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Topology and the Gradient Flow method

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Motivation

The main advantage of the **gradient flow** method compared to other smearing algorithms (e.g. APE smearing, HYP smearing or stout-link smearing) is that the damping of the UV fluctuations is governed by a differential equation which means that we have a better analytical control of the smoothing procedure. In this work, the topology of "Highly Improved Staggered Quark" (HISQ) [4] configurations of size $64^3 \times 16$ with physical light and strange quark masses ($m_l = m_s/27$) have been studied in more detail after applying the gradient flow.

Gradient Flow

By introducing an extra coordinate t, the so called **flow-time**, the smoothed field $B_\mu(t, x)$ of SU(3) gauge fields is defined by [1]

$$\dot{B}_\mu(x,t) = D_\nu G_{\nu\mu}(x,t), \qquad B_\mu(x,t)|_{t=0} = A_\mu(x),$$

$$D_\mu = \partial_\mu + [B_\mu(x,t),\ \cdot\ ]\,,$$

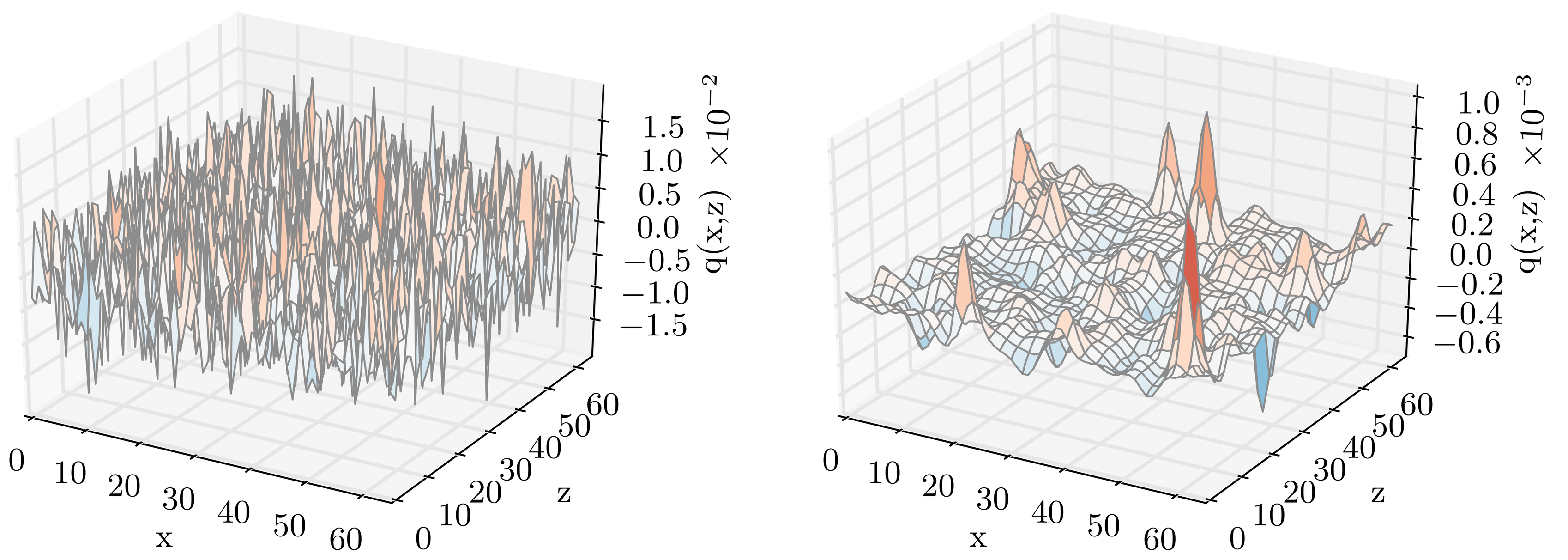
where the dot in the definition of the covariant derivative D_μ denotes a derivative with respect to the flow-time t and $G_{\nu\mu}(x,t)$ is the field strength tensor of the field $B_\mu(x,t)$. This equation drives the gauge field along the direction of steepest descent towards the stationary points of the Yang-Mills action. The flow-time has to be sufficiently large to get rid of ultraviolet noise but typically has to be smaller than $1/\Lambda^2_{QCD}$. For non-zero temperature, additionally, we restrict the flow-time such that the effective smearing radius $\sqrt{8t}$ is smaller than $1/T$.

