Non-conformal anisotropic solution in 5-dimensional Maxwell-dilaton gravity

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HISS 2017
BLTP JINR, Dubna

24.08.2017

Motivation

Holography needs:

- Natural theory with metric as solution of EOM.
- Non-zero chemical potential $\mu \neq 0$.

2016 *I. Ya. Aref'eva*

"Holography for Heavy Ions Collisions at LHC and NICA"

(arXiv:1612.08928 [hep-th])

IA's talk on Monday

$$ds^2 = L^2 \, \frac{b(z)}{z^2} \left[- \, g(z) dt^2 + dx^2 + z^{2-\frac{2}{\nu}} \left(dy_1^2 + dy_2^2 \right) + \frac{dz^2}{g(z)} \right]$$

AGG solution

$$ds^2 = \frac{L^2}{z^2}\left[-\,g(z)dt^2 + dx^2 + z^{2-\frac{2}{\nu}}\left(dy_1^2 + dy_2^2\right) + \frac{dz^2}{g(z)}\right]$$

$$\mu = 0, \qquad b(z) = 1$$

2015 *I.Ya. Aref'eva, A.A. Golubtsova*
JHEP **1504** 011 (arXiv:1410.4595 [hep-th])
2016 *I.Ya. Aref'eva, A.A. Golubtsova, E. Gourgoulhon*
JHEP **1609** 142 (arXiv:1601.06046 [hep-th])

Since phenomenally require $\mu \neq 0,\ b \neq 1,\ \nu \neq 1$ *Aref'eva 1612.08928*

OUR GOAL

$\Downarrow$

Generalizations:

$\downarrow$ $\downarrow$

$\mu \neq 0$ $\qquad b(z) \neq 1$

Action in Einstein frame, metric ansatz

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \times$$

$$\times \left[R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

$$A_\mu^{(1)} = A_t(z) \delta_\mu^0 \qquad F^{(2)} = q \; dy^1 \wedge dy^2 \qquad \phi = \phi(z)$$

$$ds^2 = L^2 \, \frac{b(z)}{z^2} \left[- \, g(z) dt^2 + dx^2 + z^{2-\frac{2}{\nu}} \left(dy_1^2 + dy_2^2 \right) + \frac{dz^2}{g(z)} \right]$$

Boundary conditions:

$$g(0) = 1, \; g(z_h) = 0 \qquad A_t(0) = \mu, \; A_t(z_h) = 0 \qquad \phi(z_h) = 0$$

$\nu = 1$: 2017 *M.-W. Li, Y. Yang, P.-H. Yuan* (arXiv:1703.09184 [hep-th])

Equations of motion

$$\phi'' + \phi'\left(\frac{g'}{g} + \frac{3b'}{2b} - \frac{\nu+2}{\nu z}\right) + \frac{z^2 A_t'^{\,2}}{2bg}\,\frac{\partial f_1}{\partial \phi} - \frac{q^2 z^{-2+\frac{4}{\nu}}}{2bg}\,\frac{\partial f_2}{\partial \phi} - \frac{b}{z^2 g}\,\frac{\partial V}{\partial \phi} = 0$$

$$A_t'' + A_t'\left(\frac{b'}{2b} + \frac{f_1'}{f_1} - \frac{2-\nu}{\nu z}\right) = 0$$

1-st: $$g'' + g'\left(\frac{3b'}{2b} - \frac{1}{z} - \frac{2}{\nu z}\right) - \frac{z^2}{b}\, f_1 A_t'^{\,2} = 0$$

2-nd: $$2g'\left(1 - \frac{1}{\nu}\right) + g\left(1 - \frac{1}{\nu}\right)\left(\frac{3b'}{b} - \frac{4}{z} - \frac{4}{\nu z}\right) + \frac{q^2 z^{-1+\frac{4}{\nu}}}{b} f_2 = 0$$

3-rd: $$b'' - \frac{3b'^{\,2}}{2b} + \frac{2b'}{z} - \frac{4b}{3\nu z^2}\left(1 - \frac{1}{\nu}\right) + \frac{b}{3}\,\phi'^{\,2} = 0$$

4-th: $$-V - \frac{z^4}{2b^2}\, A_t'^{\,2} f_1 - \frac{3z^2 b' g'}{2b^2} - \frac{3z^2 g b'^{\,2}}{b^3} + \frac{9zgb'}{2\nu b^2} + \frac{15zgb'}{2b^2} + {} + \frac{zg'}{\nu b} + \frac{2zg'}{b} + \frac{z^2 g \phi'^{\,2}}{2b} - \frac{8g}{\nu b} - \frac{4g}{b} = 0$$

