Phases of QCD, topology and axions - IV

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I Symmetries and phases
of QCD in the
Temperature, Nf space

II Results on the phase diagram

III Topology - broken phase

IV Topology - hot QCD & axions

Our setup at a glance

Hot QCD and
Nf 2+1+1 twisted mass
Wilson fermions

up,down
in the isospin limit

Physical strange

Physical charm

Setup

Fixed varying scale

For each lattice spacing we explore a range of temperatures 150MeV — 500 MeV by varying Nt

We repeat this for three different lattice spacings following ETMC T=0 simulations.

Four pion masses

Advantages: we rely on the setup of ETMC T=0 simulations. Scale is set once for all.

Disadvantages: mismatch of temperatures - need interpolation before taking the continuum limit

Number of flavours	$m_{\pi^\pm}$
$N_f = 2+1+1$	210
	260
	370
	470
$N_f = 2$	360
	430

$T=0$ (ETMC) nomenclature	β	a [fm] [6]	N_σ^3	N_τ	T [MeV]	# confs.
				5	422(17)	585
				6	351(14)	1370
				7	301(12)	341
			24^3	8	263(11)	970
A60.24	1.90	0.0936(38)		9	234(10)	577
				10	211(9)	525
				11	192(8)	227
				12	176(7)	1052
			32^3	13	162(7)	294
				14	151(6)	1988
				5	479(22)	595
				6	400(18)	345
				7	342(15)	327
				8	300(13)	233
				9	266(12)	453
B55.32	1.95	0.0823(37)	32^3	10	240(11)	295
				11	218(10)	667
				12	200(9)	1102
				13	184(8)	308
				14	171(8)	1304
				15	160(7)	456
				16	150(7)	823
				6	509(20)	403
				7	436(18)	412
				8	382(15)	416
			32^3	10	305(12)	420
D45.32	2.10	0.0646(26)		12	255(10)	380
				14	218(9)	793
				16	191(8)	626
			40^3	18	170(7)	599
			48^3	20	153(6)	582

Results I

Gluonic (butterfly) operator

+

Gradient Flow Method

Q

$$\mathcal{L}_\theta = \frac{1}{4} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) - i\theta \boxed{\frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x)},$$

$$\exp[-VF(\theta)] = \int [dA] \exp\left(-\int d^4x \mathcal{L}_\theta\right)$$

Gradient flow

Lüscher, Lüscher Weisz

Evolve the link variables in a fictitious flow time:

$$\dot{V}_{x,\mu}(t) = -g_0^2 \left[\partial_{x,\mu} S_{\mathrm{Wilson}}(V(t))\right] V_{x,\mu}(t),$$

Monitor $\langle E \rangle = \frac{1}{2 N_\tau N_\sigma^3} \sum_{x,\mu,\nu} \mathrm{Tr}[F_{\mu\nu}(x) F^{\mu\nu}(x)]$ as a function of t

Stop flowing when $t^2 \langle E \rangle \big|_{t=t_0} = 0.3$

Observables $< O(t) >$ renormalized at $\mu = 1/\sqrt{8t}$

Continuum limit of $< O(t) >$ is independent on the chosen reference value

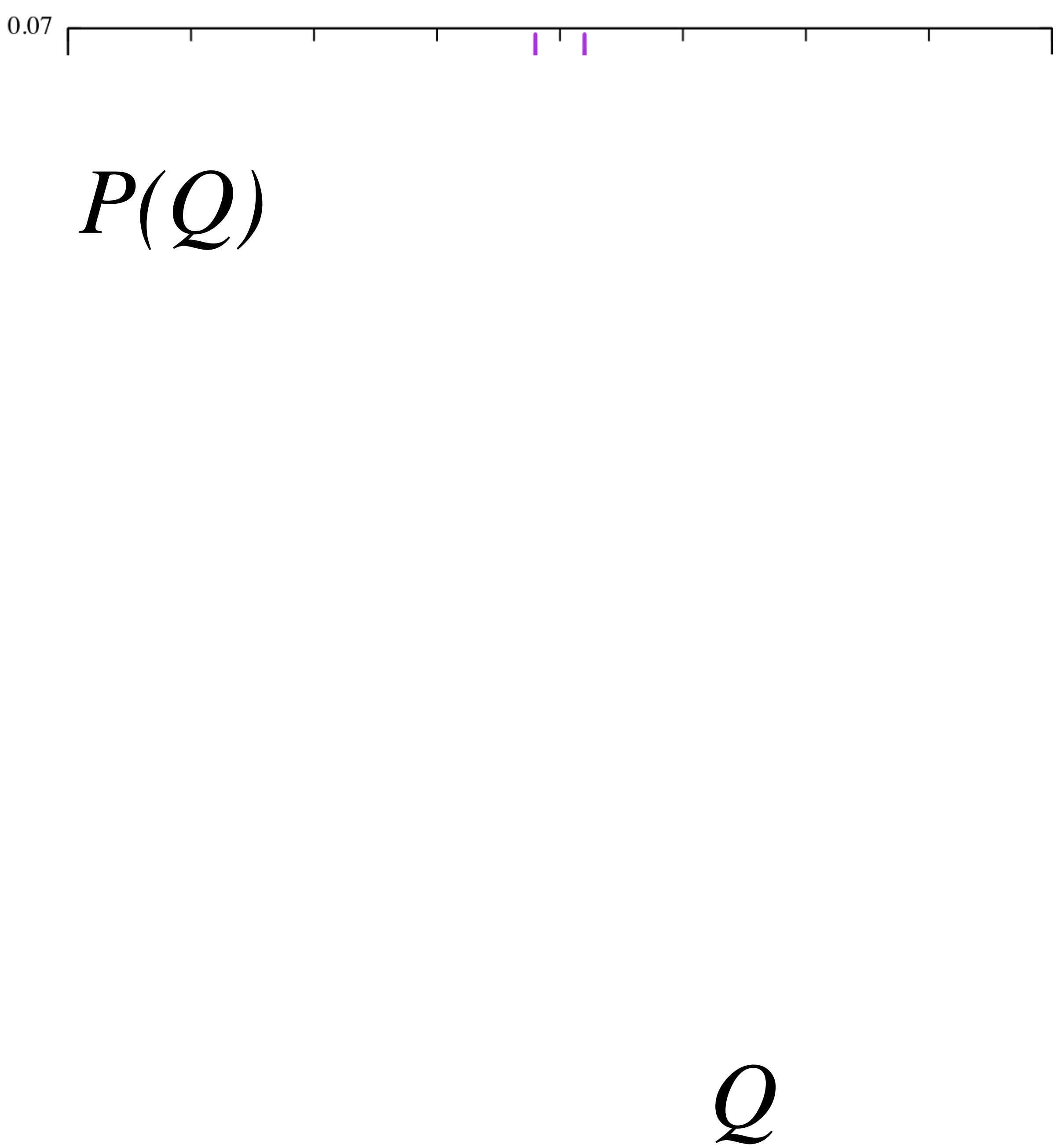
Caveat: note comments by Kanaya et al.

Distribution of the topological charge *P(Q)*

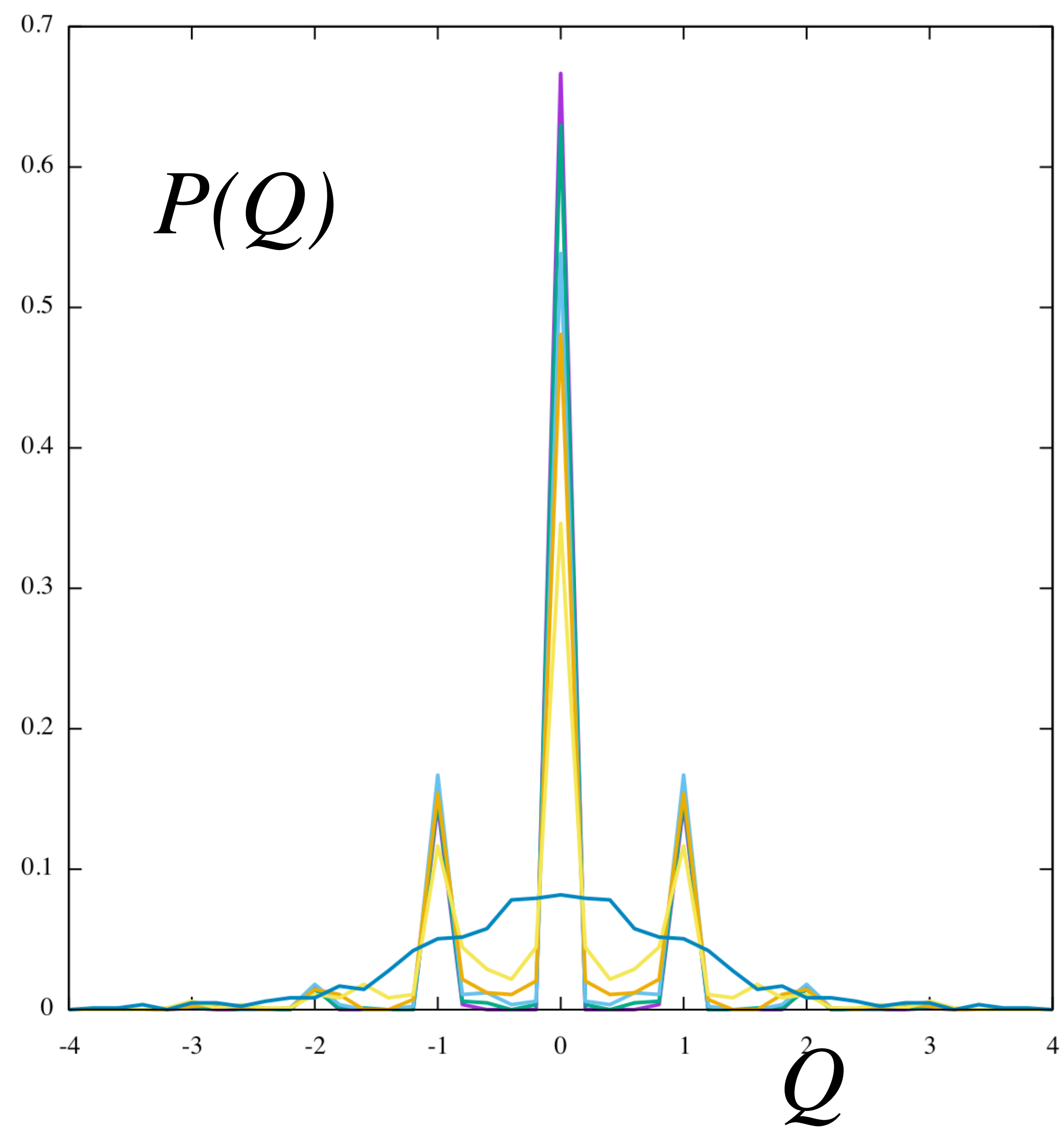
cluster around integers as cooling proceeds

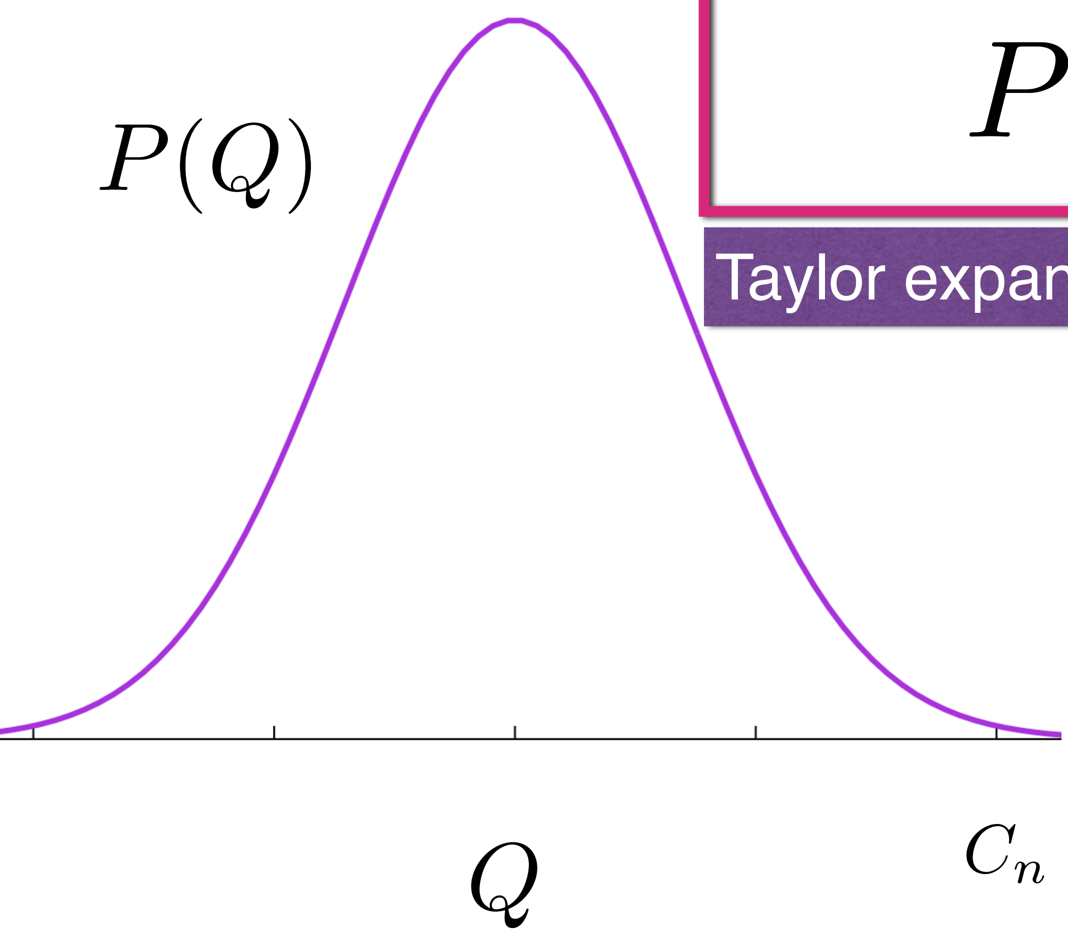
(results for a = 0.06 fm)

T=153 MeV



T=253 MeV





$P(Q)$ and $F(\theta)$

Taylor expansion, and cumulants of the topological charge distribution

$$e^{-F(\theta)} = \langle e^{i\theta Q} \rangle$$

$$P_\nu = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta\nu} e^{-F(\theta)} \qquad Q = \nu$$

$$C_n = (-1)^{n+1} \frac{1}{V} \frac{d^{2n}}{d\theta^{2n}} F(\theta)\Big|_{\theta=0} \equiv < Q^{2n} >_{conn}$$

$$F(\theta) = V \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\theta^{2n}}{(2n)!} C_n$$

$$P_\nu = \frac{e^{-\frac{\nu^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} \left[1 + \frac{1}{4!} \frac{\tau}{\sigma^2} \mathrm{He}_4 \left(\nu/\sigma \right) \right]$$

$\sigma^2 = VC_1$ and $\tau = C_2/C_1$ $\quad$ *P(Q)* is Gaussian for $V \to \infty$

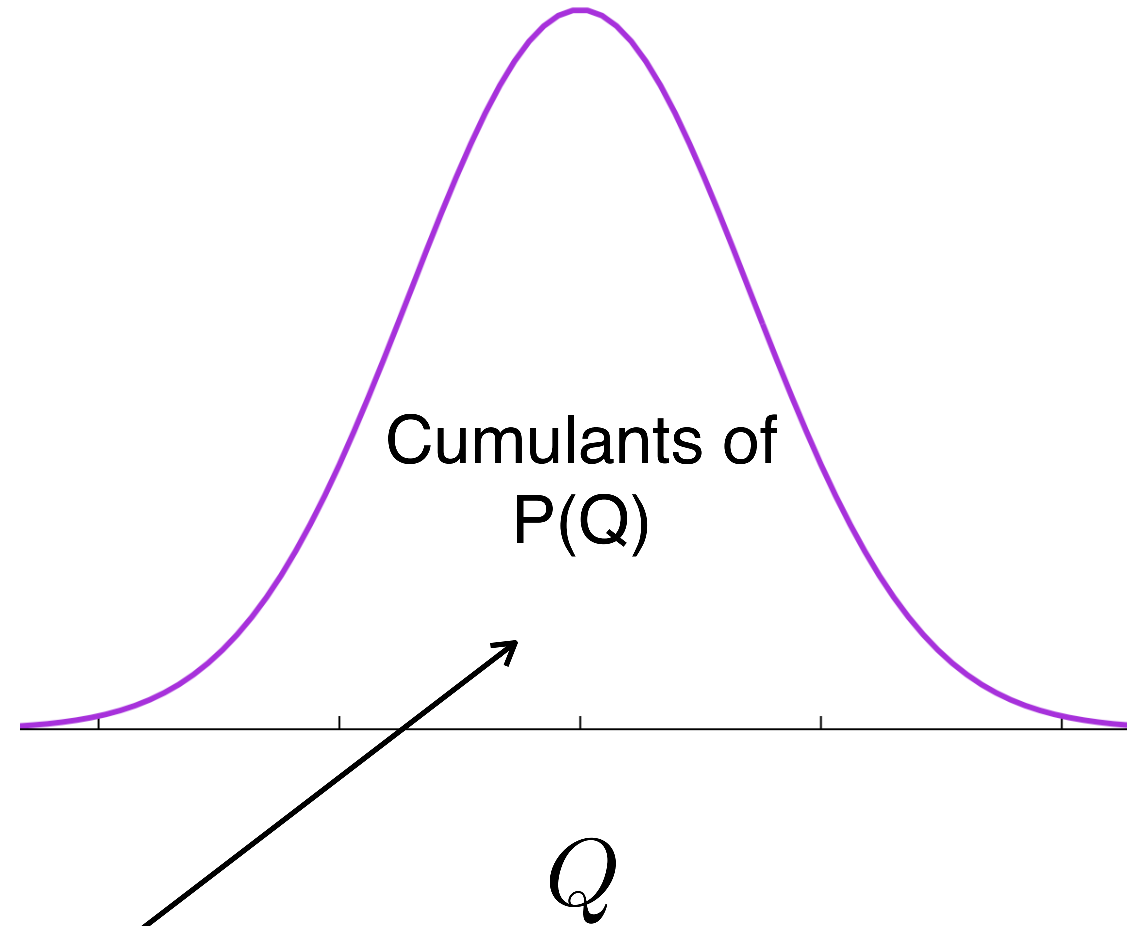
$F(\theta)$ is ‘hidden’ in *P(Q)*’s cumulants

