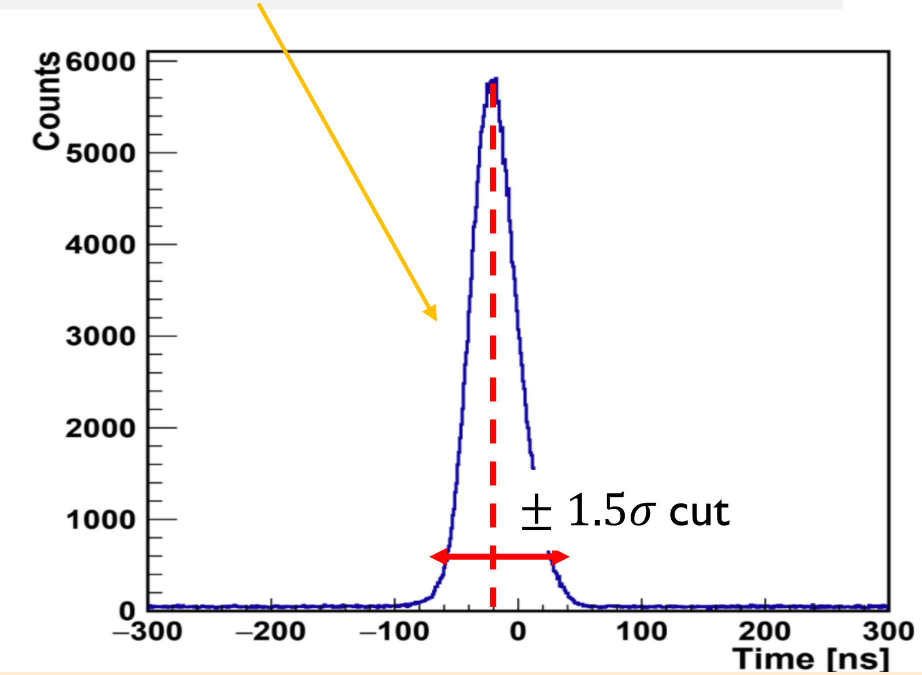
$$R_{theory}(\theta) = \frac{N\left(\phi = \frac{\pi}{2}\right)}{N(\phi = 0)} = 1 + \frac{2\sin^4\theta}{\gamma^2 - 2\gamma\sin^2\theta}$$

$$\gamma = 2 - cos\theta + (2 - cos\theta)^{-1}$$

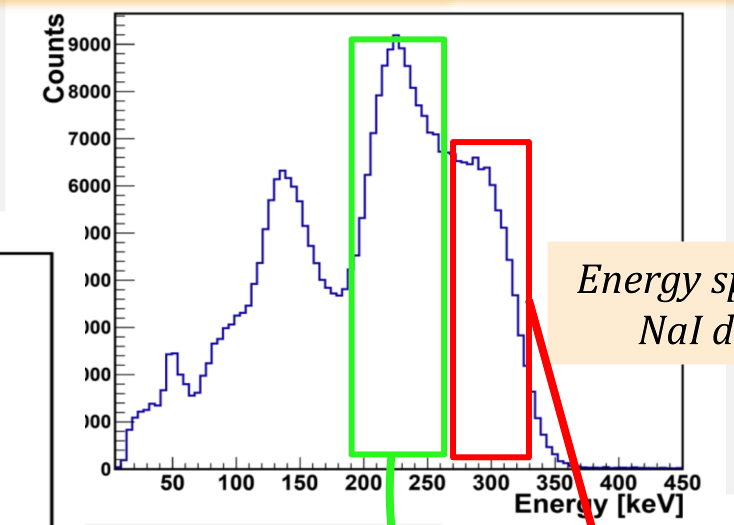
$R_{theory}(\theta = 90^0) = 2,6$ $R_{exp}(\theta = 90^0 \pm 7^0) = 2,56 \pm 0,07$