General anisotropic solution $b(z) = \exp P(z)$, $\nu \neq 1$

$$f_1 = z^{-2+\frac{2}{\nu}}$$

$$A_t = \tilde{\mu} \int_z^{z_h} e^{-\frac{P(\xi)}{2}} \xi d\xi, \quad \tilde{\mu} = \frac{\mu}{\int_0^{z_h} e^{-\frac{P(\xi)}{2}} \xi d\xi}$$

$$g = 1 + \tilde{\mu}^2 \int_0^z e^{-\frac{3P(\xi)}{2}} \left(\int_0^\xi e^{-\frac{P(\chi)}{2}} \chi d\chi \right) \xi^{1+\frac{2}{\nu}} d\xi -$$

$$- \frac{1 + \tilde{\mu}^2 \int_0^{z_h} e^{-\frac{3P(\xi)}{2}} \left(\int_0^\xi e^{-\frac{P(\chi)}{2}} \chi d\chi \right) \xi^{1+\frac{2}{\nu}} d\xi}{\int_0^{z_h} e^{-\frac{3P(\xi)}{2}} \xi^{1+\frac{2}{\nu}} d\xi} \int_0^z e^{-\frac{3P(\xi)}{2}} \xi^{1+\frac{2}{\nu}} d\xi$$

$$\phi(z) = C_5 + \int_0^z \sqrt{- 3P''(\xi) + \frac{3}{2} P'^2(\xi) - \frac{6}{\xi} P'(\xi) + 4 \frac{\nu - 1}{\xi^2 \nu^2}} d\xi$$

General anisotropic solution $b(z) = \exp(cz^2/2)$, $\nu \neq 1$

$$f_1 = z^{-2+\frac{2}{\nu}} \qquad A_t(z) = \mu\,\frac{e^{-\frac{cz^2}{4}} - e^{-\frac{cz_h^2}{4}}}{1 - e^{-\frac{cz_h^2}{4}}}$$

$$c = 0.9$$

$$g(z) = 1 - \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;\ \frac{3cz^2}{4}\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;\ \frac{3cz_h^2}{4}\right)} -$$

$$-\mu^2\, e^{\frac{cz_h^2}{2}}\,\frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;\ cz^2\right)}{4\,c^{1/\nu}\left(1 - e^{\frac{cz_h^2}{4}}\right)^2} +$$

$$+\mu^2\, e^{\frac{cz_h^2}{2}}\,\frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;\ cz_h^2\right)}{4\,c^{1/\nu}\left(1 - e^{\frac{cz_h^2}{4}}\right)^2}\,\frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;\ \frac{3cz^2}{4}\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;\ \frac{3cz_h^2}{4}\right)}$$

2006 *O. Andreev, V.I. Zakharov* PRD **74** 025023 (arXiv:0604204 [hep-ph])

Scalar field $\phi(z)$ for general solution $b(z) = \exp(cz^2/2)$, $\nu \neq 1$

$$\begin{aligned}\phi(z) = C_5 + c\sqrt{\frac{3}{8}}\,\Big\{&\sqrt{(\mathcal{A}^2 - z^2)(\mathcal{B}^2 - z^2)} + \mathcal{A}\mathcal{B}\ln(z^2) - \\ &- \frac{\mathcal{A}^2 + \mathcal{B}^2}{2}\ln\left(2\sqrt{(\mathcal{A}^2 - z^2)(\mathcal{B}^2 - z^2)} - \mathcal{A}^2 - \mathcal{B}^2 + 2z^2\right) - \\ &- \mathcal{A}\mathcal{B}\ln\left[-\left(2\mathcal{A}^2\mathcal{B}^2 - \mathcal{A}^2 z^2 - \mathcal{B}^2 z^2 + 2\mathcal{A}\mathcal{B}\sqrt{(\mathcal{A}^2 - z^2)(\mathcal{B}^2 - z^2)}\right)\right]\Big\}\end{aligned}$$

$$z \leq \mathcal{A}$$

$$c = 0.9, \ \nu = 4.5$$

$$\Downarrow$$

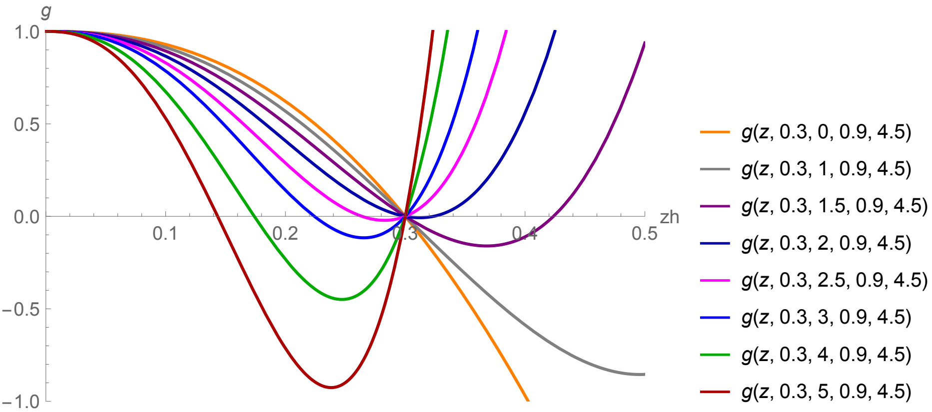
$$\mathcal{A} = 0.29406, \ \mathcal{B} = 2.565$$