In practice only the first two cumulants are accessible:

$$F(\theta, T) = 1/2\chi(T)\theta^2 s(\theta, t)$$

$$s(\theta, T)1 + b_2(T)\theta^2 + \ldots$$

$$b_2 = -\frac{<Q^4> - 3<Q^2>^2}{12<Q^2>}$$



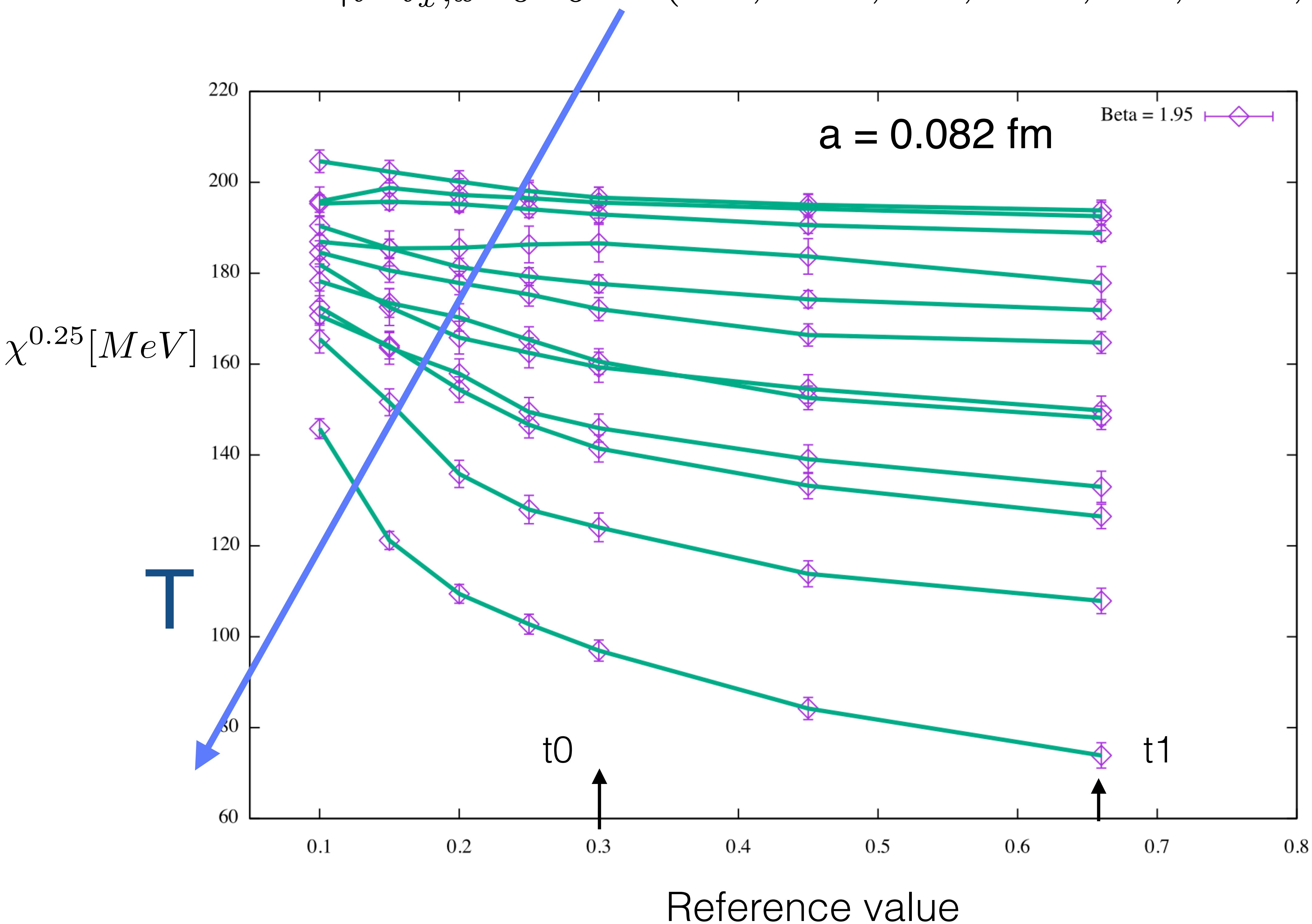
Taylor coefficients of $F(\theta, T)$

DIGA — at very high temperature — predicts

$$F(\theta, T) - F(0, T) = \chi(T)(1 - cos(\theta)) \longrightarrow b_2 = -1/12$$

Flowing towards the plateau

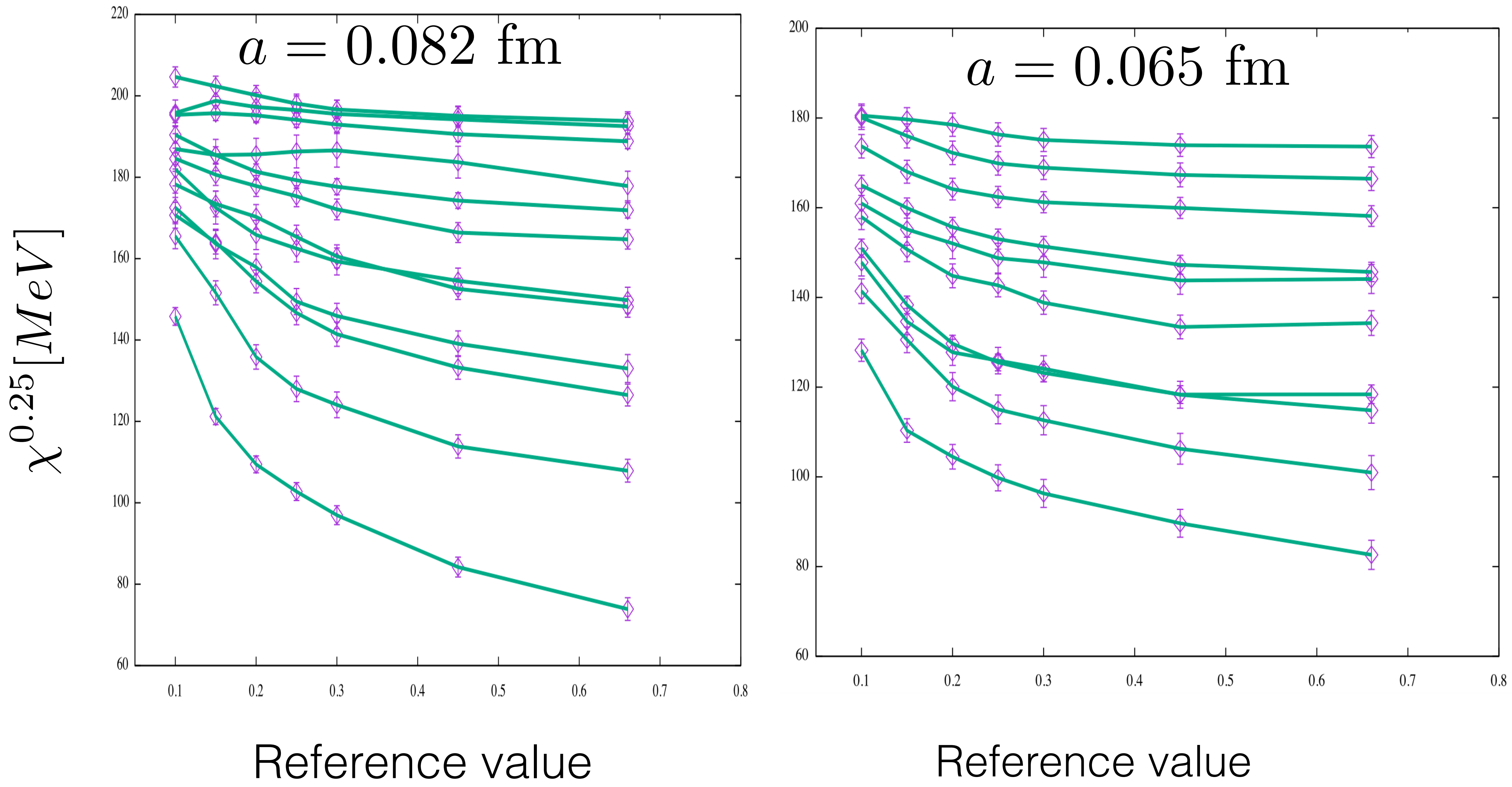
$$t^2 < E > |_{t=t_x, x=0-6} = (0.3, 0.66, 0.1, 0.15, 0.2, 0.25, 0.45)$$



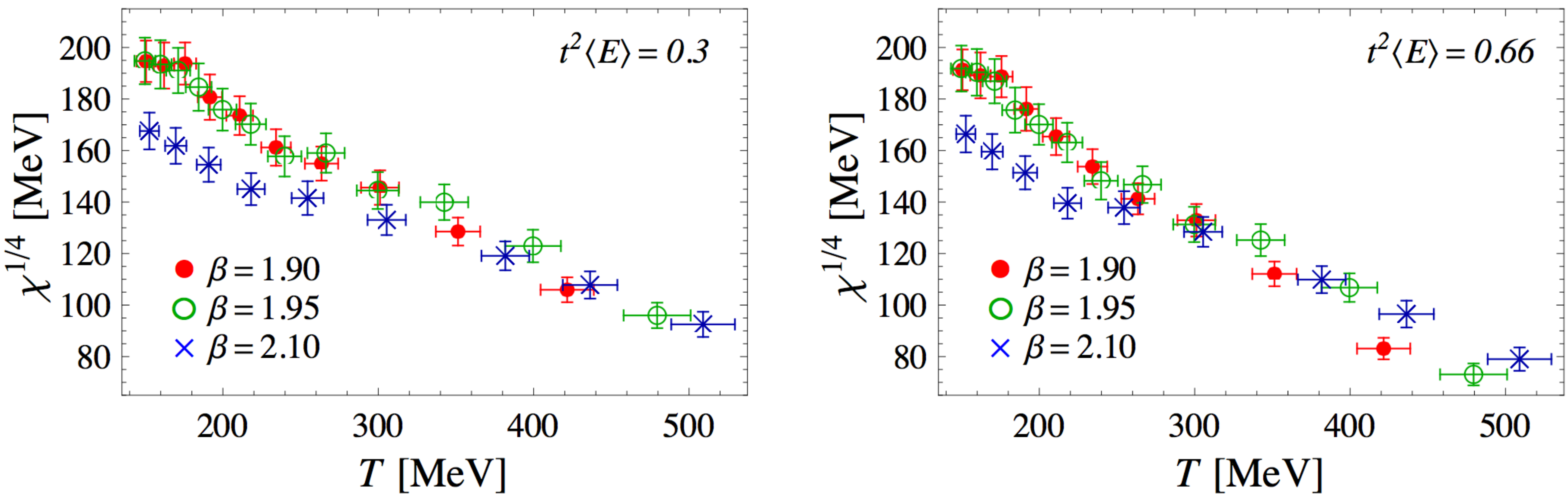
We focus on t(0.3) = t0 and t(0.66)= t1

On finer lattices, plateau is almost reached:

Gradient method coincides with cooling



Results for the topological susceptibility for $M_\pi = 270\ \mathrm{MeV}$



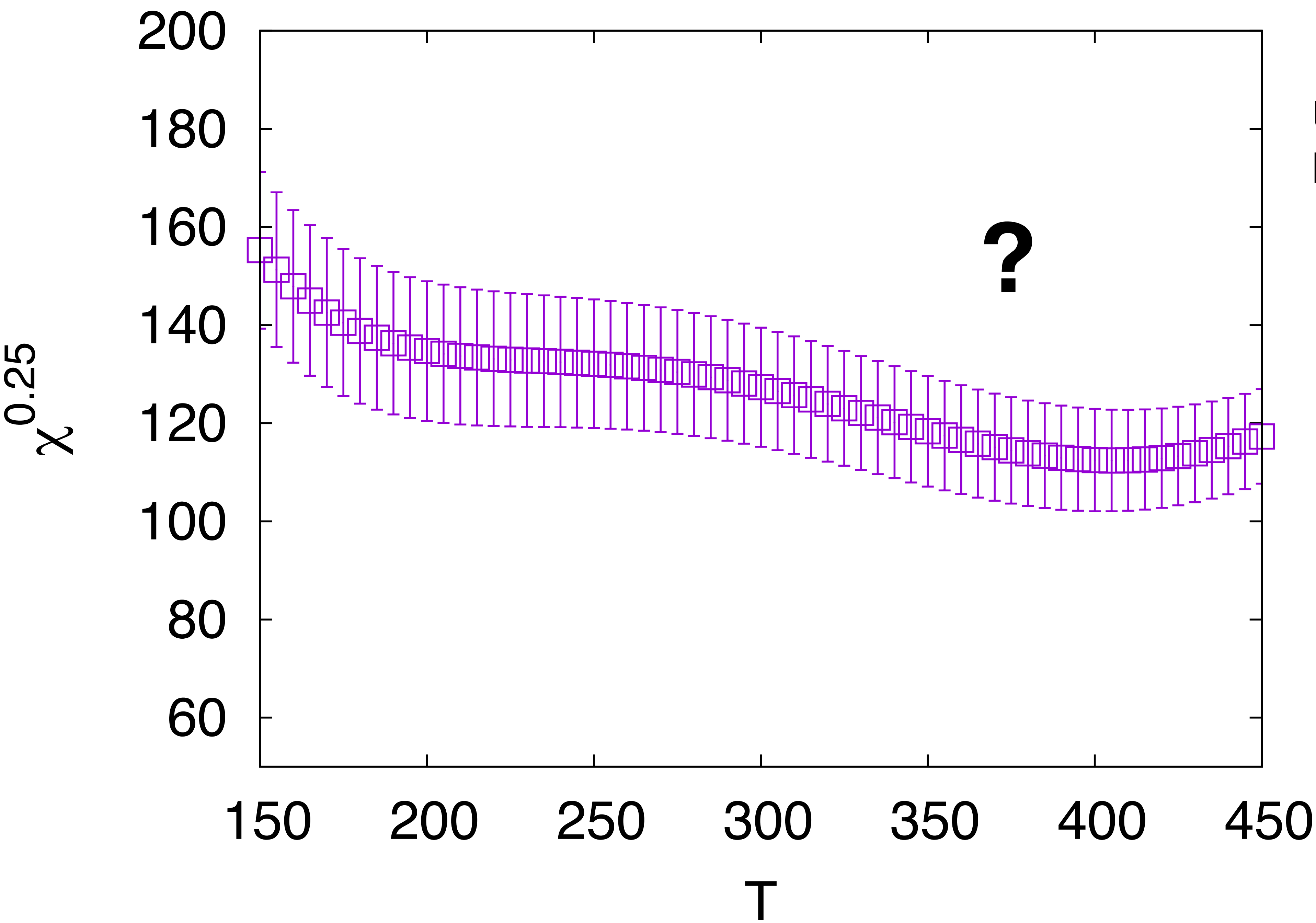
Continuum limit:

- in principle independent on flow limit
- we need to interpolate results at fixed scale to match T

$$\chi(T, m_\pi) = \lim_{a\to 0} \chi^{1/4}(T, a, m_\pi, t_x)$$

$$\chi^{1/4}(T, a, m_\pi, t_x) = \chi^{1/4}(T, m_\pi) + a^2 k(T, t_x)$$

Continuum results for $m_\pi = 370 \text{ MeV}$



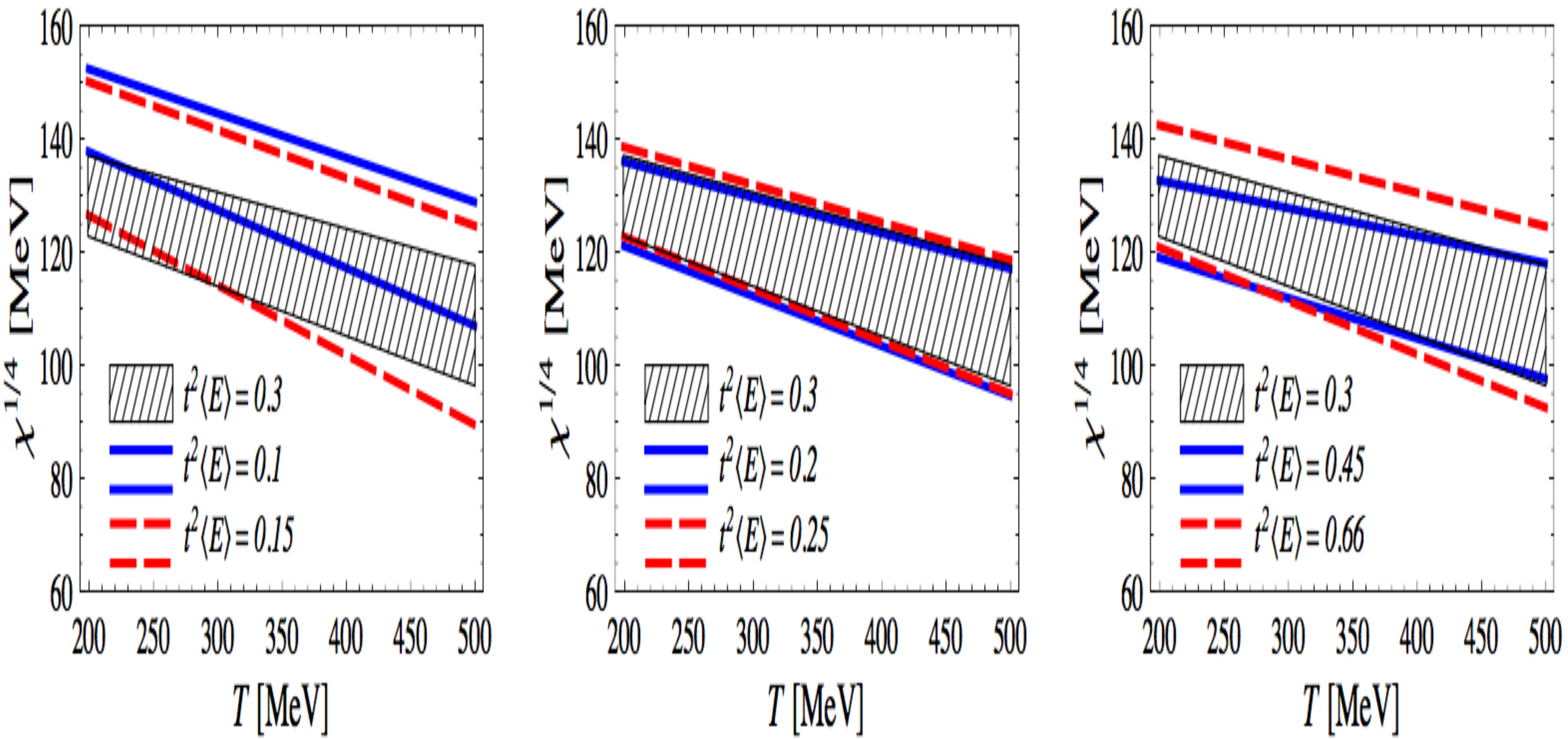
Used:
Bezier interp.

