Lecture I - Mpl

Symmetries and Phases of QCD in the T, Nf plane

Basic bibliography &
Scanned notes

Some references for the first lecture.

The aim of this short bibliographic note is to provide some basic references which cover the topics discussed in the first introductory lecture. They are not meant to be complete, rather as a help to navigate the rich literature on the various subjects touched upon. The order of the references follows the presentation, and some of the topics have been expanded in the subsequent lectures.

Symmetries of QCD are discussed in many books and reviews, for instance in the textbook by Andrei Smilga, *Lectures on Quantum Chromodynamics*,and the lecture notes by John Preskill, available on line at
`http://www.theory.caltech.edu/ preskill/notes.html`.

A classic reference for the $U_A(1)$ problem is Coleman's lecture, *The use of instantons* reprinted in *Selected Erice Lectures*, and also available on line as a preprint at
`https://lib-extopc.kek.jp/preprints/PDF/1978/7805/7805043.pdf` .

For basic aspects of topology of gauge fields, perhaps the book by A.M. Polyakov *Gauge Fields and Strings*, in particular Chapter 6 *Topology of Gauge Fields* is the 'best' reading. A very nice paper with one of the first lattice detailed studies is J. B. Kogut, J. F. Lagae and D. K. Sinclair, *Topology, fermionic zero modes and flavor singlet correlators in finite temperature QCD*, Phys. Rev. D58 (1998) 054504.

The discussion on the zero temperature pattern of symmetries in QCD is in T. Appelquist, J. Terning and L. C. R. Wijewardhana, *The Zero temperature chiral phase transition in SU(N) gauge theories*, Phys. Rev. Lett. 77 (1996) 1214. This article also reviews in a concise way the relevant aspects of the gauge fields dynamics and Banks-Zaks fixed point. The discussion of the phase diagram in the N_f, g^2 plane, as well as of the conformal phase transition and particle spectrum, may be found in the original paper by V. A. Miransky and K. Yamawaki, *Conformal phase transition in gauge theories*, Phys. Rev. D 55 (1997) 5051.

The finite temperature transition in QCD has been discussed from many different points of view at this School, see e.g. the lectures by Olaf Kaczmarek. A classic basic reference on the pure gauge systems is B. Svetitsky, *Symmetry Aspects of Finite Temperature Confinement Transitions* Phys. Rept.

132 (1986) 1. Although limited to pure gauge, it provides an excellent introduction into the general idea of the universality of critical phenomena, and their applicability in various fields of physics. The paper by J. Engels and F. Karsch *The scaling functions of the free energy density and its derivatives for the 3d O(4) model,* Phys. Rev. D **85** (2012) 094506 is a detailed analysis in a spin model performed having in mind QCD applications. The link between chiral dynamics, Ward identies and critical phenomena is discussed in A. Kocic, J. B. Kogut and M. P. Lombardo, *Universal properties of chiral symmetry breaking* Nucl. Phys. B **398** (1993) 376. In this short series of lectures *Finite temperature field theory and phase transitions,* ICTP Lect. Notes Ser. **4** (2001) I have tried to summarize the main aspects of the theory of critical phenomena which are usuful in particle physics.

The phase diagram in the plane of the temperature and number of flavors is discussed in J. Braun and H. Gies, *Chiral phase boundary of QCD at finite temperature,* JHEP **0606** (2006) 024. This paper also re-derive the conditions for chiral symmetry breaking in QCD from the perspective of functional renormalization group. One short review prepared with my collaborators summarizes various aspects of the phases of QCD, and contains more references: M. P. Lombardo, K. Miura, T. J. Nunes da Silva and E. Pallante, *One, two, zero: Scales of strong interactions,* Int. J. Mod. Phys. A **29** (2014) no.25, 1445007.

Finally, the TASI lectures by M. Dine, *TASI lectures on the strong CP problem* available on the arXives at
`https://arxiv.org/pdf/hep-ph/0011376.pdf` provide an excellent bridge between our introductory first lecture and the subject of topology and axions which will be discussed in the following.