Topological Charge

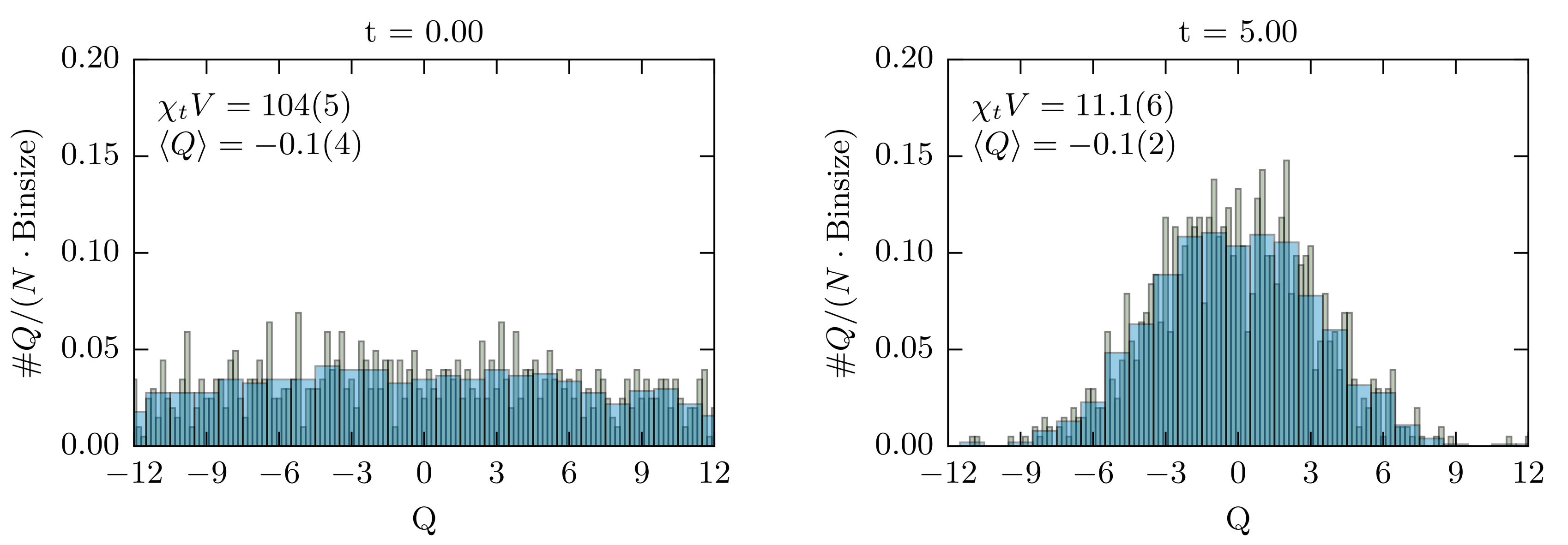
The topological charge density $q(x)$ is defined as

$$q(x) = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left\{F_{\mu\nu}(x)F_{\rho\sigma}(x)\right\}.$$

The plots show the density on the x-z plane of one HISQ configuration at $T = 0.94T_c$. It is seen that at flow-time $t = 0$ (left plot) the density is quite noisy. A calculation of the topological charge Q would not lead to a reliable value. On the other hand, at flow-time $t = 5$ (right plot), certain domains appear which can be distinguished by their topological charge density.