Very slow decay:
Nearly linear,
or power-like
with a
tiny exponent:

$$\chi(T)^{0.25} \simeq aT^{-0.26} \simeq T, T > 200 \text{ MeV}$$

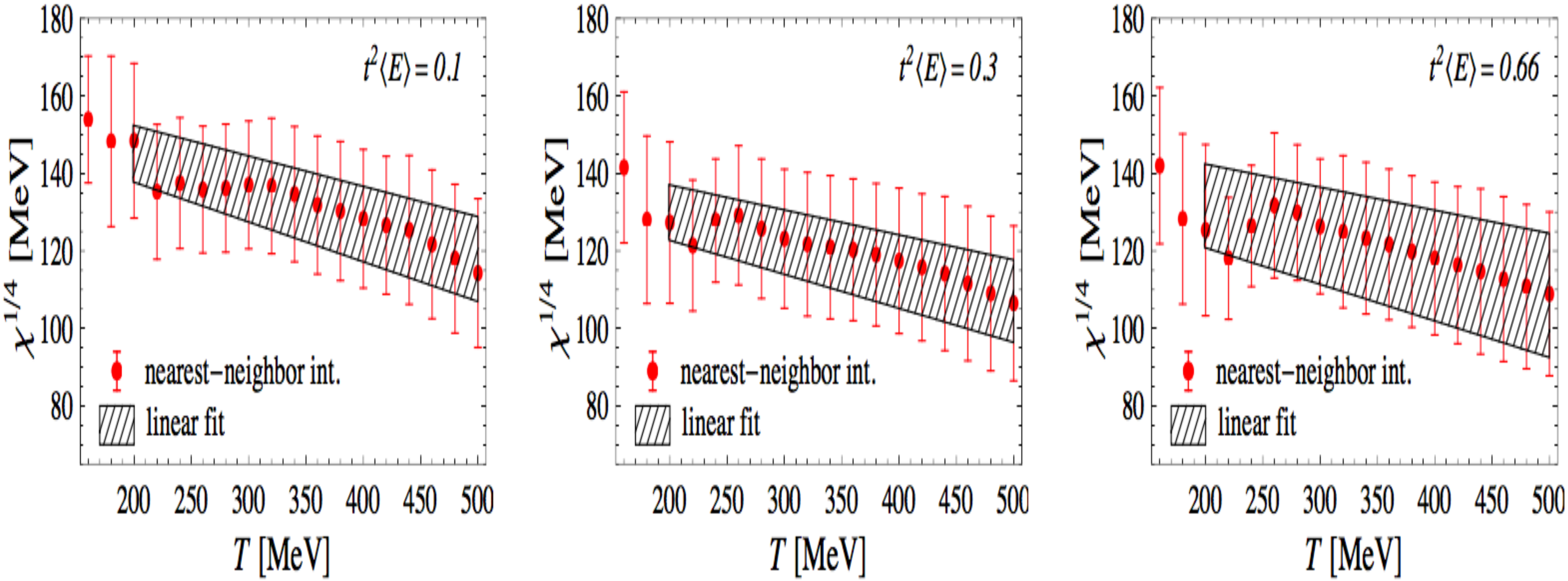
Detailed analysis for T > 200 MeV (use approx. linearity) - **1**

(In)dependence of continuum limit on flow's limit: 0.3 OK



Detailed analysis for T > 200 MeV - **2**

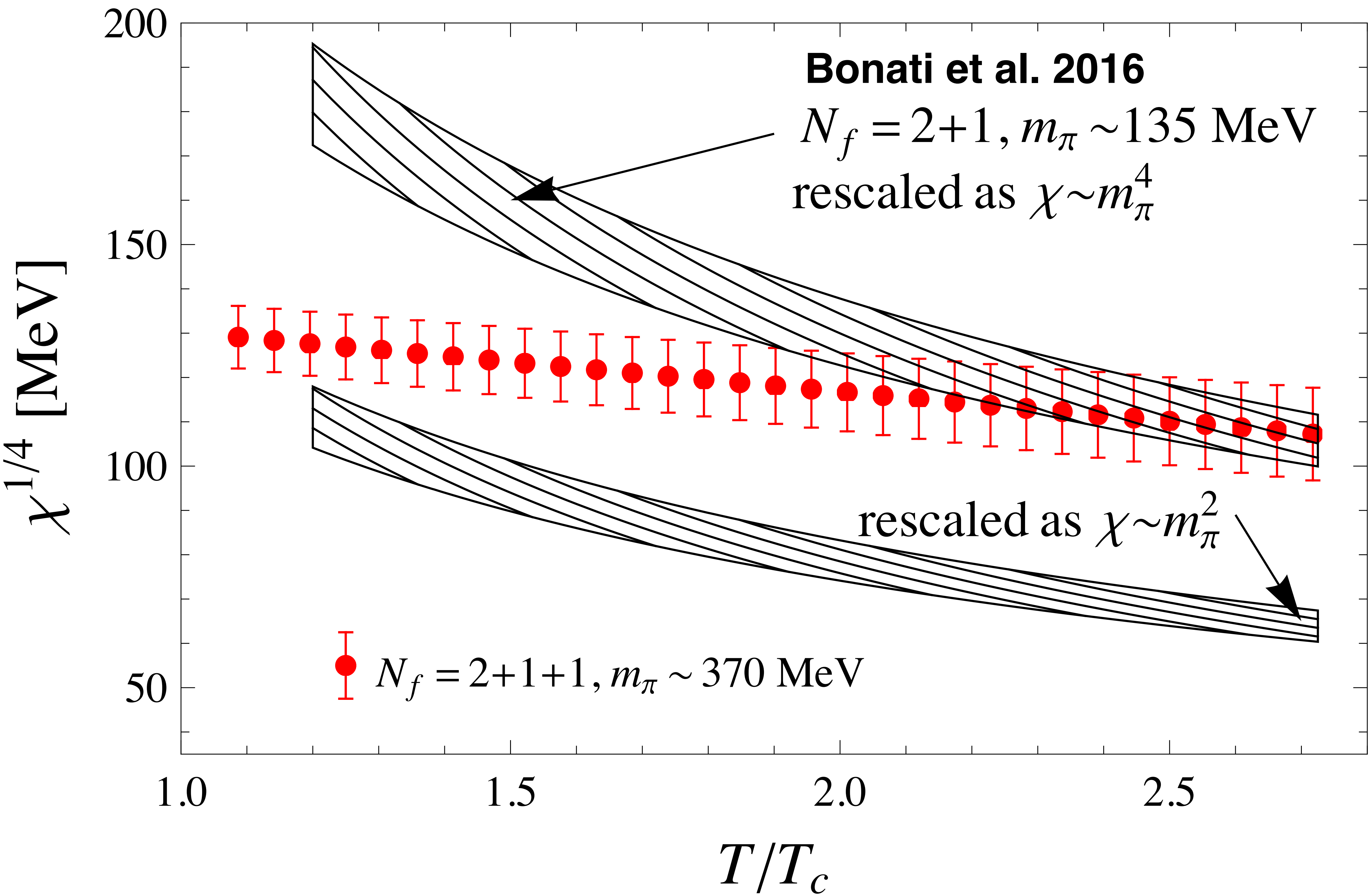
Interpolation ok.



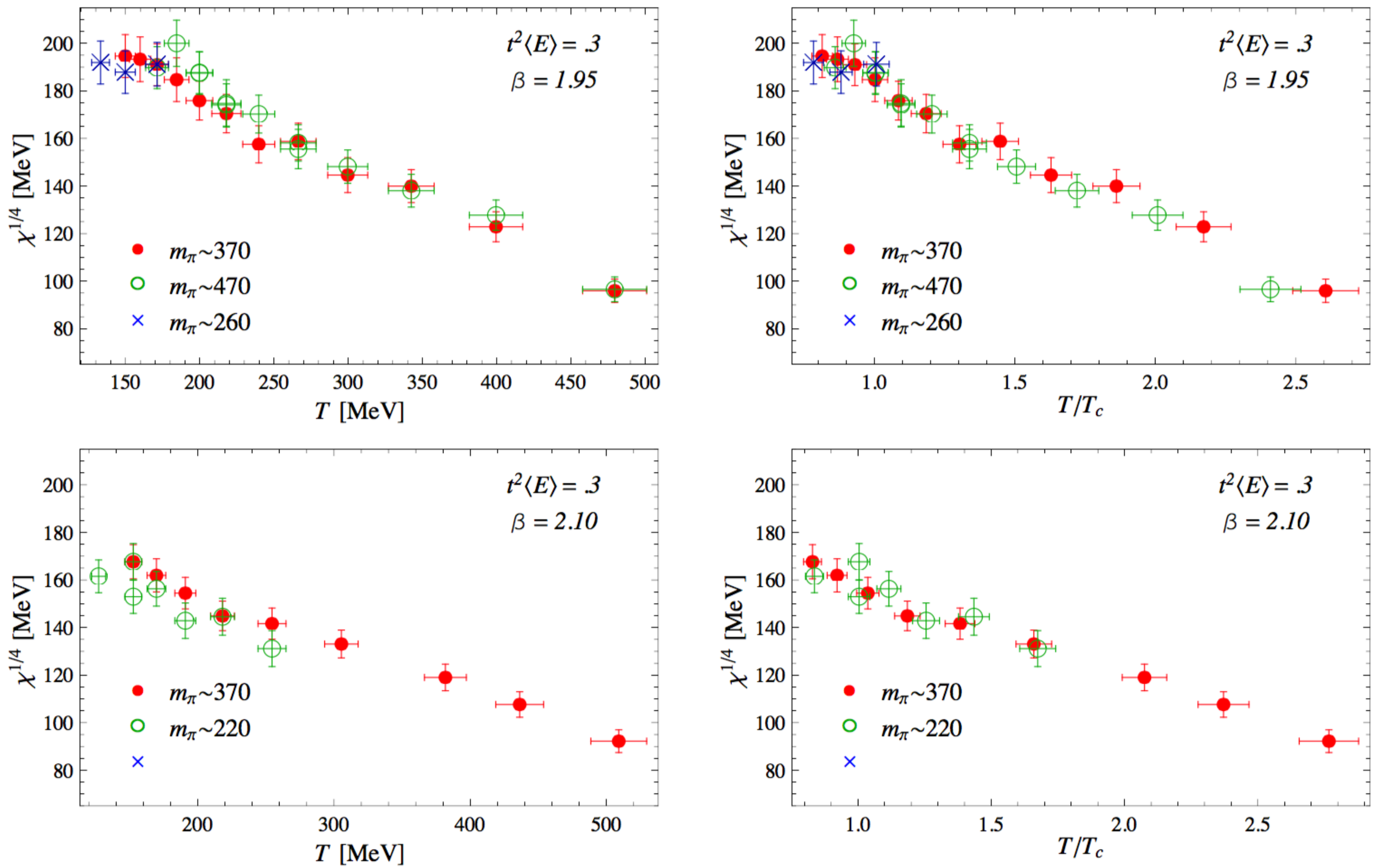
Detailed analysis for T > 200 MeV - **3**

A mass rescaling appears to work nicely

Bonati et al. 2016



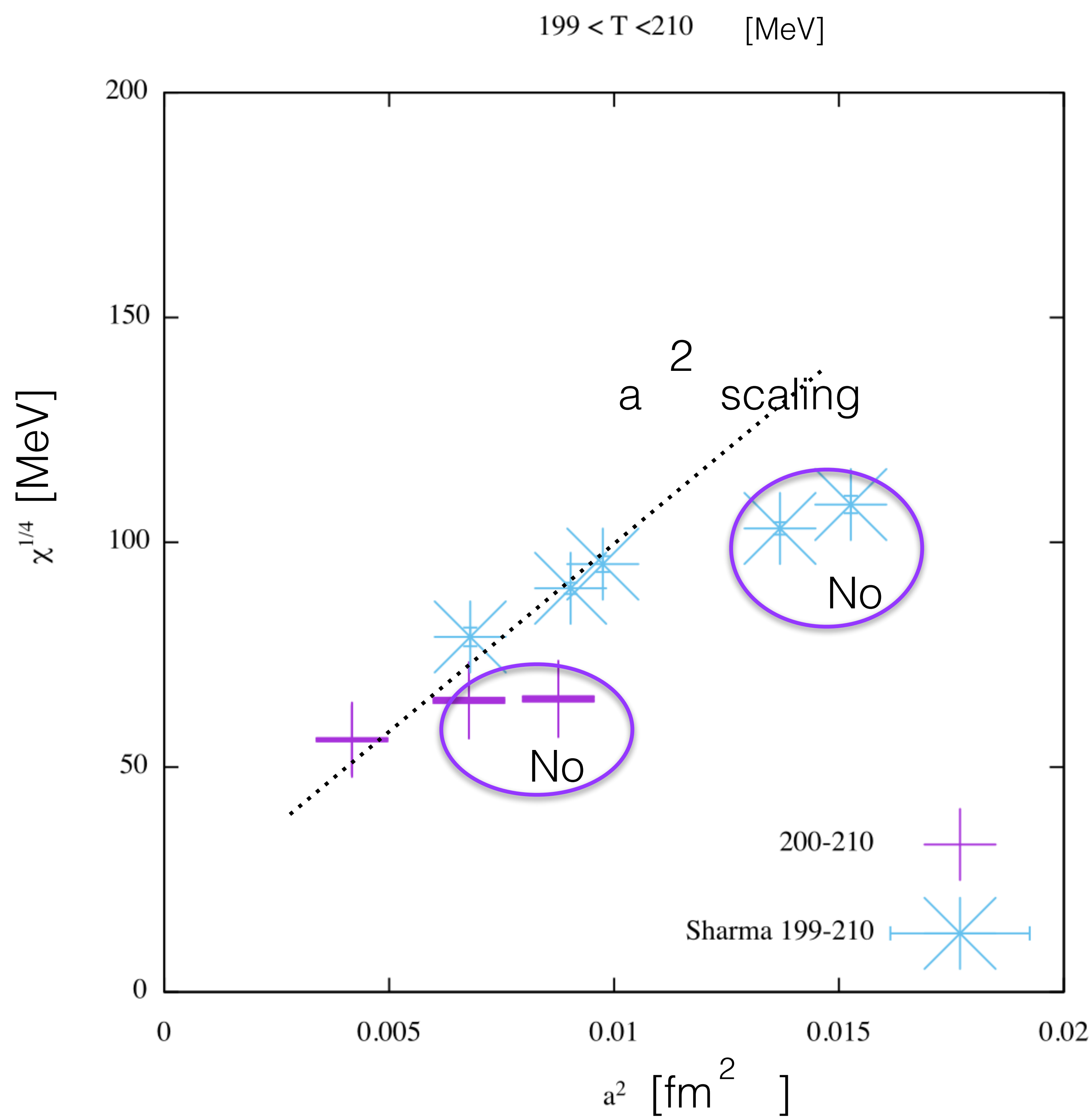
However: there is no mass dependence..