Symmetries of QCD

$$\mathcal{L} = \sum_{a=1}^{n} \left[\bar{q}_{La} \not{D} q_{La} + \bar{q}_{Ra} \not{D} q_{Ra} - m \left(\bar{q}_{La} q_{Ra} + \bar{q}_{Ra} q_{La} \right) \right]$$

$$+ \underbrace{\frac{1}{16\pi^2} \vartheta \, \mathrm{Tr}\, F_{\mu\nu} \tilde{F}^{\mu\nu}}_{\mathcal{L}(\vartheta),\ \text{to be discussed later}} + \mathcal{L}_{gauge}$$

In this form where we used $q_L \equiv \frac{1+\gamma_5}{2} q$, $q_R = \frac{1-\gamma_5}{2} q$

the symmetry $q_L \to V_L q_L$, $q_R \to V_R q_R$ $V \in U(n)$ is manifest.

at classical level the symmetry is $U_L(n) \times U_R(n)$. (for $m = 0$)

However, $q_L \to e^{i\vartheta} q_L$; $q_R \to e^{-i\vartheta} q_R$ is violated by quantum effects:

The remaining symmetry is $U_L(n) \times U_R(n) / U(1) \sim SU(n) \times SU(n) \times U_V(1)$.

This is a global and continuous symmetry.

The pseudoscalar mesons are candidate Goldstone bosons if the symmetry $SU(n) \times SU(n)$ is spontaneously broken.

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Let us have a look at the pseudoscalar mesons:

three pions $(\pi^+, \pi^-, \pi^0) \sim 140$ MeV

strange mesons: $\eta \sim 547$ MeV, the four K's ~ 400 MeV - η': 960 MeV

All the other mesons much heavier.

Plausible patterns of symmetry breaking are $SU(3) \times SU(3) \to SU(3)$ &

$SU(2) \times SU(2) \to SU(2)$.

In either cases, η' is anomalously large

To better see this let us consider a sketch of symmetry partners for $SU(2) \times SU(2) \times U_A(1)$

$$\begin{pmatrix} \vec{\pi} \\ \sigma \end{pmatrix} \downarrow$$

mass term points in σ direction

$$\begin{tikzcd}
(\bar{q}\gamma_5 \frac{\vec{\tau}}{2} q)\ \vec{\pi} \arrow[r, leftrightarrow, "SU(2)\times SU(2)"] \arrow[d, leftrightarrow, "U_A(1)"'] & \sigma\ (\bar{q}q) \arrow[d, leftrightarrow, "U_A(1)"] \\
(\bar{q}\frac{\vec{\tau}}{2} q)\ \vec{\delta} \arrow[r, leftrightarrow, "SU(2)\times SU(2)"] & \eta\ (\bar{q}\gamma_5 q)
\end{tikzcd}$$

The most natural scenario compatible with experimental observations is $SU(2)\times SU(2)$ explicitly broken by $m_{u/d}$, $SU(3)\times SU(3)$ by the same, and even more by m_s. The order parameter $\langle \bar{q}q \rangle$ also breaks $U_A(1)$. We would then expect η' to be on the same footing as the other pseudo Goldstone, which is not.

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We will discuss the 'η' problem' in connection with topology. Here we merely summarize what we know so far on $U_A(1)$:

→ It is responsible for the difference between $\vec{\pi}$ and $\vec{\delta}$, and σ and η'

→ It is <u>not</u> spontaneously broken: η' is <u>not</u> a Goldstone boson.