Selection of decoherent pairs

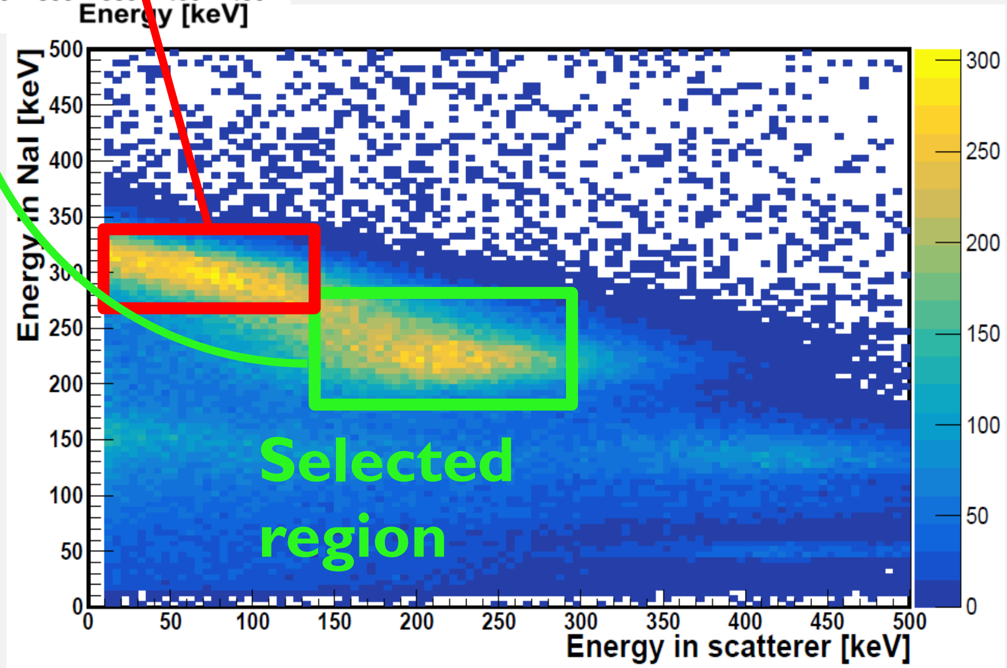
Peak corresponds to decoherent photons



Spectrum of time difference between main scatterer and intermediate scatterer (all events)



Energy spectrum in NaI detector



Deposited energy correlation between scatterer and scintillation detector (NaI) for decoherent photons

Asymmetry in angular distribution of scattered photons (two cases)

The Na-22 source is placed closer to arm with intermediate scatterer

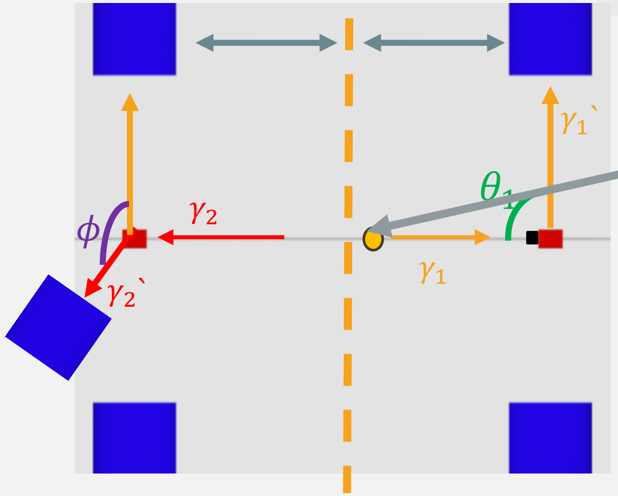
- Interaction in intermediate scatterer = decoherent pair

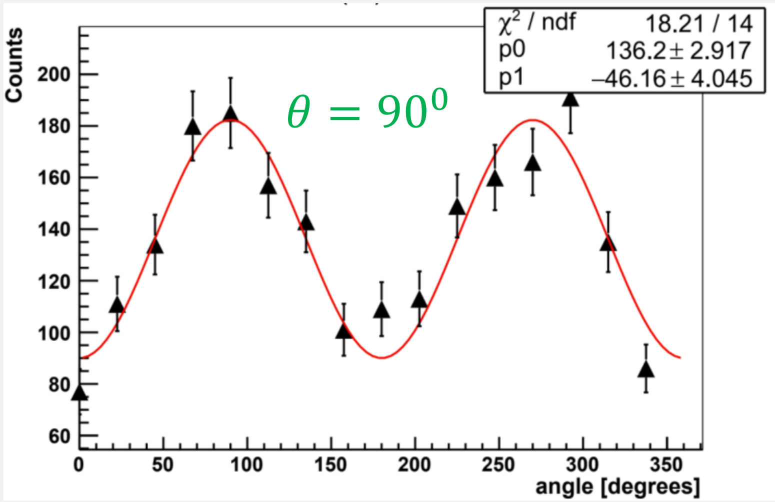
The Na-22 source is placed closer to arm without intermediate scatterer

- Interaction in intermediate scatterer = entangled pair

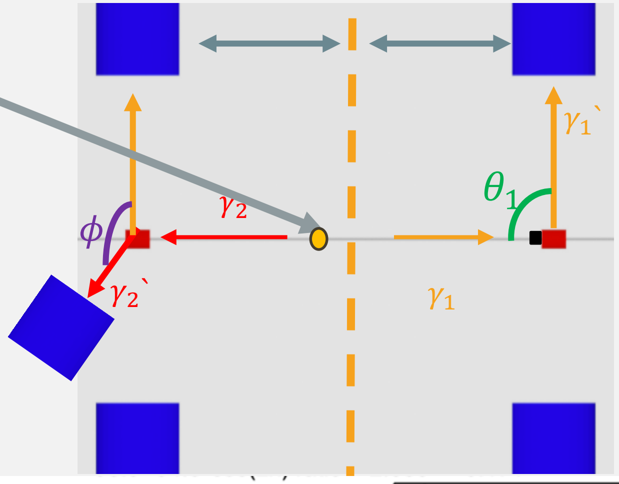
Similar scattering kinematics for both cases