Possible explanation : strong scaling violations

Comparison with BNL results

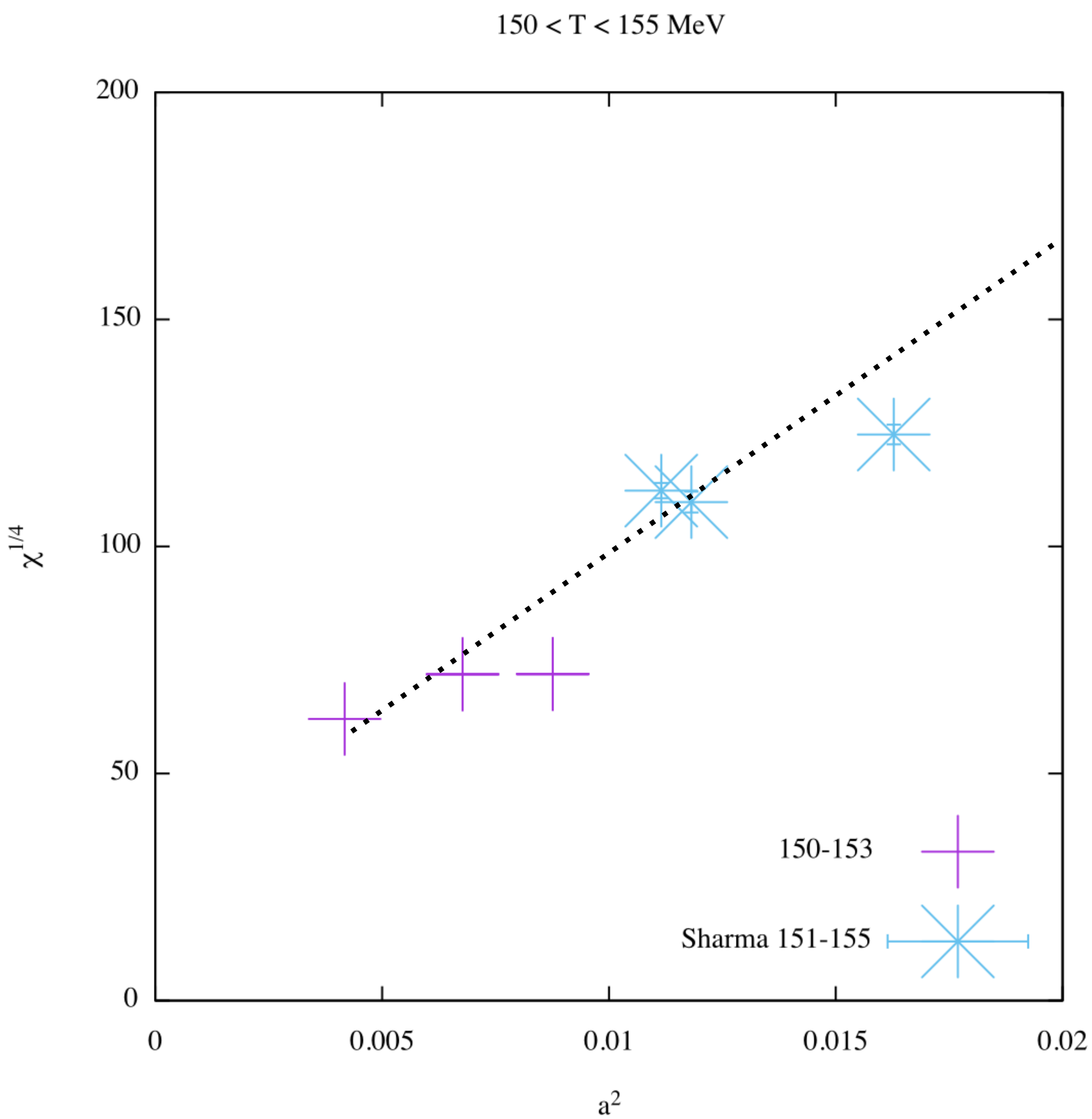
numerical data courtesy *S. Sharma*

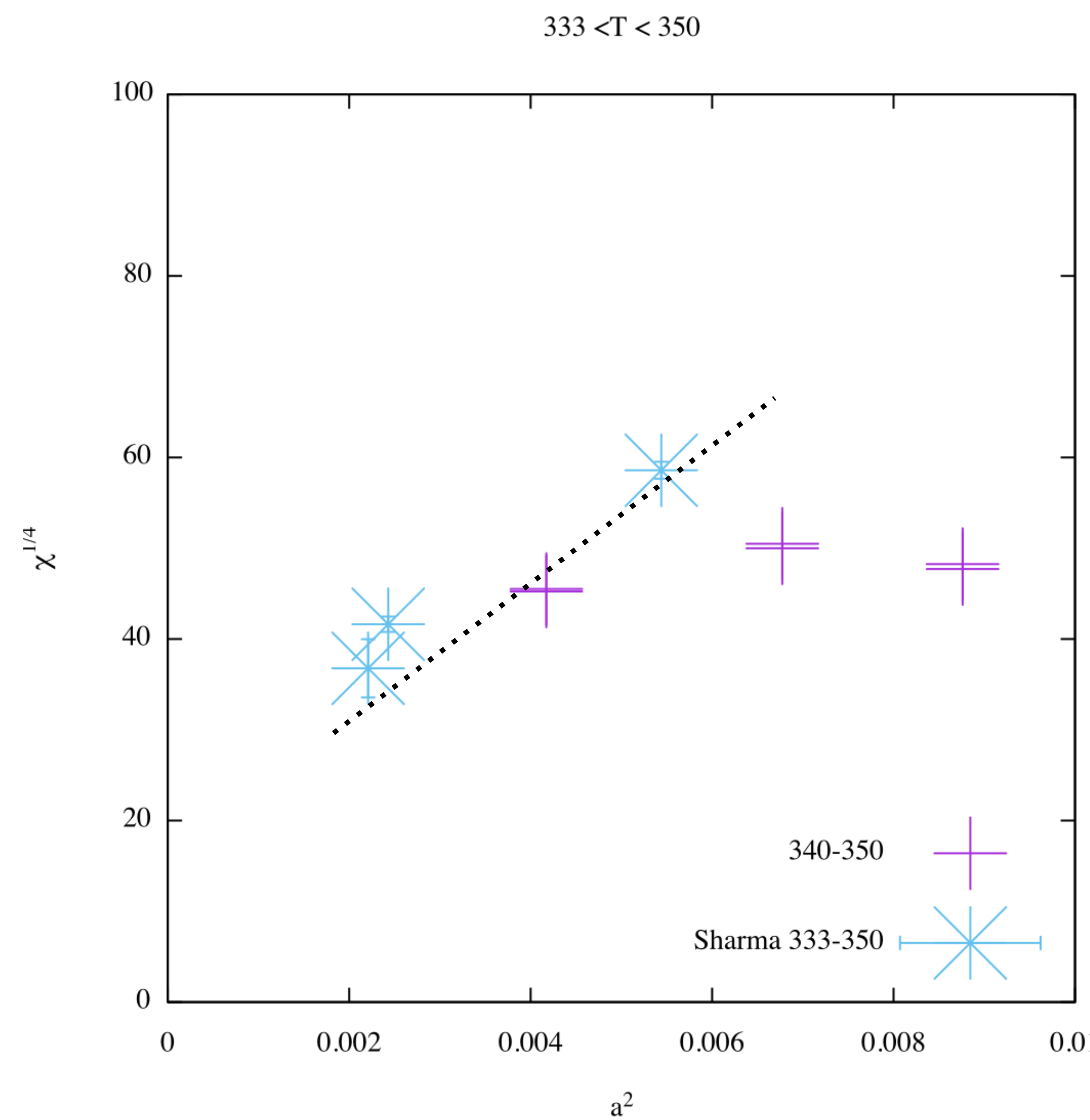


Indication of corrections to a^2 scaling on our two coarser lattices

Comparison with BNL results (contn'd)

numerical data courtesy *S. Sharma*



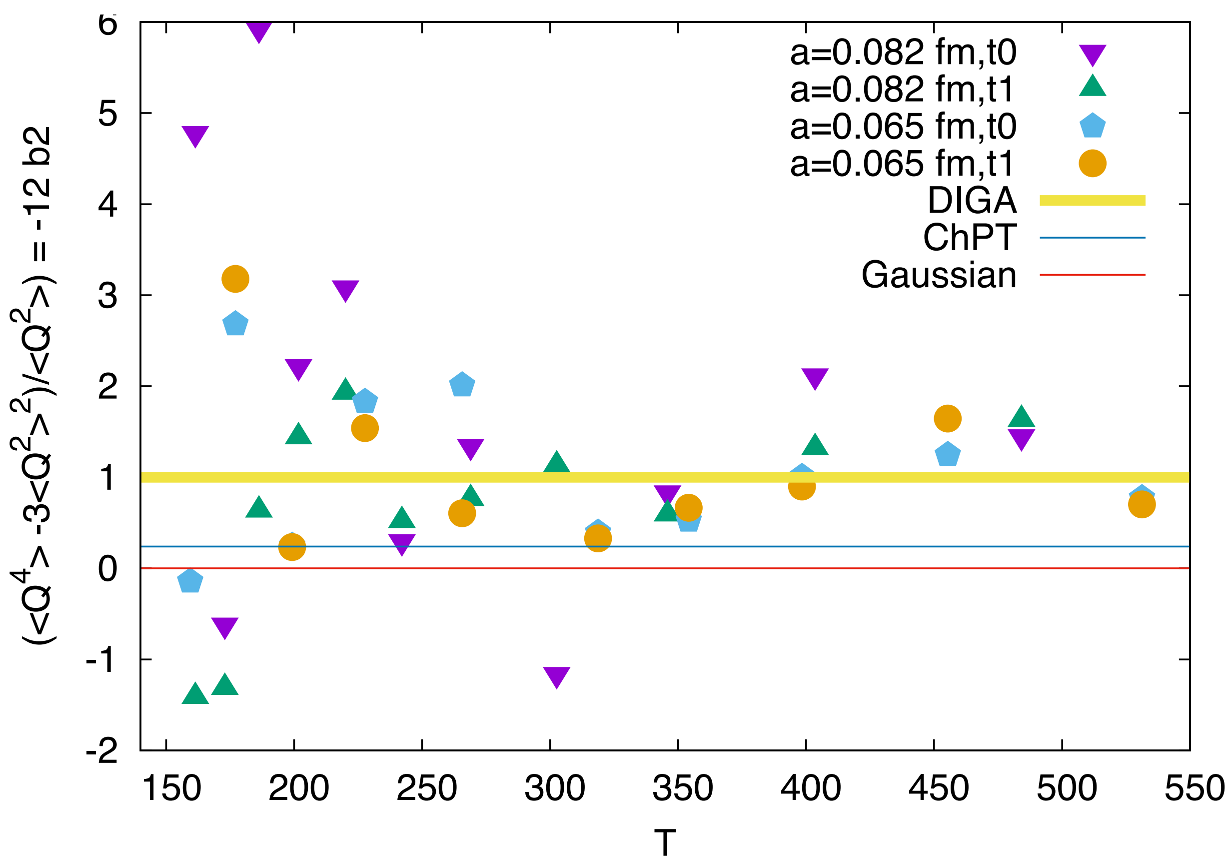


Consistent trend for other temperatures: on our finer lattice the corrections to a^2 scaling seem moderate

Instanton potential - cumulants' ratio b2

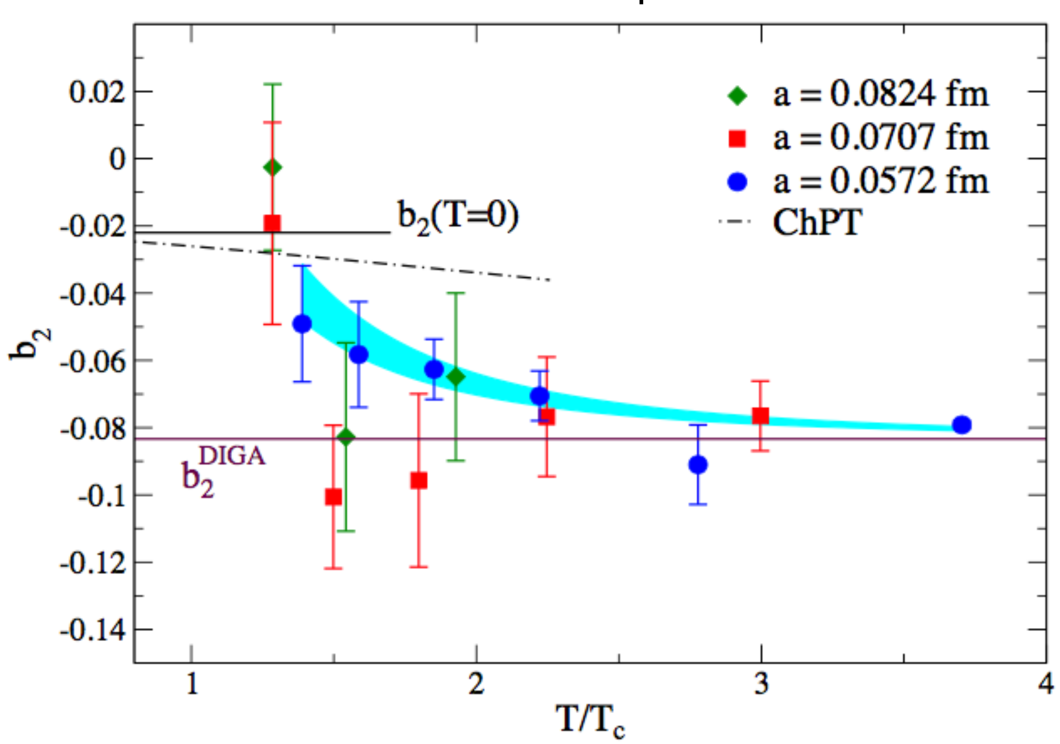
DIGA predicts

$$F(\theta, T) - F(0, T) = \chi(T)(1 - cos(\theta)) \longrightarrow b_2 = -1/12$$



b2 = -1/12

DIGA limit for T > 350 MeV



Consistent with Bonati et al.

Results II

Fermionic operator

$$n_L - n_R = Q_{top}$$

$$m \int d^4x \bar{\psi} \gamma_5 \psi = Q_{top}$$

Topological and chiral susceptibility

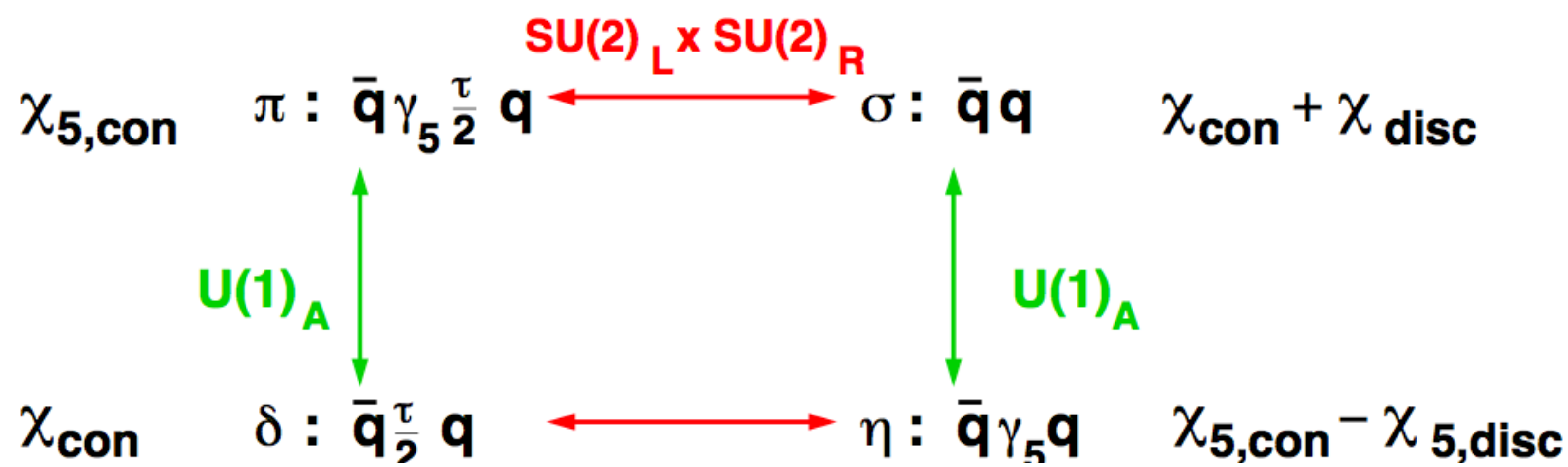
Kogut, Lagae, Sinclair 1999

HotQCD, 2012

$$\chi_{top} =< Q_{top}^2 > /V = m_l^2 \chi_{5,disc}$$

From:

$$m \int d^4x \bar{\psi}\gamma_5\psi = Q_{top}$$

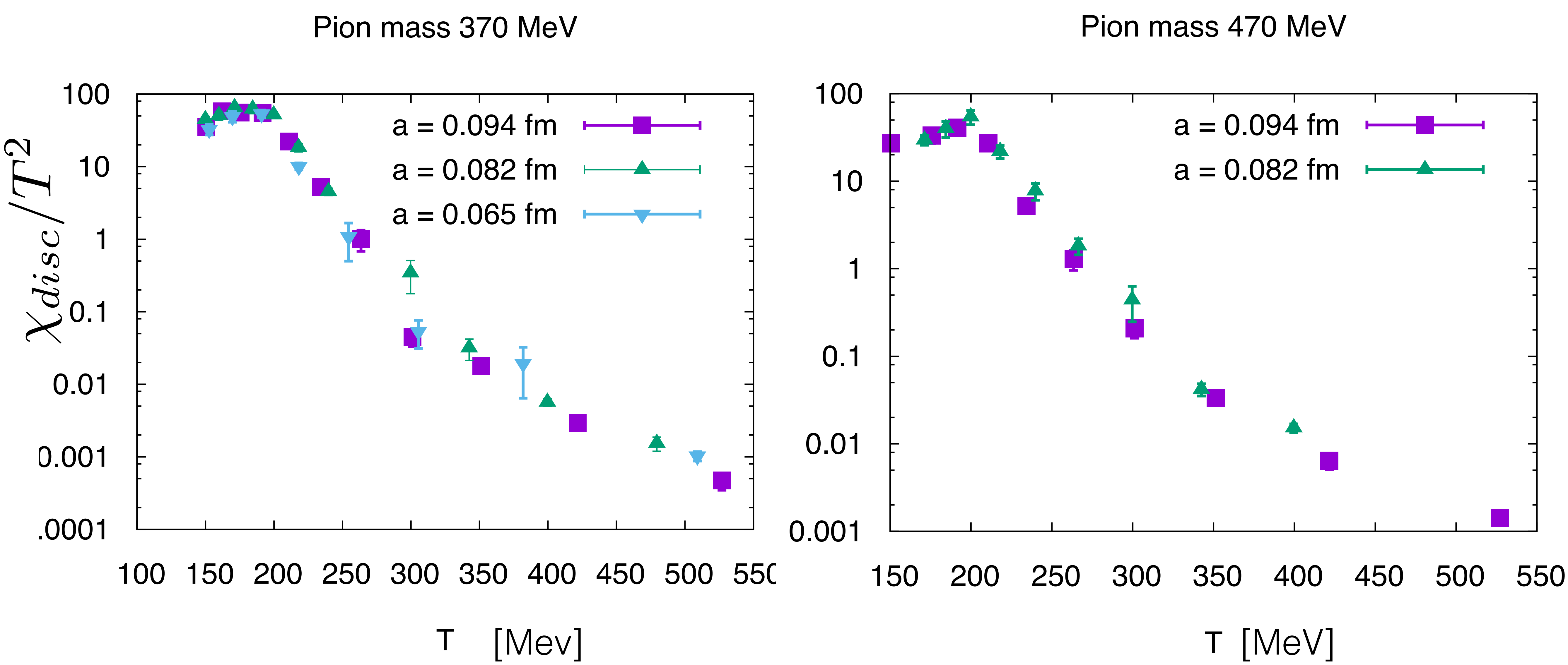


$$\chi_\pi - \chi_\delta = \chi_{\rm disc} = \chi_{5,\rm disc}\ , \qquad \text{for} \quad T \geq T_c\ ,\ m_l \to 0$$