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Let us now concentrate on $SU(n) \times SU(n)$

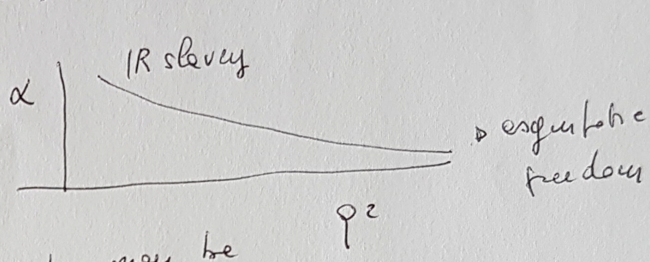
– $SU(n) \times SU(n)$ –

→ What triggers the breaking of the symmetry?

→ When and how will it be restored?

Symmetry alone does not suffice to answer these questions.

Gauge fields dynamics



Consider β function (defines α – may be computed in perturbation theory)

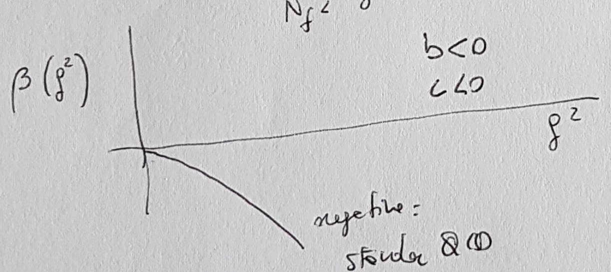
$$\alpha(\mu) = g^2(\mu)/4\pi \qquad \beta = \frac{d\alpha(\mu)}{d\ln(\mu)} \equiv -b\alpha^2 - c\alpha^3$$

$$b = \frac{1}{6\pi}\left(11 N_c - 2N_f\right)$$

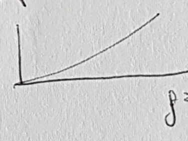
$$c = \frac{1}{24\pi^2}\left(34 N_c^2 - 10 N_c N_f - 3\frac{(N_c^2-1)}{N_c} N_f\right)$$

} scheme independent

$N_f \lesssim 8$

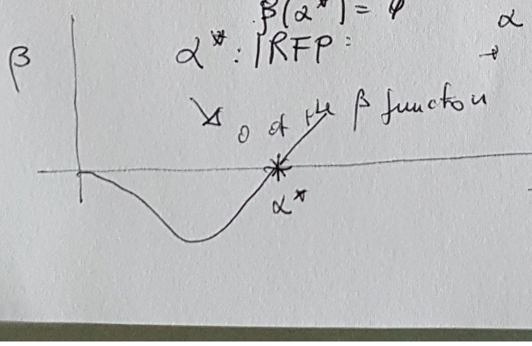


$N_f > 16$ asymptotic freedom lost

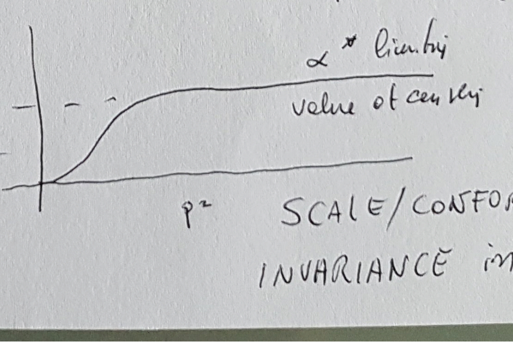


$b>0$

Intermediate case $\sim 8 < N_f < 16$ $\left.\begin{matrix} b<0 \\ c>0 \end{matrix}\right\}$ there is a zero of the β function



The coupling does not run.



SCALE/CONFORMAL INVARIANCE in I.R.