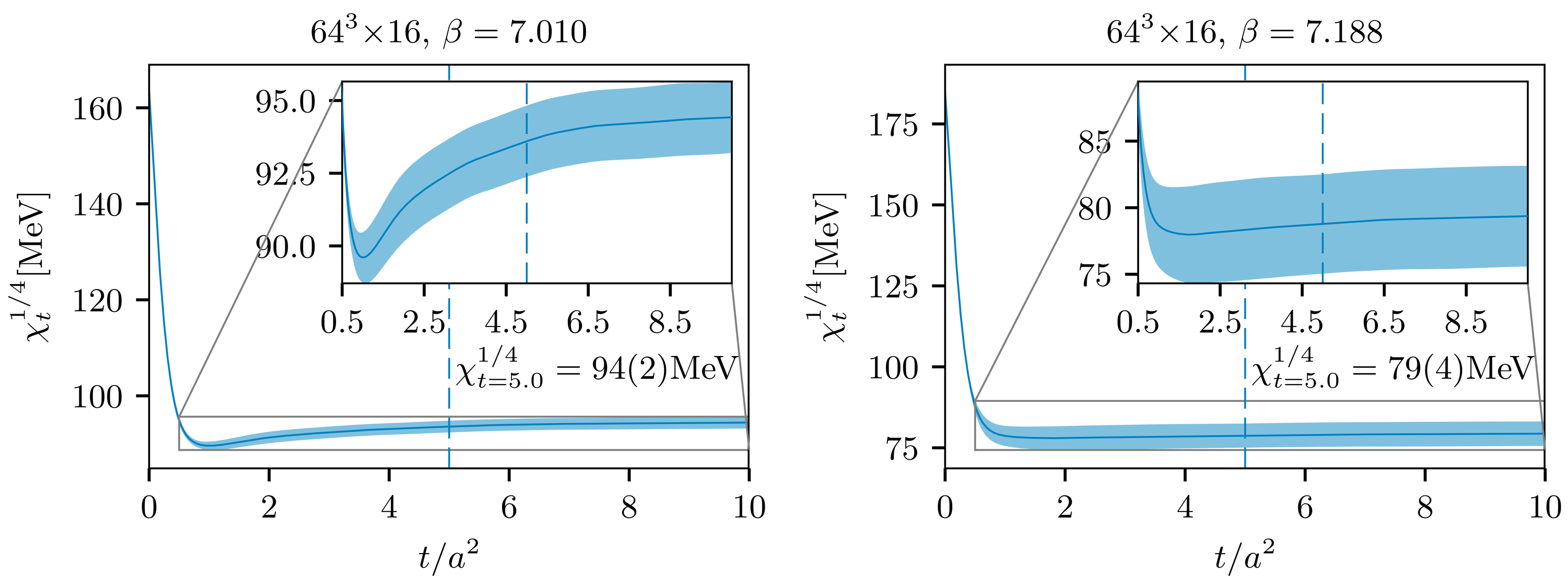


The Topological charge, $Q = \int d^4x\, q(x)$, at $T = 0.94T_c$ at zero flow-time (left plot) and after smearing the lattices till $t = 5$ (right plot). At $t = 0$ no reliable distribution is seen. On the other hand, at $t = 5$ a Gaussian distribution emerges. The variance (σ^2) and expectation value (μ^2) extracted from a Gaussian fit are close to $\sigma^2 \approx \chi_t V$ and $\mu \approx \langle Q \rangle$. Moreover the topological charge approaches close-to-integer values after applying the gradient flow.