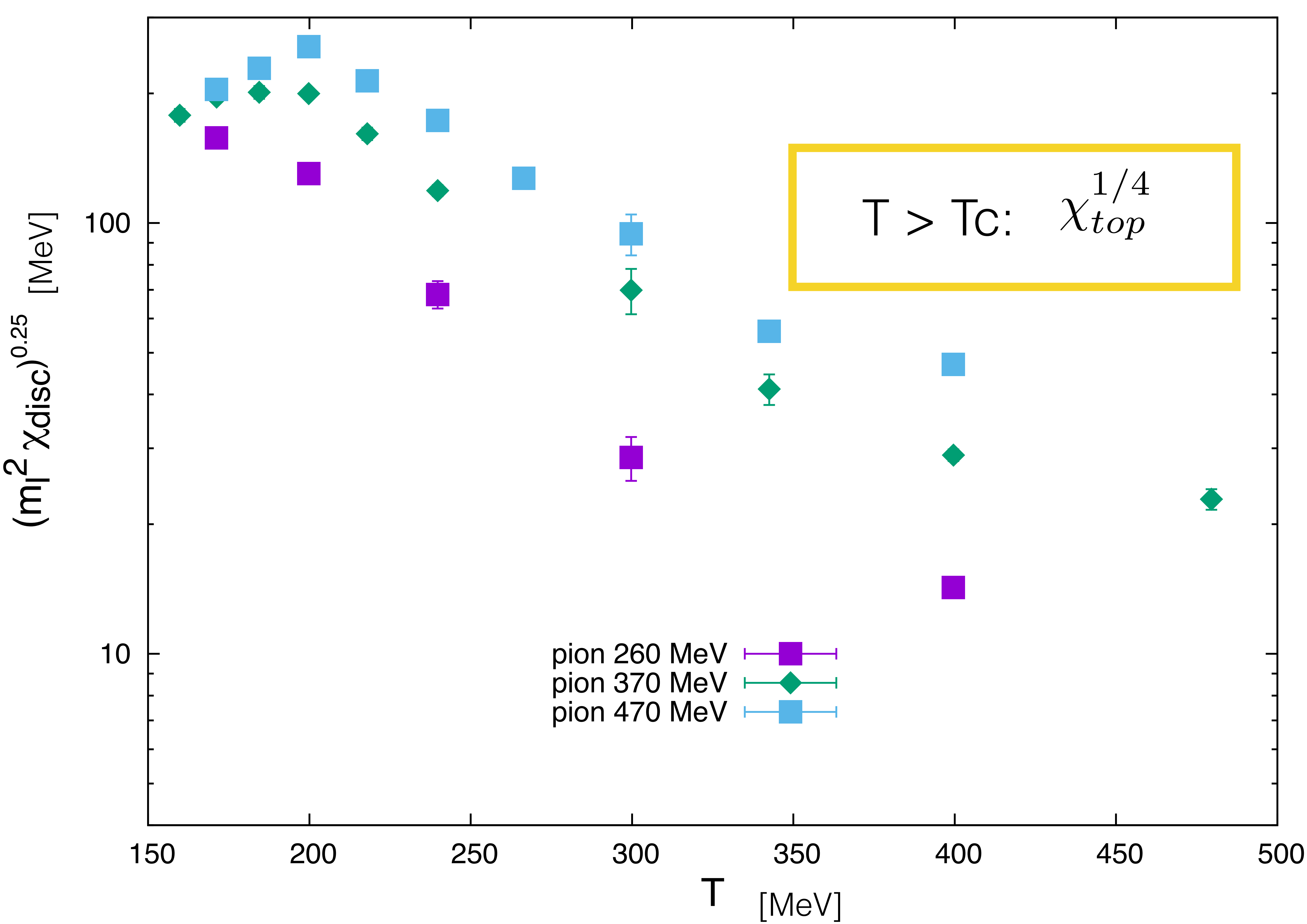
$$\chi_{top} =< Q_{top}^2 > /V = m_l^2 \chi_{disc}$$

$$T > T_{U(1)_A} \simeq T_c$$

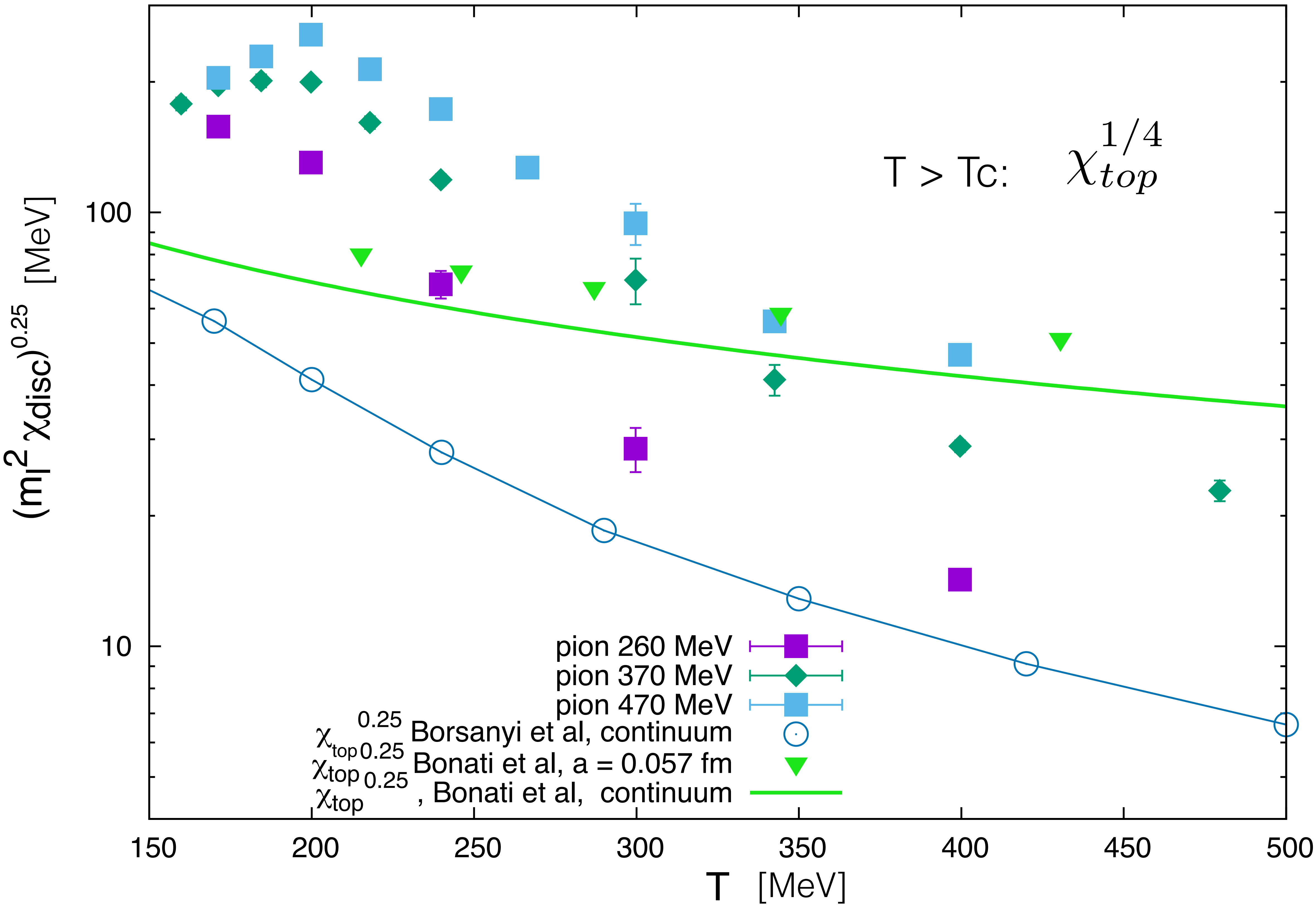
Chiral susceptibility



Within errors, no discernable spacing dependence

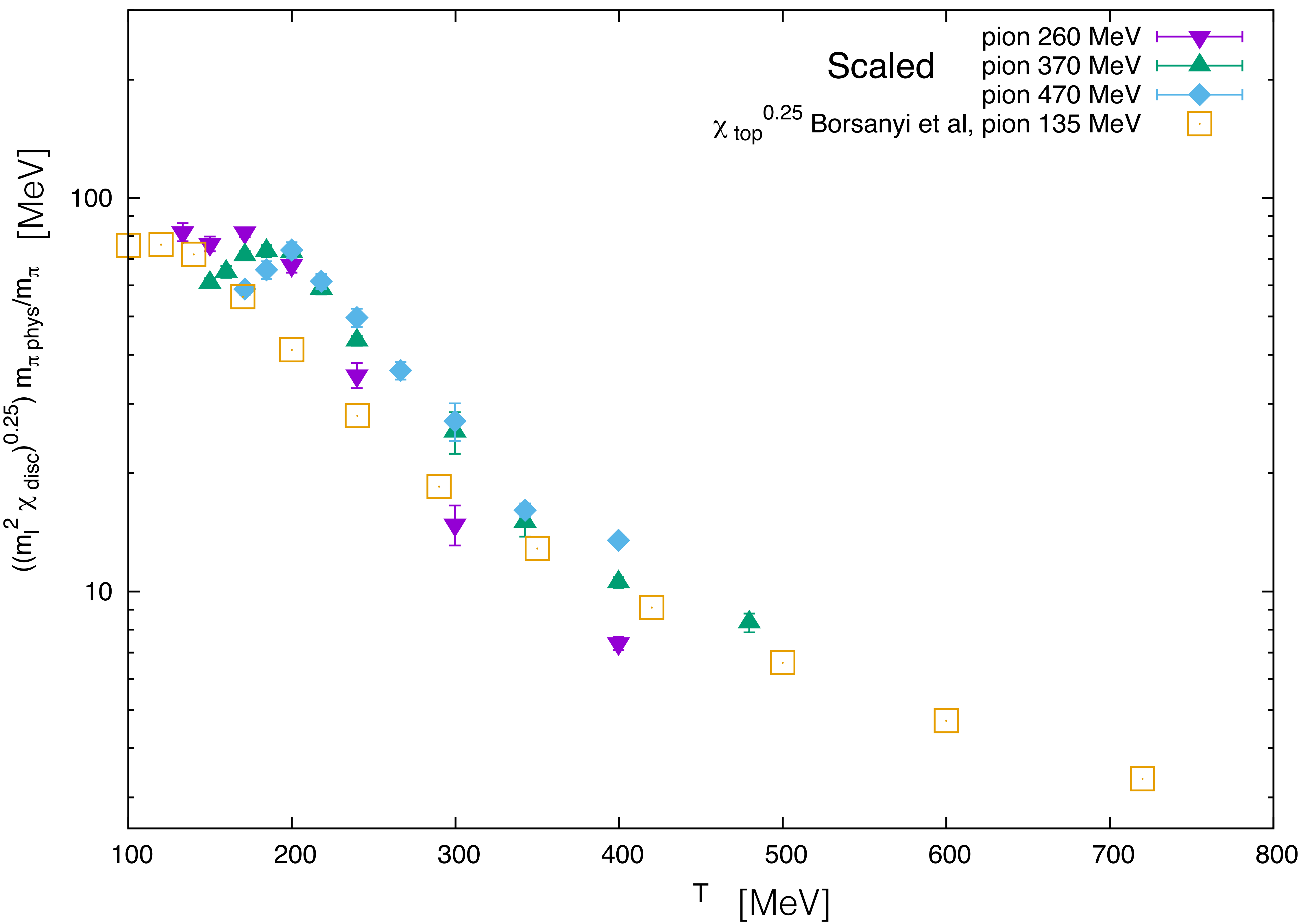


T > Tc: $\chi_{top}^{1/4}$
$(m_l^2\chi disc)^{0.25}$ [MeV]
100
10
150
200
250
300
350
400
450
500
T [MeV]
pion 260 MeV
pion 370 MeV
pion 470 MeV



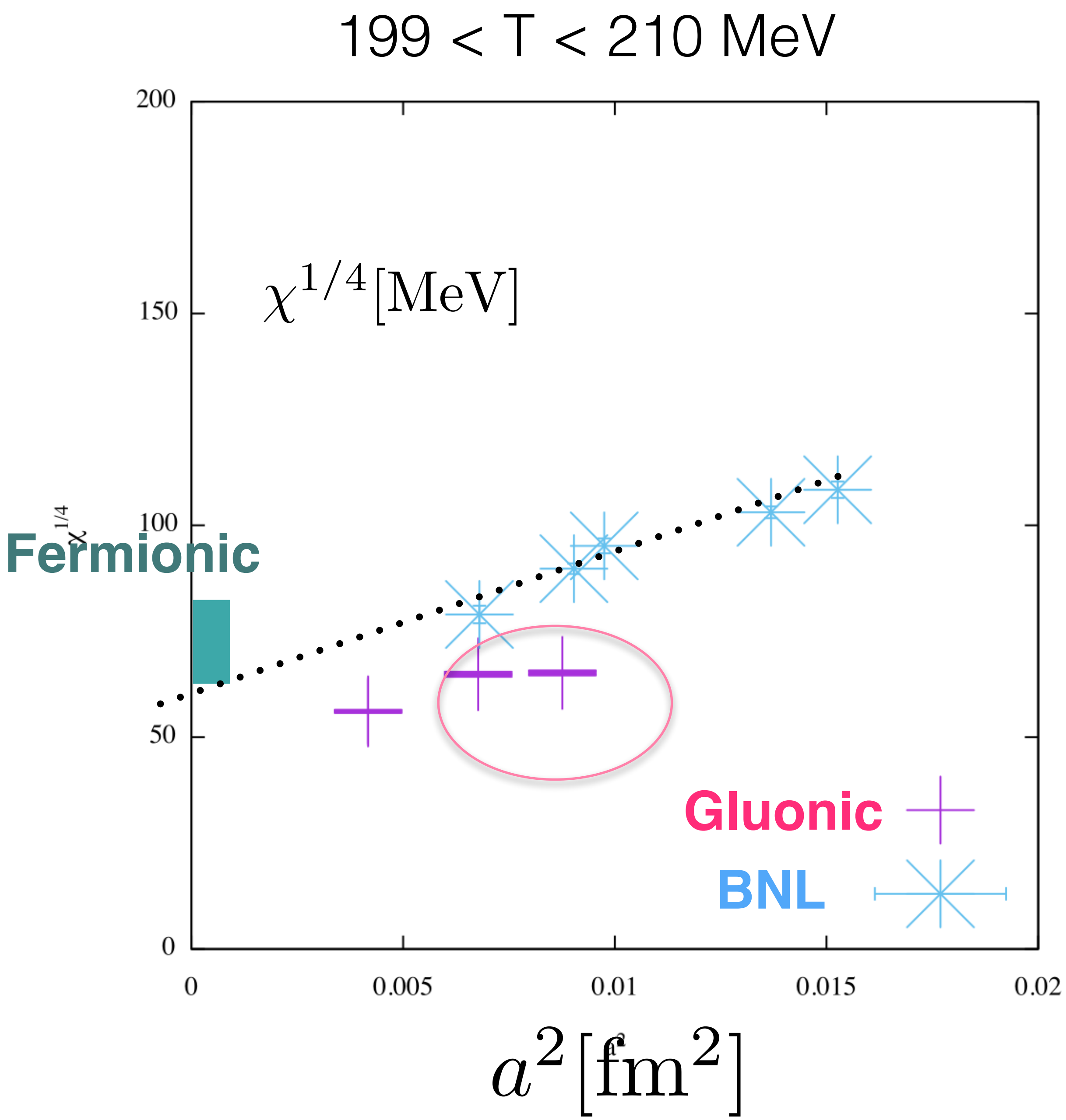
T > Tc: $\chi_{top}^{1/4}$
100
10
$(m_l^2 \chi_{disc})^{0.25}$ [MeV]
150
200
250
300
350
400
450
500
T [MeV]
pion 260 MeV
pion 370 MeV
pion 470 MeV
$\chi_{top}^{0.25}$ Borsanyi et al, continuum
$\chi_{top}^{0.25}$ Bonati et al, a = 0.057 fm
$\chi_{top}^{0.25}$, Bonati et al, continuum