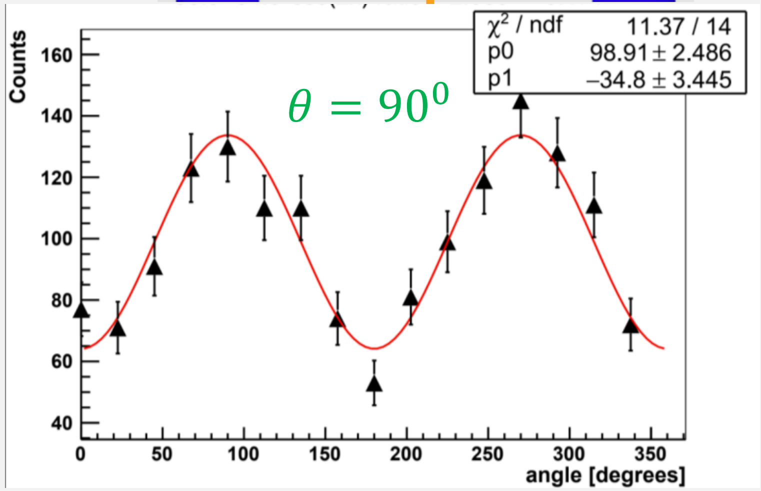
Na-22 Source (at different positions)





$R(\theta = 90^0) = 2,04 \pm 0,15$





$R(\theta = 90^0) = 2,09 \pm 0,17$

Conclusion

- Experimental setup to study the Compton scattering of entangled and decoherent annihilation photons was constructed;

- The dependence of the number of detected gammas on the angle between the scattered photons is obtained for entangled and decoherent gammas;

- The angular dependence corresponds to the theoretical predictions for the entangled photons;

- No difference in scattering kinematics of entangled and decoherent photons was found;

- As follows from the above results, the entanglement of annihilation photons cannot be proven from angular distributions;

- New methods should be developed to prove the entanglement of the annihilation photons.

Thank you for your attention

AZIMUTHAL CORRELATIONS OF SCATTERED PHOTONS

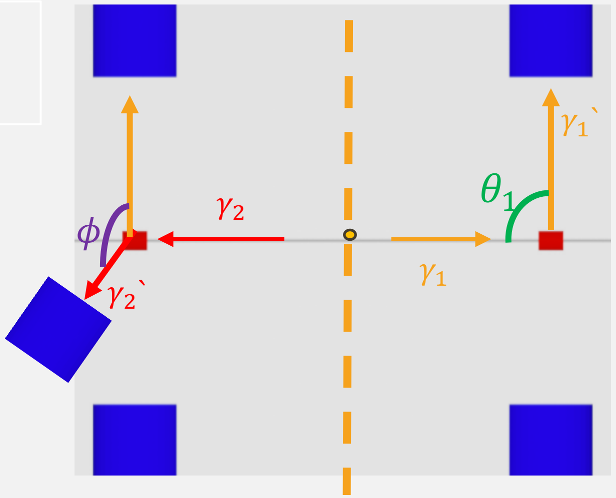
Snyder H S, Pasternack S and Hornbostel J, - 1948 Angular correlation of scattered annihilation radiation Phys. Rev. 73 440–8

$$\boldsymbol{P_{12}(E_1, E_2, \phi) = \left(\frac{d\sigma}{d\Omega_1}\right)_{NP} \left(\frac{d\sigma}{d\Omega_2}\right)_{NP} [1 - \alpha(\theta_1)\alpha(\theta_2)cos(2\phi)]}$$

Ratio of the numbers of scattered annihilation photons:

$$R_{theory}(\theta) = \frac{N\left(\phi = \frac{\pi}{2}\right)}{N(\phi = 0)} = 1 + \frac{2\sin^4\theta}{\gamma^2 - 2\gamma\sin^2\theta}; \qquad \gamma = 2 - cos\theta + (2 - cos\theta)^{-1}$$

$\boldsymbol{R = 2.83 \quad for\ \ \theta = 82^0}$ ***or*** $\boldsymbol{R = 2.6\ \ for\ \theta = 90^0}$



According to D. Bohm and Y. Aharonov (Phys. Rev. (1957) 108, 1070:
if R > 2 ⇒ gamma pair is entangled.

For decoherent photons ***R=1*** for non-entangled photons ***R<2***

The best experimental values:

H. Langhof, Zeitschrift fur Physik 160, 186-193 (1960) $\boldsymbol{R = 2.47 \pm 0.07}$

The above data confirmed (*to authors belief*) that ***the annihilation photons are entangled***!
The ***decoherent*** annihilation photons ***were not measured*** at all!

CURRENT SITUATION WITH ANNIHILATION PHOTONS

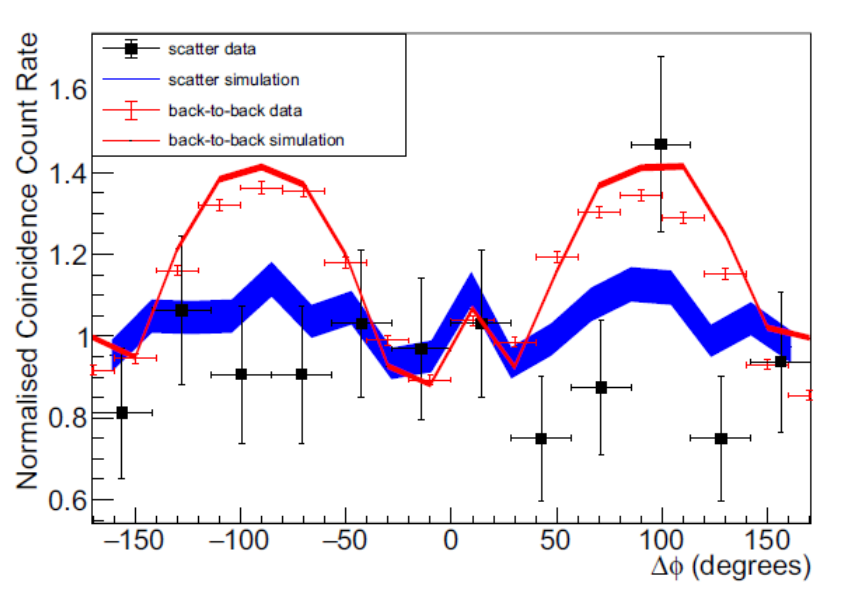
Hiesmayr B.C. and Moskal P. Witnessing entanglement in Compton scattering processes via mutually unbiased bases Sci. Rep.9 8166 (2019)