$$\Downarrow$$

$$z \leq 0.29406$$

Blackening function $g(z)$ for various μ

$z_h = 0.29,\ c = 0.9,\ \nu = 4.5$



Solution for anisotropic metric ansatz $b(z) = 1,\ \nu \neq 1$

$$f_1 = z^{-2+\frac{2}{\nu}}$$

$$A_t(z) = \mu \left(1 - \frac{z^2}{z_h^2}\right)$$

$$g(z) = 1 - \left(\frac{z}{z_h}\right)^{2+\frac{2}{\nu}} \left[1 + \frac{\mu^2 \nu z_h^{\frac{2}{\nu}}}{1+2\nu} \left(1 - \frac{z^2}{z_h^2}\right)\right]$$

$$\phi = 2\,\frac{\sqrt{\nu - 1}}{\nu}\,\log z + C_5$$

Anisotropic solution

$b(z) = 1, \ \nu \neq 1, \ A_t(z) = 0$

$$g(z) = 1 - \left(\frac{z}{z_h}\right)^{2+\frac{2}{\nu}}$$

$$f_2(z) = \frac{4z^{-4/\nu}}{q^2}\,\frac{(\nu-1)(1+3\nu+2\nu^2)}{\nu^2\ (1+2\nu)}$$

$$\phi(z) = C_5 \pm 2\,\frac{\sqrt{\nu-1}}{\nu}\,\log(z)$$

$$V(z) = -\ 2\,\frac{(1+\nu)(1+2\nu)}{\nu^2}$$

Corresponds to AGG solution

2016 *I.Ya. Aref'eva, A.A. Golubtsova and E. Gourgoulhon*

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Thermodynamics for anisotropic solution $b(z) = \exp(cz^2/2)$, $\nu \neq 1$

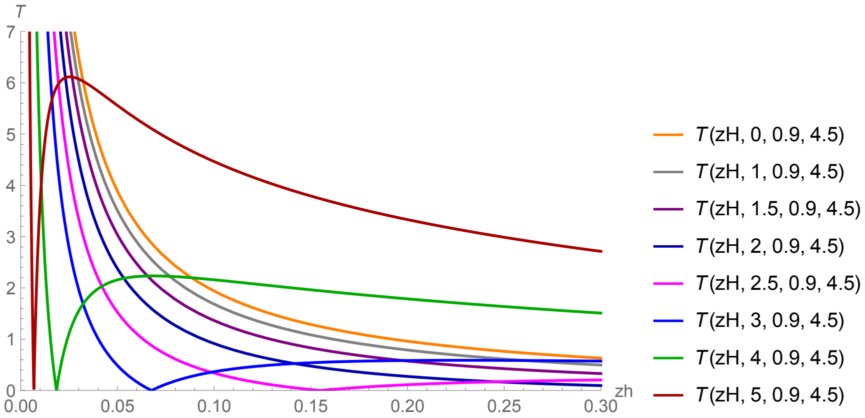
$$s = \frac{e^{\frac{3cz_h^2}{4}}}{4z_h^{1+\frac{2}{\nu}}}$$

$$T = \frac{z_h^{1+\frac{2}{\nu}}}{8\pi}\, e^{-\frac{3cz_h^2}{4}} \left| \left(\frac{3c}{2}\right)^{1+\frac{1}{\nu}} \frac{2^{1-\frac{1}{\nu}}}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\,;\ \frac{3cz_h^2}{4}\right)} + \right.$$

$$\left. + \frac{\mu^2 c\, e^{\frac{cz_h^2}{4}}}{\left(1-e^{\frac{cz_h^2}{4}}\right)^2} \left[1 - \left(\frac{3}{4}\right)^{1+\frac{1}{\nu}} e^{\frac{cz_h^2}{4}} \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\,;\ cz_h^2\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\,;\ \frac{3cz_h^2}{4}\right)}\right] \right|$$

Temperature $T(z_h)$ for various μ

$c = 0.9,\ \nu = 4.5$



Conclusions

- The AdS anisotropic black hole solution for 5-dimensional Maxwell-dilaton gravity is found.
- The AGG solution is generalized for the non-zero chemical potential case.
- Scalar field restricts the BH raduis.

To do:

- Shock-wave consideration
  (Aref'eva, Mamedov, KR, Arifulov) – in progress.
- Confinement $\leftarrow$ axion?

Thank you
for your attention