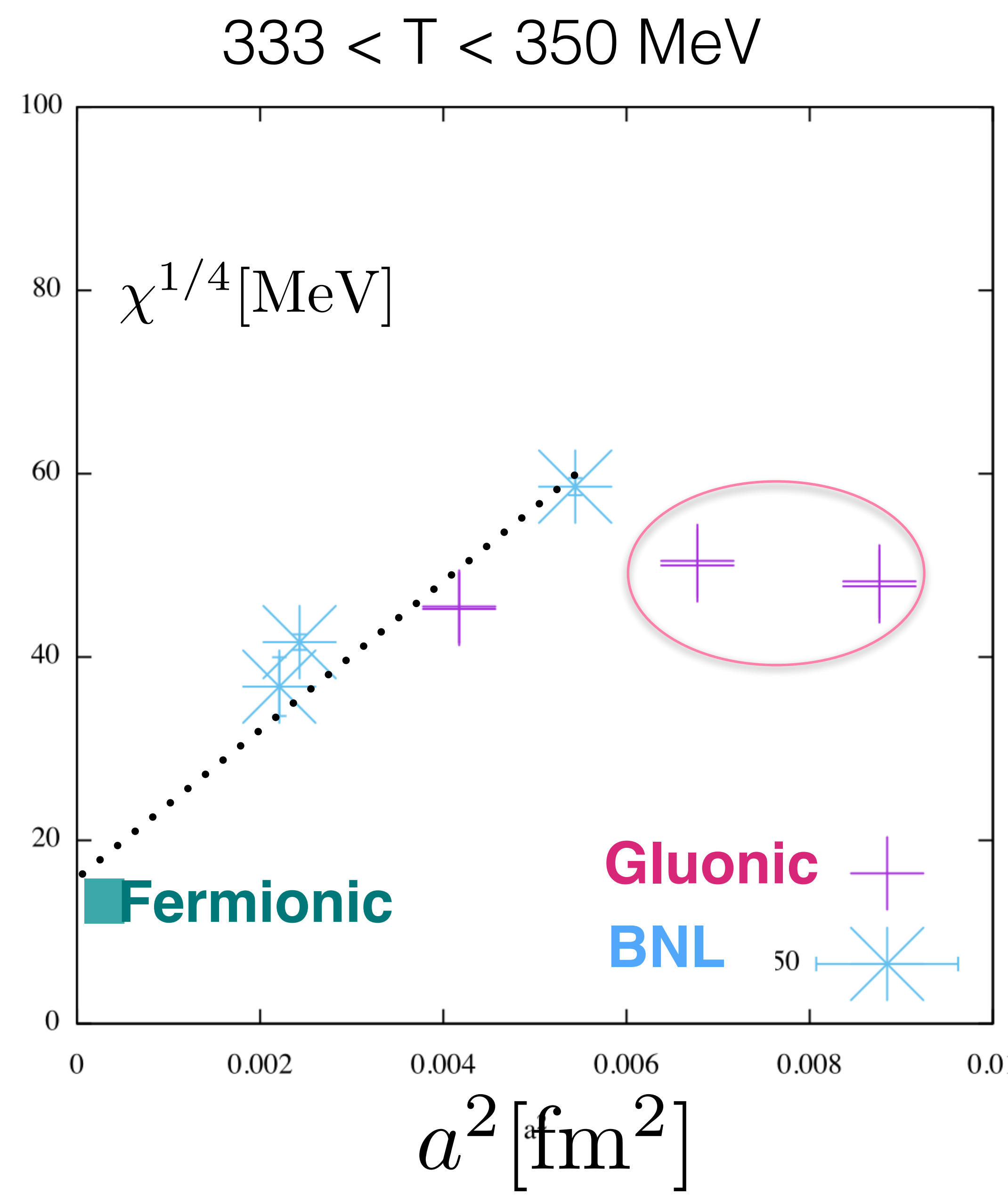
Results for physical pion mass



Comparison with BNL results including fermionic results

numerical data courtesy *S. Sharma*



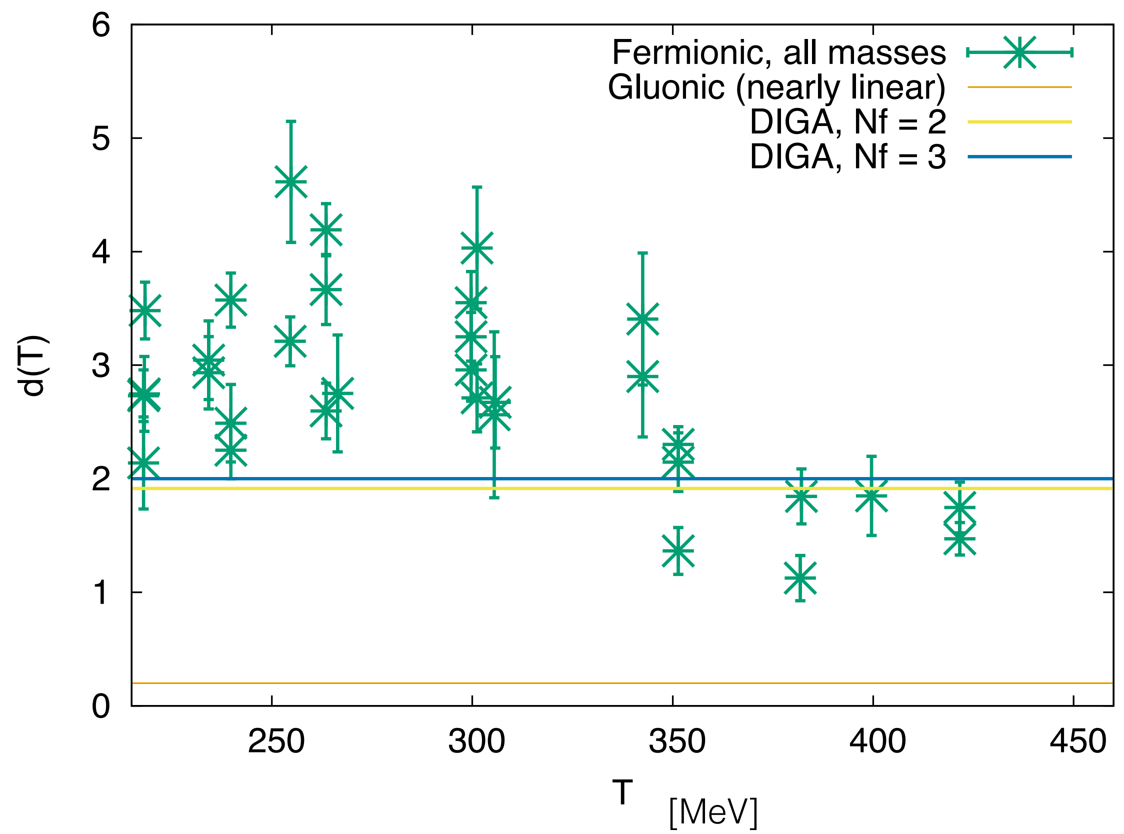


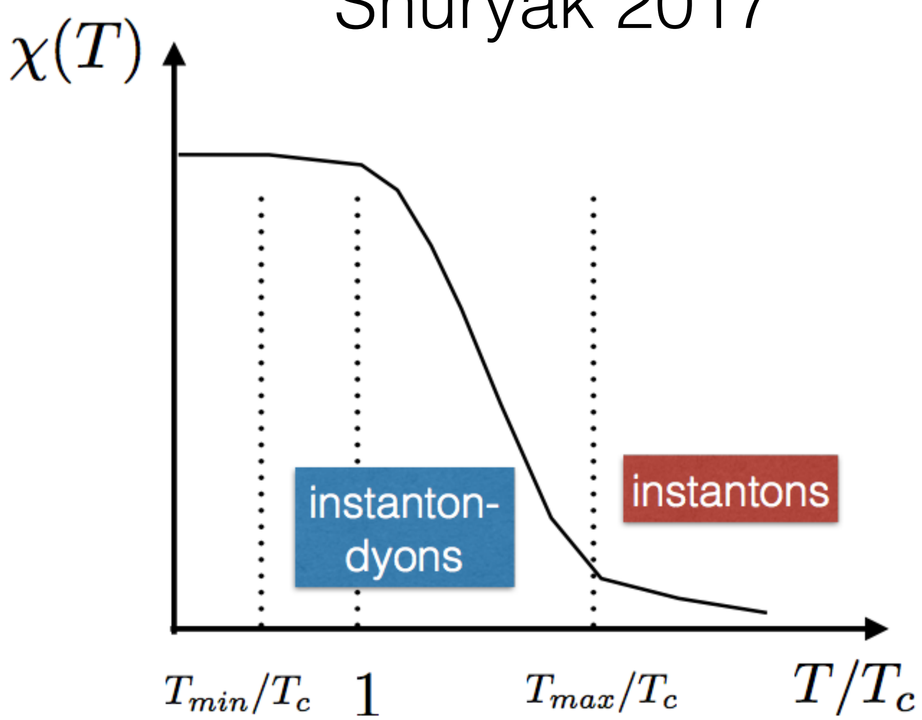
dotted lines to guide the eye

Effective exponent : $$d(T) = -T\frac{d}{dT}\ln\chi^{0.25}(T)$$

$$\chi^{0.25}(T) = aT^{-d(T)}$$

Possibly consistent with instant -dyon? Shuryak 2017



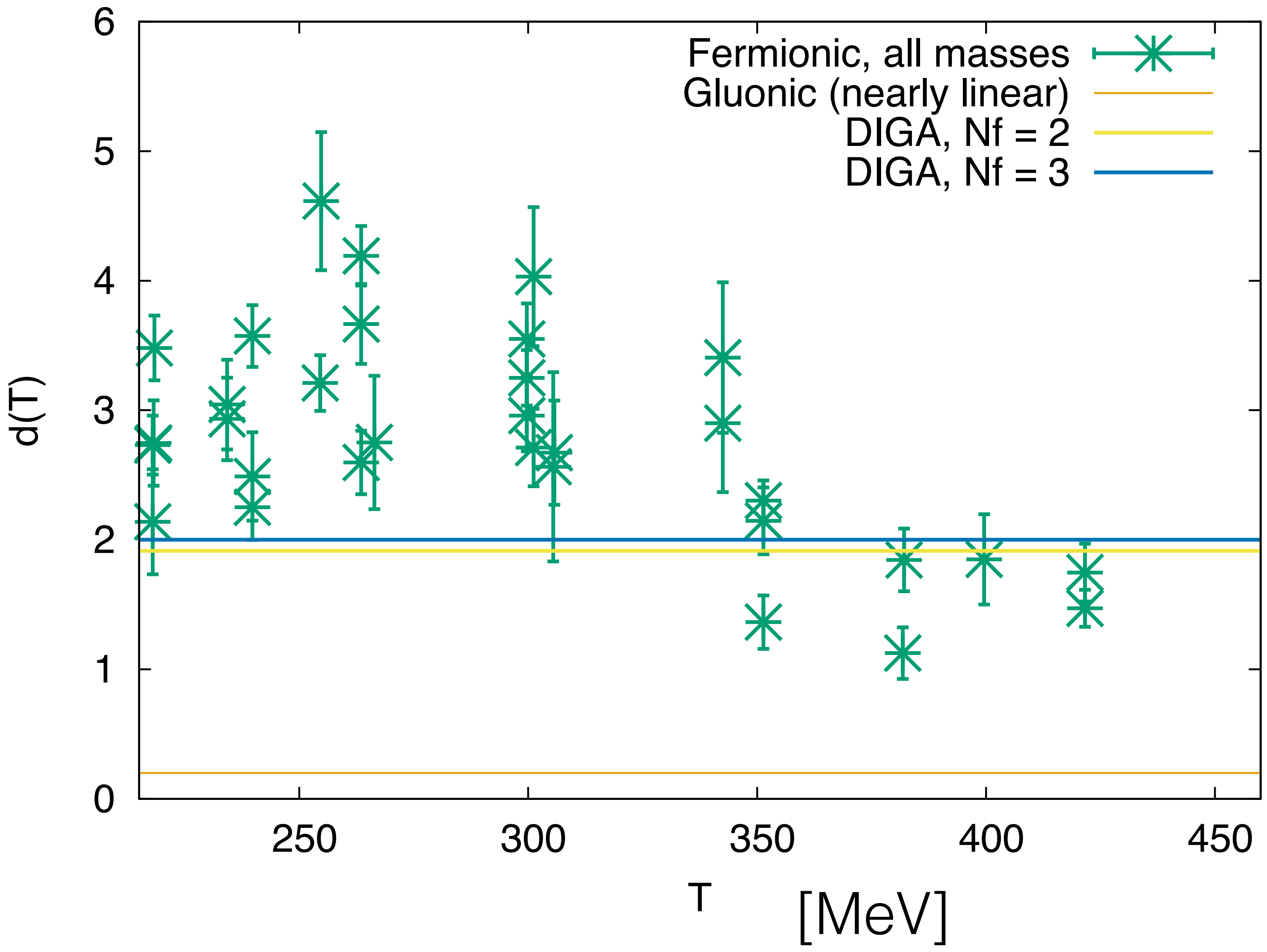


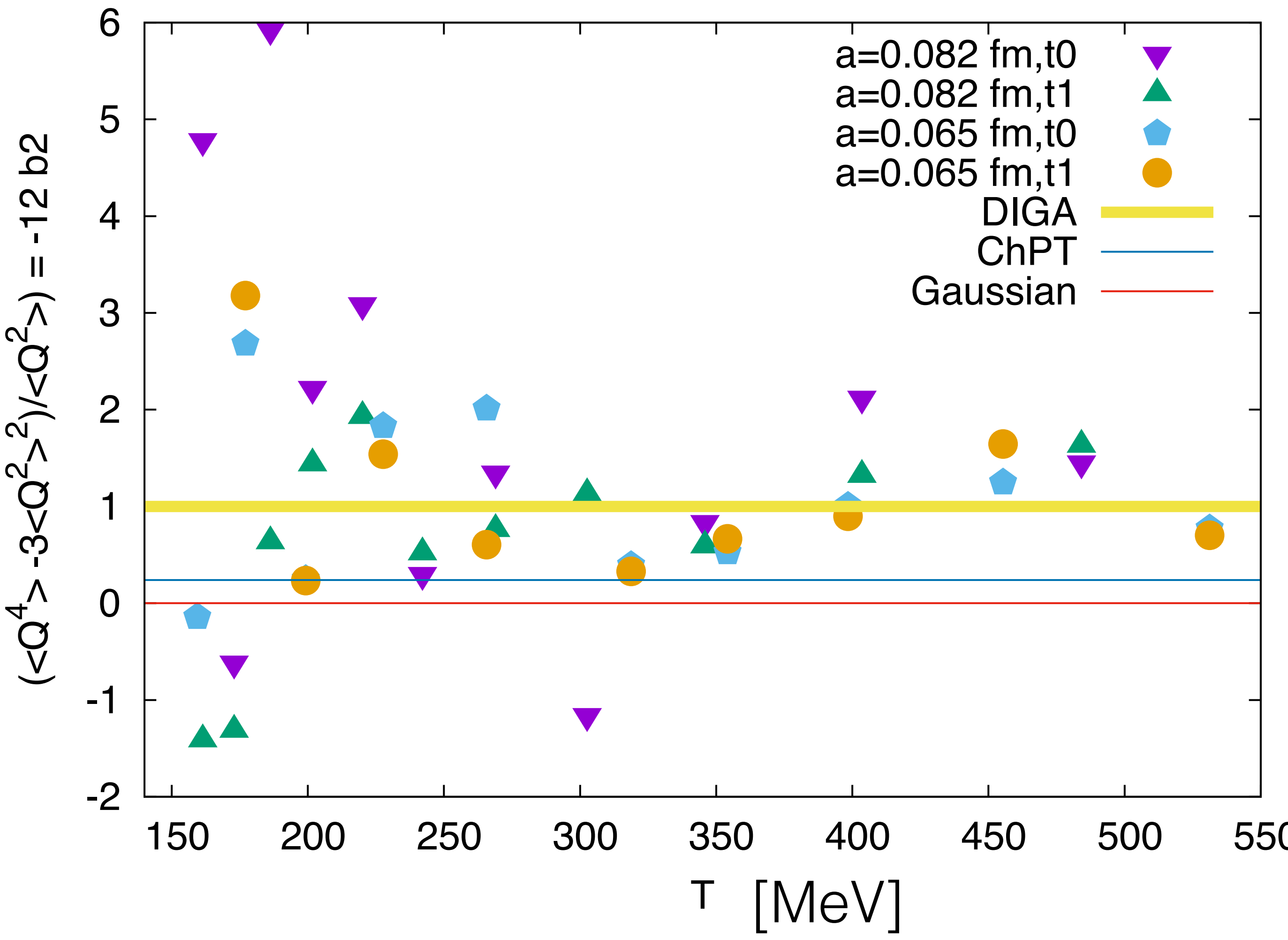
Faster decrease before DIGA sets in

Effective exponent :