The Compton scattering of annihilation photons is the same for both entangled and decoherent states. There is NO the experimental proof of the entanglement.

Peter Caradonna *et al.* Probing entanglement in Compton interactions *J. Phys. Commun.* **3** 105005 (2019)

The Compton scattering of annihilation photons is principally different for entangled and decoherent states. There is no need to prove the entanglement. But…
The measurements of decoherent photons are needed!

Watts, D.P., Bordes, J., Brown, J.R. *et al.* Photon quantum entanglement in the MeV regime and its application in PET imaging. *Nat Commun* **12,** 2646 (2021)

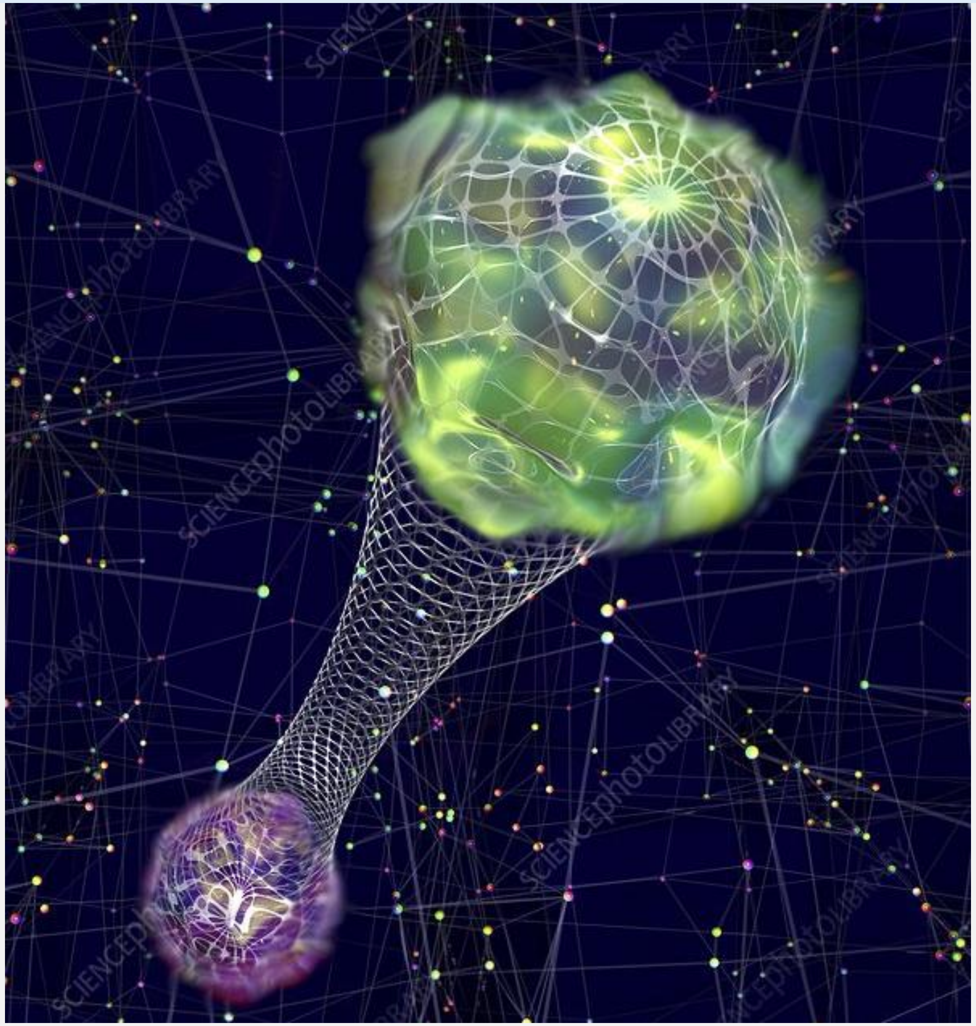


First measurement of decoherent annihilation photons was done this year with decoherent photons. The sensitivity of experimental setup and the poor statistics do not allow the comparison of Compton scattering of photons in entangled and decoherent states.