Phases in The T, N_f plane

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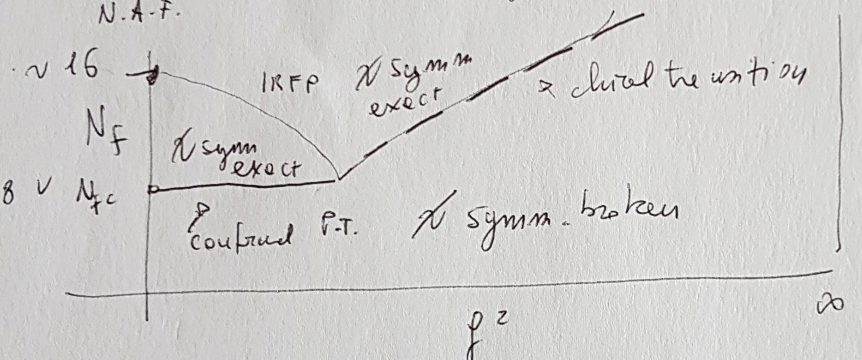
Effective potential analysis predicts a critical value for the coupling

$$\alpha_c = \frac{2\pi N_c}{N_c^2 - 1}$$ — for $\alpha > \alpha_c$ χ symm is broken

Then, if $\alpha^* > \alpha_c$: χ sym. is broken, IRFP disappears.

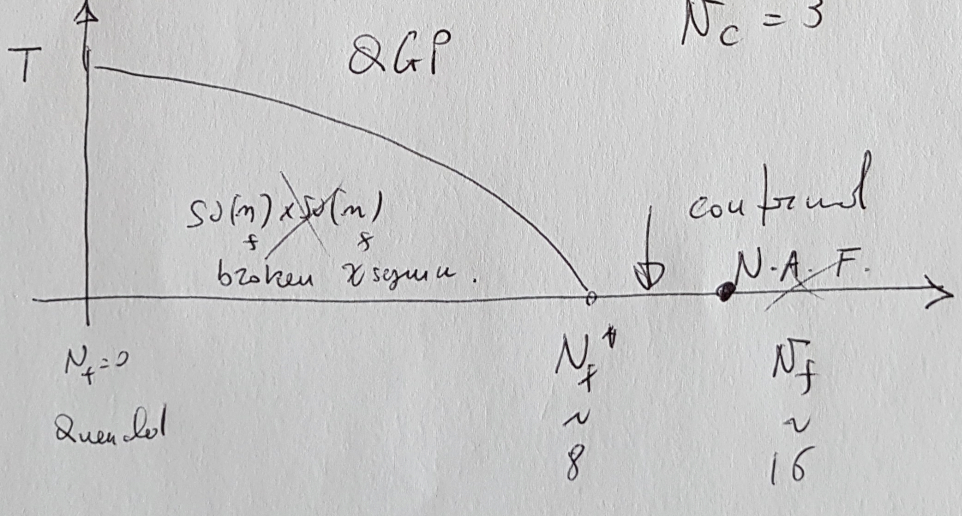
BUT: if $\alpha^* < \alpha_c$: χ symm. is realized

Phase diagram of QCD, $T=0$, in the N_f, g^2 plane:



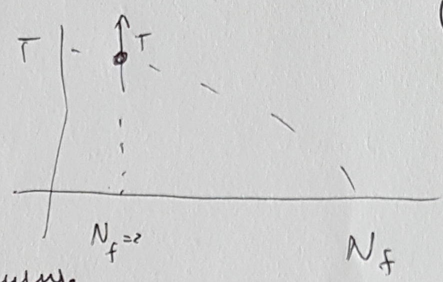
At finite T, in the N_f, T plane

$N_c = 3$



Nature of

The finite temperature transition



$N_f = 2$

$SU(2) \times SU(2) \to SU(2)$ or in isomorphic language

$O(4) \to O(3)$

Order parameter $\langle \bar\psi \psi \rangle$ _ Universal description: dictionary

	External field h	Response M	Distance from criticality t
Magnetic systems	h	M	$T - T_c$
QCD	m_q	$\langle \bar\psi \psi \rangle$	$T - T_c$
(lattice)	m_q	$\langle \bar\psi \psi \rangle$	$\beta - \beta_c$