$$\chi_{top}^{1/4} = aT^{-d(T)}$$

Same DIGA onset seen in b2 ≈ 350 MeV

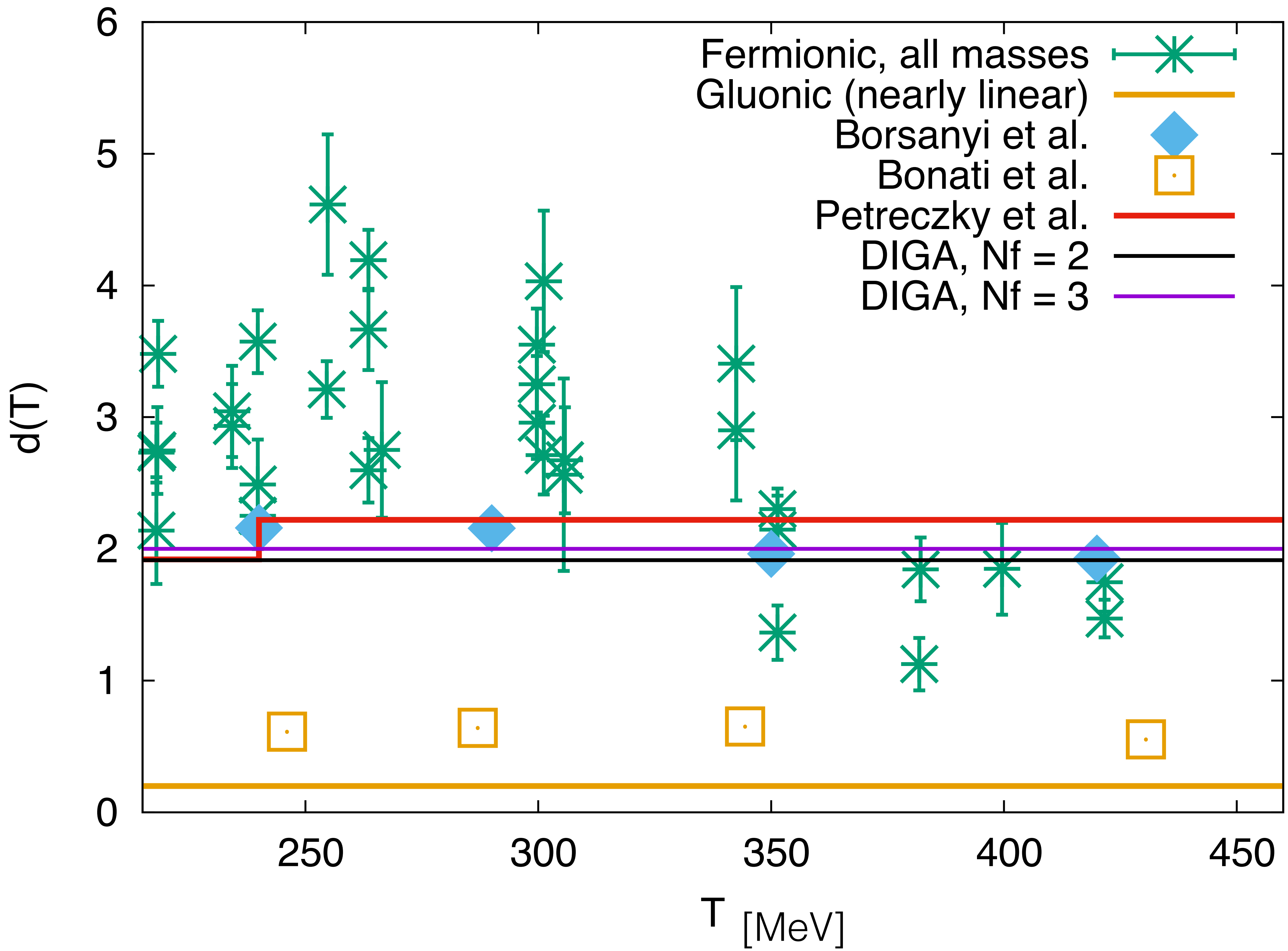


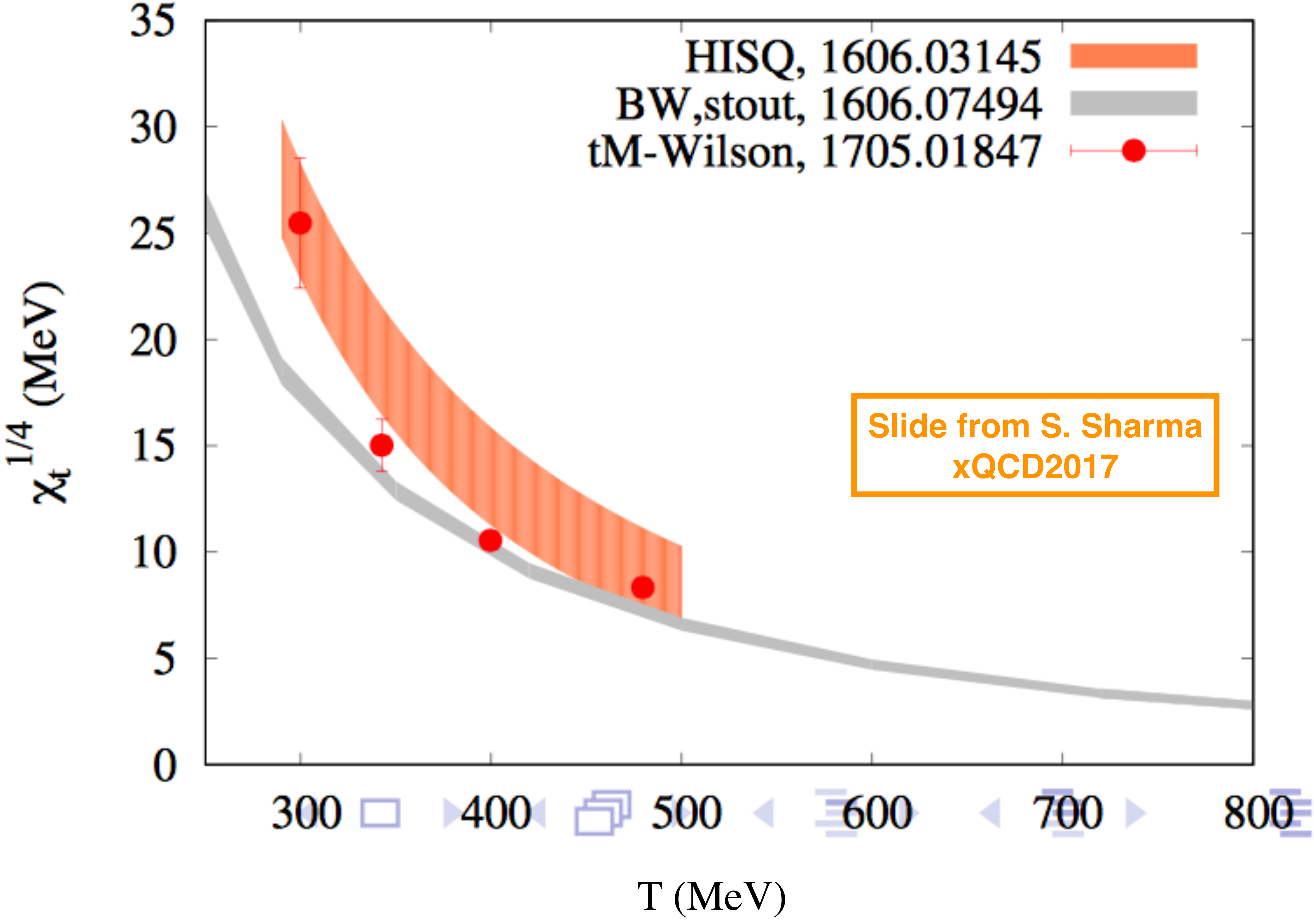


Effective exponent d(T):

Comparisons with other results :

$$\chi_{top}^{1/4} = aT^{-d(T)}$$

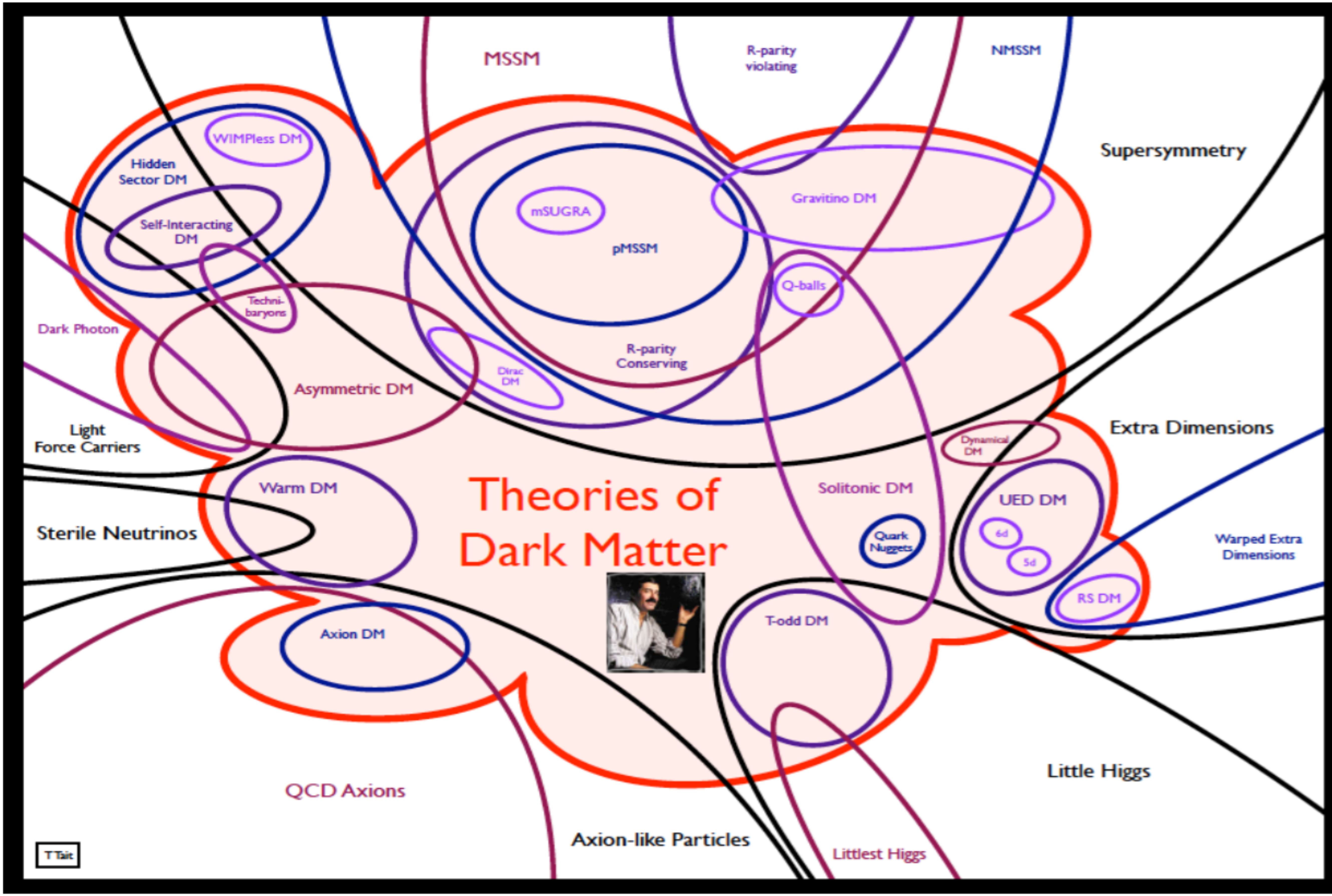



HISQ, 1606.03145
BW,stout, 1606.07494
tM-Wilson, 1705.01847
Slide from S. Sharma
xQCD2017
$\chi_t^{1/4}$ (MeV)
T (MeV)
0
5
10
15
20
25
30
35
300
400
500
600
700
800

A window on the axions

Inspiring paper:
Berkowitz, Buchoff, Rinaldi **Phys.Rev. D92 (2015) no.3, 034507**

Theory landscape (From Tim Tait, Snowmass)



The Equation of State of the Quark Gluon Plasma paves the way to Cosmology

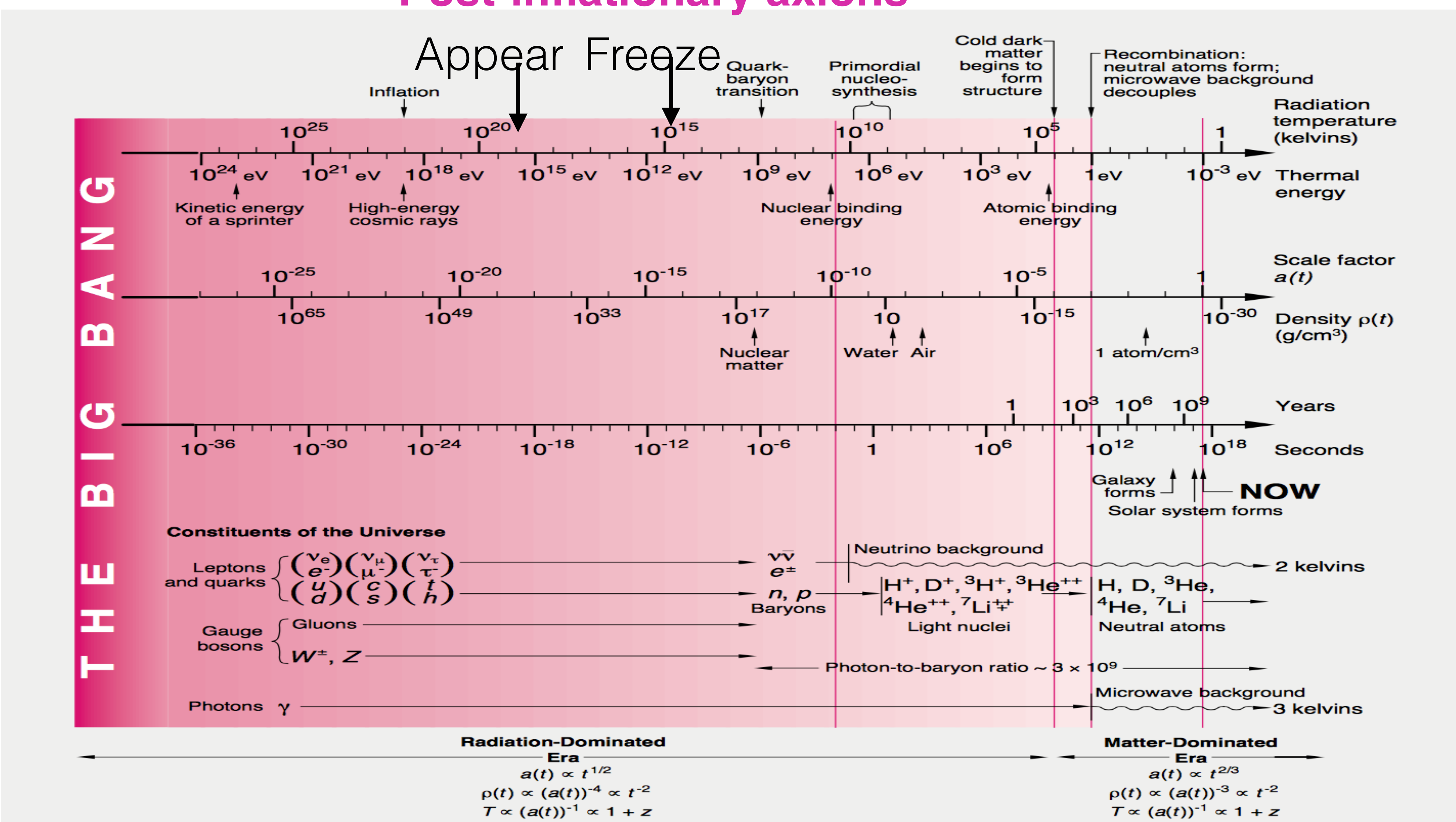
Cold Dark Matter candidates might have been created after the inflation

Several CDM candidates are highly speculative - but one, **the axion**, is

Theoretically well motivated in QCD
Amenable to quantitative estimates once QCD topological properties are known:

$$m_a(T) = \sqrt{\chi(T)}/f_a$$

Post-inflationary axions



Axions 'must' be there: solution to the strong CP problem

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}.$$

Ammitted but $\theta < 10^{-9}$

$$Q = \int d^4x \, \frac{g^2}{32\pi^2}\mathrm{tr}F\tilde{F}$$

Postulate axions, coupled to Q:

$$\mathcal{L}_{\mathrm{axions}} = \frac{1}{2}(\partial_\mu a)^2 + \left(\frac{a}{f_a} + \theta\right)\frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}]\exp\left(-T\sum_t d^3x\, \mathcal{L}_{QCD}(\theta)\right) = \exp[-VF(\theta, T)]$$

Axion potential

$$m_a^2(T)f_a^2 = \left.\frac{\partial^2 F(\theta, T)}{\partial\theta^2}\right|_{\theta=0} \equiv \chi(T),$$