New experiment is needed to test the theoretical

Main goal

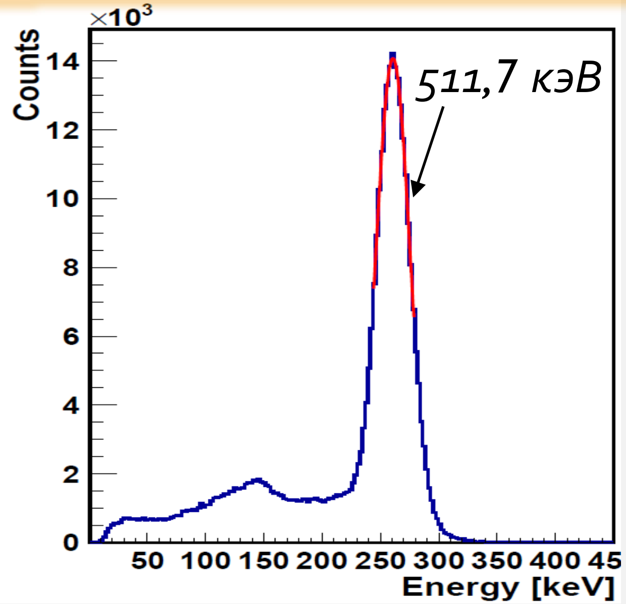
Quantum entanglement is a phenomenon, when quantum states of several objects are bound and can be described with one wave function.



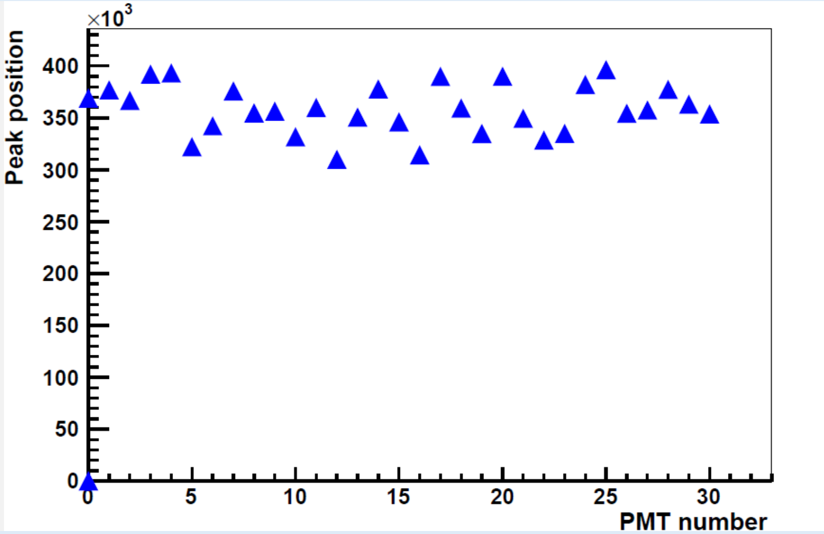
Main goal is to:
Compare the scattering kinematics in Compton scattering of entangled and decoherent annihilation photons. Decoherence is the transition from entangled to mixed quantum state as a result of interaction with matter.

In the past there were several experiments dedicated to studying the kinematics of entangled photons scattering.
Research of comparison of scattering kinematics of entangled and decoherent photons was not conducted in the past.
Theoretical works dedicated to the topic predict controversial results.

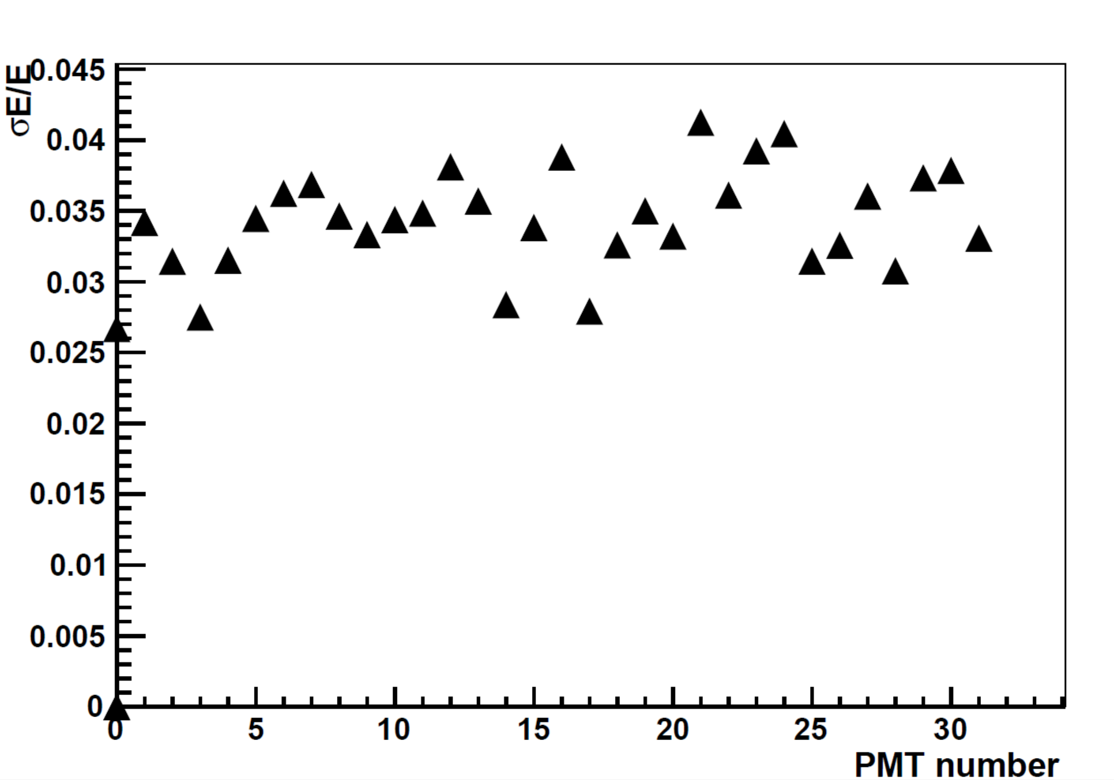
Calibration of energy resolution of scintillation detectors (NaI)