EoS contains definition of the critical exponents:

$$h = M^\delta f\left(\frac{t}{M^{1/\beta}}\right) \quad - \quad f(0) = 1$$

at $t = 0$ $\quad h = M^\delta$

at $h = 0$ $\quad M \propto t^\beta$

Consider continuous symmetry, separate longitudinal and transverse components:

$$h_a = M_a M^{\delta - 1} f\left(t / M^{1/\beta}\right)$$

$$\chi^{-1}_{ab} = \partial h_a / \partial M_b$$

$$\chi_T = \frac{M}{h} \leftarrow \text{Ward identity} \qquad \chi_L = \frac{\partial M}{\partial h}$$

At the critical point $t = 0$

$$\frac{\chi_T}{\chi_L} = \delta$$

In QCD longitud. and transverse modes are π^0 and σ (6)
(still $N_f = 2$)

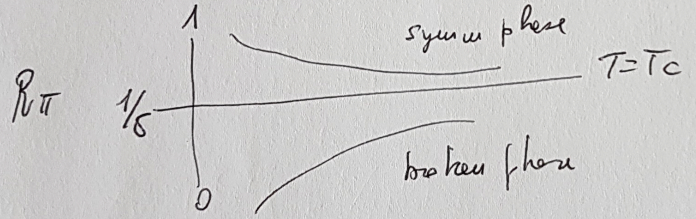
$$\chi_\sigma \equiv \int \langle \bar\psi\psi(x)\;\bar\psi\psi(0)\rangle\, d^4x$$
$$\chi_\pi \equiv \int \langle \bar\psi\gamma_5\psi(x)\;\bar\psi\gamma_5\psi(0)\rangle\, d^4x$$

$$\chi_\pi \equiv \langle\bar\psi\psi\rangle/m \qquad \chi_\sigma \equiv \frac{\partial\langle\bar\psi\psi\rangle}{\partial m}$$
$$\Downarrow \qquad\qquad \Downarrow$$
$$\frac{Z_\pi}{M_\pi^2} \qquad\qquad Z_\sigma/M_\sigma^2$$

$$R_\pi^{(t,m)} \equiv \chi_\pi^{-1}/\chi_\sigma^{-1} \approx M_\pi^2/M_\sigma^2$$

$$R_\pi(0,m) \equiv 1/\delta$$

$$\frac{1}{R_\pi(t,m)} \equiv \delta - \frac{\nu}{\beta}\,\frac{f'(x)}{f(x)}$$



Mass dependence at Tc

$$\langle\bar\psi\psi\rangle\Big|_{\substack{t=0\\T=T_c}} \propto m^{1/\delta} \qquad \chi_\sigma \equiv \frac{\partial\langle\bar\psi\psi\rangle}{\partial m} \propto m^{1/\delta-1}$$

$T > T_c$: analytic $\propto$ $\langle\bar\psi\psi\rangle \propto m$ $\quad \chi_\sigma \propto$ cte

at $m = 0$ Temperature dependence

$$\chi_\sigma \propto t^{-\gamma} \qquad T > T_c$$

approx. the same also for small masses

– QCD with physical quark masses

→ The scaling function f characterizes a universality class

– Which universality class in QCD finite T transition?

If $SU(2) \times SU(2) \to SU(2)$ 2nd order [may still be first]

If $U(2) \times U(2) \to U(2)$ (i.e. if $U_A(1)$ is restored at T_c) 1st order

If $SU(3) \times SU(3) \to SU(3)$ 1st order.

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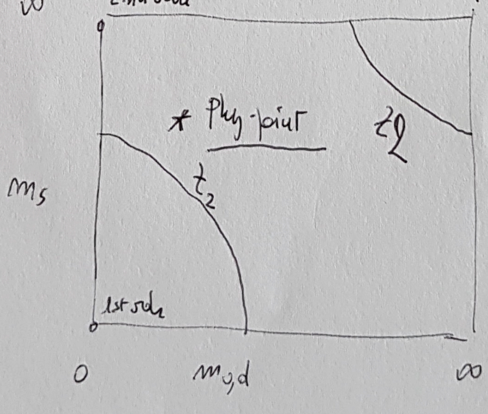
Has to be studied numerically –

If we accept the conventional scenario $N_f = 2$ 2nd order, $N_f = 3$ 1st T there are different possibilities according to the value of the quark mass



1st order PURE GAUGE

to be checked!