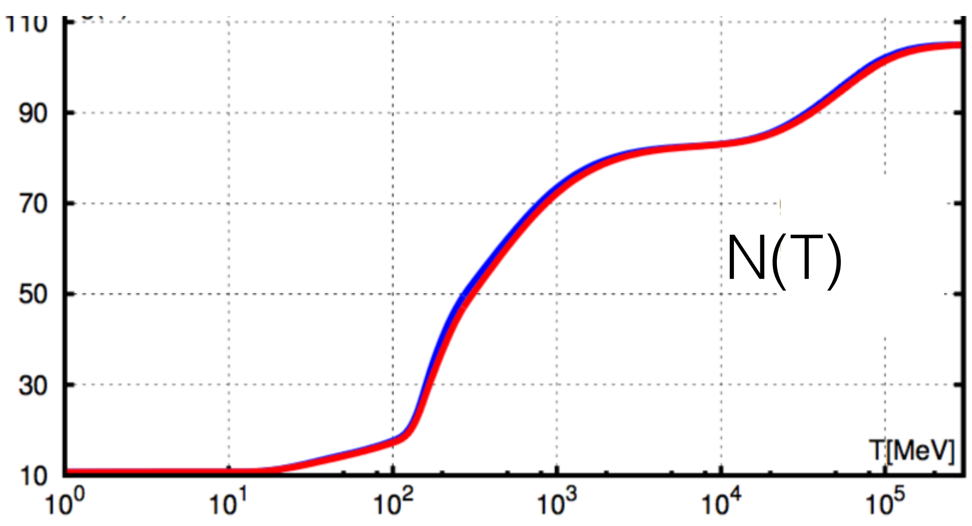
Time from Big Bang

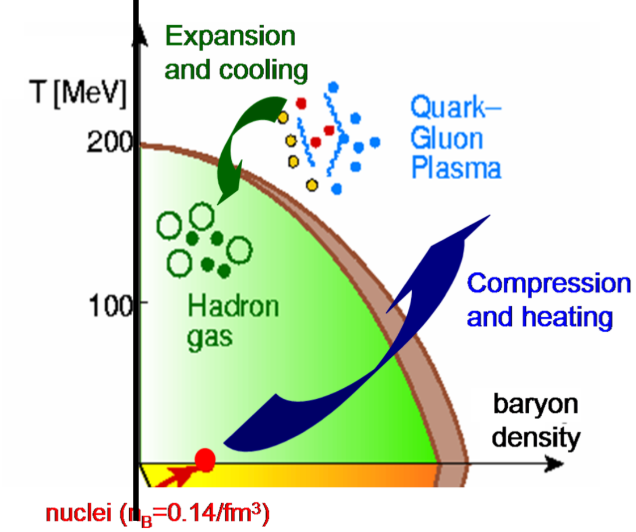
Temperatures

150 MeV < T < 500 MeV

..and beyond

Temperature and Time from BigBang are linked by the Equation of State





Temperature

Time from Big Bang

Hubble parameter

$$H(T) \simeq T^2/M_P$$

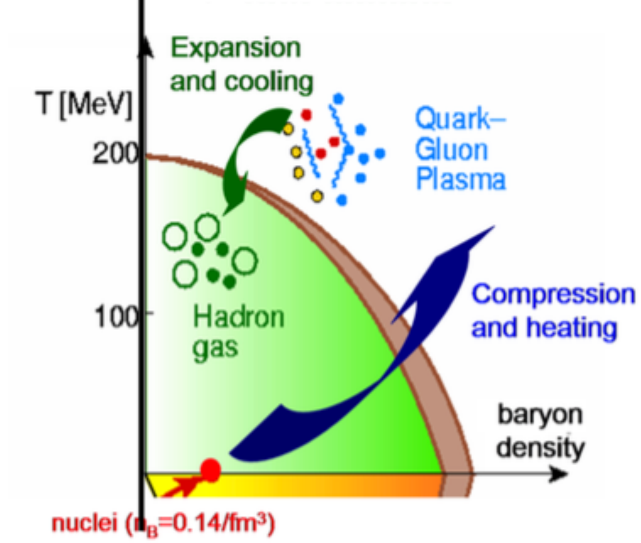
Axions's freezout

$$3H(T) = m_a(T)$$

$$m_a(T) = \sqrt{\chi(T)}/f_a$$

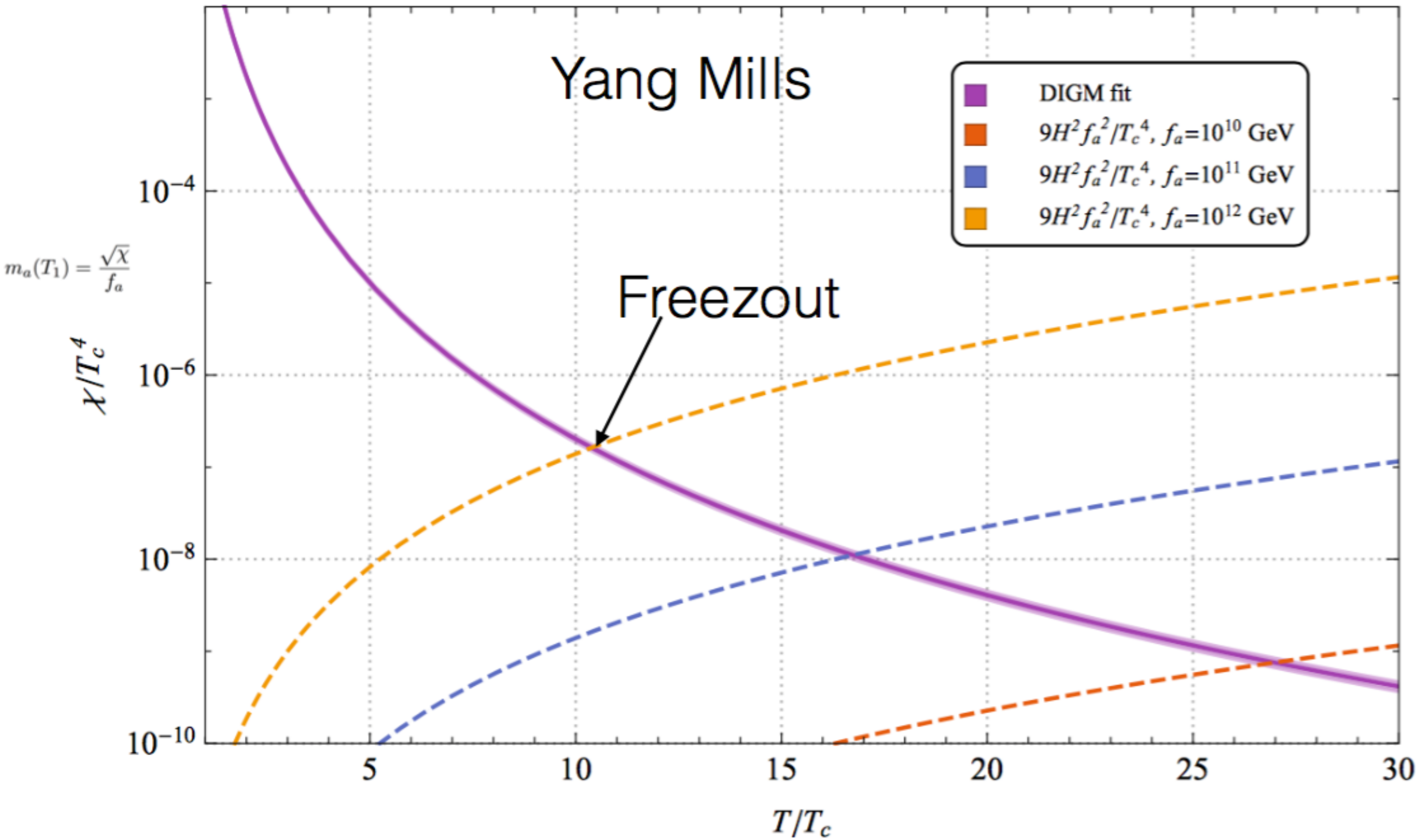
Quark Gluon Plasma: Topology

Axions' mass and density today



Axion freezout : $3H(T) = m_a(T) = \sqrt{\chi(T)}/f_a$

Berkowitz Buchoff Rinaldi 2015



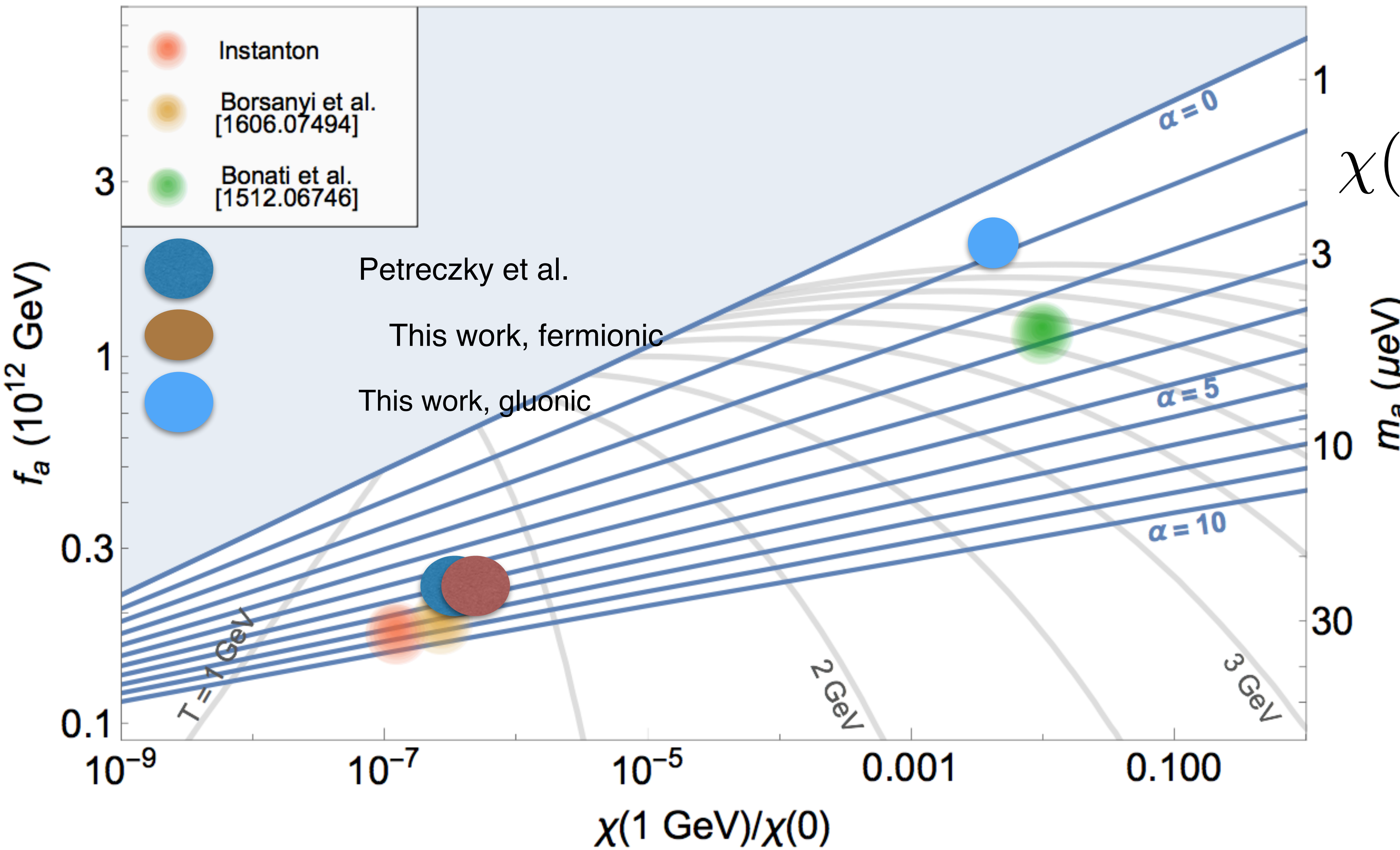
Axion density at freezout controls axion density today

Needed assumption on fraction of DM made of axions

Assume: Axions make all of Dark Matter

$\chi(T) \propto 1/T^\alpha$

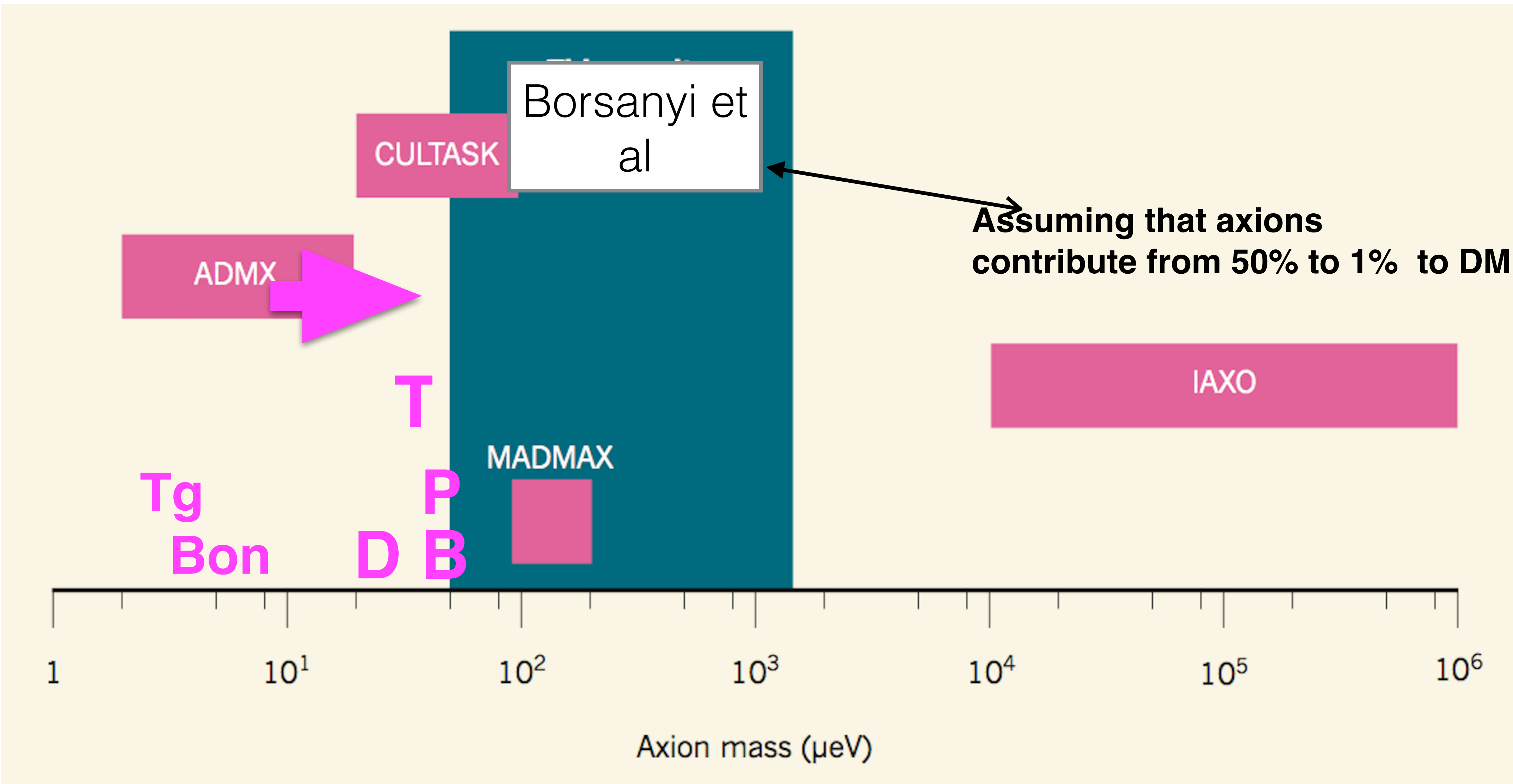
$T > 1\ \text{GeV}$



PhD Thesis, G. Grilli di Cortona, Sissa 2016 (advisor G. Villadoro)

Lower limits on the axion mass assuming that axions make 100% of DM:

Tg: This work, gluonic; Bon: Bonati et al.; D: DIGA, B: Borsanyi et al.,
P: Petreczky et al., T: this work, fermionic



Updated from Nature N&V

Summary and open points I

-*Gluonic operator with gradient flow method:*

Strong lattice artifacts for a > 0.06 fm. The results for a = 0.06 compare well with BNL results, where a^2 corrections are still visible. No reliable continuum limit for the topological susceptibility.

b2 is approaching the DIGA value for T > 300 MeV on all the lattices, possibly due to a cancellation of lattice artifacts

-*Fermionic operator:*

Residual lattice artifacts below statistical errors, allowing a continuum limit estimate. The results for T> 300 are broadly consistent with others once rescaled to the physical pion mass, and confirm the DIGA behavior

We observe a faster decrease closer to Tc, in agreement with recent instanton-dyons predictions. This feature has not been seen in other studies

Summary and open points II

-What next for Topology and QGP phenomenology

- All in all, there is an emerging evidence that the QGP behaves as a DIGA for T > 300 MeV, but such evidence only comes from the exponent and b2.Can this agreement be accidental?

The behavior around Tc is still under scrutiny, and should be clarified to better understand the approach to DIGA, and the nature of the medium produced at the LHC.

-What next for the lattice

Twisted mass Wilson fermions seem to perform well for topology: very little spacing effects for the fermionic operator, access to the cumulants even on coarse lattices.

Needless to say, simulations for smaller masses, and finer lattices would be most useful, and in view of the positive features of these fermions very worthwhile.The disconnected susceptibilities should be measured as well.

-What next for Axions ?

THANK YOU!