Energy spectrum of ^{22}Na in scintillation detector



Peak position for all scintillation detectors of the setup (NaI)



Energy resolutions of scintillation detectors (NaI)

Energy resolution FWHM/E of implemented PMTs claimed by Hamamatsu is nearly equal to 8% for the peak.

$$\frac{FWHM}{E} = 2{,}355 \cdot \frac{\sigma_E}{E} \cong 2{,}355 \cdot 0{,}034 = 0{,}08$$

As we can see, the resolution of our PNTs is equal to that number

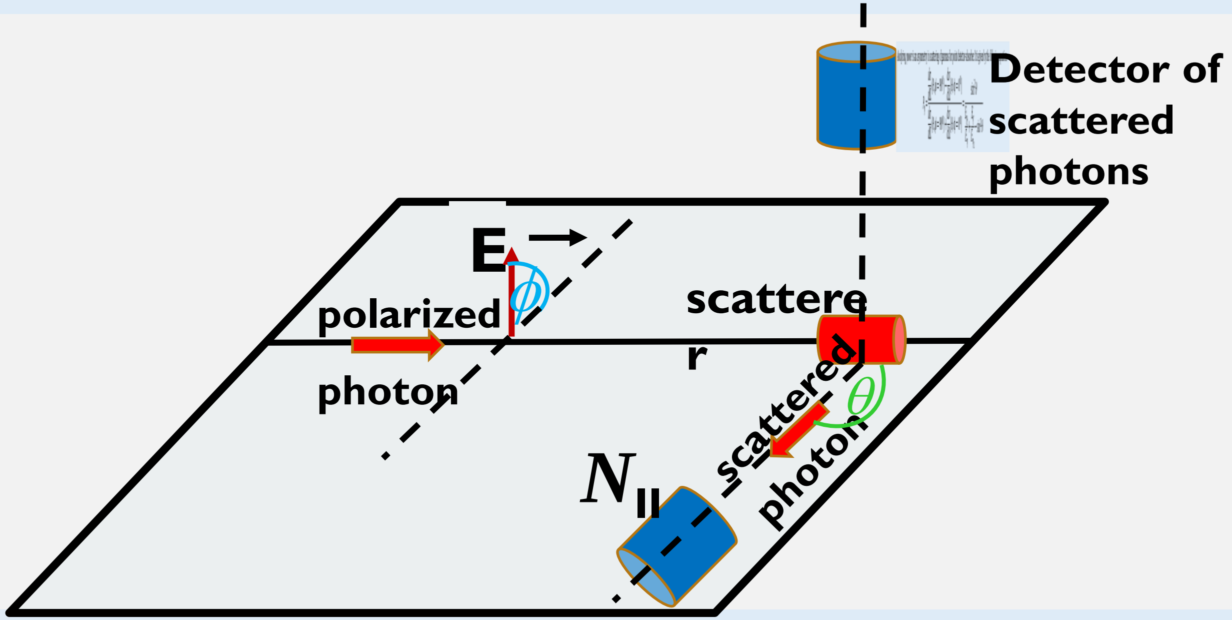
Compton polarimeter

Differential cross-section of Compton is given by Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega}(\theta,\phi) = \frac{1}{2}\cdot\frac{e^2}{m_e c^2}\cdot\frac{E_{\gamma_1}^2}{E_\gamma^2}\cdot\left(\frac{E_{\gamma_1}}{E_\gamma}+\frac{E_\gamma}{E_{\gamma_1}} - 2\,\boldsymbol{\sin^2\theta\cos^2\phi}\right)$$