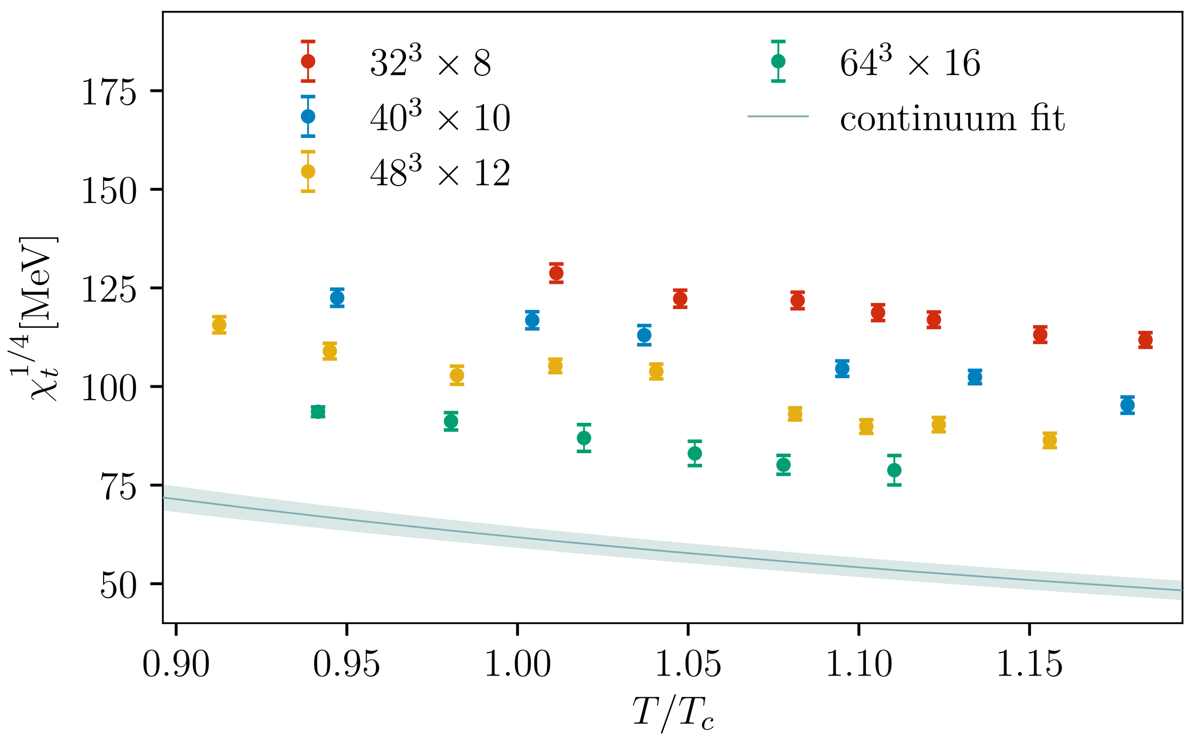


Topological Susceptibility

Topological susceptibility, $\chi_t = \frac{\langle Q^2 \rangle - \langle Q \rangle^2}{V}$, with respect to the flow-time t at $T = 0.94T_c$ (left plot) and $T = 1.11T_c$ (right plot). After the first few gradient flow steps, the susceptibility drops to a plateau which indicates that a large part of the UV-fluctuations are already smeared out in this regime. At the first glance it appears that after the drop the value remains in a specific plateau. However, as it is seen in the sub plots, the susceptibility is continuously increasing. Since the coarser lattice (left plot) shows a larger slope than the finer lattice (right plot) this growing of χ_t at flow-times $t > 0$ might depend on the lattice spacing.



A combined continuum extrapolation of χ_t has been performed in the temperature range $T \in [135\mathrm{MeV}, 185\mathrm{MeV}]$ using additional values from ref. [2] with quark masses $m_l = m_s/20$. Since these values have been obtained at different quark masses we have to treat them properly. The susceptibility is supposed to be proportional to the square of the pion mass, $\chi^{1/4} \sim \sqrt{m_\pi}$ [3]. Therefore, we rescaled the values from ref. [2] with $\sqrt{140/160}$.



Conclusion

We have analyzed the effect of the gradient flow method on the topology which allowed for a cleaner determination of the topological charge and susceptibility.

The result of the continuum extrapolation has to be considered with care, since the susceptibilities which have been used for the extrapolation were obtained at different quark masses. Although we have scaled them according to the expected dependence on the pion mass, additional results for physical quark masses on different lattice spacings may allow for a better continuum extrapolation for 2+1-flavor QCD at physical quark masses.

References

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[2] P. Petreczky et al. "The topological susceptibility in finite temperature QCD and axion cosmology". In: *Phys. Lett.* B762 (2016), pp. 498–505.

[3] G. S. Bali et al. "Quark mass effects on the topological susceptibility in QCD". In: *Phys. Rev.* D64 (2001), p. 054502.

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