Pure gauge.

Why first order?

For $m_{u,d,s} = \infty$ there is no notion of chiral symmetry.
The YM theory with N_c (3) colors enjoy a center symmetry (global) $\mathbb{Z}_{N_c} = \mathbb{Z}_3$. The order parameter is the Polyakov loop $\langle L \rangle$

Besides being the order parameter of the global $\mathbb{Z}_3$ symmetry the Polyakov loop is also related to the free energy of an isolated quark: $e^{-F_q} = \langle L \rangle$. then

$$\langle L \rangle = \begin{cases} 0 \Rightarrow F_q = \infty, & \mathbb{Z}_3 \text{ symmetric, confinement} \\ \neq 0 \quad F_q \text{ finite}, & \mathbb{Z}_3 \text{ broken, deconfinement} \end{cases}$$

Hence, deconfinement $\leftrightarrow$ $\mathbb{Z}_3$ breaking ($\rightarrow$ in 3d) $\rightarrow$ 1st order transition

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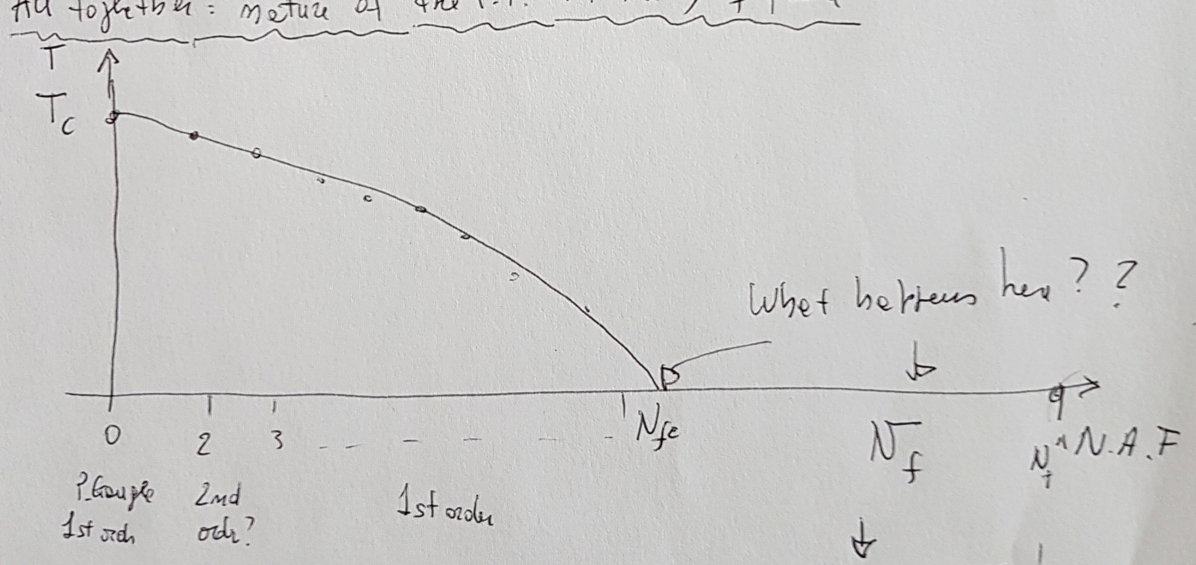
All together: nature of the P.T. in the T, N_f plane



Remember, IRFP!
The theory is IR conformal: simple LANDAU Ginzburg ~~does~~ is not applicable -

We discuss next the $T=0$ axis, from QCD-like to the CONFORMAL WINDOW