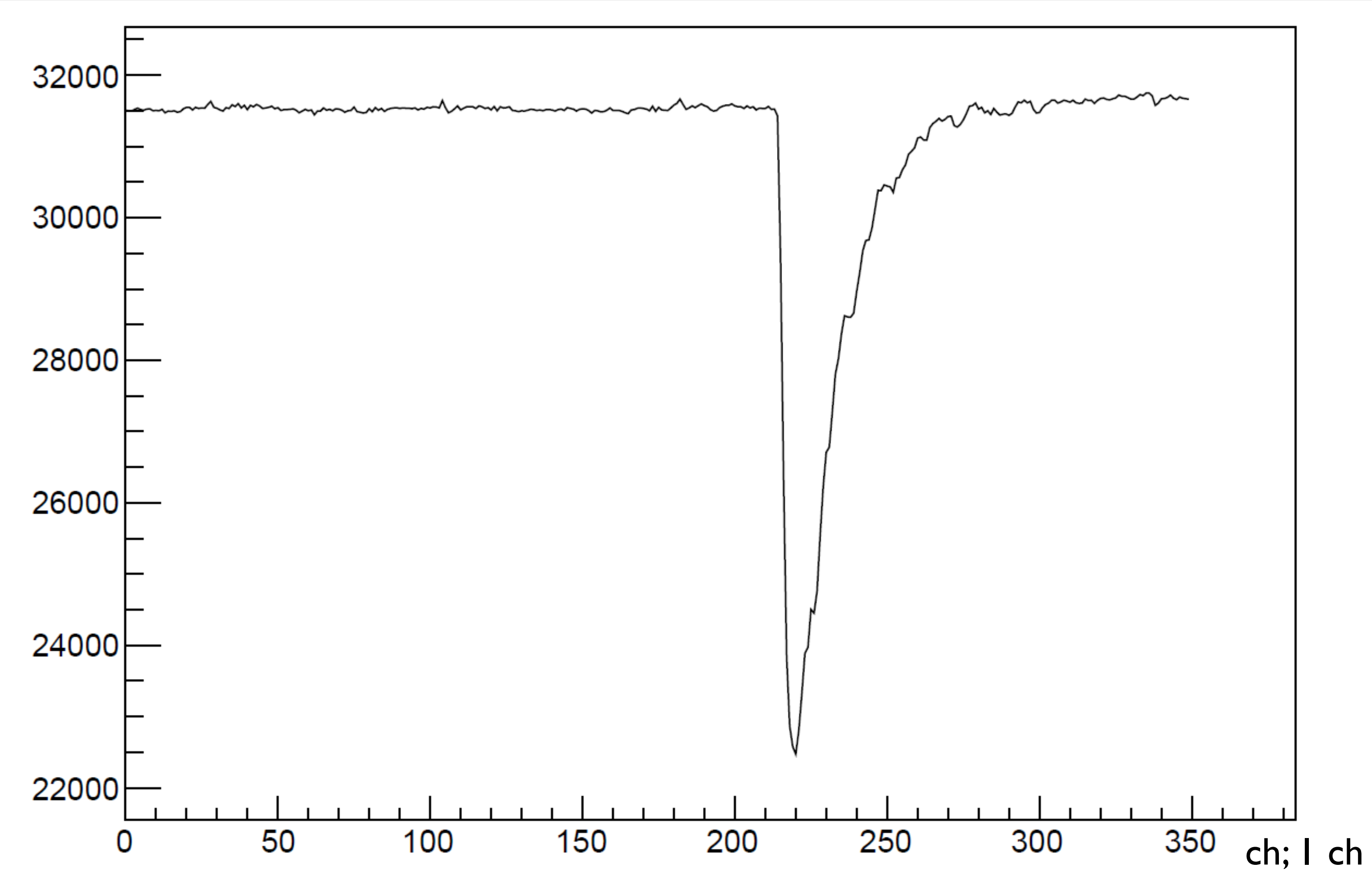
Analyzing this equation we can come to several conclusions:

- Photons scatter predominantly perpendicularly to polarization plane.
- **By registering scattered photon the initial photons polarization can be determined.**



Analyzing power is an asymmetry in scattering of gammas for point detector-absorber. It is given by the following equation:

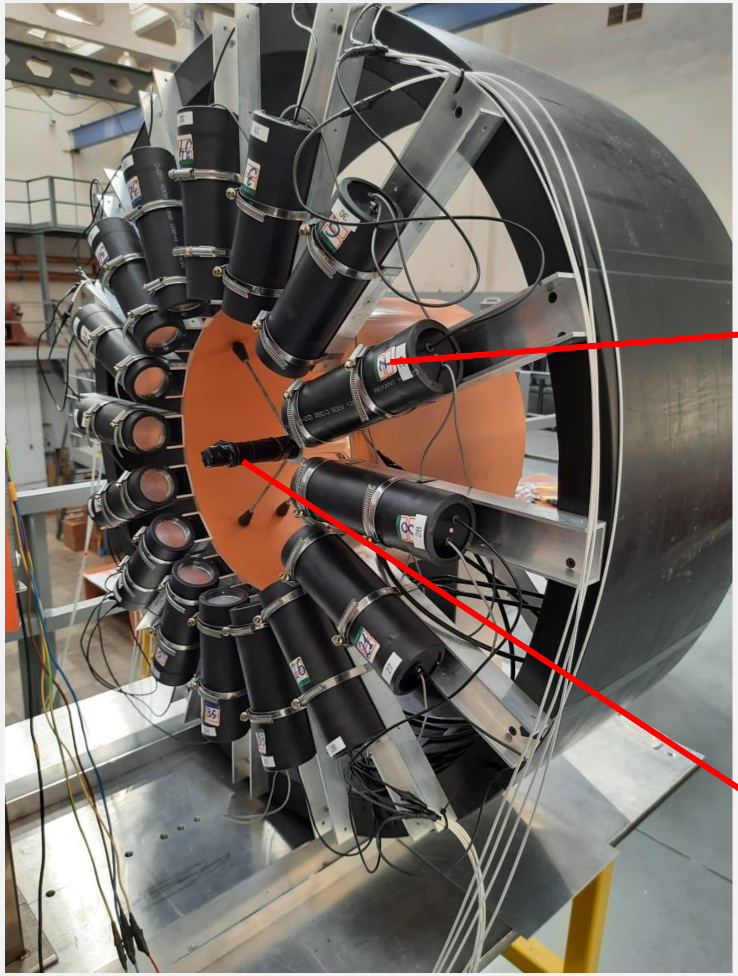
$$P_\gamma = \frac{\frac{d\sigma}{d\Omega}(\theta,\phi=90^0) - \frac{d\sigma}{d\Omega}(\theta,\phi=0^0)}{\frac{d\sigma}{d\Omega}(\theta,\phi=90^0) + \frac{d\sigma}{d\Omega}(\theta,\phi=0^0)} = \frac{\sin^2\theta}{\frac{E_{\gamma_1}}{E_\gamma}+\frac{E_\gamma}{E_{\gamma_1}} - \sin^2\theta}$$

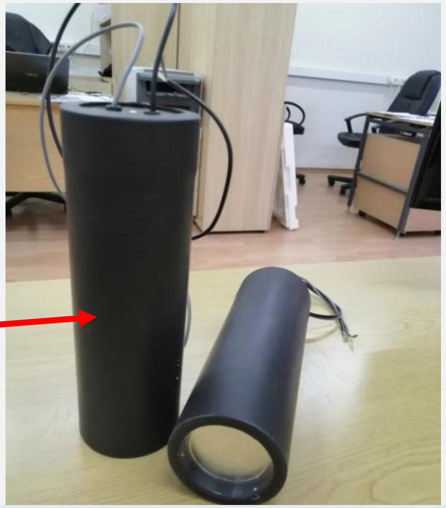


32000
30000
28000
26000
24000
22000
0
50
100
150
200
250
300
350

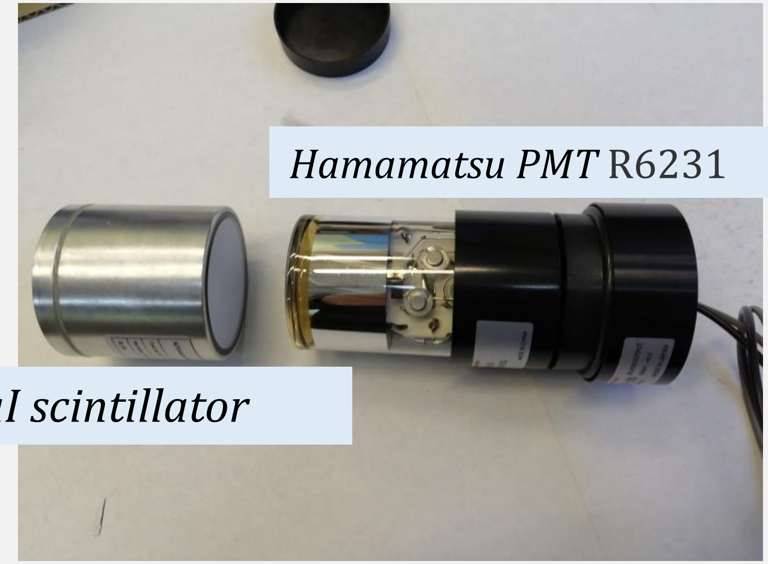
ch; 1 ch = 16 ns

Experimental Setup





Scintillation detectors (NaI)



Hamamatsu PMT R6231

NaI scintillator

Scatterer

Characteristics of Hamamatsu PMT R6231

- *Operating voltage 1 kV*
- *Typical gain* $2{,}7 \cdot 10^5$
- *Dynamic region[300 nm,650nm] with sensitivity peak in 420 nm.*

Asymmetry in angular distribution of scattered photons

Using results presented in article M. H. L. Pryce and J. C. Ward, “Angular correlation effects with annihilation radiation.” for double differential cross-section a probability of simultaneous detection of photons with energies $E_{1,2}$, and scattering angles $\theta_{1,2}$,$\phi_{1,2}$:

$$\boldsymbol{P_{12}(E_1, E_2, \phi) = \left(\frac{d\sigma}{d\Omega_1}\right)_{NP} \left(\frac{d\sigma}{d\Omega_2}\right)_{NP} [\,1 - \alpha(\theta_1)\alpha(\theta_2)cos(2\phi)]}$$

$$\left(\frac{d\sigma}{d\Omega_i}\right)_{NP} = \frac{r_e^2 \epsilon_i^2}{2}\left[\epsilon_i + \frac{1}{\epsilon_i} - \sin^2\theta_i\right]; \alpha(\theta_i) = P_{\gamma_i} = \frac{\sin^2\theta_i}{\epsilon_i + \frac{1}{\epsilon_i} - \sin^2\theta_i}$$

$\left(\frac{d\sigma}{d\Omega_i}\right)_{NP}$ - differential cross-section of scattering of unpolarized photon. ϵ – ratio of scattered photon energy to initial photons energy.

$$\frac{d\sigma}{d\Omega_i} = \left(\frac{d\sigma}{d\Omega_i}\right)_{NP} \cdot [1 - \alpha(\theta_i)\cos(2\phi_i)]$$

ϕ_i - angle between a polarization vector of incident photon and direction of scattered photon. $\boldsymbol{\phi = \Delta\phi_i}$